Computer algebra independent integration tests

4-Trig-functions/4.7-Miscellaneous/4.7.1-c-trig-^m-d-trig-^n

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May 23, 2020 Compiled on May 23, 2020 at 7:12am

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [254]. This is test number [135].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (254)	% 0. (0)
Mathematica	% 99.21 (252)	% 0.79 (2)
Maple	% 88.19 (224)	% 11.81 (30)
Maxima	% 62.6 (159)	% 37.4 (95)
Fricas	% 82.28 (209)	% 17.72 (45)
Sympy	% 14.96 (38)	% 85.04 (216)
Giac	% 59.84 (152)	% 40.16 (102)

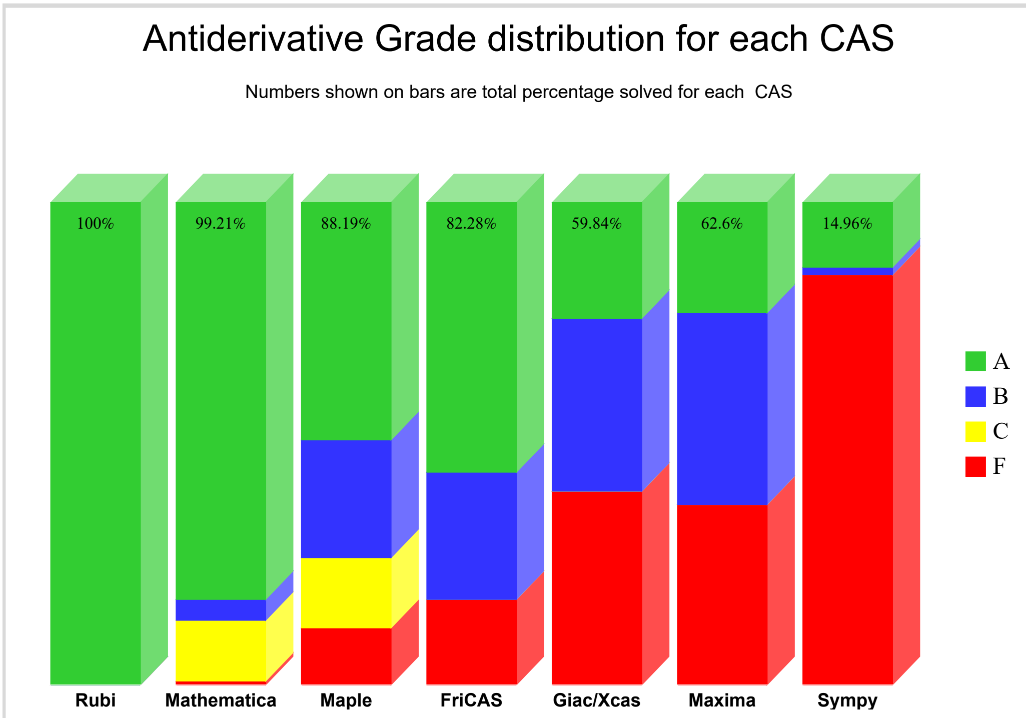
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

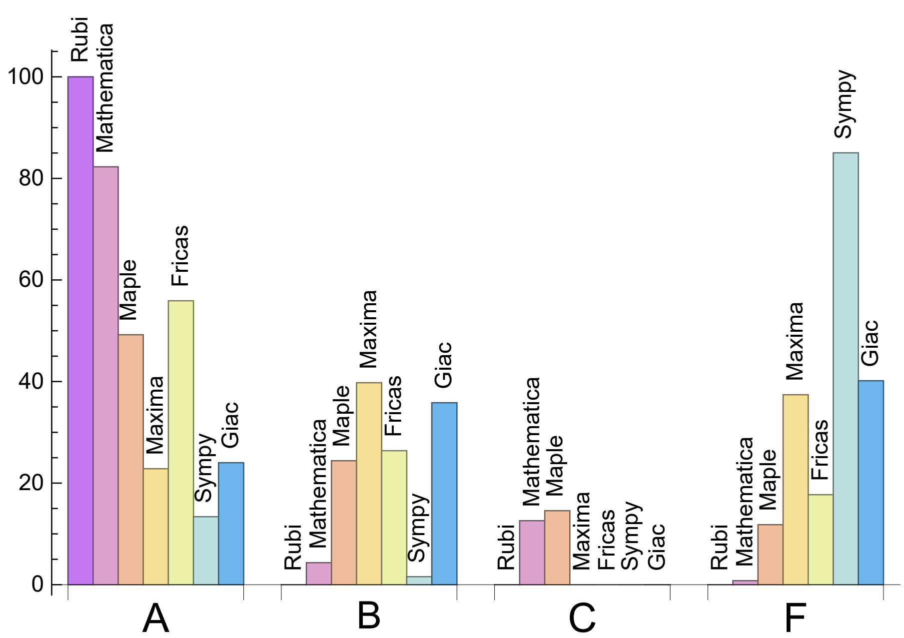
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	82.28	4.33	12.6	0.79
Maple	49.21	24.41	14.57	11.81
Maxima	22.83	39.76	0.	37.4
Fricas	55.91	26.38	0.	17.72
Sympy	13.39	1.57	0.	85.04
Giac	24.02	35.83	0.	40.16

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.08	72.75	1.	60.	1.
Mathematica	0.61	99.96	1.48	62.	0.95
Maple	9.47	7 1.38135 10	191436.	97.	1.58
Maxima	1.36	948.84	14.02	167.	8.62
Fricas	0.51	291.19	4.79	193.	3.36
Sympy	47.68	388.24	7.55	283.	6.47
Giac	1.83	863.59	14.55	130.	2.79

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {123, 124, 125, 126, 127, 128, 187, 188, 189, 191, 209, 219, 223}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

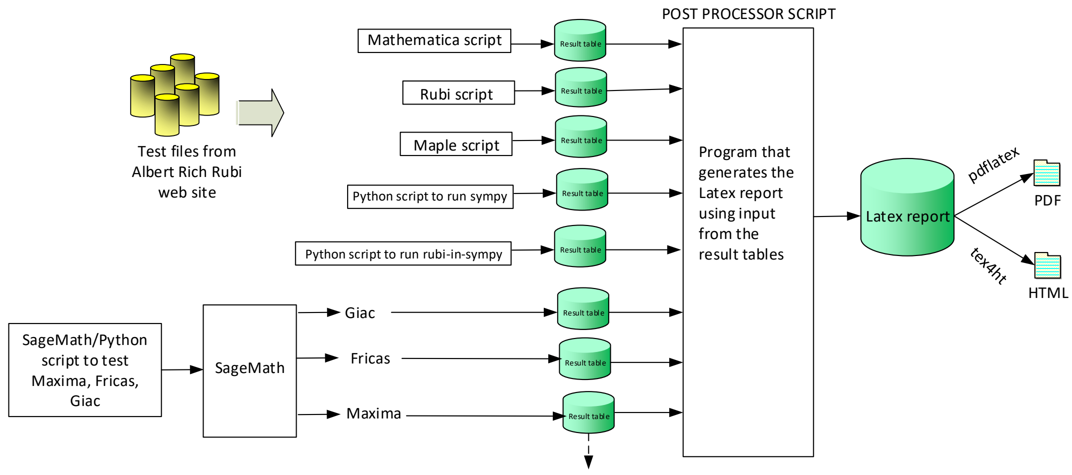
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 70, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 129, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 190, 191, 192, 193, 194, 195, 197, 198, 199, 200, 202, 203, 204, 206, 207, 208, 209, 210, 211, 212, 213, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 229, 230, 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 247, 252, 253, 254 }

B grade: { 11, 40, 42, 44, 61, 72, 136, 138, 140, 158, 160 }

C grade: { 10, 12, 32, 41, 43, 69, 71, 123, 124, 125, 126, 127, 128, 137, 139, 159, 187, 188, 189, 196, 214, 227, 228, 231, 232, 233, 234, 242, 248, 249, 250, 251 }

F grade: { 201, 205}

2.1.3 Maple

A grade: { 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 16, 18, 19, 20, 21, 22, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 87, 105, 106, 107, 108, 112, 114, 130, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 144, 146, 147, 148, 149, 150, 152, 153, 154, 156, 157, 158, 159, 160, 175, 190, 192, 193, 194, 202, 203, 204, 206, 207, 208, 210, 211, 212, 220, 221, 222, 224, 225, 226, 229, 238, 239, 240, 243, 244, 245, 246 }

B grade: { 1, 3, 13, 15, 17, 23, 27, 73, 74, 75, 76, 77, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 109, 111, 113, 129, 131, 141, 143, 145, 151, 155, 161, 162, 163, 164, 165, 170, 171, 172, 173, 174, 176, 178, 179, 180, 195, 196, 197, 198, 199, 200, 213, 214, 215, 216, 217, 218, 227, 228, 241, 242 }

C grade: { 78, 79, 93, 97, 98, 99, 100, 101, 102, 103, 104, 115, 116, 117, 118, 119, 121, 122, 166, 167, 168, 181, 182, 185, 186, 230, 231, 232, 233, 234, 235, 247, 248, 249, 250, 251, 252 }

F grade: { 80, 81, 94, 95, 96, 110, 120, 123, 124, 125, 126, 127, 128, 169, 177, 183, 184, 187, 188, 189, 191, 201, 205, 209, 219, 223, 236, 237, 253, 254 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 13, 14, 15, 16, 17, 23, 24, 25, 26, 33, 34, 35, 36, 37, 38, 39, 40, 45, 46, 47, 48, 49, 50, 51, 59, 60, 61, 62, 63, 64, 66, 129, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 145, 151, 152, 153, 154, 190, 194, 212, 240

B grade: { 8, 9, 10, 11, 12, 18, 19, 20, 21, 22, 27, 28, 29, 30, 31, 32, 41, 42, 43, 44, 52, 53, 54, 55, 56, 57, 58, 65, 67, 68, 69, 70, 71, 72, 136, 137, 138, 139, 140, 146, 147, 148, 149, 150, 155, 156, 157, 158, 159, 160, 192, 193, 195, 196, 197, 198, 199, 200, 202, 203, 204, 206, 207, 208, 210, 211, 213, 214, 215, 216, 217, 218, 220, 221, 222, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 238, 239, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252 }

C grade: { }

F grade: { 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 191, 201, 205, 209, 219, 223, 236, 237, 253, 254 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 72, 77, 78, 79, 80, 94, 95, 96, 101, 102, 103, 104, 115, 116, 120, 121, 122, 129, 130, 131, 132, 133, 134, 135, 140, 141, 142, 143, 144, 145, 146, 147, 149, 151, 152, 153, 154, 155, 156, 157, 165, 166, 167, 168, 182, 183, 184, 190, 192, 193, 194, 195, 196, 197, 199, 202, 203, 204, 206, 207, 208, 210, 211, 212, 213, 215, 216, 217, 218, 220, 221, 222, 224, 225, 226, 227, 229, 238, 239, 240, 241, 242, 243, 244, 245, 246 }

B grade: { 8, 9, 20, 22, 27, 30, 41, 42, 43, 53, 55, 57, 69, 70, 71, 73, 74, 75, 76, 89, 90, 91, 92, 93, 97, 98, 99, 100, 117, 118, 119, 136, 137, 138, 139, 148, 150, 158, 159, 160, 161, 162, 163, 164, 177, 178, 179, 180, 181, 185, 186, 198, 200, 214, 228, 230, 231, 232, 233, 234, 235, 247, 248, 249, 250, 251, 252 }

C grade: { }

F grade: { 81, 82, 83, 84, 85, 86, 87, 88, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 123, 124, 125, 126, 127, 128, 169, 170, 171, 172, 173, 174, 175, 176, 187, 188, 189, 191, 201, 205, 209, 219, 223, 236, 237, 253, 254 }

2.1.6 Sympy

A grade: { 4, 5, 6, 7, 15, 16, 17, 26, 27, 132, 133, 134, 135, 143, 144, 145, 154, 155, 192, 193, 194, 202, 203, 206, 210, 211, 212, 220, 221, 224, 238, 239, 240, 245 }

B grade: { 195, 213, 227, 241 }

C grade: { }

F grade: { 1, 2, 3, 8, 9, 10, 11, 12, 13, 14, 18, 19, 20, 21, 22, 23, 24, 25, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 136, 137, 138, 139, 140, 141, 142, 146, 147, 148, 149, 150, 151, 152, 153, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 196, 197, 198, 199, 200, 201, 204, 205, 207, 208, 209, 214, 215, 216, 217, 218, 219, 222, 223, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 253, 254 }

2.1.7 Giac

A grade: { 2, 4, 6, 7, 14, 16, 24, 26, 34, 36, 38, 40, 45, 47, 49, 51, 60, 62, 64, 66, 68, 130, 132, 134, 135, 136, 137, 139, 142, 144, 147, 149, 152, 154, 157, 159, 190, 192, 193, 194, 202, 203, 204, 206, 207, 208, 210, 211, 212, 220, 221, 222, 224, 225, 226, 238, 239, 240, 244, 245, 246 }

B grade: { 1, 3, 5, 8, 9, 10, 11, 12, 13, 15, 17, 18, 19, 20, 21, 22, 23, 25, 27, 28, 29, 30, 31, 32, 33, 35, 37, 39, 41, 42, 43, 44, 46, 48, 50, 52, 53, 54, 55, 56, 57, 58, 59, 61, 63, 65, 67, 69, 70, 71, 129, 131, 133, 138, 140, 141, 143, 145, 146, 148, 150, 151, 153, 155, 156, 158, 160, 195, 196, 197, 198, 199, 200, 213, 214, 215, 216, 217, 218, 227, 228, 229, 233, 234, 235, 241, 242, 243, 250, 251, 252 }

C grade: { }

F grade: { 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 191, 201, 205, 209, 219, 223, 230, 231, 232, 236, 237, 247, 248, 249, 253, 254 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	47	111	123	238	0	149
normalized size	1	1.	0.77	1.82	2.02	3.9	0.	2.44
time (sec)	N/A	0.057	0.466	0.082	1.237	0.53	0.	1.522

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	47	97	108	132	0	130
normalized size	1	1.	0.77	1.59	1.77	2.16	0.	2.13
time (sec)	N/A	0.058	0.325	0.023	1.175	0.522	0.	1.438

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	37	83	93	174	0	111
normalized size	1	1.	0.8	1.8	2.02	3.78	0.	2.41
time (sec)	N/A	0.055	0.251	0.012	1.213	0.504	0.	1.38

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	37	69	78	96	163	92
normalized size	1	1.	0.8	1.5	1.7	2.09	3.54	2.
time (sec)	N/A	0.055	0.146	0.013	1.258	0.49	65.169	1.237

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	55	63	108	126	73
normalized size	1	1.	0.87	1.77	2.03	3.48	4.06	2.35
time (sec)	N/A	0.05	0.09	0.012	1.188	0.481	16.938	1.276

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	41	49	62	92	54
normalized size	1	1.	0.87	1.32	1.58	2.	2.97	1.74
time (sec)	N/A	0.05	0.066	0.01	1.231	0.479	4.359	1.294

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	15	27	35	57	51	35
normalized size	1	1.	0.5	0.9	1.17	1.9	1.7	1.17
time (sec)	N/A	0.011	0.032	0.007	1.099	0.471	1.125	1.306

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	20	155	76	0	235
normalized size	1	1.	1.	1.43	11.07	5.43	0.	16.79
time (sec)	N/A	0.017	0.005	0.036	1.79	0.485	0.	1.52

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	50	36	319	154	0	556
normalized size	1	1.	1.79	1.29	11.39	5.5	0.	19.86
time (sec)	N/A	0.038	0.039	0.032	1.228	0.495	0.	1.643

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	29	55	1091	230	0	1918
normalized size	1	1.	0.59	1.12	22.27	4.69	0.	39.14
time (sec)	N/A	0.063	0.017	0.037	1.882	0.5	0.	2.205

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	205	78	2935	302	0	4088
normalized size	1	1.	3.11	1.18	44.47	4.58	0.	61.94
time (sec)	N/A	0.067	0.445	0.036	1.391	0.505	0.	4.72

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	31	97	4169	378	0	7623
normalized size	1	1.	0.35	1.09	46.84	4.25	0.	85.65
time (sec)	N/A	0.069	0.03	0.038	2.318	0.523	0.	13.169

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	68	86	97	122	0	115
normalized size	1	1.	1.55	1.95	2.2	2.77	0.	2.61
time (sec)	N/A	0.069	0.342	0.022	1.192	0.504	0.	1.449

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	62	75	88	180	0	100
normalized size	1	1.	0.82	0.99	1.16	2.37	0.	1.32
time (sec)	N/A	0.068	0.193	0.027	1.128	0.511	0.	1.354

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	48	58	68	88	362	77
normalized size	1	1.	1.66	2.	2.34	3.03	12.48	2.66
time (sec)	N/A	0.059	0.114	0.01	1.084	0.483	63.208	1.378

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	40	47	55	117	231	62
normalized size	1	1.	0.82	0.96	1.12	2.39	4.71	1.27
time (sec)	N/A	0.056	0.072	0.009	1.116	0.489	17.447	1.346

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	30	35	58	133	39
normalized size	1	1.	1.	2.	2.33	3.87	8.87	2.6
time (sec)	N/A	0.033	0.005	0.009	1.197	0.474	4.491	1.4

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	74	36	0	128
normalized size	1	1.	1.	0.93	5.29	2.57	0.	9.14
time (sec)	N/A	0.026	0.011	0.022	1.13	0.498	0.	1.507

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	72	47	0	205
normalized size	1	1.	1.	0.92	5.54	3.62	0.	15.77
time (sec)	N/A	0.035	0.008	0.091	1.113	0.453	0.	1.477

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	36	27	865	155	0	991
normalized size	1	1.	1.2	0.9	28.83	5.17	0.	33.03
time (sec)	N/A	0.048	0.036	0.036	1.122	0.499	0.	1.634

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	48	51	416	109	0	1396
normalized size	1	1.	1.14	1.21	9.9	2.6	0.	33.24
time (sec)	N/A	0.063	0.054	0.057	1.075	0.47	0.	1.699

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	56	69	4271	289	0	3587
normalized size	1	1.	0.93	1.15	71.18	4.82	0.	59.78
time (sec)	N/A	0.071	0.241	0.036	1.531	0.517	0.	2.58

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	37	97	108	204	0	130
normalized size	1	1.	0.8	2.11	2.35	4.43	0.	2.83
time (sec)	N/A	0.063	0.374	0.012	1.062	0.515	0.	1.592

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	47	83	93	130	0	111
normalized size	1	1.	0.77	1.36	1.52	2.13	0.	1.82
time (sec)	N/A	0.069	0.226	0.011	1.218	0.503	0.	1.429

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	55	63	138	0	73
normalized size	1	1.	0.87	1.77	2.03	4.45	0.	2.35
time (sec)	N/A	0.06	0.142	0.01	1.176	0.489	0.	1.284

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	37	55	63	95	202	73
normalized size	1	1.	0.8	1.2	1.37	2.07	4.39	1.59
time (sec)	N/A	0.064	0.109	0.01	1.149	0.485	73.172	1.307

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	41	46	81	117	54
normalized size	1	1.	1.	2.73	3.07	5.4	7.8	3.6
time (sec)	N/A	0.033	0.007	0.009	1.076	0.471	19.165	1.278

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	32	167	99	0	834
normalized size	1	1.	0.96	1.14	5.96	3.54	0.	29.79
time (sec)	N/A	0.037	0.012	0.026	1.757	0.503	0.	1.611

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	112	30	0	431
normalized size	1	1.	1.	1.08	8.62	2.31	0.	33.15
time (sec)	N/A	0.035	0.009	0.02	1.149	0.465	0.	1.619

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	38	38	648	163	0	1500
normalized size	1	1.	1.12	1.12	19.06	4.79	0.	44.12
time (sec)	N/A	0.041	0.011	0.066	1.72	0.492	0.	2.188

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	61	49	1332	194	0	2854
normalized size	1	1.	1.42	1.14	30.98	4.51	0.	66.37
time (sec)	N/A	0.053	0.026	0.035	1.294	0.502	0.	2.825

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	29	76	2437	262	0	5443
normalized size	1	1.	0.41	1.09	34.81	3.74	0.	77.76
time (sec)	N/A	0.072	0.035	0.036	2.067	0.509	0.	5.79

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	119	71	123	136	0	365
normalized size	1	1.	1.95	1.16	2.02	2.23	0.	5.98
time (sec)	N/A	0.061	0.094	0.036	1.068	0.538	0.	1.936

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	48	97	108	207	0	62
normalized size	1	1.	0.79	1.59	1.77	3.39	0.	1.02
time (sec)	N/A	0.061	0.214	0.06	1.224	0.52	0.	1.826

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	89	53	93	99	0	275
normalized size	1	1.	1.93	1.15	2.02	2.15	0.	5.98
time (sec)	N/A	0.056	0.058	0.029	1.15	0.503	0.	1.596

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	38	69	78	142	0	49
normalized size	1	1.	0.83	1.5	1.7	3.09	0.	1.07
time (sec)	N/A	0.057	0.098	0.056	1.163	0.495	0.	1.403

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	59	35	63	63	0	186
normalized size	1	1.	1.9	1.13	2.03	2.03	0.	6.
time (sec)	N/A	0.052	0.038	0.029	1.14	0.482	0.	1.4

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	28	41	49	84	0	35
normalized size	1	1.	0.9	1.32	1.58	2.71	0.	1.13
time (sec)	N/A	0.051	0.048	0.047	1.201	0.477	0.	1.348

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	31	31	0	70
normalized size	1	1.	1.	0.93	2.07	2.07	0.	4.67
time (sec)	N/A	0.037	0.007	0.019	1.172	0.472	0.	1.263

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	23	12	15	24	0	15
normalized size	1	1.	2.09	1.09	1.36	2.18	0.	1.36
time (sec)	N/A	0.017	0.008	0.018	1.077	0.468	0.	1.303

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	29	34	315	136	0	653
normalized size	1	1.	1.04	1.21	11.25	4.86	0.	23.32
time (sec)	N/A	0.041	0.017	0.03	1.8	0.491	0.	1.601

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	143	57	1315	262	0	1791
normalized size	1	1.	2.92	1.16	26.84	5.35	0.	36.55
time (sec)	N/A	0.059	0.259	0.032	1.088	0.5	0.	1.927

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	31	76	2403	342	0	4095
normalized size	1	1.	0.47	1.15	36.41	5.18	0.	62.05
time (sec)	N/A	0.063	0.021	0.035	1.95	0.505	0.	4.398

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	268	99	5192	417	0	7393
normalized size	1	1.	3.01	1.11	58.34	4.69	0.	83.07
time (sec)	N/A	0.072	0.512	0.037	1.788	0.519	0.	12.218

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	85	111	117	244	0	128
normalized size	1	1.	0.55	0.72	0.75	1.57	0.	0.83
time (sec)	N/A	0.171	0.248	0.06	1.115	0.546	0.	1.89

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	48	53	97	97	0	247
normalized size	1	1.	1.09	1.2	2.2	2.2	0.	5.61
time (sec)	N/A	0.065	0.172	0.027	1.081	0.511	0.	1.679

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	62	83	88	177	0	101
normalized size	1	1.	0.56	0.75	0.79	1.59	0.	0.91
time (sec)	N/A	0.12	0.198	0.052	1.199	0.513	0.	1.557

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	48	35	68	61	0	188
normalized size	1	1.	1.66	1.21	2.34	2.1	0.	6.48
time (sec)	N/A	0.057	0.124	0.025	1.121	0.49	0.	1.451

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	40	55	58	115	0	74
normalized size	1	1.	0.67	0.92	0.97	1.92	0.	1.23
time (sec)	N/A	0.074	0.099	0.05	1.199	0.485	0.	1.39

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	35	28	0	93
normalized size	1	1.	1.	1.08	2.69	2.15	0.	7.15
time (sec)	N/A	0.041	0.006	0.02	1.099	0.462	0.	1.339

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	20	28	24	53	0	39
normalized size	1	1.	0.95	1.33	1.14	2.52	0.	1.86
time (sec)	N/A	0.034	0.024	0.025	1.126	0.476	0.	1.297

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	20	13	109	36	0	74
normalized size	1	1.	1.67	1.08	9.08	3.	0.	6.17
time (sec)	N/A	0.02	0.015	0.02	1.217	0.484	0.	1.308

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	34	27	886	176	0	1098
normalized size	1	1.	1.13	0.9	29.53	5.87	0.	36.6
time (sec)	N/A	0.042	0.052	0.03	1.225	0.5	0.	1.47

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	48	51	416	136	0	1457
normalized size	1	1.	1.14	1.21	9.9	3.24	0.	34.69
time (sec)	N/A	0.059	0.08	0.032	1.208	0.467	0.	1.546

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	54	69	4304	367	0	3791
normalized size	1	1.	0.9	1.15	71.73	6.12	0.	63.18
time (sec)	N/A	0.067	0.36	0.035	1.456	0.521	0.	2.237

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	90	87	1656	228	0	4651
normalized size	1	1.	1.25	1.21	23.	3.17	0.	64.6
time (sec)	N/A	0.07	0.054	0.038	1.257	0.482	0.	3.423

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	76	111	10328	514	0	8605
normalized size	1	1.	0.84	1.23	114.76	5.71	0.	95.61
time (sec)	N/A	0.082	0.406	0.037	2.621	0.535	0.	7.276

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	132	123	3659	324	0	10094
normalized size	1	1.	1.29	1.21	35.87	3.18	0.	98.96
time (sec)	N/A	0.084	0.07	0.041	1.654	0.517	0.	13.248

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	119	71	123	142	0	424
normalized size	1	1.	1.95	1.16	2.02	2.33	0.	6.95
time (sec)	N/A	0.072	0.146	0.04	1.062	0.57	0.	2.309

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	58	107	123	246	0	76
normalized size	1	1.	0.76	1.41	1.62	3.24	0.	1.
time (sec)	N/A	0.077	0.329	0.058	1.075	0.53	0.	2.153

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	104	53	108	104	0	335
normalized size	1	1.	2.26	1.15	2.35	2.26	0.	7.28
time (sec)	N/A	0.067	0.098	0.032	1.058	0.523	0.	1.869

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	48	79	93	176	0	62
normalized size	1	1.	0.79	1.3	1.52	2.89	0.	1.02
time (sec)	N/A	0.071	0.167	0.029	1.104	0.511	0.	1.733

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	35	63	63	0	246
normalized size	1	1.	0.87	1.13	2.03	2.03	0.	7.94
time (sec)	N/A	0.061	0.146	0.029	1.041	0.492	0.	1.651

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	37	51	63	116	0	49
normalized size	1	1.	0.8	1.11	1.37	2.52	0.	1.07
time (sec)	N/A	0.066	0.108	0.029	1.037	0.484	0.	1.365

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	46	32	0	100
normalized size	1	1.	1.	0.93	3.07	2.13	0.	6.67
time (sec)	N/A	0.044	0.01	0.023	1.068	0.48	0.	1.422

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	28	22	31	55	0	30
normalized size	1	1.	1.04	0.81	1.15	2.04	0.	1.11
time (sec)	N/A	0.039	0.011	0.025	1.066	0.474	0.	1.351

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	44	34	124	112	0	77
normalized size	1	1.	1.83	1.42	5.17	4.67	0.	3.21
time (sec)	N/A	0.043	0.026	0.029	1.052	0.504	0.	1.25

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	113	28	0	18
normalized size	1	1.	1.	1.27	10.27	2.55	0.	1.64
time (sec)	N/A	0.026	0.01	0.019	1.038	0.454	0.	1.204

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	31	47	1126	254	0	2830
normalized size	1	1.	0.72	1.09	26.19	5.91	0.	65.81
time (sec)	N/A	0.049	0.018	0.032	1.782	0.502	0.	2.102

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	129	78	3020	374	0	5096
normalized size	1	1.	1.84	1.11	43.14	5.34	0.	72.8
time (sec)	N/A	0.073	4.633	0.036	1.343	0.509	0.	4.403

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	31	97	4178	458	0	8529
normalized size	1	1.	0.38	1.2	51.58	5.65	0.	105.3
time (sec)	N/A	0.073	0.035	0.039	2.32	0.516	0.	11.886

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	278	120	5762	529	0	0
normalized size	1	1.	2.48	1.07	51.45	4.72	0.	0.
time (sec)	N/A	0.087	0.834	0.04	1.94	0.532	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	98	183661406	0	803	0	0
normalized size	1	1.	0.72	6 1.35045 10	0.	5.9	0.	0.
time (sec)	N/A	0.094	0.288	59.26	0.	0.564	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	86	65166864	0	771	0	0
normalized size	1	1.	0.78	592426.	0.	7.01	0.	0.
time (sec)	N/A	0.068	0.194	15.386	0.	0.556	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	72	6219390	0	737	0	0
normalized size	1	1.	0.86	74040.4	0.	8.77	0.	0.
time (sec)	N/A	0.043	0.072	1.171	0.	0.538	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	50	15979424	0	657	0	0
normalized size	1	1.	0.86	275507.	0.	11.33	0.	0.
time (sec)	N/A	0.021	0.049	1.424	0.	0.526	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	22	59131270	0	107	0	0
normalized size	1	1.	0.96	6 2.57092 10	0.	4.65	0.	0.
time (sec)	N/A	0.018	0.018	8.56	0.	0.487	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	43	597	0	184	0	0
normalized size	1	1.	0.81	11.26	0.	3.47	0.	0.
time (sec)	N/A	0.04	0.103	18.474	0.	0.51	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	52	308	0	231	0	0
normalized size	1	1.	0.66	3.9	0.	2.92	0.	0.
time (sec)	N/A	0.059	0.189	257.353	0.	0.515	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	67	0	0	300	0	0
normalized size	1	1.	0.64	0.	0.	2.86	0.	0.
time (sec)	N/A	0.083	0.146	180.	0.	0.526	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	96	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.394	180.	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	66	278672995	0	0	0	0
normalized size	1	1.	0.96	6 4.03874 10	0.	0.	0.	0.
time (sec)	N/A	0.047	0.217	103.249	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	76	172329442	0	0	0	0
normalized size	1	1.	1.1	6 2.49753 10	0.	0.	0.	0.
time (sec)	N/A	0.047	0.345	48.767	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	34	16542587	0	0	0	0
normalized size	1	1.	0.85	413565.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.083	3.85	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	75	53350427	0	0	0	0
normalized size	1	1.	1.88	6 1.33376 10	0.	0.	0.	0.
time (sec)	N/A	0.037	0.245	8.632	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	41	83117872	0	0	0	0
normalized size	1	1.	0.91	6 1.84706 10	0.	0.	0.	0.
time (sec)	N/A	0.037	0.096	9.329	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	83	123	0	0	0	0
normalized size	1	1.	1.73	2.56	0.	0.	0.	0.
time (sec)	N/A	0.038	0.19	11.461	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	66	227	0	0	0	0
normalized size	1	1.	0.86	2.95	0.	0.	0.	0.
time (sec)	N/A	0.046	0.815	58.739	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	98	322248039	0	805	0	0
normalized size	1	1.	0.72	6 2.36947 10	0.	5.92	0.	0.
time (sec)	N/A	0.101	0.34	127.454	0.	0.569	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	86	47430975	0	774	0	0
normalized size	1	1.	0.78	431191.	0.	7.04	0.	0.
time (sec)	N/A	0.078	0.219	15.251	0.	0.551	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	74	155734626	0	740	0	0
normalized size	1	1.	0.88	6 1.85398 10	0.	8.81	0.	0.
time (sec)	N/A	0.054	0.141	33.727	0.	0.547	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	72	149404972	0	815	0	0
normalized size	1	1.	0.89	6 1.84451 10	0.	10.06	0.	0.
time (sec)	N/A	0.078	0.091	30.918	0.	0.536	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	727	0	132	0	0
normalized size	1	1.	0.96	25.96	0.	4.71	0.	0.
time (sec)	N/A	0.028	0.054	44.19	0.	0.493	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	35	0	0	147	0	0
normalized size	1	1.	0.64	0.	0.	2.67	0.	0.
time (sec)	N/A	0.049	0.096	180.	0.	0.503	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	55	0	0	213	0	0
normalized size	1	1.	0.68	0.	0.	2.63	0.	0.
time (sec)	N/A	0.069	0.107	180.	0.	0.517	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	62	0	0	263	0	0
normalized size	1	1.	0.58	0.	0.	2.46	0.	0.
time (sec)	N/A	0.092	0.163	180.	0.	0.536	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	98	973	0	806	0	0
normalized size	1	1.	0.72	7.15	0.	5.93	0.	0.
time (sec)	N/A	0.123	0.322	4.362	0.	0.579	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	86	243	0	768	0	0
normalized size	1	1.	0.78	2.21	0.	6.98	0.	0.
time (sec)	N/A	0.095	0.19	2.911	0.	0.557	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	70	362	0	734	0	0
normalized size	1	1.	0.86	4.47	0.	9.06	0.	0.
time (sec)	N/A	0.072	0.086	1.805	0.	0.542	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	52	157	0	657	0	0
normalized size	1	1.	0.98	2.96	0.	12.4	0.	0.
time (sec)	N/A	0.047	0.041	1.042	0.	0.52	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	23	308	0	103	0	0
normalized size	1	1.	0.96	12.83	0.	4.29	0.	0.
time (sec)	N/A	0.024	0.047	1.546	0.	0.491	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	43	194	0	192	0	0
normalized size	1	1.	0.81	3.66	0.	3.62	0.	0.
time (sec)	N/A	0.064	0.1	2.469	0.	0.498	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	52	481	0	267	0	0
normalized size	1	1.	0.66	6.09	0.	3.38	0.	0.
time (sec)	N/A	0.086	0.124	5.486	0.	0.514	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	67	222	0	320	0	0
normalized size	1	1.	0.64	2.11	0.	3.05	0.	0.
time (sec)	N/A	0.108	0.138	30.944	0.	0.53	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	66	204	0	0	0	0
normalized size	1	1.	0.62	1.92	0.	0.	0.	0.
time (sec)	N/A	0.058	0.295	7.493	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	76	139	0	0	0	0
normalized size	1	1.	0.72	1.31	0.	0.	0.	0.
time (sec)	N/A	0.059	0.258	5.161	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	34	137	0	0	0	0
normalized size	1	1.	0.45	1.83	0.	0.	0.	0.
time (sec)	N/A	0.047	0.078	3.971	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	73	111	0	0	0	0
normalized size	1	1.	1.04	1.59	0.	0.	0.	0.
time (sec)	N/A	0.048	0.86	2.622	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	37	176	0	0	0	0
normalized size	1	1.	0.84	4.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.127	2.096	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	82	0	0	0	0	0
normalized size	1	1.	1.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.951	180.	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	64	227	0	0	0	0
normalized size	1	1.	0.83	2.95	0.	0.	0.	0.
time (sec)	N/A	0.047	0.58	6.056	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	66	154	0	0	0	0
normalized size	1	1.	0.86	2.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.461	9.525	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	85	240	0	0	0	0
normalized size	1	1.	0.8	2.26	0.	0.	0.	0.
time (sec)	N/A	0.058	0.814	30.493	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	86	167	0	0	0	0
normalized size	1	1.	0.81	1.58	0.	0.	0.	0.
time (sec)	N/A	0.059	0.344	47.27	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	100	441	0	802	0	0
normalized size	1	1.	0.53	2.32	0.	4.22	0.	0.
time (sec)	N/A	0.186	0.435	26.75	0.	0.574	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	84	973	0	772	0	0
normalized size	1	1.	0.51	5.93	0.	4.71	0.	0.
time (sec)	N/A	0.156	0.22	19.125	0.	0.56	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	70	243	0	737	0	0
normalized size	1	1.	0.55	1.91	0.	5.8	0.	0.
time (sec)	N/A	0.131	0.13	7.616	0.	0.54	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	68	542	0	814	0	0
normalized size	1	1.	0.65	5.21	0.	7.83	0.	0.
time (sec)	N/A	0.102	0.098	6.01	0.	0.53	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	192	0	140	0	0
normalized size	1	1.	0.96	6.86	0.	5.	0.	0.
time (sec)	N/A	0.028	0.049	3.405	0.	0.488	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	35	0	0	198	0	0
normalized size	1	1.	0.64	0.	0.	3.6	0.	0.
time (sec)	N/A	0.053	0.092	180.	0.	0.498	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	55	222	0	281	0	0
normalized size	1	1.	0.68	2.74	0.	3.47	0.	0.
time (sec)	N/A	0.092	0.116	9.663	0.	0.507	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	62	560	0	356	0	0
normalized size	1	1.	0.58	5.23	0.	3.33	0.	0.
time (sec)	N/A	0.116	0.091	18.636	0.	0.527	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	602	0	0	0	0	0
normalized size	1	1.	7.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	5.625	1.214	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	602	0	0	0	0	0
normalized size	1	1.	7.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	3.596	0.848	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	82	82	152	0	0	0	0	0
normalized size	1	1.	1.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.268	0.957	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	72	72	254	0	0	0	0	0
normalized size	1	1.	3.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.889	0.517	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	938	0	0	0	0	0
normalized size	1	1.	11.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	5.515	0.326	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	2421	0	0	0	0	0
normalized size	1	1.	28.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	14.423	0.345	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	47	111	123	136	0	149
normalized size	1	1.	0.77	1.82	2.02	2.23	0.	2.44
time (sec)	N/A	0.06	0.454	0.024	1.082	0.538	0.	1.664

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	47	97	108	205	0	130
normalized size	1	1.	0.77	1.59	1.77	3.36	0.	2.13
time (sec)	N/A	0.061	0.315	0.036	1.121	0.519	0.	1.528

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	37	83	93	99	0	111
normalized size	1	1.	0.8	1.8	2.02	2.15	0.	2.41
time (sec)	N/A	0.055	0.264	0.023	1.027	0.504	0.	1.386

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	37	69	78	142	162	92
normalized size	1	1.	0.8	1.5	1.7	3.09	3.52	2.
time (sec)	N/A	0.054	0.131	0.027	1.052	0.504	76.047	1.265

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	55	63	62	128	73
normalized size	1	1.	0.87	1.77	2.03	2.	4.13	2.35
time (sec)	N/A	0.05	0.09	0.019	1.011	0.478	22.704	1.245

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	41	49	84	90	54
normalized size	1	1.	0.87	1.32	1.58	2.71	2.9	1.74
time (sec)	N/A	0.048	0.063	0.022	1.033	0.478	6.312	1.252

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	15	27	35	31	53	35
normalized size	1	1.	0.5	0.9	1.17	1.03	1.77	1.17
time (sec)	N/A	0.011	0.006	0.014	1.067	0.483	1.329	1.251

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	42	22	113	93	0	22
normalized size	1	1.	3.	1.57	8.07	6.64	0.	1.57
time (sec)	N/A	0.016	0.012	0.019	1.158	0.491	0.	1.193

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	29	34	315	136	0	51
normalized size	1	1.	1.04	1.21	11.25	4.86	0.	1.82
time (sec)	N/A	0.038	0.018	0.023	1.818	0.498	0.	1.235

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	143	57	1315	262	0	189
normalized size	1	1.	2.92	1.16	26.84	5.35	0.	3.86
time (sec)	N/A	0.055	0.255	0.027	1.255	0.499	0.	1.275

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	31	76	2403	343	0	97
normalized size	1	1.	0.47	1.15	36.41	5.2	0.	1.47
time (sec)	N/A	0.059	0.024	0.027	2.169	0.515	0.	1.301

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	268	99	5192	419	0	282
normalized size	1	1.	3.01	1.11	58.34	4.71	0.	3.17
time (sec)	N/A	0.067	0.489	0.032	1.863	0.519	0.	1.348

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	68	86	97	96	0	115
normalized size	1	1.	1.55	1.95	2.2	2.18	0.	2.61
time (sec)	N/A	0.067	0.402	0.021	1.15	0.511	0.	1.415

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	62	75	88	177	0	100
normalized size	1	1.	0.82	0.99	1.16	2.33	0.	1.32
time (sec)	N/A	0.064	0.189	0.03	1.156	0.511	0.	1.341

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	48	58	68	61	359	77
normalized size	1	1.	1.71	2.07	2.43	2.18	12.82	2.75
time (sec)	N/A	0.057	0.122	0.017	1.11	0.493	174.245	1.231

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	40	47	58	116	231	62
normalized size	1	1.	0.82	0.96	1.18	2.37	4.71	1.27
time (sec)	N/A	0.054	0.094	0.023	1.201	0.491	28.56	1.285

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	30	35	31	131	39
normalized size	1	1.	1.	2.	2.33	2.07	8.73	2.6
time (sec)	N/A	0.033	0.006	0.013	1.121	0.475	5.989	1.29

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	22	13	111	39	0	76
normalized size	1	1.	1.57	0.93	7.93	2.79	0.	5.43
time (sec)	N/A	0.025	0.015	0.014	1.18	0.491	0.	1.281

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	72	49	0	18
normalized size	1	1.	1.	0.92	5.54	3.77	0.	1.38
time (sec)	N/A	0.033	0.013	0.02	1.15	0.458	0.	1.194

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	34	27	886	177	0	161
normalized size	1	1.	1.13	0.9	29.53	5.9	0.	5.37
time (sec)	N/A	0.047	0.044	0.023	1.202	0.5	0.	1.285

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	48	51	416	136	0	47
normalized size	1	1.	1.14	1.21	9.9	3.24	0.	1.12
time (sec)	N/A	0.06	0.052	0.03	1.137	0.469	0.	1.264

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	54	69	4304	369	0	313
normalized size	1	1.	0.9	1.15	71.73	6.15	0.	5.22
time (sec)	N/A	0.068	0.367	0.027	1.507	0.517	0.	1.296

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	37	97	108	103	0	130
normalized size	1	1.	0.8	2.11	2.35	2.24	0.	2.83
time (sec)	N/A	0.061	0.39	0.024	1.126	0.519	0.	1.408

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	47	83	93	174	0	111
normalized size	1	1.	0.77	1.36	1.52	2.85	0.	1.82
time (sec)	N/A	0.065	0.212	0.03	1.166	0.511	0.	1.213

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	55	63	62	0	73
normalized size	1	1.	0.87	1.77	2.03	2.	0.	2.35
time (sec)	N/A	0.056	0.136	0.017	1.106	0.499	0.	1.3

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	37	55	63	115	202	73
normalized size	1	1.	0.8	1.2	1.37	2.5	4.39	1.59
time (sec)	N/A	0.06	0.092	0.026	1.097	0.481	77.694	1.222

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	41	46	31	117	54
normalized size	1	1.	1.	2.73	3.07	2.07	7.8	3.6
time (sec)	N/A	0.033	0.008	0.014	1.121	0.491	19.201	1.216

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	46	34	124	115	0	77
normalized size	1	1.	1.64	1.21	4.43	4.11	0.	2.75
time (sec)	N/A	0.035	0.019	0.022	1.121	0.5	0.	1.244

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	113	31	0	18
normalized size	1	1.	1.	1.08	8.69	2.38	0.	1.38
time (sec)	N/A	0.035	0.012	0.017	1.061	0.459	0.	1.232

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	79	40	753	203	0	124
normalized size	1	1.	2.32	1.18	22.15	5.97	0.	3.65
time (sec)	N/A	0.041	0.015	0.048	1.206	0.493	0.	1.315

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	31	47	1126	254	0	70
normalized size	1	1.	0.72	1.09	26.19	5.91	0.	1.63
time (sec)	N/A	0.051	0.018	0.027	2.416	0.502	0.	1.233

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	195	78	3020	375	0	220
normalized size	1	1.	2.79	1.11	43.14	5.36	0.	3.14
time (sec)	N/A	0.07	0.338	0.03	1.495	0.507	0.	1.312

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	98	199880296	0	806	0	0
normalized size	1	1.	0.72	6 1.46971 10	0.	5.93	0.	0.
time (sec)	N/A	0.091	0.357	61.386	0.	0.573	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	86	74316378	0	768	0	0
normalized size	1	1.	0.78	675603.	0.	6.98	0.	0.
time (sec)	N/A	0.067	0.186	18.977	0.	0.551	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	70	5339854	0	736	0	0
normalized size	1	1.	0.83	63569.7	0.	8.76	0.	0.
time (sec)	N/A	0.043	0.097	1.177	0.	0.533	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	52	622763	0	657	0	0
normalized size	1	1.	0.9	10737.3	0.	11.33	0.	0.
time (sec)	N/A	0.021	0.047	0.355	0.	0.53	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	23	55926091	0	108	0	0
normalized size	1	1.	0.96	6 2.33025 10	0.	4.5	0.	0.
time (sec)	N/A	0.018	0.02	6.793	0.	0.491	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	43	194	0	193	0	0
normalized size	1	1.	0.81	3.66	0.	3.64	0.	0.
time (sec)	N/A	0.038	0.102	6.571	0.	0.504	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	52	481	0	269	0	0
normalized size	1	1.	0.66	6.09	0.	3.41	0.	0.
time (sec)	N/A	0.059	0.121	89.464	0.	0.514	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	67	222	0	320	0	0
normalized size	1	1.	0.64	2.11	0.	3.05	0.	0.
time (sec)	N/A	0.08	0.143	183.381	0.	0.535	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	96	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.385	180.	0.	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	66	336654858	0	0	0	0
normalized size	1	1.	0.96	6 4.87906 10	0.	0.	0.	0.
time (sec)	N/A	0.045	0.208	110.597	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	76	174364218	0	0	0	0
normalized size	1	1.	1.1	6 2.52702 10	0.	0.	0.	0.
time (sec)	N/A	0.045	0.348	33.345	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	34	24847123	0	0	0	0
normalized size	1	1.	0.85	621178.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.058	3.975	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	76	59635246	0	0	0	0
normalized size	1	1.	1.9	6 1.49088 10	0.	0.	0.	0.
time (sec)	N/A	0.035	0.864	8.96	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	39	94273592	0	0	0	0
normalized size	1	1.	0.85	6 2.04943 10	0.	0.	0.	0.
time (sec)	N/A	0.036	0.114	9.786	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	82	123	0	0	0	0
normalized size	1	1.	1.71	2.56	0.	0.	0.	0.
time (sec)	N/A	0.036	1.007	11.904	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	64	227	0	0	0	0
normalized size	1	1.	0.83	2.95	0.	0.	0.	0.
time (sec)	N/A	0.046	0.562	71.1	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	99	0	0	803	0	0
normalized size	1	1.	0.73	0.	0.	5.9	0.	0.
time (sec)	N/A	0.099	0.328	180.	0.	0.562	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	84	88762396	0	774	0	0
normalized size	1	1.	0.76	806931.	0.	7.04	0.	0.
time (sec)	N/A	0.076	0.182	19.511	0.	0.55	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	73	190984194	0	738	0	0
normalized size	1	1.	0.87	6 2.27362 10	0.	8.79	0.	0.
time (sec)	N/A	0.052	0.11	38.924	0.	0.545	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	70	179366588	0	815	0	0
normalized size	1	1.	0.85	6 2.1874 10	0.	9.94	0.	0.
time (sec)	N/A	0.076	0.089	29.262	0.	0.541	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	192	0	142	0	0
normalized size	1	1.	0.96	6.86	0.	5.07	0.	0.
time (sec)	N/A	0.027	0.054	52.142	0.	0.49	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	35	482	0	200	0	0
normalized size	1	1.	0.64	8.76	0.	3.64	0.	0.
time (sec)	N/A	0.047	0.093	213.665	0.	0.501	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	55	0	0	282	0	0
normalized size	1	1.	0.68	0.	0.	3.48	0.	0.
time (sec)	N/A	0.068	0.114	180.	0.	0.506	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	62	0	0	358	0	0
normalized size	1	1.	0.58	0.	0.	3.35	0.	0.
time (sec)	N/A	0.09	0.094	180.	0.	0.526	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	29	98	0	455	0	0
normalized size	1	1.	0.94	3.16	0.	14.68	0.	0.
time (sec)	N/A	0.013	0.025	0.046	0.	0.508	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	99	0	455	0	0
normalized size	1	1.	1.	3.96	0.	18.2	0.	0.
time (sec)	N/A	0.03	0.016	0.046	0.	0.508	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	2472	0	0	0	0	0
normalized size	1	1.	29.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	13.303	0.992	0.	0.	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	890	0	0	0	0	0
normalized size	1	1.	10.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	7.969	0.875	0.	0.	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	149	0	0	0	0	0
normalized size	1	1.	1.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.244	0.914	0.	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	37	69	78	95	0	92
normalized size	1	1.	0.8	1.5	1.7	2.07	0.	2.
time (sec)	N/A	0.097	0.163	0.023	1.223	0.491	0.	1.858

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	293	293	209	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.848	0.857	1.304	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	86	84	1237	267	921	113
normalized size	1	1.	0.95	0.92	13.59	2.93	10.12	1.24
time (sec)	N/A	0.076	0.511	0.027	1.606	0.508	133.383	1.137

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	69	57	559	171	401	76
normalized size	1	1.	1.11	0.92	9.02	2.76	6.47	1.23
time (sec)	N/A	0.053	0.714	0.016	1.339	0.494	10.725	1.116

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	54	99	153	54
normalized size	1	1.	1.	0.93	1.26	2.3	3.56	1.26
time (sec)	N/A	0.035	0.187	0.016	1.249	0.475	3.613	1.141

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	325	146	77	335	319
normalized size	1	1.	1.	12.5	5.62	2.96	12.88	12.27
time (sec)	N/A	0.027	0.141	0.201	1.337	0.496	18.406	1.211

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	90	890	613	203	0	471
normalized size	1	1.	2.5	24.72	17.03	5.64	0.	13.08
time (sec)	N/A	0.033	0.097	0.446	1.283	0.502	0.	1.136

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	120	539	113	0	196
normalized size	1	1.	0.9	3.08	13.82	2.9	0.	5.03
time (sec)	N/A	0.044	0.19	0.602	1.193	0.47	0.	1.173

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	14880	2394	377	0	2998
normalized size	1	1.	1.	222.09	35.73	5.63	0.	44.75
time (sec)	N/A	0.046	0.553	2.466	1.655	0.516	0.	1.242

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	58	321	1453	193	0	406
normalized size	1	1.	0.97	5.35	24.22	3.22	0.	6.77
time (sec)	N/A	0.047	0.374	1.888	1.574	0.481	0.	1.157

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	79	97954	5237	543	0	10847
normalized size	1	1.	0.84	1042.06	55.71	5.78	0.	115.39
time (sec)	N/A	0.058	1.165	10.075	2.231	0.532	0.	1.32

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	410	410	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.973	0.375	1.504	0.	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	80	63	501	155	410	82
normalized size	1	1.	1.18	0.93	7.37	2.28	6.03	1.21
time (sec)	N/A	0.054	0.351	0.017	1.149	0.477	18.268	1.138

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	106	83	837	263	1027	108
normalized size	1	1.	1.2	0.94	9.51	2.99	11.67	1.23
time (sec)	N/A	0.065	0.792	0.027	1.247	0.512	40.937	1.168

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	158	133	1839	429	0	174
normalized size	1	1.	1.1	0.92	12.77	2.98	0.	1.21
time (sec)	N/A	0.097	1.645	0.023	1.461	0.545	0.	1.166

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	600	600	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.722	0.542	1.488	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	91	90	1065	246	957	120
normalized size	1	1.	0.94	0.93	10.98	2.54	9.87	1.24
time (sec)	N/A	0.075	0.518	0.024	1.388	0.499	88.981	1.132

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	153	127	1836	427	0	167
normalized size	1	1.	1.11	0.92	13.3	3.09	0.	1.21
time (sec)	N/A	0.098	1.672	0.022	1.559	0.534	0.	1.117

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	177	184	3526	652	0	244
normalized size	1	1.	0.91	0.94	18.08	3.34	0.	1.25
time (sec)	N/A	0.132	1.657	0.046	2.036	0.574	0.	1.121

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	277	277	202	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.589	0.975	0.698	0.	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	87	84	1231	239	923	113
normalized size	1	1.	0.96	0.92	13.53	2.63	10.14	1.24
time (sec)	N/A	0.066	0.509	0.019	1.284	0.512	79.467	1.16

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	71	57	559	162	401	76
normalized size	1	1.	1.15	0.92	9.02	2.61	6.47	1.23
time (sec)	N/A	0.047	0.781	0.016	1.212	0.491	12.126	1.139

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	54	100	155	54
normalized size	1	1.	1.	0.93	1.26	2.33	3.6	1.26
time (sec)	N/A	0.036	0.196	0.014	1.178	0.474	2.872	1.135

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	563	99	74	435	213
normalized size	1	1.	1.	20.85	3.67	2.74	16.11	7.89
time (sec)	N/A	0.018	0.142	0.201	1.259	0.498	169.994	1.161

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	88	888	522	186	0	335
normalized size	1	1.	2.59	26.12	15.35	5.47	0.	9.85
time (sec)	N/A	0.027	0.091	0.424	2.042	0.504	0.	1.178

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	34	150	528	109	0	235
normalized size	1	1.	0.89	3.95	13.89	2.87	0.	6.18
time (sec)	N/A	0.037	0.174	0.595	1.181	0.49	0.	1.197

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	64	14825	1922	258	0	668
normalized size	1	1.	0.96	221.27	28.69	3.85	0.	9.97
time (sec)	N/A	0.042	0.431	2.318	2.161	0.527	0.	1.201

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	48	381	1450	142	0	441
normalized size	1	1.	0.81	6.46	24.58	2.41	0.	7.47
time (sec)	N/A	0.051	0.339	1.819	1.191	0.495	0.	1.246

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	78	97703	4180	294	0	1021
normalized size	1	1.	0.83	1039.39	44.47	3.13	0.	10.86
time (sec)	N/A	0.07	1.003	9.186	2.661	0.548	0.	1.214

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	386	386	242	0	0	0	0	0
normalized size	1	1.	0.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.7	1.786	0.751	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	76	63	501	155	410	82
normalized size	1	1.	1.12	0.93	7.37	2.28	6.03	1.21
time (sec)	N/A	0.051	0.784	0.023	1.213	0.489	9.755	1.151

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	108	83	837	246	1027	108
normalized size	1	1.	1.23	0.94	9.51	2.8	11.67	1.23
time (sec)	N/A	0.068	0.782	0.028	1.21	0.509	151.378	1.127

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	158	133	1839	393	0	174
normalized size	1	1.	1.1	0.92	12.77	2.73	0.	1.21
time (sec)	N/A	0.096	1.826	0.036	1.416	0.527	0.	1.141

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	568	568	329	0	0	0	0	0
normalized size	1	1.	0.58	0.	0.	0.	0.	0.
time (sec)	N/A	1.18	24.644	0.762	0.	0.	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	90	90	1060	247	964	120
normalized size	1	1.	0.93	0.93	10.93	2.55	9.94	1.24
time (sec)	N/A	0.068	0.523	0.02	1.267	0.505	88.4	1.127

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	153	127	1836	404	0	167
normalized size	1	1.	1.11	0.92	13.3	2.93	0.	1.21
time (sec)	N/A	0.096	1.641	0.023	1.43	0.54	0.	1.141

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	176	184	3526	591	0	244
normalized size	1	1.	0.9	0.94	18.08	3.03	0.	1.25
time (sec)	N/A	0.124	1.602	0.029	1.889	0.588	0.	1.133

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	58	325	143	77	335	651
normalized size	1	1.	2.15	12.04	5.3	2.85	12.41	24.11
time (sec)	N/A	0.017	0.173	0.184	1.113	0.501	10.342	1.228

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	90	1062	608	203	0	1326
normalized size	1	1.	2.57	30.34	17.37	5.8	0.	37.89
time (sec)	N/A	0.028	0.096	0.452	1.31	0.517	0.	1.332

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	35	55	533	113	0	441
normalized size	1	1.	0.92	1.45	14.03	2.97	0.	11.61
time (sec)	N/A	0.043	0.197	0.544	1.122	0.469	0.	1.262

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	70	186	1386	1015	0	0
normalized size	1	1.	0.97	2.58	19.25	14.1	0.	0.
time (sec)	N/A	0.071	0.363	0.115	1.976	0.565	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	109	143	702	857	0	0
normalized size	1	1.	2.48	3.25	15.95	19.48	0.	0.
time (sec)	N/A	0.036	0.1	0.083	1.992	0.539	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	94	99	177	509	0	0
normalized size	1	1.	3.24	3.41	6.1	17.55	0.	0.
time (sec)	N/A	0.016	0.049	0.066	1.914	0.517	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	93	95	142	531	0	305
normalized size	1	1.	3.21	3.28	4.9	18.31	0.	10.52
time (sec)	N/A	0.016	0.053	0.085	1.164	0.531	0.	1.193

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	111	143	826	859	0	779
normalized size	1	1.	2.41	3.11	17.96	18.67	0.	16.93
time (sec)	N/A	0.04	0.097	0.089	1.335	0.553	0.	1.21

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	71	184	1693	987	0	1175
normalized size	1	1.	0.96	2.49	22.88	13.34	0.	15.88
time (sec)	N/A	0.072	0.351	0.104	1.249	0.565	0.	1.332

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	116	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	1.787	0.181	0.	0.	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	260	0	0	0	0	0
normalized size	1	1.	1.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	3.595	0.208	0.	0.	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	85	84	1234	243	933	113
normalized size	1	1.	0.93	0.92	13.56	2.67	10.25	1.24
time (sec)	N/A	0.065	0.486	0.027	1.455	0.511	136.5	1.124

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	69	57	562	147	401	76
normalized size	1	1.	1.11	0.92	9.06	2.37	6.47	1.23
time (sec)	N/A	0.045	0.713	0.023	1.248	0.504	12.625	1.119

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	54	99	153	54
normalized size	1	1.	1.	0.93	1.26	2.3	3.56	1.26
time (sec)	N/A	0.033	0.181	0.019	1.073	0.473	2.412	1.091

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	461	100	73	435	594
normalized size	1	1.	1.	17.73	3.85	2.81	16.73	22.85
time (sec)	N/A	0.013	0.121	0.199	1.182	0.533	109.34	1.215

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	89	1049	528	185	0	1810
normalized size	1	1.	2.54	29.97	15.09	5.29	0.	51.71
time (sec)	N/A	0.029	0.089	0.451	1.868	0.536	0.	1.38

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	35	56	516	108	0	425
normalized size	1	1.	0.92	1.47	13.58	2.84	0.	11.18
time (sec)	N/A	0.039	0.191	0.556	1.129	0.485	0.	1.174

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	158	133	1839	370	0	174
normalized size	1	1.	1.1	0.92	12.77	2.57	0.	1.21
time (sec)	N/A	0.089	1.639	0.036	1.573	0.533	0.	1.125

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	105	83	837	234	1027	108
normalized size	1	1.	1.19	0.94	9.51	2.66	11.67	1.23
time (sec)	N/A	0.069	0.733	0.031	1.279	0.511	35.085	1.128

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	176	184	3529	535	0	244
normalized size	1	1.	0.9	0.94	18.1	2.74	0.	1.25
time (sec)	N/A	0.135	1.63	0.046	1.879	0.592	0.	1.117

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	70	181	1386	986	0	0
normalized size	1	1.	0.97	2.51	19.25	13.69	0.	0.
time (sec)	N/A	0.078	0.363	0.099	2.031	0.571	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	111	149	710	857	0	0
normalized size	1	1.	2.41	3.24	15.43	18.63	0.	0.
time (sec)	N/A	0.039	0.097	0.085	1.969	0.536	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	93	97	177	531	0	0
normalized size	1	1.	3.1	3.23	5.9	17.7	0.	0.
time (sec)	N/A	0.018	0.06	0.066	1.862	0.531	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	94	93	142	510	0	316
normalized size	1	1.	3.24	3.21	4.9	17.59	0.	10.9
time (sec)	N/A	0.021	0.051	0.079	1.182	0.519	0.	1.16

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	112	145	828	856	0	846
normalized size	1	1.	2.43	3.15	18.	18.61	0.	18.39
time (sec)	N/A	0.041	0.099	0.095	1.329	0.544	0.	1.23

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	71	179	1693	1040	0	1300
normalized size	1	1.	0.97	2.45	23.19	14.25	0.	17.81
time (sec)	N/A	0.077	0.336	0.104	1.341	0.56	0.	1.384

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	142	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.119	1.884	0.175	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	108	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.12	1.742	0.204	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [201] had the largest ratio of [0.5882]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.	18	0.167
2	A	4	3	1.	18	0.167
3	A	4	3	1.	18	0.167
4	A	4	3	1.	18	0.167
5	A	4	3	1.	18	0.167
6	A	4	3	1.	18	0.167
7	A	1	1	1.	16	0.062
8	A	2	2	1.	16	0.125
9	A	4	4	1.	18	0.222
10	A	5	5	1.	18	0.278
11	A	6	5	1.	18	0.278
12	A	7	5	1.	18	0.278
13	A	5	4	1.	20	0.2
14	A	6	5	1.	20	0.25
15	A	4	3	1.	20	0.15
16	A	5	5	1.	20	0.25
17	A	3	3	1.	18	0.167
18	A	2	2	1.	18	0.111
19	A	3	3	1.	20	0.15
20	A	4	3	1.	20	0.15
21	A	4	3	1.	20	0.15
22	A	5	4	1.	20	0.2
23	A	4	3	1.	20	0.15
24	A	4	3	1.	20	0.15
25	A	4	3	1.	20	0.15
26	A	4	3	1.	20	0.15
27	A	3	3	1.	18	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
28	A	4	4	1.	18	0.222
29	A	3	3	1.	20	0.15
30	A	3	3	1.	20	0.15
31	A	5	4	1.	20	0.2
32	A	6	5	1.	20	0.25
33	A	4	3	1.	18	0.167
34	A	4	3	1.	18	0.167
35	A	4	3	1.	18	0.167
36	A	4	3	1.	18	0.167
37	A	4	3	1.	18	0.167
38	A	4	3	1.	18	0.167
39	A	3	3	1.	18	0.167
40	A	2	2	1.	16	0.125
41	A	4	4	1.	16	0.25
42	A	5	5	1.	18	0.278
43	A	6	5	1.	18	0.278
44	A	7	5	1.	18	0.278
45	A	9	4	1.	20	0.2
46	A	5	4	1.	20	0.2
47	A	7	4	1.	20	0.2
48	A	4	3	1.	20	0.15
49	A	5	4	1.	20	0.2
50	A	3	3	1.	20	0.15
51	A	3	3	1.	20	0.15
52	A	2	2	1.	18	0.111
53	A	4	3	1.	18	0.167
54	A	4	3	1.	20	0.15
55	A	5	4	1.	20	0.2
56	A	4	3	1.	20	0.15
57	A	5	4	1.	20	0.2
58	A	4	3	1.	20	0.15
59	A	4	3	1.	20	0.15
60	A	4	3	1.	20	0.15
61	A	4	3	1.	20	0.15
62	A	4	3	1.	20	0.15
63	A	4	3	1.	20	0.15
64	A	4	3	1.	20	0.15
65	A	3	3	1.	20	0.15
66	A	3	2	1.	20	0.1
67	A	4	4	1.	20	0.2
68	A	3	3	1.	18	0.167
69	A	5	4	1.	18	0.222
70	A	6	5	1.	20	0.25
71	A	6	5	1.	20	0.25

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
72	A	8	5	1.	20	0.25
73	A	4	3	1.	20	0.15
74	A	3	3	1.	20	0.15
75	A	2	2	1.	20	0.1
76	A	1	1	1.	20	0.05
77	A	1	1	1.	20	0.05
78	A	2	2	1.	20	0.1
79	A	3	3	1.	20	0.15
80	A	4	3	1.	20	0.15
81	A	4	3	1.	22	0.136
82	A	3	3	1.	22	0.136
83	A	3	3	1.	22	0.136
84	A	2	2	1.	22	0.091
85	A	2	2	1.	22	0.091
86	A	2	2	1.	22	0.091
87	A	2	2	1.	22	0.091
88	A	3	3	1.	22	0.136
89	A	4	4	1.	22	0.182
90	A	3	3	1.	22	0.136
91	A	2	2	1.	22	0.091
92	A	3	3	1.	22	0.136
93	A	1	1	1.	22	0.045
94	A	2	2	1.	22	0.091
95	A	3	3	1.	22	0.136
96	A	4	4	1.	22	0.182
97	A	5	4	1.	20	0.2
98	A	4	4	1.	20	0.2
99	A	3	3	1.	20	0.15
100	A	2	2	1.	20	0.1
101	A	1	1	1.	20	0.05
102	A	3	3	1.	20	0.15
103	A	4	4	1.	20	0.2
104	A	5	4	1.	20	0.2
105	A	4	3	1.	22	0.136
106	A	4	3	1.	22	0.136
107	A	3	3	1.	22	0.136
108	A	3	3	1.	22	0.136
109	A	2	2	1.	22	0.091
110	A	2	2	1.	22	0.091
111	A	3	3	1.	22	0.136
112	A	3	3	1.	22	0.136
113	A	4	3	1.	22	0.136
114	A	4	3	1.	22	0.136
115	A	7	5	1.	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
116	A	6	5	1.	22	0.227
117	A	5	5	1.	22	0.227
118	A	4	4	1.	22	0.182
119	A	1	1	1.	22	0.045
120	A	2	2	1.	22	0.091
121	A	4	4	1.	22	0.182
122	A	5	5	1.	22	0.227
123	A	2	2	1.	20	0.1
124	A	2	2	1.	20	0.1
125	A	2	2	1.	18	0.111
126	A	2	2	1.	18	0.111
127	A	2	2	1.	20	0.1
128	A	2	2	1.	20	0.1
129	A	4	3	1.	18	0.167
130	A	4	3	1.	18	0.167
131	A	4	3	1.	18	0.167
132	A	4	3	1.	18	0.167
133	A	4	3	1.	18	0.167
134	A	4	3	1.	18	0.167
135	A	1	1	1.	16	0.062
136	A	2	2	1.	16	0.125
137	A	4	4	1.	18	0.222
138	A	5	5	1.	18	0.278
139	A	6	5	1.	18	0.278
140	A	7	5	1.	18	0.278
141	A	5	4	1.	20	0.2
142	A	6	5	1.	20	0.25
143	A	4	3	1.	20	0.15
144	A	5	5	1.	20	0.25
145	A	3	3	1.	18	0.167
146	A	2	2	1.	18	0.111
147	A	3	3	1.	20	0.15
148	A	4	3	1.	20	0.15
149	A	4	3	1.	20	0.15
150	A	5	4	1.	20	0.2
151	A	4	3	1.	20	0.15
152	A	4	3	1.	20	0.15
153	A	4	3	1.	20	0.15
154	A	4	3	1.	20	0.15
155	A	3	3	1.	18	0.167
156	A	4	4	1.	18	0.222
157	A	3	3	1.	20	0.15
158	A	3	3	1.	20	0.15
159	A	5	4	1.	20	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
160	A	6	5	1.	20	0.25
161	A	4	3	1.	20	0.15
162	A	3	3	1.	20	0.15
163	A	2	2	1.	20	0.1
164	A	1	1	1.	20	0.05
165	A	1	1	1.	20	0.05
166	A	2	2	1.	20	0.1
167	A	3	3	1.	20	0.15
168	A	4	3	1.	20	0.15
169	A	4	3	1.	22	0.136
170	A	3	3	1.	22	0.136
171	A	3	3	1.	22	0.136
172	A	2	2	1.	22	0.091
173	A	2	2	1.	22	0.091
174	A	2	2	1.	22	0.091
175	A	2	2	1.	22	0.091
176	A	3	3	1.	22	0.136
177	A	4	4	1.	22	0.182
178	A	3	3	1.	22	0.136
179	A	2	2	1.	22	0.091
180	A	3	3	1.	22	0.136
181	A	1	1	1.	22	0.045
182	A	2	2	1.	22	0.091
183	A	3	3	1.	22	0.136
184	A	4	4	1.	22	0.182
185	A	1	1	1.	11	0.091
186	A	2	2	1.	11	0.182
187	A	2	2	1.	20	0.1
188	A	2	2	1.	20	0.1
189	A	2	2	1.	18	0.111
190	A	4	3	1.	28	0.107
191	A	10	5	1.	15	0.333
192	A	6	2	1.	15	0.133
193	A	5	2	1.	15	0.133
194	A	4	2	1.	13	0.154
195	A	3	3	1.	13	0.231
196	A	4	4	1.	15	0.267
197	A	5	5	1.	15	0.333
198	A	5	5	1.	15	0.333
199	A	5	4	1.	15	0.267
200	A	6	5	1.	15	0.333
201	A	15	10	1.	17	0.588
202	A	5	2	1.	15	0.133
203	A	6	2	1.	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
204	A	8	2	1.	17	0.118
205	A	18	5	1.	17	0.294
206	A	6	2	1.	15	0.133
207	A	8	2	1.	17	0.118
208	A	10	2	1.	17	0.118
209	A	8	4	1.	15	0.267
210	A	6	2	1.	15	0.133
211	A	5	2	1.	15	0.133
212	A	4	2	1.	13	0.154
213	A	3	3	1.	13	0.231
214	A	4	4	1.	15	0.267
215	A	5	5	1.	15	0.333
216	A	5	5	1.	15	0.333
217	A	5	4	1.	15	0.267
218	A	6	5	1.	15	0.333
219	A	11	7	1.	17	0.412
220	A	5	2	1.	15	0.133
221	A	6	2	1.	17	0.118
222	A	8	2	1.	17	0.118
223	A	14	4	1.	17	0.235
224	A	6	2	1.	15	0.133
225	A	8	2	1.	17	0.118
226	A	10	2	1.	17	0.118
227	A	3	3	1.	13	0.231
228	A	4	4	1.	15	0.267
229	A	5	5	1.	15	0.333
230	A	9	7	1.	15	0.467
231	A	6	6	1.	15	0.4
232	A	3	3	1.	13	0.231
233	A	3	3	1.	13	0.231
234	A	6	6	1.	15	0.4
235	A	9	7	1.	15	0.467
236	A	6	3	1.	13	0.231
237	A	6	3	1.	13	0.231
238	A	6	2	1.	15	0.133
239	A	5	2	1.	15	0.133
240	A	4	2	1.	13	0.154
241	A	3	3	1.	13	0.231
242	A	4	4	1.	15	0.267
243	A	5	5	1.	15	0.333
244	A	8	2	1.	17	0.118
245	A	6	2	1.	17	0.118
246	A	10	2	1.	17	0.118
247	A	9	7	1.	15	0.467

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
248	A	6	6	1.	15	0.4
249	A	3	3	1.	13	0.231
250	A	3	3	1.	13	0.231
251	A	6	6	1.	15	0.4
252	A	9	7	1.	15	0.467
253	A	6	3	1.	13	0.231
254	A	6	3	1.	13	0.231

Chapter 3

Listing of integrals

3.1 $\int \sin(a+bx)\sin^7(2a+2bx)\,dx$

Optimal. Leaf size=61

$$-\frac{128\sin^{15}(a+bx)}{15b}+\frac{384\sin^{13}(a+bx)}{13b}-\frac{384\sin^{11}(a+bx)}{11b}+\frac{128\sin^9(a+bx)}{9b}$$

```
[Out] (128*Sin[a + b*x]^9)/(9*b) - (384*Sin[a + b*x]^11)/(11*b) + (384*Sin[a + b*
x]^13)/(13*b) - (128*Sin[a + b*x]^15)/(15*b)
```

Rubi [A] time = 0.0574345, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2564, 270}

$$-\frac{128\sin^{15}(a+bx)}{15b}+\frac{384\sin^{13}(a+bx)}{13b}-\frac{384\sin^{11}(a+bx)}{11b}+\frac{128\sin^9(a+bx)}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (128*Sin[a + b*x]^9)/(9*b) - (384*Sin[a + b*x]^11)/(11*b) + (384*Sin[a + b*
x]^13)/(13*b) - (128*Sin[a + b*x]^15)/(15*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sin(a+bx)\sin^7(2a+2bx)\,dx &= 128\int \cos^7(a+bx)\sin^8(a+bx)\,dx \\
&= \frac{128\,\text{Subst}\left(\int x^8\left(1-x^2\right)^3\,dx, x, \sin(a+bx)\right)}{b} \\
&= \frac{128\,\text{Subst}\left(\int \left(x^8-3x^{10}+3x^{12}-x^{14}\right)dx, x, \sin(a+bx)\right)}{b} \\
&= \frac{128\sin^9(a+bx)}{9b}-\frac{384\sin^{11}(a+bx)}{11b}+\frac{384\sin^{13}(a+bx)}{13b}-\frac{128\sin^{15}(a+bx)}{15b}
\end{aligned}
$$

Mathematica [A] time = 0.46577, size = 47, normalized size = 0.77

$$\frac{4\sin^9(a+bx)(10755\cos(2(a+bx))+3366\cos(4(a+bx))+429\cos(6(a+bx))+8330)}{6435b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (4*(8330 + 10755*Cos[2*(a + b*x)] + 3366*Cos[4*(a + b*x)] + 429*Cos[6*(a +
b*x)])*Sin[a + b*x]^9)/(6435*b)
```

Maple [B] time = 0.082, size = 111, normalized size = 1.8

$$\frac{35\sin(bx+a)}{128b}-\frac{35\sin(3bx+3a)}{384b}-\frac{21\sin(5bx+5a)}{640b}+\frac{3\sin(7bx+7a)}{128b}+\frac{7\sin(9bx+9a)}{1152b}-\frac{7\sin(11bx+11a)}{1408b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^7,x)
```

```
[Out] 35/128*sin(b*x+a)/b-35/384*sin(3*b*x+3*a)/b-21/640/b*sin(5*b*x+5*a)+3/128/b
*sin(7*b*x+7*a)+7/1152/b*sin(9*b*x+9*a)-7/1408/b*sin(11*b*x+11*a)-1/1664/b*
sin(13*b*x+13*a)+1/1920/b*sin(15*b*x+15*a)
```

Maxima [A] time = 1.23679, size = 123, normalized size = 2.02

$$\frac{429\sin(15bx+15a)-495\sin(13bx+13a)-4095\sin(11bx+11a)+5005\sin(9bx+9a)+19305\sin(7bx+7a)}{823680b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="maxima")
```

```
[Out] 1/823680*(429*sin(15*b*x + 15*a) - 495*sin(13*b*x + 13*a) - 4095*sin(11*b*x
 + 11*a) + 5005*sin(9*b*x + 9*a) + 19305*sin(7*b*x + 7*a) - 27027*sin(5*b*x
 + 5*a) - 75075*sin(3*b*x + 3*a) + 225225*sin(b*x + a))/b
```

Fricas [A] time = 0.52975, size = 238, normalized size = 3.9

$$\frac{128 \left(429 \cos(bx+a)^{14} - 1518 \cos(bx+a)^{12} + 1854 \cos(bx+a)^{10} - 800 \cos(bx+a)^{8} + 5 \cos(bx+a)^{6} + 6 \cos(bx$$
$$6435\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="fricas")
```

```
[Out] 128/6435*(429*cos(b*x + a)^14 - 1518*cos(b*x + a)^12 + 1854*cos(b*x + a)^10
 - 800*cos(b*x + a)^8 + 5*cos(b*x + a)^6 + 6*cos(b*x + a)^4 + 8*cos(b*x + a
)^2 + 16)*sin(b*x + a)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**7,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.52236, size = 149, normalized size = 2.44

$$\frac{\sin(15\,bx+15\,a)}{1920\,b} - \frac{\sin(13\,bx+13\,a)}{1664\,b} - \frac{7\,\sin(11\,bx+11\,a)}{1408\,b} + \frac{7\,\sin(9\,bx+9\,a)}{1152\,b} + \frac{3\,\sin(7\,bx+7\,a)}{128\,b} - \frac{21\,\sin(5\,bx+}{640\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="giac")
```

```
[Out] 1/1920*sin(15*b*x + 15*a)/b - 1/1664*sin(13*b*x + 13*a)/b - 7/1408*sin(11*b
*x + 11*a)/b + 7/1152*sin(9*b*x + 9*a)/b + 3/128*sin(7*b*x + 7*a)/b - 21/64
0*sin(5*b*x + 5*a)/b - 35/384*sin(3*b*x + 3*a)/b + 35/128*sin(b*x + a)/b
```

3.2 $\int \sin(a+bx)\sin^6(2a+2bx)\,dx$

Optimal. Leaf size=61

$$\frac{64\cos^{13}(a+bx)}{13b}-\frac{192\cos^{11}(a+bx)}{11b}+\frac{64\cos^9(a+bx)}{3b}-\frac{64\cos^7(a+bx)}{7b}$$

```
[Out] (-64*Cos[a + b*x]^7)/(7*b) + (64*Cos[a + b*x]^9)/(3*b) - (192*Cos[a + b*x]^
11)/(11*b) + (64*Cos[a + b*x]^13)/(13*b)
```

Rubi [A] time = 0.0580589, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2565, 270}

$$\frac{64\cos^{13}(a+bx)}{13b}-\frac{192\cos^{11}(a+bx)}{11b}+\frac{64\cos^9(a+bx)}{3b}-\frac{64\cos^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

```
[Out] (-64*Cos[a + b*x]^7)/(7*b) + (64*Cos[a + b*x]^9)/(3*b) - (192*Cos[a + b*x]^
11)/(11*b) + (64*Cos[a + b*x]^13)/(13*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\sin^6(2a+2bx)\,dx &= 64\int \cos^6(a+bx)\sin^7(a+bx)\,dx\\ &= -\frac{64\,\text{Subst}\left(\int x^6\left(1-x^2\right)^3\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{64\,\text{Subst}\left(\int \left(x^6-3x^8+3x^{10}-x^{12}\right)\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{64\cos^7(a+bx)}{7b}+\frac{64\cos^9(a+bx)}{3b}-\frac{192\cos^{11}(a+bx)}{11b}+\frac{64\cos^{13}(a+bx)}{13b}\end{aligned}$$

Mathematica [A] time = 0.325048, size = 47, normalized size = 0.77

$$\frac{2\cos^7(a+bx)(6377\cos(2(a+bx))-1890\cos(4(a+bx))+231\cos(6(a+bx))-5230)}{3003b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

```
[Out] (2*Cos[a + b*x]^7*(-5230 + 6377*Cos[2*(a + b*x)] - 1890*Cos[4*(a + b*x)] +
231*Cos[6*(a + b*x)]))/(3003*b)
```

Maple [A] time = 0.023, size = 97, normalized size = 1.6

$$-\frac{5\cos(bx+a)}{16b}-\frac{5\cos(3bx+3a)}{64b}+\frac{3\cos(5bx+5a)}{64b}+\frac{3\cos(7bx+7a)}{224b}-\frac{\cos(9bx+9a)}{96b}-\frac{\cos(11bx+11a)}{704b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^6,x)
```

```
[Out] -5/16*cos(b*x+a)/b-5/64*cos(3*b*x+3*a)/b+3/64*cos(5*b*x+5*a)/b+3/224*cos(7*
b*x+7*a)/b-1/96*cos(9*b*x+9*a)/b-1/704*cos(11*b*x+11*a)/b+1/832*cos(13*b*x+
13*a)/b
```

Maxima [A] time = 1.17533, size = 108, normalized size = 1.77

$$\frac{231\cos(13bx+13a)-273\cos(11bx+11a)-2002\cos(9bx+9a)+2574\cos(7bx+7a)+9009\cos(5bx+5a}{192192b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="maxima")
```

```
[Out] 1/192192*(231*cos(13*b*x + 13*a) - 273*cos(11*b*x + 11*a) - 2002*cos(9*b*x
+ 9*a) + 2574*cos(7*b*x + 7*a) + 9009*cos(5*b*x + 5*a) - 15015*cos(3*b*x +
3*a) - 60060*cos(b*x + a))/b
```

Fricas [A] time = 0.522097, size = 132, normalized size = 2.16

$$\frac{64\left(231\cos(bx+a)^{13}-819\cos(bx+a)^{11}+1001\cos(bx+a)^{9}-429\cos(bx+a)^{7}\right)}{3003b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="fricas")
```

```
[Out] 64/3003*(231*cos(b*x + a)^13 - 819*cos(b*x + a)^11 + 1001*cos(b*x + a)^9 -
429*cos(b*x + a)^7)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.43786, size = 130, normalized size = 2.13

$$\frac{\cos\left(13\,bx+13\,a\right)}{832\,b}-\frac{\cos\left(11\,bx+11\,a\right)}{704\,b}-\frac{\cos\left(9\,bx+9\,a\right)}{96\,b}+\frac{3\,\cos\left(7\,bx+7\,a\right)}{224\,b}+\frac{3\,\cos\left(5\,bx+5\,a\right)}{64\,b}-\frac{5\,\cos\left(3\,bx+3\,a\right)}{64\,b}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="giac")
```

```
[Out] 1/832*cos(13*b*x + 13*a)/b - 1/704*cos(11*b*x + 11*a)/b - 1/96*cos(9*b*x +
9*a)/b + 3/224*cos(7*b*x + 7*a)/b + 3/64*cos(5*b*x + 5*a)/b - 5/64*cos(3*b*
x + 3*a)/b - 5/16*cos(b*x + a)/b
```

3.3 $\int \sin(a+bx)\sin^5(2a+2bx)\,dx$

Optimal. Leaf size=46

$$\frac{32\sin^{11}(a+bx)}{11b}-\frac{64\sin^9(a+bx)}{9b}+\frac{32\sin^7(a+bx)}{7b}$$

```
[Out] (32*Sin[a + b*x]^7)/(7*b) - (64*Sin[a + b*x]^9)/(9*b) + (32*Sin[a + b*x]^11
)/(11*b)
```

Rubi [A] time = 0.0546081, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2564, 270}

$$\frac{32\sin^{11}(a+bx)}{11b}-\frac{64\sin^9(a+bx)}{9b}+\frac{32\sin^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (32*Sin[a + b*x]^7)/(7*b) - (64*Sin[a + b*x]^9)/(9*b) + (32*Sin[a + b*x]^11
)/(11*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\sin^5(2a+2bx)\,dx &= 32\int \cos^5(a+bx)\sin^6(a+bx)\,dx\\ &= \frac{32\,\mathrm{Subst}\left(\int x^6\left(1-x^2\right)^2\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{32\,\mathrm{Subst}\left(\int \left(x^6-2x^8+x^{10}\right)\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{32\sin^7(a+bx)}{7b}-\frac{64\sin^9(a+bx)}{9b}+\frac{32\sin^{11}(a+bx)}{11b}\end{aligned}$$

Mathematica [A] time = 0.251399, size = 37, normalized size = 0.8

$$\frac{4\sin^7(a+bx)(364\cos(2(a+bx))+63\cos(4(a+bx))+365)}{693b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (4*(365 + 364*Cos[2*(a + b*x)] + 63*Cos[4*(a + b*x)])*Sin[a + b*x]^7)/(693*
b)
```

Maple [B] time = 0.012, size = 83, normalized size = 1.8

$$\frac{5\sin(bx+a)}{16b}-\frac{5\sin(3bx+3a)}{48b}-\frac{\sin(5bx+5a)}{32b}+\frac{5\sin(7bx+7a)}{224b}+\frac{\sin(9bx+9a)}{288b}-\frac{\sin(11bx+11a)}{352b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^5,x)
```

```
[Out] 5/16*sin(b*x+a)/b-5/48*sin(3*b*x+3*a)/b-1/32/b*sin(5*b*x+5*a)+5/224/b*sin(7
*b*x+7*a)+1/288/b*sin(9*b*x+9*a)-1/352/b*sin(11*b*x+11*a)
```

Maxima [A] time = 1.21303, size = 93, normalized size = 2.02

$$-\frac{63\sin(11bx+11a)-77\sin(9bx+9a)-495\sin(7bx+7a)+693\sin(5bx+5a)+2310\sin(3bx+3a)-6930\sin}{22176b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] -1/22176*(63*sin(11*b*x + 11*a) - 77*sin(9*b*x + 9*a) - 495*sin(7*b*x + 7*a
) + 693*sin(5*b*x + 5*a) + 2310*sin(3*b*x + 3*a) - 6930*sin(b*x + a))/b
```

Fricas [A] time = 0.504112, size = 174, normalized size = 3.78

$$-\frac{32\left(63\cos(bx+a)^{10}-161\cos(bx+a)^{8}+113\cos(bx+a)^{6}-3\cos(bx+a)^{4}-4\cos(bx+a)^{2}-8\right)\sin(bx+a)}{693b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] -32/693*(63*cos(b*x + a)^10 - 161*cos(b*x + a)^8 + 113*cos(b*x + a)^6 - 3*c
os(b*x + a)^4 - 4*cos(b*x + a)^2 - 8)*sin(b*x + a)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.38042, size = 111, normalized size = 2.41

$$-\frac{\sin\left(11\,bx+11\,a\right)}{352\,b}+\frac{\sin\left(9\,bx+9\,a\right)}{288\,b}+\frac{5\,\sin\left(7\,bx+7\,a\right)}{224\,b}-\frac{\sin\left(5\,bx+5\,a\right)}{32\,b}-\frac{5\,\sin\left(3\,bx+3\,a\right)}{48\,b}+\frac{5\,\sin\left(bx+a\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] -1/352*sin(11*b*x + 11*a)/b + 1/288*sin(9*b*x + 9*a)/b + 5/224*sin(7*b*x +
7*a)/b - 1/32*sin(5*b*x + 5*a)/b - 5/48*sin(3*b*x + 3*a)/b + 5/16*sin(b*x +
 a)/b
```

3.4 $\int \sin(a+bx)\sin^4(2a+2bx)\,dx$

Optimal. Leaf size=46

$$-\frac{16\cos^9(a+bx)}{9b}+\frac{32\cos^7(a+bx)}{7b}-\frac{16\cos^5(a+bx)}{5b}$$

```
[Out] (-16*Cos[a + b*x]^5)/(5*b) + (32*Cos[a + b*x]^7)/(7*b) - (16*Cos[a + b*x]^9
)/(9*b)
```

Rubi [A] time = 0.055045, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2565, 270}

$$-\frac{16\cos^9(a+bx)}{9b}+\frac{32\cos^7(a+bx)}{7b}-\frac{16\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (-16*Cos[a + b*x]^5)/(5*b) + (32*Cos[a + b*x]^7)/(7*b) - (16*Cos[a + b*x]^9
)/(9*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \sin(a+bx)\sin^4(2a+2bx)\,dx &= 16\int \cos^4(a+bx)\sin^5(a+bx)\,dx\\
&= -\frac{16\,\mathrm{Subst}\left(\int x^4\left(1-x^2\right)^2\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{16\,\mathrm{Subst}\left(\int \left(x^4-2x^6+x^8\right)\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{16\cos^5(a+bx)}{5b}+\frac{32\cos^7(a+bx)}{7b}-\frac{16\cos^9(a+bx)}{9b}
\end{aligned}$$

Mathematica [A] time = 0.14622, size = 37, normalized size = 0.8

$$\frac{2\cos^5(a+bx)(220\cos(2(a+bx))-35\cos(4(a+bx))-249)}{315b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (2*Cos[a + b*x]^5*(-249 + 220*Cos[2*(a + b*x)] - 35*Cos[4*(a + b*x)]))/(315
*b)
```

Maple [A] time = 0.013, size = 69, normalized size = 1.5

$$-\frac{3\cos(bx+a)}{8b}-\frac{\cos(3bx+3a)}{12b}+\frac{\cos(5bx+5a)}{20b}+\frac{\cos(7bx+7a)}{112b}-\frac{\cos(9bx+9a)}{144b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^4,x)
```

```
[Out] -3/8*cos(b*x+a)/b-1/12*cos(3*b*x+3*a)/b+1/20*cos(5*b*x+5*a)/b+1/112*cos(7*b
*x+7*a)/b-1/144*cos(9*b*x+9*a)/b
```

Maxima [A] time = 1.25769, size = 78, normalized size = 1.7

$$-\frac{35\cos(9bx+9a)-45\cos(7bx+7a)-252\cos(5bx+5a)+420\cos(3bx+3a)+1890\cos(bx+a)}{5040b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] -1/5040*(35*cos(9*b*x + 9*a) - 45*cos(7*b*x + 7*a) - 252*cos(5*b*x + 5*a) +
 420*cos(3*b*x + 3*a) + 1890*cos(b*x + a))/b
```

Fricas [A] time = 0.490088, size = 96, normalized size = 2.09

$$-\frac{16\left(35\cos(bx+a)^9-90\cos(bx+a)^7+63\cos(bx+a)^5\right)}{315b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] -16/315*(35*cos(b*x + a)^9 - 90*cos(b*x + a)^7 + 63*cos(b*x + a)^5)/b
```

Sympy [A] time = 65.1689, size = 163, normalized size = 3.54

$$\begin{cases} -\frac{104\sin(a+bx)\sin^3(2a+2bx)\cos(2a+2bx)}{315b}-\frac{64\sin(a+bx)\sin(2a+2bx)\cos^3(2a+2bx)}{315b}-\frac{107\sin^4(2a+2bx)\cos(a+bx)}{315b}-\frac{16\sin^2(2a+2bx)\cos(a+bx}{21b} \\ x\sin(a)\sin^4(2a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**4,x)
```

```
[Out] Piecewise((-104*sin(a + b*x)*sin(2*a + 2*b*x)**3*cos(2*a + 2*b*x)/(315*b) -
 64*sin(a + b*x)*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**3/(315*b) - 107*sin(2*a
 + 2*b*x)**4*cos(a + b*x)/(315*b) - 16*sin(2*a + 2*b*x)**2*cos(a + b*x)*cos
(2*a + 2*b*x)**2/(21*b) - 128*cos(a + b*x)*cos(2*a + 2*b*x)**4/(315*b), Ne(
b, 0)), (x*sin(a)*sin(2*a)**4, True))
```

Giac [A] time = 1.23711, size = 92, normalized size = 2.

$$-\frac{\cos(9\,bx+9\,a)}{144\,b}+\frac{\cos(7\,bx+7\,a)}{112\,b}+\frac{\cos(5\,bx+5\,a)}{20\,b}-\frac{\cos(3\,bx+3\,a)}{12\,b}-\frac{3\,\cos(bx+a)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] -1/144*cos(9*b*x + 9*a)/b + 1/112*cos(7*b*x + 7*a)/b + 1/20*cos(5*b*x + 5*a
)/b - 1/12*cos(3*b*x + 3*a)/b - 3/8*cos(b*x + a)/b
```

3.5 $\int \sin(a+bx)\sin^3(2a+2bx)\,dx$

Optimal. Leaf size=31

$$\frac{8\sin^5(a+bx)}{5b}-\frac{8\sin^7(a+bx)}{7b}$$

```
[Out] (8*Sin[a + b*x]^5)/(5*b) - (8*Sin[a + b*x]^7)/(7*b)
```

Rubi [A] time = 0.0496947, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2564, 14}

$$\frac{8\sin^5(a+bx)}{5b}-\frac{8\sin^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^3,x]

[Out] (8*Sin[a + b*x]^5)/(5*b) - (8*Sin[a + b*x]^7)/(7*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\sin^3(2a+2bx)\,dx &= 8\int \cos^3(a+bx)\sin^4(a+bx)\,dx\\ &= \frac{8\,\text{Subst}\left(\int x^4\left(1-x^2\right)\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{8\,\text{Subst}\left(\int \left(x^4-x^6\right)\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{8\sin^5(a+bx)}{5b}-\frac{8\sin^7(a+bx)}{7b}\end{aligned}$$

Mathematica [A] time = 0.0902121, size = 27, normalized size = 0.87

$$\frac{4\sin^5(a+bx)(5\cos(2(a+bx))+9)}{35b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (4*(9 + 5*Cos[2*(a + b*x)])*Sin[a + b*x]^5)/(35*b)
```

Maple [A] time = 0.012, size = 55, normalized size = 1.8

$$\frac{3\sin(bx+a)}{8b}-\frac{\sin(3bx+3a)}{8b}-\frac{\sin(5bx+5a)}{40b}+\frac{\sin(7bx+7a)}{56b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^3,x)
```

```
[Out] 3/8*sin(b*x+a)/b-1/8*sin(3*b*x+3*a)/b-1/40/b*sin(5*b*x+5*a)+1/56/b*sin(7*b*
x+7*a)
```

Maxima [A] time = 1.18821, size = 63, normalized size = 2.03

$$\frac{5\sin(7bx+7a)-7\sin(5bx+5a)-35\sin(3bx+3a)+105\sin(bx+a)}{280b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 1/280*(5*sin(7*b*x + 7*a) - 7*sin(5*b*x + 5*a) - 35*sin(3*b*x + 3*a) + 105*
sin(b*x + a))/b
```

Fricas [A] time = 0.481122, size = 108, normalized size = 3.48

$$\frac{8\left(5\cos(bx+a)^6-8\cos(bx+a)^4+\cos(bx+a)^2+2\right)\sin(bx+a)}{35b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] 8/35*(5*cos(b*x + a)^6 - 8*cos(b*x + a)^4 + cos(b*x + a)^2 + 2)*sin(b*x + a
)/b
```

Sympy [A] time = 16.9376, size = 126, normalized size = 4.06

$$\begin{cases} -\frac{22\sin(a+bx)\sin^2(2a+2bx)\cos(2a+2bx)}{35b}-\frac{16\sin(a+bx)\cos^3(2a+2bx)}{35b}+\frac{9\sin^3(2a+2bx)\cos(a+bx)}{35b}+\frac{8\sin(2a+2bx)\cos(a+bx)\cos^2(2a+2bx)}{35b} & \text{for} \\ x\sin(a)\sin^3(2a) & \text{oth} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**3,x)
```

```
[Out] Piecewise((-22*sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)/(35*b) - 1
6*sin(a + b*x)*cos(2*a + 2*b*x)**3/(35*b) + 9*sin(2*a + 2*b*x)**3*cos(a + b
*x)/(35*b) + 8*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**2/(35*b), Ne
(b, 0)), (x*sin(a)*sin(2*a)**3, True))
```

Giac [B] time = 1.27638, size = 73, normalized size = 2.35

$$\frac{\sin\left(7\,bx+7\,a\right)}{56\,b}-\frac{\sin\left(5\,bx+5\,a\right)}{40\,b}-\frac{\sin\left(3\,bx+3\,a\right)}{8\,b}+\frac{3\,\sin\left(bx+a\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] 1/56*sin(7*b*x + 7*a)/b - 1/40*sin(5*b*x + 5*a)/b - 1/8*sin(3*b*x + 3*a)/b
+ 3/8*sin(b*x + a)/b
```

3.6 $\int \sin(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=31

$$\frac{4\cos^5(a+bx)}{5b}-\frac{4\cos^3(a+bx)}{3b}$$

```
[Out] (-4*Cos[a + b*x]^3)/(3*b) + (4*Cos[a + b*x]^5)/(5*b)
```

Rubi [A] time = 0.0496542, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2565, 14}

$$\frac{4\cos^5(a+bx)}{5b}-\frac{4\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (-4*Cos[a + b*x]^3)/(3*b) + (4*Cos[a + b*x]^5)/(5*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \sin(a+bx)\sin^2(2a+2bx)\,dx &= 4\int \cos^2(a+bx)\sin^3(a+bx)\,dx\\
&= -\frac{4\,\mathrm{Subst}\left(\int x^2\left(1-x^2\right)dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{4\,\mathrm{Subst}\left(\int \left(x^2-x^4\right)dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{4\cos^3(a+bx)}{3b}+\frac{4\cos^5(a+bx)}{5b}
\end{aligned}$$

Mathematica [A] time = 0.0662624, size = 27, normalized size = 0.87

$$\frac{2\cos^3(a+bx)(3\cos(2(a+bx))-7)}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (2*Cos[a + b*x]^3*(-7 + 3*Cos[2*(a + b*x)]))/(15*b)
```

Maple [A] time = 0.01, size = 41, normalized size = 1.3

$$-\frac{\cos(bx+a)}{2b}-\frac{\cos(3bx+3a)}{12b}+\frac{\cos(5bx+5a)}{20b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^2,x)
```

```
[Out] -1/2*cos(b*x+a)/b-1/12*cos(3*b*x+3*a)/b+1/20*cos(5*b*x+5*a)/b
```

Maxima [A] time = 1.23104, size = 49, normalized size = 1.58

$$\frac{3\cos(5bx+5a)-5\cos(3bx+3a)-30\cos(bx+a)}{60b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] 1/60*(3*cos(5*b*x + 5*a) - 5*cos(3*b*x + 3*a) - 30*cos(b*x + a))/b
```

Fricas [A] time = 0.478896, size = 62, normalized size = 2.

$$\frac{4\left(3\cos(bx+a)^5-5\cos(bx+a)^3\right)}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] 4/15*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)/b
```

Sympy [A] time = 4.35895, size = 92, normalized size = 2.97

$$\begin{cases}-\frac{4\sin(a+bx)\sin(2a+2bx)\cos(2a+2bx)}{15b}-\frac{7\sin^2(2a+2bx)\cos(a+bx)}{15b}-\frac{8\cos(a+bx)\cos^2(2a+2bx)}{15b} & \text{for } b\neq 0\\ x\sin(a)\sin^2(2a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**2,x)
```

```
[Out] Piecewise((-4*sin(a + b*x)*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)/(15*b) - 7*sin
(2*a + 2*b*x)**2*cos(a + b*x)/(15*b) - 8*cos(a + b*x)*cos(2*a + 2*b*x)**2/(
15*b), Ne(b, 0)), (x*sin(a)*sin(2*a)**2, True))
```

Giac [A] time = 1.29399, size = 54, normalized size = 1.74

$$\frac{\cos(5\,bx+5\,a)}{20\,b}-\frac{\cos(3\,bx+3\,a)}{12\,b}-\frac{\cos(bx+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] 1/20*cos(5*b*x + 5*a)/b - 1/12*cos(3*b*x + 3*a)/b - 1/2*cos(b*x + a)/b
```

3.7 $\int \sin(a+bx)\sin(2a+2bx)\,dx$

Optimal. Leaf size=30

$$\frac{\sin(a+bx)}{2b}-\frac{\sin(3a+3bx)}{6b}$$

```
[Out] Sin[a + b*x]/(2*b) - Sin[3*a + 3*b*x]/(6*b)
```

Rubi [A] time = 0.011129, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4282}

$$\frac{\sin(a+bx)}{2b}-\frac{\sin(3a+3bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x],x]

[Out] Sin[a + b*x]/(2*b) - Sin[3*a + 3*b*x]/(6*b)
```

Rule 4282

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sin(a+bx)\sin(2a+2bx)\,dx=\frac{\sin(a+bx)}{2b}-\frac{\sin(3a+3bx)}{6b}$$

Mathematica [A] time = 0.0319888, size = 15, normalized size = 0.5

$$\frac{2\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x],x]

[Out] (2*Sin[a + b*x]^3)/(3*b)
```

Maple [A] time = 0.007, size = 27, normalized size = 0.9

$$\frac{\sin(bx+a)}{2b}-\frac{\sin(3bx+3a)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a),x)
```

```
[Out] 1/2*sin(b*x+a)/b-1/6*sin(3*b*x+3*a)/b
```

Maxima [A] time = 1.09884, size = 35, normalized size = 1.17

$$-\frac{\sin\left(3\,bx+3\,a\right)}{6\,b}+\frac{\sin\left(bx+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -1/6*sin(3*b*x + 3*a)/b + 1/2*sin(b*x + a)/b
```

Fricas [A] time = 0.47103, size = 57, normalized size = 1.9

$$-\frac{2\left(\cos\left(bx+a\right)^{2}-1\right)\sin\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] -2/3*(cos(b*x + a)^2 - 1)*sin(b*x + a)/b
```

Sympy [A] time = 1.12469, size = 51, normalized size = 1.7

$$\begin{cases} -\frac{2\sin\left(a+bx\right)\cos\left(2a+2bx\right)}{3b}+\frac{\sin\left(2a+2bx\right)\cos\left(a+bx\right)}{3b} & \text{for}\ b\neq 0\\ x\sin\left(a\right)\sin\left(2a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a),x)
```

```
[Out] Piecewise((-2*sin(a + b*x)*cos(2*a + 2*b*x)/(3*b) + sin(2*a + 2*b*x)*cos(a
+ b*x)/(3*b), Ne(b, 0)), (x*sin(a)*sin(2*a), True))
```

Giac [A] time = 1.30566, size = 35, normalized size = 1.17

$$-\frac{\sin\left(3\,bx+3\,a\right)}{6\,b}+\frac{\sin\left(bx+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] -1/6*sin(3*b*x + 3*a)/b + 1/2*sin(b*x + a)/b
```

3.8 $\int \csc(2a+2bx)\sin(a+bx)\,dx$

Optimal. Leaf size=14

$$\frac{\tanh^{-1}(\sin(a+bx))}{2b}$$

```
[Out] ArcTanh[Sin[a + b*x]]/(2*b)
```

Rubi [A] time = 0.0169135, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4288, 3770}

$$\frac{\tanh^{-1}(\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]*Sin[a + b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(2*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc(2a+2bx)\sin(a+bx)\,dx &= \frac{1}{2}\int \sec(a+bx)\,dx\\ &= \frac{\tanh^{-1}(\sin(a+bx))}{2b}\end{aligned}$$

Mathematica [A] time = 0.0045653, size = 14, normalized size = 1.

$$\frac{\tanh^{-1}(\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]*Sin[a + b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(2*b)
```

Maple [A] time = 0.036, size = 20, normalized size = 1.4

$$\frac{\ln\left(\sec\left(bx+a\right)+\tan\left(bx+a\right)\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)*sin(b*x+a),x)
```

```
[Out] 1/2/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [B] time = 1.78992, size = 155, normalized size = 11.07

$$-\frac{\log\left(\frac{\cos(bx+2a)^2+\cos(a)^2-2\cos(a)\sin(bx+2a)+\sin(bx+2a)^2+2\cos(bx+2a)\sin(a)+\sin(a)^2}{\cos(bx+2a)^2+\cos(a)^2+2\cos(a)\sin(bx+2a)+\sin(bx+2a)^2-2\cos(bx+2a)\sin(a)+\sin(a)^2}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x +
 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2
+ 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + si
n(a)^2))/b
```

Fricas [B] time = 0.484673, size = 76, normalized size = 5.43

$$\frac{\log\left(\sin\left(bx+a\right)+1\right)-\log\left(-\sin\left(bx+a\right)+1\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.51967, size = 235, normalized size = 16.79

$$\log\left(\left|\tan\left(\frac{1}{2}bx+2a\right)\tan\left(\frac{1}{2}a\right)^3+3\tan\left(\frac{1}{2}bx+2a\right)\tan\left(\frac{1}{2}a\right)^2-\tan\left(\frac{1}{2}a\right)^3-3\tan\left(\frac{1}{2}bx+2a\right)\tan\left(\frac{1}{2}a\right)+3\tan\left(\frac{1}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 3*tan(1/2*b*x + 2*a)*tan(1/2
*a)^2 - tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 - t
an(1/2*b*x + 2*a) + 3*tan(1/2*a) - 1)) - log(abs(tan(1/2*b*x + 2*a)*tan(1/2
*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 + tan(1/2*a)^3 - 3*tan(1/2*b*x +
2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a) - 1)))
/b
```

3.9 $\int \csc^2(2a+2bx)\sin(a+bx)\,dx$

Optimal. Leaf size=28

$$\frac{\sec(a+bx)}{4b}-\frac{\tanh^{-1}(\cos(a+bx))}{4b}$$

```
[Out] -ArcTanh[Cos[a + b*x]]/(4*b) + Sec[a + b*x]/(4*b)
```

Rubi [A] time = 0.0382959, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4288, 2622, 321, 207}

$$\frac{\sec(a+bx)}{4b}-\frac{\tanh^{-1}(\cos(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^2*Sin[a + b*x],x]

[Out] -ArcTanh[Cos[a + b*x]]/(4*b) + Sec[a + b*x]/(4*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^2(2a+2bx)\sin(a+bx)\,dx = \frac{1}{4}\int \csc(a+bx)\sec^2(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{4b}$$
$$= \frac{\sec(a+bx)}{4b} + \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{4b}$$
$$= -\frac{\tanh^{-1}(\cos(a+bx))}{4b} + \frac{\sec(a+bx)}{4b}$$

Mathematica [A] time = 0.0385351, size = 50, normalized size = 1.79

$$\frac{\sec(a+bx)}{4b} + \frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{4b} - \frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^2*Sin[a + b*x],x]
```

```
[Out] -Log[Cos[(a + b*x)/2]]/(4*b) + Log[Sin[(a + b*x)/2]]/(4*b) + Sec[a + b*x]/(
4*b)
```

Maple [A] time = 0.032, size = 36, normalized size = 1.3

$$\frac{1}{4\,b\cos(bx+a)} + \frac{\ln(\csc(bx+a) - \cot(bx+a))}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^2*sin(b*x+a),x)
```

```
[Out] 1/4/b/cos(b*x+a)+1/4/b*ln(csc(b*x+a)-cot(b*x+a))
```

Maxima [B] time = 1.22817, size = 319, normalized size = 11.39

$$4\cos(2bx+2a)\cos(bx+a) - \left(\cos(2bx+2a)^2 + \sin(2bx+2a)^2 + 2\cos(2bx+2a) + 1\right)\log\left(\cos(bx)^2 + 2\cos(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/8*(4*cos(2*b*x + 2*a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*
a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^
2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + (cos(2*b*x + 2*a)^2 + sin(
2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a)
 + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*sin(2*b*x + 2*
a)*sin(b*x + a) + 4*cos(b*x + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a
)^2 + 2*b*cos(2*b*x + 2*a) + b)
```

Fricas [B] time = 0.494509, size = 154, normalized size = 5.5

$$-\frac{\cos\left(bx+a\right)\log\left(\frac{1}{2}\,\cos\left(bx+a\right)+\frac{1}{2}\right)-\cos\left(bx+a\right)\log\left(-\frac{1}{2}\,\cos\left(bx+a\right)+\frac{1}{2}\right)-2}{8\,b\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8*(cos(b*x + a)*log(1/2*cos(b*x + a) + 1/2) - cos(b*x + a)*log(-1/2*cos(
b*x + a) + 1/2) - 2)/(b*cos(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**2*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.64272, size = 556, normalized size = 19.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(2*(6*tan(1/2*b*x + 2*a)*tan(1/2*a)^11 - tan(1/2*a)^12 - 2*tan(1/2*b*x
 + 2*a)*tan(1/2*a)^9 + 12*tan(1/2*a)^10 - 36*tan(1/2*b*x + 2*a)*tan(1/2*a)^
7 + 27*tan(1/2*a)^8 - 36*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - 2*tan(1/2*b*x +
2*a)*tan(1/2*a)^3 - 27*tan(1/2*a)^4 + 6*tan(1/2*b*x + 2*a)*tan(1/2*a) - 12*
tan(1/2*a)^2 + 1)/((tan(1/2*b*x + 2*a)^2*tan(1/2*a)^6 - 15*tan(1/2*b*x + 2*
a)^2*tan(1/2*a)^4 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - tan(1/2*a)^6 + 15*
tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2 - 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 15
*tan(1/2*a)^4 - tan(1/2*b*x + 2*a)^2 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a) - 1
5*tan(1/2*a)^2 + 1)*(tan(1/2*a)^6 - 15*tan(1/2*a)^4 + 15*tan(1/2*a)^2 - 1))
 + log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a
) + 3*tan(1/2*a)^2 - 1)) - log(abs(3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - tan(
1/2*a)^3 - tan(1/2*b*x + 2*a) + 3*tan(1/2*a))))/b
```

3.10 $\int \csc^3(2a+2bx)\sin(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{3\csc(a+bx)}{16b}+\frac{3\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\csc(a+bx)\sec^2(a+bx)}{16b}$$

```
[Out] (3*ArcTanh[Sin[a + b*x]])/(16*b) - (3*Csc[a + b*x])/(16*b) + (Csc[a + b*x]*
Sec[a + b*x]^2)/(16*b)
```

Rubi [A] time = 0.062518, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {4288, 2621, 288, 321, 207}

$$-\frac{3\csc(a+bx)}{16b}+\frac{3\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\csc(a+bx)\sec^2(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^3*Sin[a + b*x],x]
```

```
[Out] (3*ArcTanh[Sin[a + b*x]])/(16*b) - (3*Csc[a + b*x])/(16*b) + (Csc[a + b*x]*
Sec[a + b*x]^2)/(16*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \csc^3(2a+2bx)\sin(a+bx)\,dx &= \frac{1}{8}\int \csc^2(a+bx)\sec^3(a+bx)\,dx \\
&= -\frac{\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \csc(a+bx)\right)}{8b} \\
&= \frac{\csc(a+bx)\sec^2(a+bx)}{16b} - \frac{3\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{16b} \\
&= -\frac{3\csc(a+bx)}{16b} + \frac{\csc(a+bx)\sec^2(a+bx)}{16b} - \frac{3\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{16b} \\
&= \frac{3\tanh^{-1}(\sin(a+bx))}{16b} - \frac{3\csc(a+bx)}{16b} + \frac{\csc(a+bx)\sec^2(a+bx)}{16b}
\end{aligned}$$

Mathematica [C] time = 0.0166838, size = 29, normalized size = 0.59

$$-\frac{\csc(a+bx)\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},\sin^2(a+bx)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^3*Sin[a + b*x],x]
```

```
[Out] -(Csc[a + b*x]*Hypergeometric2F1[-1/2, 2, 1/2, Sin[a + b*x]^2])/(8*b)
```

Maple [A] time = 0.037, size = 55, normalized size = 1.1

$$\frac{1}{16\,b\sin(bx+a)(\cos(bx+a))^2} - \frac{3}{16\,b\sin(bx+a)} + \frac{3\,\ln(\sec(bx+a)+\tan(bx+a))}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^3*sin(b*x+a),x)
```

```
[Out] 1/16/b/sin(b*x+a)/cos(b*x+a)^2-3/16/b/sin(b*x+a)+3/16/b*ln(sec(b*x+a)+tan(b
*x+a))
```

Maxima [B] time = 1.88189, size = 1091, normalized size = 22.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/32*(4*(3*sin(5*b*x + 5*a) + 2*sin(3*b*x + 3*a) + 3*sin(b*x + a))*cos(6*b*
x + 6*a) - 12*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(5*b*x + 5*a) + 4*(2
*sin(3*b*x + 3*a) + 3*sin(b*x + a))*cos(4*b*x + 4*a) - 3*(2*(cos(4*b*x + 4*
a) - cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) + cos(6*b*x + 6*a)^2 - 2*(cos(2
*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + cos(2*b*x + 2*a)^2
 + 2*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + sin(6*b*x + 6
*a)^2 + sin(4*b*x + 4*a)^2 - 2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + sin(2*b*
x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*c
os(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^
2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^
2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(3*cos(5*b*x + 5*a) + 2*cos(3*
b*x + 3*a) + 3*cos(b*x + a))*sin(6*b*x + 6*a) + 12*(cos(4*b*x + 4*a) - cos(
2*b*x + 2*a) - 1)*sin(5*b*x + 5*a) - 4*(2*cos(3*b*x + 3*a) + 3*cos(b*x + a)
)*sin(4*b*x + 4*a) - 8*(cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a) + 8*cos(3*b*
x + 3*a)*sin(2*b*x + 2*a) + 12*cos(b*x + a)*sin(2*b*x + 2*a) - 12*cos(2*b*x
 + 2*a)*sin(b*x + a) - 12*sin(b*x + a))/(b*cos(6*b*x + 6*a)^2 + b*cos(4*b*x
 + 4*a)^2 + b*cos(2*b*x + 2*a)^2 + b*sin(6*b*x + 6*a)^2 + b*sin(4*b*x + 4*a
)^2 - 2*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b*sin(2*b*x + 2*a)^2 + 2*(b*c
os(4*b*x + 4*a) - b*cos(2*b*x + 2*a) - b)*cos(6*b*x + 6*a) - 2*(b*cos(2*b*x
 + 2*a) + b)*cos(4*b*x + 4*a) + 2*b*cos(2*b*x + 2*a) + 2*(b*sin(4*b*x + 4*a
) - b*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + b)
```

Fricas [A] time = 0.500022, size = 230, normalized size = 4.69

$$\frac{3\cos(bx+a)^2\log(\sin(bx+a)+1)\sin(bx+a) - 3\cos(bx+a)^2\log(-\sin(bx+a)+1)\sin(bx+a) - 6\cos(bx+a}{32\,b\cos(bx+a)^2\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/32*(3*cos(b*x + a)^2*log(sin(b*x + a) + 1)*sin(b*x + a) - 3*cos(b*x + a)^
2*log(-sin(b*x + a) + 1)*sin(b*x + a) - 6*cos(b*x + a)^2 + 2)/(b*cos(b*x +
a)^2*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**3*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.20544, size = 1918, normalized size = 39.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16*((tan(1/2*b*x + 2*a)*tan(1/2*a)^12 - 12*tan(1/2*b*x + 2*a)*tan(1/2*a)
^10 + 6*tan(1/2*a)^11 - 27*tan(1/2*b*x + 2*a)*tan(1/2*a)^8 - 2*tan(1/2*a)^9
 - 36*tan(1/2*a)^7 + 27*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 - 36*tan(1/2*a)^5 +
 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - 2*tan(1/2*a)^3 - tan(1/2*b*x + 2*a) +
 6*tan(1/2*a))/((3*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 - tan(1/2*b*x + 2*a)*t
an(1/2*a)^6 - 10*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 + 15*tan(1/2*b*x + 2*a)*
tan(1/2*a)^4 - 3*tan(1/2*a)^5 + 3*tan(1/2*b*x + 2*a)^2*tan(1/2*a) - 15*tan(
1/2*b*x + 2*a)*tan(1/2*a)^2 + 10*tan(1/2*a)^3 + tan(1/2*b*x + 2*a) - 3*tan(
1/2*a))*(3*tan(1/2*a)^5 - 10*tan(1/2*a)^3 + 3*tan(1/2*a))) + 2*(tan(1/2*b*x
 + 2*a)^3*tan(1/2*a)^24 + 30*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^22 - 6*tan(1/2
*b*x + 2*a)^2*tan(1/2*a)^23 + tan(1/2*b*x + 2*a)*tan(1/2*a)^24 - 756*tan(1/
2*b*x + 2*a)^3*tan(1/2*a)^20 + 614*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^21 - 114
*tan(1/2*b*x + 2*a)*tan(1/2*a)^22 + 6*tan(1/2*a)^23 + 2058*tan(1/2*b*x + 2*
a)^3*tan(1/2*a)^18 - 4578*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^19 + 1932*tan(1/2
*b*x + 2*a)*tan(1/2*a)^20 - 182*tan(1/2*a)^21 - 27*tan(1/2*b*x + 2*a)^3*tan
(1/2*a)^16 + 6210*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^17 - 7462*tan(1/2*b*x + 2
*a)*tan(1/2*a)^18 + 1554*tan(1/2*a)^19 - 9396*tan(1/2*b*x + 2*a)^3*tan(1/2*
a)^14 + 15588*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^15 - 2331*tan(1/2*b*x + 2*a)*
tan(1/2*a)^16 - 2178*tan(1/2*a)^17 - 21924*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^
13 + 26028*tan(1/2*b*x + 2*a)*tan(1/2*a)^14 - 5668*tan(1/2*a)^15 + 9396*tan
(1/2*b*x + 2*a)^3*tan(1/2*a)^10 - 21924*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^11
+ 6468*tan(1/2*a)^13 + 27*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^8 + 15588*tan(1/2
*b*x + 2*a)^2*tan(1/2*a)^9 - 26028*tan(1/2*b*x + 2*a)*tan(1/2*a)^10 + 6468*
tan(1/2*a)^11 - 2058*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^6 + 6210*tan(1/2*b*x +
 2*a)^2*tan(1/2*a)^7 + 2331*tan(1/2*b*x + 2*a)*tan(1/2*a)^8 - 5668*tan(1/2*
a)^9 + 756*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^4 - 4578*tan(1/2*b*x + 2*a)^2*ta
n(1/2*a)^5 + 7462*tan(1/2*b*x + 2*a)*tan(1/2*a)^6 - 2178*tan(1/2*a)^7 - 30*
tan(1/2*b*x + 2*a)^3*tan(1/2*a)^2 + 614*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 -
 1932*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 1554*tan(1/2*a)^5 - tan(1/2*b*x + 2
*a)^3 - 6*tan(1/2*b*x + 2*a)^2*tan(1/2*a) + 114*tan(1/2*b*x + 2*a)*tan(1/2*
a)^2 - 182*tan(1/2*a)^3 - tan(1/2*b*x + 2*a) + 6*tan(1/2*a))/((tan(1/2*a)^1
2 - 30*tan(1/2*a)^10 + 255*tan(1/2*a)^8 - 452*tan(1/2*a)^6 + 255*tan(1/2*a)
^4 - 30*tan(1/2*a)^2 + 1)*(tan(1/2*b*x + 2*a)^2*tan(1/2*a)^6 - 15*tan(1/2*b
*x + 2*a)^2*tan(1/2*a)^4 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - tan(1/2*a)^
6 + 15*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2 - 40*tan(1/2*b*x + 2*a)*tan(1/2*a)
^3 + 15*tan(1/2*a)^4 - tan(1/2*b*x + 2*a)^2 + 12*tan(1/2*b*x + 2*a)*tan(1/2
*a) - 15*tan(1/2*a)^2 + 1)^2) - 3*log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 +
 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*ta
n(1/2*a) + 3*tan(1/2*a)^2 - tan(1/2*b*x + 2*a) + 3*tan(1/2*a) - 1)) + 3*log
(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 +
tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 + tan(1/2*b
*x + 2*a) - 3*tan(1/2*a) - 1)))/b
```

3.11 $\int \csc^4(2a+2bx)\sin(a+bx)\,dx$

Optimal. Leaf size=66

$$\frac{5\sec^3(a+bx)}{96b}+\frac{5\sec(a+bx)}{32b}-\frac{5\tanh^{-1}(\cos(a+bx))}{32b}-\frac{\csc^2(a+bx)\sec^3(a+bx)}{32b}$$

```
[Out] (-5*ArcTanh[Cos[a + b*x]])/(32*b) + (5*Sec[a + b*x])/(32*b) + (5*Sec[a + b*
x]^3)/(96*b) - (Csc[a + b*x]^2*Sec[a + b*x]^3)/(32*b)
```

Rubi [A] time = 0.0674468, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {4288, 2622, 288, 302, 207}

$$\frac{5\sec^3(a+bx)}{96b}+\frac{5\sec(a+bx)}{32b}-\frac{5\tanh^{-1}(\cos(a+bx))}{32b}-\frac{\csc^2(a+bx)\sec^3(a+bx)}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^4*Sin[a + b*x],x]
```

```
[Out] (-5*ArcTanh[Cos[a + b*x]])/(32*b) + (5*Sec[a + b*x])/(32*b) + (5*Sec[a + b*
x]^3)/(96*b) - (Csc[a + b*x]^2*Sec[a + b*x]^3)/(32*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^4(2a+2bx)\sin(a+bx)\,dx = \frac{1}{16}\int \csc^3(a+bx)\sec^4(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^2}\,dx, x, \sec(a+bx)\right)}{16b}$$
$$= -\frac{\csc^2(a+bx)\sec^3(a+bx)}{32b} + \frac{5\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{32b}$$
$$= -\frac{\csc^2(a+bx)\sec^3(a+bx)}{32b} + \frac{5\,\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \sec(a+bx)\right)}{32b}$$
$$= \frac{5\sec(a+bx)}{32b} + \frac{5\sec^3(a+bx)}{96b} - \frac{\csc^2(a+bx)\sec^3(a+bx)}{32b} + \frac{5\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x,\right.}{32b}$$
$$= -\frac{5\tanh^{-1}(\cos(a+bx))}{32b} + \frac{5\sec(a+bx)}{32b} + \frac{5\sec^3(a+bx)}{96b} - \frac{\csc^2(a+bx)\sec^3(a+bx}{32b}$$

Mathematica [B] time = 0.445169, size = 205, normalized size = 3.11

$$\frac{\csc^8(a+bx)\left(-40\cos(2(a+bx))+13\cos(3(a+bx))-30\cos(4(a+bx))+13\cos(5(a+bx))+15\cos(3(a+bx))\log\left(\cos\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^4*Sin[a + b*x],x]
```

```
[Out] (Csc[a + b*x]^8*(22 - 40*Cos[2*(a + b*x)] + 13*Cos[3*(a + b*x)] - 30*Cos[4*
(a + b*x)] + 13*Cos[5*(a + b*x)] + 15*Cos[3*(a + b*x)]*Log[Cos[(a + b*x)/2]
] + 15*Cos[5*(a + b*x)]*Log[Cos[(a + b*x)/2]] - 15*Cos[3*(a + b*x)]*Log[Sin
[(a + b*x)/2]] - 15*Cos[5*(a + b*x)]*Log[Sin[(a + b*x)/2]] + Cos[a + b*x]*(
-26 - 30*Log[Cos[(a + b*x)/2]] + 30*Log[Sin[(a + b*x)/2]])))/(24*b*(Csc[(a
+ b*x)/2]^2 - Sec[(a + b*x)/2]^2)^3)
```

Maple [A] time = 0.036, size = 78, normalized size = 1.2

$$\frac{1}{48\,b\,(\sin(bx+a))^2\,(\cos(bx+a))^3} - \frac{5}{96\,b\,(\sin(bx+a))^2\cos(bx+a)} + \frac{5}{32\,b\cos(bx+a)} + \frac{5\,\ln(\csc(bx+a)-\cot(bx+}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^4*sin(b*x+a),x)
```

```
[Out] 1/48/b/sin(b*x+a)^2/cos(b*x+a)^3-5/96/b/sin(b*x+a)^2/cos(b*x+a)+5/32/b/cos(
b*x+a)+5/32/b*ln(csc(b*x+a)-cot(b*x+a))
```

Maxima [B] time = 1.39144, size = 2935, normalized size = 44.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^4*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/192*(4*(15*cos(9*b*x + 9*a) + 20*cos(7*b*x + 7*a) - 22*cos(5*b*x + 5*a) +
 20*cos(3*b*x + 3*a) + 15*cos(b*x + a))*cos(10*b*x + 10*a) + 60*(cos(8*b*x
+ 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) + 1)*co
s(9*b*x + 9*a) + 4*(20*cos(7*b*x + 7*a) - 22*cos(5*b*x + 5*a) + 20*cos(3*b*
x + 3*a) + 15*cos(b*x + a))*cos(8*b*x + 8*a) - 80*(2*cos(6*b*x + 6*a) + 2*c
os(4*b*x + 4*a) - cos(2*b*x + 2*a) - 1)*cos(7*b*x + 7*a) + 8*(22*cos(5*b*x
+ 5*a) - 20*cos(3*b*x + 3*a) - 15*cos(b*x + a))*cos(6*b*x + 6*a) + 88*(2*co
s(4*b*x + 4*a) - cos(2*b*x + 2*a) - 1)*cos(5*b*x + 5*a) - 40*(4*cos(3*b*x +
 3*a) + 3*cos(b*x + a))*cos(4*b*x + 4*a) + 80*(cos(2*b*x + 2*a) + 1)*cos(3*
b*x + 3*a) + 60*cos(2*b*x + 2*a)*cos(b*x + a) - 15*(2*(cos(8*b*x + 8*a) - 2
*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) + 1)*cos(10*b*x +
 10*a) + cos(10*b*x + 10*a)^2 - 2*(2*cos(6*b*x + 6*a) + 2*cos(4*b*x + 4*a)
- cos(2*b*x + 2*a) - 1)*cos(8*b*x + 8*a) + cos(8*b*x + 8*a)^2 + 4*(2*cos(4*
b*x + 4*a) - cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) + 4*cos(6*b*x + 6*a)^2
- 4*(cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*cos(4*b*x + 4*a)^2 + cos(2*
b*x + 2*a)^2 + 2*(sin(8*b*x + 8*a) - 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a
) + sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + sin(10*b*x + 10*a)^2 - 2*(2*sin(
6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + si
n(8*b*x + 8*a)^2 + 4*(2*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(6*b*x + 6*
a) + 4*sin(6*b*x + 6*a)^2 + 4*sin(4*b*x + 4*a)^2 - 4*sin(4*b*x + 4*a)*sin(2
*b*x + 2*a) + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x)^2 +
 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2)
+ 15*(2*(cos(8*b*x + 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + cos(2
*b*x + 2*a) + 1)*cos(10*b*x + 10*a) + cos(10*b*x + 10*a)^2 - 2*(2*cos(6*b*x
 + 6*a) + 2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) - 1)*cos(8*b*x + 8*a) + cos
(8*b*x + 8*a)^2 + 4*(2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) - 1)*cos(6*b*x +
 6*a) + 4*cos(6*b*x + 6*a)^2 - 4*(cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) +
4*cos(4*b*x + 4*a)^2 + cos(2*b*x + 2*a)^2 + 2*(sin(8*b*x + 8*a) - 2*sin(6*b
*x + 6*a) - 2*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + sin
(10*b*x + 10*a)^2 - 2*(2*sin(6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - sin(2*b*x
+ 2*a))*sin(8*b*x + 8*a) + sin(8*b*x + 8*a)^2 + 4*(2*sin(4*b*x + 4*a) - sin
(2*b*x + 2*a))*sin(6*b*x + 6*a) + 4*sin(6*b*x + 6*a)^2 + 4*sin(4*b*x + 4*a)
^2 - 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x
 + 2*a) + 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2
*sin(b*x)*sin(a) + sin(a)^2) + 4*(15*sin(9*b*x + 9*a) + 20*sin(7*b*x + 7*a)
 - 22*sin(5*b*x + 5*a) + 20*sin(3*b*x + 3*a) + 15*sin(b*x + a))*sin(10*b*x
+ 10*a) + 60*(sin(8*b*x + 8*a) - 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) +
sin(2*b*x + 2*a))*sin(9*b*x + 9*a) + 4*(20*sin(7*b*x + 7*a) - 22*sin(5*b*x
+ 5*a) + 20*sin(3*b*x + 3*a) + 15*sin(b*x + a))*sin(8*b*x + 8*a) - 80*(2*si
n(6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(7*b*x + 7*a) +
8*(22*sin(5*b*x + 5*a) - 20*sin(3*b*x + 3*a) - 15*sin(b*x + a))*sin(6*b*x +
 6*a) + 88*(2*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(5*b*x + 5*a) - 40*(4
*sin(3*b*x + 3*a) + 3*sin(b*x + a))*sin(4*b*x + 4*a) + 80*sin(3*b*x + 3*a)*
sin(2*b*x + 2*a) + 60*sin(2*b*x + 2*a)*sin(b*x + a) + 60*cos(b*x + a))/(b*c
os(10*b*x + 10*a)^2 + b*cos(8*b*x + 8*a)^2 + 4*b*cos(6*b*x + 6*a)^2 + 4*b*c
os(4*b*x + 4*a)^2 + b*cos(2*b*x + 2*a)^2 + b*sin(10*b*x + 10*a)^2 + b*sin(8
*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 + 4*b*sin(4*b*x + 4*a)^2 - 4*b*sin(4
*b*x + 4*a)*sin(2*b*x + 2*a) + b*sin(2*b*x + 2*a)^2 + 2*(b*cos(8*b*x + 8*a)
 - 2*b*cos(6*b*x + 6*a) - 2*b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a) + b)*co
s(10*b*x + 10*a) - 2*(2*b*cos(6*b*x + 6*a) + 2*b*cos(4*b*x + 4*a) - b*cos(2
*b*x + 2*a) - b)*cos(8*b*x + 8*a) + 4*(2*b*cos(4*b*x + 4*a) - b*cos(2*b*x +
 2*a) - b)*cos(6*b*x + 6*a) - 4*(b*cos(2*b*x + 2*a) + b)*cos(4*b*x + 4*a) +
 2*b*cos(2*b*x + 2*a) + 2*(b*sin(8*b*x + 8*a) - 2*b*sin(6*b*x + 6*a) - 2*b*
sin(4*b*x + 4*a) + b*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 2*(2*b*sin(6*b*
x + 6*a) + 2*b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + 4*
```

```
(2*b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + b)
```

Fricas [A] time = 0.504879, size = 302, normalized size = 4.58

$$\frac{30 \cos(bx+a)^4 - 20 \cos(bx+a)^2 - 15\left(\cos(bx+a)^5 - \cos(bx+a)^3\right)\log\left(\frac{1}{2}\cos(bx+a) + \frac{1}{2}\right) + 15\left(\cos(bx+a)^5 - \cos\right.}{192\left(b\cos(bx+a)^5 - b\cos(bx+a)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^4*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/192*(30*cos(b*x + a)^4 - 20*cos(b*x + a)^2 - 15*(cos(b*x + a)^5 - cos(b*x
 + a)^3)*log(1/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^5 - cos(b*x + a)^3)
*log(-1/2*cos(b*x + a) + 1/2) - 4)/(b*cos(b*x + a)^5 - b*cos(b*x + a)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**4*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 4.71955, size = 4088, normalized size = 61.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^4*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/384*(3*(6*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^23 - tan(1/2*b*x + 2*a)^2*tan(
1/2*a)^24 - 74*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^21 + 60*tan(1/2*b*x + 2*a)^2
*tan(1/2*a)^22 - 6*tan(1/2*b*x + 2*a)*tan(1/2*a)^23 + 798*tan(1/2*b*x + 2*a
)^3*tan(1/2*a)^19 - 924*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^20 + 290*tan(1/2*b*
x + 2*a)*tan(1/2*a)^21 - 18*tan(1/2*a)^22 - 1170*tan(1/2*b*x + 2*a)^3*tan(1
/2*a)^17 + 3892*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^18 - 2310*tan(1/2*b*x + 2*a
)*tan(1/2*a)^19 + 336*tan(1/2*a)^20 - 3188*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^
15 + 1467*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^16 + 3186*tan(1/2*b*x + 2*a)*tan(
1/2*a)^17 - 1190*tan(1/2*a)^18 + 2604*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^13 -
12744*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^14 + 8148*tan(1/2*b*x + 2*a)*tan(1/2*
a)^15 - 288*tan(1/2*a)^16 + 2604*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^11 - 10332
*tan(1/2*b*x + 2*a)*tan(1/2*a)^13 + 4428*tan(1/2*a)^14 - 3188*tan(1/2*b*x +
 2*a)^3*tan(1/2*a)^9 + 12744*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^10 - 10332*tan
(1/2*b*x + 2*a)*tan(1/2*a)^11 - 1170*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^7 - 14
67*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^8 + 8148*tan(1/2*b*x + 2*a)*tan(1/2*a)^9
 - 4428*tan(1/2*a)^10 + 798*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^5 - 3892*tan(1/
2*b*x + 2*a)^2*tan(1/2*a)^6 + 3186*tan(1/2*b*x + 2*a)*tan(1/2*a)^7 + 288*ta
n(1/2*a)^8 - 74*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^3 + 924*tan(1/2*b*x + 2*a)^
```

```
2*tan(1/2*a)^4 - 2310*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 + 1190*tan(1/2*a)^6 +
 6*tan(1/2*b*x + 2*a)^3*tan(1/2*a) - 60*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2 +
 290*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 336*tan(1/2*a)^4 + tan(1/2*b*x + 2*a
)^2 - 6*tan(1/2*b*x + 2*a)*tan(1/2*a) + 18*tan(1/2*a)^2)/((9*tan(1/2*a)^10
- 60*tan(1/2*a)^8 + 118*tan(1/2*a)^6 - 60*tan(1/2*a)^4 + 9*tan(1/2*a)^2)*(3
*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 - tan(1/2*b*x + 2*a)*tan(1/2*a)^6 - 10*t
an(1/2*b*x + 2*a)^2*tan(1/2*a)^3 + 15*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 - 3*t
an(1/2*a)^5 + 3*tan(1/2*b*x + 2*a)^2*tan(1/2*a) - 15*tan(1/2*b*x + 2*a)*tan
(1/2*a)^2 + 10*tan(1/2*a)^3 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a))^2) + 16*(5
4*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^35 - 9*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^36
 - 2610*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^33 + 1620*tan(1/2*b*x + 2*a)^4*tan(
1/2*a)^34 - 252*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^35 + 12*tan(1/2*b*x + 2*a)^
2*tan(1/2*a)^36 + 52920*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^31 - 58455*tan(1/2*
b*x + 2*a)^4*tan(1/2*a)^32 + 19956*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^33 - 280
8*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^34 + 198*tan(1/2*b*x + 2*a)*tan(1/2*a)^35
 - 7*tan(1/2*a)^36 - 573912*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^29 + 942984*tan
(1/2*b*x + 2*a)^4*tan(1/2*a)^30 - 503568*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^31
 + 112068*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^32 - 11298*tan(1/2*b*x + 2*a)*tan
(1/2*a)^33 + 468*tan(1/2*a)^34 + 3241488*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^27
 - 7636140*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^28 + 5941584*tan(1/2*b*x + 2*a)^
3*tan(1/2*a)^29 - 1893936*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^30 + 257976*tan(1
/2*b*x + 2*a)*tan(1/2*a)^31 - 12969*tan(1/2*a)^32 - 6862752*tan(1/2*b*x + 2
*a)^5*tan(1/2*a)^25 + 27168408*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^26 - 3237094
4*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^27 + 15361632*tan(1/2*b*x + 2*a)^2*tan(1/
2*a)^28 - 2923992*tan(1/2*b*x + 2*a)*tan(1/2*a)^29 + 188808*tan(1/2*a)^30 -
 1663560*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^23 - 24990420*tan(1/2*b*x + 2*a)^4
*tan(1/2*a)^24 + 67201056*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^25 - 54063504*tan
(1/2*b*x + 2*a)^2*tan(1/2*a)^26 + 16195152*tan(1/2*b*x + 2*a)*tan(1/2*a)^27
 - 1484964*tan(1/2*a)^28 + 20436072*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^21 - 57
467160*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^22 + 13860720*tan(1/2*b*x + 2*a)^3*t
an(1/2*a)^23 + 50019840*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^24 - 33931872*tan(1
/2*b*x + 2*a)*tan(1/2*a)^25 + 5559192*tan(1/2*a)^26 - 14627700*tan(1/2*b*x
+ 2*a)^5*tan(1/2*a)^19 + 112971402*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^20 - 204
830640*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^21 + 114114960*tan(1/2*b*x + 2*a)^2*
tan(1/2*a)^22 - 7584840*tan(1/2*b*x + 2*a)*tan(1/2*a)^23 - 4921020*tan(1/2*
a)^24 - 14627700*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^17 + 150157800*tan(1/2*b*x
 + 2*a)^3*tan(1/2*a)^19 - 227062440*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^20 + 10
2261096*tan(1/2*b*x + 2*a)*tan(1/2*a)^21 - 11686680*tan(1/2*a)^22 + 2043607
2*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^15 - 112971402*tan(1/2*b*x + 2*a)^4*tan(1
/2*a)^16 + 150157800*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^17 - 74262420*tan(1/2*
b*x + 2*a)*tan(1/2*a)^19 + 22278726*tan(1/2*a)^20 - 1663560*tan(1/2*b*x + 2
*a)^5*tan(1/2*a)^13 + 57467160*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^14 - 2048306
40*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^15 + 227062440*tan(1/2*b*x + 2*a)^2*tan(
1/2*a)^16 - 74262420*tan(1/2*b*x + 2*a)*tan(1/2*a)^17 - 6862752*tan(1/2*b*x
 + 2*a)^5*tan(1/2*a)^11 + 24990420*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^12 + 138
60720*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^13 - 114114960*tan(1/2*b*x + 2*a)^2*t
an(1/2*a)^14 + 102261096*tan(1/2*b*x + 2*a)*tan(1/2*a)^15 - 22278726*tan(1/
2*a)^16 + 3241488*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^9 - 27168408*tan(1/2*b*x
+ 2*a)^4*tan(1/2*a)^10 + 67201056*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^11 - 5001
9840*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^12 - 7584840*tan(1/2*b*x + 2*a)*tan(1/
2*a)^13 + 11686680*tan(1/2*a)^14 - 573912*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^7
 + 7636140*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^8 - 32370944*tan(1/2*b*x + 2*a)^
3*tan(1/2*a)^9 + 54063504*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^10 - 33931872*tan
(1/2*b*x + 2*a)*tan(1/2*a)^11 + 4921020*tan(1/2*a)^12 + 52920*tan(1/2*b*x +
 2*a)^5*tan(1/2*a)^5 - 942984*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^6 + 5941584*t
an(1/2*b*x + 2*a)^3*tan(1/2*a)^7 - 15361632*tan(1/2*b*x + 2*a)^2*tan(1/2*a)
^8 + 16195152*tan(1/2*b*x + 2*a)*tan(1/2*a)^9 - 5559192*tan(1/2*a)^10 - 261
0*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^3 + 58455*tan(1/2*b*x + 2*a)^4*tan(1/2*a)
^4 - 503568*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^5 + 1893936*tan(1/2*b*x + 2*a)^
```

```
2*tan(1/2*a)^6 - 2923992*tan(1/2*b*x + 2*a)*tan(1/2*a)^7 + 1484964*tan(1/2*
a)^8 + 54*tan(1/2*b*x + 2*a)^5*tan(1/2*a) - 1620*tan(1/2*b*x + 2*a)^4*tan(1
/2*a)^2 + 19956*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^3 - 112068*tan(1/2*b*x + 2*
a)^2*tan(1/2*a)^4 + 257976*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - 188808*tan(1/2
*a)^6 + 9*tan(1/2*b*x + 2*a)^4 - 252*tan(1/2*b*x + 2*a)^3*tan(1/2*a) + 2808
*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2 - 11298*tan(1/2*b*x + 2*a)*tan(1/2*a)^3
+ 12969*tan(1/2*a)^4 - 12*tan(1/2*b*x + 2*a)^2 + 198*tan(1/2*b*x + 2*a)*tan
(1/2*a) - 468*tan(1/2*a)^2 + 7)/((tan(1/2*a)^18 - 45*tan(1/2*a)^16 + 720*ta
n(1/2*a)^14 - 4728*tan(1/2*a)^12 + 10890*tan(1/2*a)^10 - 10890*tan(1/2*a)^8
 + 4728*tan(1/2*a)^6 - 720*tan(1/2*a)^4 + 45*tan(1/2*a)^2 - 1)*(tan(1/2*b*x
 + 2*a)^2*tan(1/2*a)^6 - 15*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^4 + 12*tan(1/2*
b*x + 2*a)*tan(1/2*a)^5 - tan(1/2*a)^6 + 15*tan(1/2*b*x + 2*a)^2*tan(1/2*a)
^2 - 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 - tan(1/2*b*x + 2
*a)^2 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)^3) + 60*log
(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*
tan(1/2*a)^2 - 1)) - 60*log(abs(3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - tan(1/2
*a)^3 - tan(1/2*b*x + 2*a) + 3*tan(1/2*a))))/b
```

3.12 $\int \csc^5(2a+2bx)\sin(a+bx)\,dx$

Optimal. Leaf size=89

$$-\frac{35\csc^3(a+bx)}{768b}-\frac{35\csc(a+bx)}{256b}+\frac{35\tanh^{-1}(\sin(a+bx))}{256b}+\frac{\csc^3(a+bx)\sec^4(a+bx)}{128b}+\frac{7\csc^3(a+bx)\sec^2(a+b}{256b}$$

```
[Out] (35*ArcTanh[Sin[a + b*x]])/(256*b) - (35*Csc[a + b*x])/(256*b) - (35*Csc[a
+ b*x]^3)/(768*b) + (7*Csc[a + b*x]^3*Sec[a + b*x]^2)/(256*b) + (Csc[a + b*
x]^3*Sec[a + b*x]^4)/(128*b)
```

Rubi [A] time = 0.0694721, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {4288, 2621, 288, 302, 207}

$$-\frac{35\csc^3(a+bx)}{768b}-\frac{35\csc(a+bx)}{256b}+\frac{35\tanh^{-1}(\sin(a+bx))}{256b}+\frac{\csc^3(a+bx)\sec^4(a+bx)}{128b}+\frac{7\csc^3(a+bx)\sec^2(a+b}{256b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^5*Sin[a + b*x],x]
```


```
[Out] (35*ArcTanh[Sin[a + b*x]])/(256*b) - (35*Csc[a + b*x])/(256*b) - (35*Csc[a
+ b*x]^3)/(768*b) + (7*Csc[a + b*x]^3*Sec[a + b*x]^2)/(256*b) + (Csc[a + b*
x]^3*Sec[a + b*x]^4)/(128*b)
```


Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^5(2a+2bx)\sin(a+bx)\,dx = \frac{1}{32}\int \csc^4(a+bx)\sec^5(a+bx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{x^8}{\left(-1+x^2\right)^3}\,dx, x, \csc(a+bx)\right)}{32b}$$
$$= \frac{\csc^3(a+bx)\sec^4(a+bx)}{128b} - \frac{7\,\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^2}\,dx, x, \csc(a+bx)\right)}{128b}$$
$$= \frac{7\csc^3(a+bx)\sec^2(a+bx)}{256b} + \frac{\csc^3(a+bx)\sec^4(a+bx)}{128b} - \frac{35\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \text{cs}\right.}{256b}$$
$$= \frac{7\csc^3(a+bx)\sec^2(a+bx)}{256b} + \frac{\csc^3(a+bx)\sec^4(a+bx)}{128b} - \frac{35\,\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+}\right.\right.}{256b}$$
$$= -\frac{35\csc(a+bx)}{256b} - \frac{35\csc^3(a+bx)}{768b} + \frac{7\csc^3(a+bx)\sec^2(a+bx)}{256b} + \frac{\csc^3(a+bx)\sec^4(}{128b}$$
$$= \frac{35\tanh^{-1}(\sin(a+bx))}{256b} - \frac{35\csc(a+bx)}{256b} - \frac{35\csc^3(a+bx)}{768b} + \frac{7\csc^3(a+bx)\sec^2(a+}{256b}$$

Mathematica [C] time = 0.030472, size = 31, normalized size = 0.35

$$-\frac{\csc^3(a+bx)\text{Hypergeometric2F1}\left(-\frac{3}{2},3,-\frac{1}{2},\sin^2(a+bx)\right)}{96b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^5*Sin[a + b*x],x]
```

```
[Out] -(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 3, -1/2, Sin[a + b*x]^2])/(96*b)
```

Maple [A] time = 0.038, size = 97, normalized size = 1.1

$$\frac{1}{128\,b\left(\sin(bx+a)\right)^3\left(\cos(bx+a)\right)^4} - \frac{7}{384\,b\left(\sin(bx+a)\right)^3\left(\cos(bx+a)\right)^2} + \frac{35}{768\,b\sin(bx+a)\left(\cos(bx+a)\right)^2} - \frac{3}{256\,b\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^5*sin(b*x+a),x)
```

```
[Out] 1/128/b/sin(b*x+a)^3/cos(b*x+a)^4-7/384/b/sin(b*x+a)^3/cos(b*x+a)^2+35/768/
b/sin(b*x+a)/cos(b*x+a)^2-35/256/b/sin(b*x+a)+35/256/b*ln(sec(b*x+a)+tan(b*
x+a))
```

Maxima [B] time = 2.31794, size = 4169, normalized size = 46.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^5*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/1536*(4*(105*sin(13*b*x + 13*a) + 70*sin(11*b*x + 11*a) - 329*sin(9*b*x +
 9*a) - 204*sin(7*b*x + 7*a) - 329*sin(5*b*x + 5*a) + 70*sin(3*b*x + 3*a) +
 105*sin(b*x + a))*cos(14*b*x + 14*a) - 420*(sin(12*b*x + 12*a) - 3*sin(10*
b*x + 10*a) - 3*sin(8*b*x + 8*a) + 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a)
- sin(2*b*x + 2*a))*cos(13*b*x + 13*a) + 4*(70*sin(11*b*x + 11*a) - 329*sin
(9*b*x + 9*a) - 204*sin(7*b*x + 7*a) - 329*sin(5*b*x + 5*a) + 70*sin(3*b*x
+ 3*a) + 105*sin(b*x + a))*cos(12*b*x + 12*a) + 280*(3*sin(10*b*x + 10*a) +
 3*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) + sin(2*b*x +
 2*a))*cos(11*b*x + 11*a) + 12*(329*sin(9*b*x + 9*a) + 204*sin(7*b*x + 7*a)
 + 329*sin(5*b*x + 5*a) - 70*sin(3*b*x + 3*a) - 105*sin(b*x + a))*cos(10*b*
x + 10*a) - 1316*(3*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4
*a) + sin(2*b*x + 2*a))*cos(9*b*x + 9*a) + 12*(204*sin(7*b*x + 7*a) + 329*s
in(5*b*x + 5*a) - 70*sin(3*b*x + 3*a) - 105*sin(b*x + a))*cos(8*b*x + 8*a)
+ 816*(3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(7*b*
x + 7*a) - 84*(47*sin(5*b*x + 5*a) - 10*sin(3*b*x + 3*a) - 15*sin(b*x + a))
*cos(6*b*x + 6*a) + 1316*(3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(5*b*x
+ 5*a) + 420*(2*sin(3*b*x + 3*a) + 3*sin(b*x + a))*cos(4*b*x + 4*a) - 105*(
2*(cos(12*b*x + 12*a) - 3*cos(10*b*x + 10*a) - 3*cos(8*b*x + 8*a) + 3*cos(6
*b*x + 6*a) + 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) - 1)*cos(14*b*x + 14*a)
 + cos(14*b*x + 14*a)^2 - 2*(3*cos(10*b*x + 10*a) + 3*cos(8*b*x + 8*a) - 3*
cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) + 1)*cos(12*b*x +
12*a) + cos(12*b*x + 12*a)^2 + 6*(3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) -
 3*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) + 1)*cos(10*b*x + 10*a) + 9*cos(10*b
*x + 10*a)^2 - 6*(3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a
) - 1)*cos(8*b*x + 8*a) + 9*cos(8*b*x + 8*a)^2 + 6*(3*cos(4*b*x + 4*a) - co
s(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) + 9*cos(6*b*x + 6*a)^2 - 6*(cos(2*b*x
+ 2*a) + 1)*cos(4*b*x + 4*a) + 9*cos(4*b*x + 4*a)^2 + cos(2*b*x + 2*a)^2 +
2*(sin(12*b*x + 12*a) - 3*sin(10*b*x + 10*a) - 3*sin(8*b*x + 8*a) + 3*sin(6
*b*x + 6*a) + 3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(14*b*x + 14*a) + s
in(14*b*x + 14*a)^2 - 2*(3*sin(10*b*x + 10*a) + 3*sin(8*b*x + 8*a) - 3*sin(
6*b*x + 6*a) - 3*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(12*b*x + 12*a) +
sin(12*b*x + 12*a)^2 + 6*(3*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) - 3*sin(4
*b*x + 4*a) + sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + 9*sin(10*b*x + 10*a)^2
 - 6*(3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(8*b*x
 + 8*a) + 9*sin(8*b*x + 8*a)^2 + 6*(3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*
sin(6*b*x + 6*a) + 9*sin(6*b*x + 6*a)^2 + 9*sin(4*b*x + 4*a)^2 - 6*sin(4*b*
x + 4*a)*sin(2*b*x + 2*a) + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*lo
g((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2
 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos
(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)
) - 4*(105*cos(13*b*x + 13*a) + 70*cos(11*b*x + 11*a) - 329*cos(9*b*x + 9*a
) - 204*cos(7*b*x + 7*a) - 329*cos(5*b*x + 5*a) + 70*cos(3*b*x + 3*a) + 105
*cos(b*x + a))*sin(14*b*x + 14*a) + 420*(cos(12*b*x + 12*a) - 3*cos(10*b*x
+ 10*a) - 3*cos(8*b*x + 8*a) + 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) - co
s(2*b*x + 2*a) - 1)*sin(13*b*x + 13*a) - 4*(70*cos(11*b*x + 11*a) - 329*cos
(9*b*x + 9*a) - 204*cos(7*b*x + 7*a) - 329*cos(5*b*x + 5*a) + 70*cos(3*b*x
+ 3*a) + 105*cos(b*x + a))*sin(12*b*x + 12*a) - 280*(3*cos(10*b*x + 10*a) +
 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) + cos(2*b*x +
 2*a) + 1)*sin(11*b*x + 11*a) - 12*(329*cos(9*b*x + 9*a) + 204*cos(7*b*x +
7*a) + 329*cos(5*b*x + 5*a) - 70*cos(3*b*x + 3*a) - 105*cos(b*x + a))*sin(1
0*b*x + 10*a) + 1316*(3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) - 3*cos(4*b*x
```

```
 + 4*a) + cos(2*b*x + 2*a) + 1)*sin(9*b*x + 9*a) - 12*(204*cos(7*b*x + 7*a)
 + 329*cos(5*b*x + 5*a) - 70*cos(3*b*x + 3*a) - 105*cos(b*x + a))*sin(8*b*x
 + 8*a) - 816*(3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) -
 1)*sin(7*b*x + 7*a) + 84*(47*cos(5*b*x + 5*a) - 10*cos(3*b*x + 3*a) - 15*c
os(b*x + a))*sin(6*b*x + 6*a) - 1316*(3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a)
 - 1)*sin(5*b*x + 5*a) - 420*(2*cos(3*b*x + 3*a) + 3*cos(b*x + a))*sin(4*b*
x + 4*a) - 280*(cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a) + 280*cos(3*b*x + 3*
a)*sin(2*b*x + 2*a) + 420*cos(b*x + a)*sin(2*b*x + 2*a) - 420*cos(2*b*x + 2
*a)*sin(b*x + a) - 420*sin(b*x + a))/(b*cos(14*b*x + 14*a)^2 + b*cos(12*b*x
 + 12*a)^2 + 9*b*cos(10*b*x + 10*a)^2 + 9*b*cos(8*b*x + 8*a)^2 + 9*b*cos(6*
b*x + 6*a)^2 + 9*b*cos(4*b*x + 4*a)^2 + b*cos(2*b*x + 2*a)^2 + b*sin(14*b*x
 + 14*a)^2 + b*sin(12*b*x + 12*a)^2 + 9*b*sin(10*b*x + 10*a)^2 + 9*b*sin(8*
b*x + 8*a)^2 + 9*b*sin(6*b*x + 6*a)^2 + 9*b*sin(4*b*x + 4*a)^2 - 6*b*sin(4*
b*x + 4*a)*sin(2*b*x + 2*a) + b*sin(2*b*x + 2*a)^2 + 2*(b*cos(12*b*x + 12*a
) - 3*b*cos(10*b*x + 10*a) - 3*b*cos(8*b*x + 8*a) + 3*b*cos(6*b*x + 6*a) +
3*b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2*a) - b)*cos(14*b*x + 14*a) - 2*(3*b*
cos(10*b*x + 10*a) + 3*b*cos(8*b*x + 8*a) - 3*b*cos(6*b*x + 6*a) - 3*b*cos(
4*b*x + 4*a) + b*cos(2*b*x + 2*a) + b)*cos(12*b*x + 12*a) + 6*(3*b*cos(8*b*
x + 8*a) - 3*b*cos(6*b*x + 6*a) - 3*b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a)
 + b)*cos(10*b*x + 10*a) - 6*(3*b*cos(6*b*x + 6*a) + 3*b*cos(4*b*x + 4*a) -
 b*cos(2*b*x + 2*a) - b)*cos(8*b*x + 8*a) + 6*(3*b*cos(4*b*x + 4*a) - b*cos
(2*b*x + 2*a) - b)*cos(6*b*x + 6*a) - 6*(b*cos(2*b*x + 2*a) + b)*cos(4*b*x
+ 4*a) + 2*b*cos(2*b*x + 2*a) + 2*(b*sin(12*b*x + 12*a) - 3*b*sin(10*b*x +
10*a) - 3*b*sin(8*b*x + 8*a) + 3*b*sin(6*b*x + 6*a) + 3*b*sin(4*b*x + 4*a)
- b*sin(2*b*x + 2*a))*sin(14*b*x + 14*a) - 2*(3*b*sin(10*b*x + 10*a) + 3*b*
sin(8*b*x + 8*a) - 3*b*sin(6*b*x + 6*a) - 3*b*sin(4*b*x + 4*a) + b*sin(2*b*
x + 2*a))*sin(12*b*x + 12*a) + 6*(3*b*sin(8*b*x + 8*a) - 3*b*sin(6*b*x + 6*
a) - 3*b*sin(4*b*x + 4*a) + b*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 6*(3*b
*sin(6*b*x + 6*a) + 3*b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*sin(8*b*x +
8*a) + 6*(3*b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + b)
```

Fricas [A] time = 0.522941, size = 378, normalized size = 4.25

$$-\frac{210 \cos(bx+a)^6 - 280 \cos(bx+a)^4 - 105 \left(\cos(bx+a)^6 - \cos(bx+a)^4\right) \log(\sin(bx+a)+1)\sin(bx+a) + 105 \left(\cos \right.}{1536 \left(b\cos(bx+a)^6 - b\cos(bx+a)^4\right) \sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^5*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/1536*(210*cos(b*x + a)^6 - 280*cos(b*x + a)^4 - 105*(cos(b*x + a)^6 - co
s(b*x + a)^4)*log(sin(b*x + a) + 1)*sin(b*x + a) + 105*(cos(b*x + a)^6 - co
s(b*x + a)^4)*log(-sin(b*x + a) + 1)*sin(b*x + a) + 42*cos(b*x + a)^2 + 12)
/((b*cos(b*x + a)^6 - b*cos(b*x + a)^4)*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**5*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 13.1691, size = 7623, normalized size = 85.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^5*sin(b*x+a),x, algorithm="giac")

[Out] -1/768*((27*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^34 - 9*tan(1/2*b*x + 2*a)^4*tan
(1/2*a)^35 + tan(1/2*b*x + 2*a)^3*tan(1/2*a)^36 + 2574*tan(1/2*b*x + 2*a)^5
*tan(1/2*a)^32 - 1563*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^33 + 234*tan(1/2*b*x
+ 2*a)^3*tan(1/2*a)^34 + 9*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^35 - 74706*tan(1
/2*b*x + 2*a)^5*tan(1/2*a)^30 + 88434*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^31 -
32229*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^32 + 3669*tan(1/2*b*x + 2*a)^2*tan(1/
2*a)^33 + 27*tan(1/2*b*x + 2*a)*tan(1/2*a)^34 + 598014*tan(1/2*b*x + 2*a)^5
*tan(1/2*a)^28 - 1170882*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^29 + 730596*tan(1/
2*b*x + 2*a)^3*tan(1/2*a)^30 - 178020*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^31 +
14238*tan(1/2*b*x + 2*a)*tan(1/2*a)^32 + 54*tan(1/2*a)^33 - 1958598*tan(1/2
*b*x + 2*a)^5*tan(1/2*a)^26 + 6055062*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^27 -
5907096*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^28 + 2329236*tan(1/2*b*x + 2*a)^2*t
an(1/2*a)^29 - 370194*tan(1/2*b*x + 2*a)*tan(1/2*a)^30 + 17442*tan(1/2*a)^3
1 + 1859910*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^24 - 12207078*tan(1/2*b*x + 2*a
)^4*tan(1/2*a)^25 + 19818828*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^26 - 12113616*
tan(1/2*b*x + 2*a)^2*tan(1/2*a)^27 + 2973582*tan(1/2*b*x + 2*a)*tan(1/2*a)^
28 - 240066*tan(1/2*a)^29 + 3993750*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^22 - 18
79110*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^23 - 18713760*tan(1/2*b*x + 2*a)^3*ta
n(1/2*a)^24 + 24499800*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^25 - 9845190*tan(1/2
*b*x + 2*a)*tan(1/2*a)^26 + 1208842*tan(1/2*a)^27 - 8882730*tan(1/2*b*x + 2
*a)^5*tan(1/2*a)^20 + 38331030*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^21 - 4109310
0*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^22 + 3908700*tan(1/2*b*x + 2*a)^2*tan(1/2
*a)^23 + 9319830*tan(1/2*b*x + 2*a)*tan(1/2*a)^24 - 2402946*tan(1/2*a)^25 -
 29609100*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^19 + 87376914*tan(1/2*b*x + 2*a)^
3*tan(1/2*a)^20 - 76707084*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^21 + 20211030*ta
n(1/2*b*x + 2*a)*tan(1/2*a)^22 - 319590*tan(1/2*a)^23 + 8882730*tan(1/2*b*x
 + 2*a)^5*tan(1/2*a)^16 - 29609100*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^17 + 588
47130*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^19 - 44106714*tan(1/2*b*x + 2*a)*tan(
1/2*a)^20 + 7594374*tan(1/2*a)^21 - 3993750*tan(1/2*b*x + 2*a)^5*tan(1/2*a)
^14 + 38331030*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^15 - 87376914*tan(1/2*b*x +
2*a)^3*tan(1/2*a)^16 + 58847130*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^17 - 618579
0*tan(1/2*a)^19 - 1859910*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^12 - 1879110*tan(
1/2*b*x + 2*a)^4*tan(1/2*a)^13 + 41093100*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^1
4 - 76707084*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^15 + 44106714*tan(1/2*b*x + 2*
a)*tan(1/2*a)^16 - 6185790*tan(1/2*a)^17 + 1958598*tan(1/2*b*x + 2*a)^5*tan
(1/2*a)^10 - 12207078*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^11 + 18713760*tan(1/2
*b*x + 2*a)^3*tan(1/2*a)^12 + 3908700*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^13 -
20211030*tan(1/2*b*x + 2*a)*tan(1/2*a)^14 + 7594374*tan(1/2*a)^15 - 598014*
tan(1/2*b*x + 2*a)^5*tan(1/2*a)^8 + 6055062*tan(1/2*b*x + 2*a)^4*tan(1/2*a)
^9 - 19818828*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^10 + 24499800*tan(1/2*b*x + 2
*a)^2*tan(1/2*a)^11 - 9319830*tan(1/2*b*x + 2*a)*tan(1/2*a)^12 - 319590*tan
(1/2*a)^13 + 74706*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^6 - 1170882*tan(1/2*b*x
+ 2*a)^4*tan(1/2*a)^7 + 5907096*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^8 - 1211361
6*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^9 + 9845190*tan(1/2*b*x + 2*a)*tan(1/2*a)
^10 - 2402946*tan(1/2*a)^11 - 2574*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^4 + 8843
4*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^5 - 730596*tan(1/2*b*x + 2*a)^3*tan(1/2*a
)^6 + 2329236*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^7 - 2973582*tan(1/2*b*x + 2*a
)*tan(1/2*a)^8 + 1208842*tan(1/2*a)^9 - 27*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^
2 - 1563*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^3 + 32229*tan(1/2*b*x + 2*a)^3*tan
(1/2*a)^4 - 178020*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 + 370194*tan(1/2*b*x +
```

```
 2*a)*tan(1/2*a)^6 - 240066*tan(1/2*a)^7 - 9*tan(1/2*b*x + 2*a)^4*tan(1/2*a
) - 234*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^2 + 3669*tan(1/2*b*x + 2*a)^2*tan(1
/2*a)^3 - 14238*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 17442*tan(1/2*a)^5 - tan(
1/2*b*x + 2*a)^3 + 9*tan(1/2*b*x + 2*a)^2*tan(1/2*a) - 27*tan(1/2*b*x + 2*a
)*tan(1/2*a)^2 + 54*tan(1/2*a)^3)/((27*tan(1/2*a)^15 - 270*tan(1/2*a)^13 +
981*tan(1/2*a)^11 - 1540*tan(1/2*a)^9 + 981*tan(1/2*a)^7 - 270*tan(1/2*a)^5
 + 27*tan(1/2*a)^3)*(3*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 - tan(1/2*b*x + 2*
a)*tan(1/2*a)^6 - 10*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 + 15*tan(1/2*b*x + 2
*a)*tan(1/2*a)^4 - 3*tan(1/2*a)^5 + 3*tan(1/2*b*x + 2*a)^2*tan(1/2*a) - 15*
tan(1/2*b*x + 2*a)*tan(1/2*a)^2 + 10*tan(1/2*a)^3 + tan(1/2*b*x + 2*a) - 3*
tan(1/2*a))^3) + 6*(13*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^48 - 174*tan(1/2*b*x
 + 2*a)^7*tan(1/2*a)^46 + 162*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^47 - 5*tan(1/
2*b*x + 2*a)^5*tan(1/2*a)^48 - 21522*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^44 + 1
3014*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^45 - 2514*tan(1/2*b*x + 2*a)^5*tan(1/2
*a)^46 + 234*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^47 - 5*tan(1/2*b*x + 2*a)^3*ta
n(1/2*a)^48 + 942746*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^42 - 1100574*tan(1/2*b
*x + 2*a)^6*tan(1/2*a)^43 + 453762*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^44 - 872
34*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^45 + 8718*tan(1/2*b*x + 2*a)^3*tan(1/2*a
)^46 - 474*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^47 + 13*tan(1/2*b*x + 2*a)*tan(1
/2*a)^48 - 17981508*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^40 + 30574566*tan(1/2*b
*x + 2*a)^6*tan(1/2*a)^41 - 18985626*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^42 + 5
680314*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^43 - 906750*tan(1/2*b*x + 2*a)^3*tan
(1/2*a)^44 + 80834*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^45 - 3918*tan(1/2*b*x +
2*a)*tan(1/2*a)^46 + 78*tan(1/2*a)^47 + 193013886*tan(1/2*b*x + 2*a)^7*tan(
1/2*a)^38 - 441566530*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^39 + 374555556*tan(1/
2*b*x + 2*a)^5*tan(1/2*a)^40 - 153629730*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^41
 + 33577926*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^42 - 4014234*tan(1/2*b*x + 2*a)
^2*tan(1/2*a)^43 + 252270*tan(1/2*b*x + 2*a)*tan(1/2*a)^44 - 6614*tan(1/2*a
)^45 - 1202084806*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^36 + 3626590890*tan(1/2*b
*x + 2*a)^6*tan(1/2*a)^37 - 4076849790*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^38 +
 2214510870*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^39 - 633112668*tan(1/2*b*x + 2*
a)^3*tan(1/2*a)^40 + 97266546*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^41 - 7629702*
tan(1/2*b*x + 2*a)*tan(1/2*a)^42 + 243198*tan(1/2*a)^43 + 4119245478*tan(1/
2*b*x + 2*a)^7*tan(1/2*a)^34 - 16824081618*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^
35 + 25189696662*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^36 - 18102012318*tan(1/2*b
*x + 2*a)^4*tan(1/2*a)^37 + 6776146338*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^38 -
 1334518022*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^39 + 130749372*tan(1/2*b*x + 2*
a)*tan(1/2*a)^40 - 5045814*tan(1/2*a)^41 - 6836731551*tan(1/2*b*x + 2*a)^7*
tan(1/2*a)^32 + 41573675178*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^33 - 8662916611
8*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^34 + 84079436982*tan(1/2*b*x + 2*a)^4*tan
(1/2*a)^35 - 41858006762*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^36 + 10786903998*t
an(1/2*b*x + 2*a)^2*tan(1/2*a)^37 - 1346794722*tan(1/2*b*x + 2*a)*tan(1/2*a
)^38 + 64179762*tan(1/2*a)^39 + 267313972*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^3
0 - 37264510188*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^31 + 143316996039*tan(1/2*b
*x + 2*a)^5*tan(1/2*a)^32 - 208138130542*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^33
 + 144563703930*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^34 - 50381554230*tan(1/2*b*
x + 2*a)^2*tan(1/2*a)^35 + 8311517370*tan(1/2*b*x + 2*a)*tan(1/2*a)^36 - 50
7068874*tan(1/2*a)^37 + 17351751084*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^28 - 56
502600420*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^29 - 5710349876*tan(1/2*b*x + 2*a
)^5*tan(1/2*a)^30 + 186100588644*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^31 - 23940
5365305*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^32 + 125386031934*tan(1/2*b*x + 2*a
)^2*tan(1/2*a)^33 - 29001178170*tan(1/2*b*x + 2*a)*tan(1/2*a)^34 + 23928889
14*tan(1/2*a)^35 - 22757749308*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^26 + 1644199
19188*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^27 - 363804236364*tan(1/2*b*x + 2*a)^
5*tan(1/2*a)^28 + 282793219500*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^29 + 8521880
716*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^30 - 111637405764*tan(1/2*b*x + 2*a)^2*
tan(1/2*a)^31 + 48133667169*tan(1/2*b*x + 2*a)*tan(1/2*a)^32 - 6017063802*t
an(1/2*a)^33 - 98616913668*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^25 + 47878440044
4*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^26 - 821763522748*tan(1/2*b*x + 2*a)^4*ta
```

n(1/2*a)^27 + 607022398644*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^28 - 17113731793
2*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^29 - 1237805836*tan(1/2*b*x + 2*a)*tan(1/
2*a)^30 + 5308170444*tan(1/2*a)^31 + 22757749308*tan(1/2*b*x + 2*a)^7*tan(1
/2*a)^22 - 98616913668*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^23 + 492829728300*ta
n(1/2*b*x + 2*a)^4*tan(1/2*a)^25 - 795918156804*tan(1/2*b*x + 2*a)^3*tan(1/
2*a)^26 + 492162548604*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^27 - 121726145364*ta
n(1/2*b*x + 2*a)*tan(1/2*a)^28 + 8269813092*tan(1/2*a)^29 - 17351751084*tan
(1/2*b*x + 2*a)^7*tan(1/2*a)^20 + 164419919188*tan(1/2*b*x + 2*a)^6*tan(1/2
*a)^21 - 478784400444*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^22 + 492829728300*tan
(1/2*b*x + 2*a)^4*tan(1/2*a)^23 - 293938021260*tan(1/2*b*x + 2*a)^2*tan(1/2
*a)^25 + 158215581828*tan(1/2*b*x + 2*a)*tan(1/2*a)^26 - 23358680596*tan(1/
2*a)^27 - 267313972*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^18 - 56502600420*tan(1/
2*b*x + 2*a)^6*tan(1/2*a)^19 + 363804236364*tan(1/2*b*x + 2*a)^5*tan(1/2*a)
^20 - 821763522748*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^21 + 795918156804*tan(1/
2*b*x + 2*a)^3*tan(1/2*a)^22 - 293938021260*tan(1/2*b*x + 2*a)^2*tan(1/2*a)
^23 + 13852570212*tan(1/2*a)^25 + 6836731551*tan(1/2*b*x + 2*a)^7*tan(1/2*a
)^16 - 37264510188*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^17 + 5710349876*tan(1/2*
b*x + 2*a)^5*tan(1/2*a)^18 + 282793219500*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^1
9 - 607022398644*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^20 + 492162548604*tan(1/2*
b*x + 2*a)^2*tan(1/2*a)^21 - 158215581828*tan(1/2*b*x + 2*a)*tan(1/2*a)^22
+ 13852570212*tan(1/2*a)^23 - 4119245478*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^14
 + 41573675178*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^15 - 143316996039*tan(1/2*b*
x + 2*a)^5*tan(1/2*a)^16 + 186100588644*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^17
- 8521880716*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^18 - 171137317932*tan(1/2*b*x
+ 2*a)^2*tan(1/2*a)^19 + 121726145364*tan(1/2*b*x + 2*a)*tan(1/2*a)^20 - 23
358680596*tan(1/2*a)^21 + 1202084806*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^12 - 1
6824081618*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^13 + 86629166118*tan(1/2*b*x + 2
*a)^5*tan(1/2*a)^14 - 208138130542*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^15 + 239
405365305*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^16 - 111637405764*tan(1/2*b*x + 2
*a)^2*tan(1/2*a)^17 + 1237805836*tan(1/2*b*x + 2*a)*tan(1/2*a)^18 + 8269813
092*tan(1/2*a)^19 - 193013886*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^10 + 36265908
90*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^11 - 25189696662*tan(1/2*b*x + 2*a)^5*ta
n(1/2*a)^12 + 84079436982*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^13 - 144563703930
*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^14 + 125386031934*tan(1/2*b*x + 2*a)^2*tan
(1/2*a)^15 - 48133667169*tan(1/2*b*x + 2*a)*tan(1/2*a)^16 + 5308170444*tan(
1/2*a)^17 + 17981508*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^8 - 441566530*tan(1/2*
b*x + 2*a)^6*tan(1/2*a)^9 + 4076849790*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^10 -
 18102012318*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^11 + 41858006762*tan(1/2*b*x +
 2*a)^3*tan(1/2*a)^12 - 50381554230*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^13 + 29
001178170*tan(1/2*b*x + 2*a)*tan(1/2*a)^14 - 6017063802*tan(1/2*a)^15 - 942
746*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^6 + 30574566*tan(1/2*b*x + 2*a)^6*tan(1
/2*a)^7 - 374555556*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^8 + 2214510870*tan(1/2*
b*x + 2*a)^4*tan(1/2*a)^9 - 6776146338*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^10 +
 10786903998*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^11 - 8311517370*tan(1/2*b*x +
2*a)*tan(1/2*a)^12 + 2392888914*tan(1/2*a)^13 + 21522*tan(1/2*b*x + 2*a)^7*
tan(1/2*a)^4 - 1100574*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^5 + 18985626*tan(1/2
*b*x + 2*a)^5*tan(1/2*a)^6 - 153629730*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^7 +
633112668*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^8 - 1334518022*tan(1/2*b*x + 2*a)
^2*tan(1/2*a)^9 + 1346794722*tan(1/2*b*x + 2*a)*tan(1/2*a)^10 - 507068874*t
an(1/2*a)^11 + 174*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^2 + 13014*tan(1/2*b*x +
2*a)^6*tan(1/2*a)^3 - 453762*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^4 + 5680314*ta
n(1/2*b*x + 2*a)^4*tan(1/2*a)^5 - 33577926*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^
6 + 97266546*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^7 - 130749372*tan(1/2*b*x + 2*
a)*tan(1/2*a)^8 + 64179762*tan(1/2*a)^9 - 13*tan(1/2*b*x + 2*a)^7 + 162*tan
(1/2*b*x + 2*a)^6*tan(1/2*a) + 2514*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^2 - 872
34*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^3 + 906750*tan(1/2*b*x + 2*a)^3*tan(1/2*
a)^4 - 4014234*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 + 7629702*tan(1/2*b*x + 2*
a)*tan(1/2*a)^6 - 5045814*tan(1/2*a)^7 + 5*tan(1/2*b*x + 2*a)^5 + 234*tan(1
/2*b*x + 2*a)^4*tan(1/2*a) - 8718*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^2 + 80834

```
*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 - 252270*tan(1/2*b*x + 2*a)*tan(1/2*a)^4
 + 243198*tan(1/2*a)^5 + 5*tan(1/2*b*x + 2*a)^3 - 474*tan(1/2*b*x + 2*a)^2*
tan(1/2*a) + 3918*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - 6614*tan(1/2*a)^3 - 13*
tan(1/2*b*x + 2*a) + 78*tan(1/2*a))/((tan(1/2*a)^24 - 60*tan(1/2*a)^22 + 14
10*tan(1/2*a)^20 - 16204*tan(1/2*a)^18 + 92655*tan(1/2*a)^16 - 245880*tan(1
/2*a)^14 + 336156*tan(1/2*a)^12 - 245880*tan(1/2*a)^10 + 92655*tan(1/2*a)^8
 - 16204*tan(1/2*a)^6 + 1410*tan(1/2*a)^4 - 60*tan(1/2*a)^2 + 1)*(tan(1/2*b
*x + 2*a)^2*tan(1/2*a)^6 - 15*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^4 + 12*tan(1/
2*b*x + 2*a)*tan(1/2*a)^5 - tan(1/2*a)^6 + 15*tan(1/2*b*x + 2*a)^2*tan(1/2*
a)^2 - 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 - tan(1/2*b*x +
 2*a)^2 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)^4) - 105*
log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2
 - tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 - tan(1/
2*b*x + 2*a) + 3*tan(1/2*a) - 1)) + 105*log(abs(tan(1/2*b*x + 2*a)*tan(1/2*
a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 + tan(1/2*a)^3 - 3*tan(1/2*b*x + 2
*a)*tan(1/2*a) + 3*tan(1/2*a)^2 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a) - 1)))/
b
```

3.13 $\int \sin^2(a+bx)\sin^5(2a+2bx)\,dx$

Optimal. Leaf size=44

$$\frac{8\sin^{12}(a+bx)}{3b}-\frac{32\sin^{10}(a+bx)}{5b}+\frac{4\sin^8(a+bx)}{b}$$

```
[Out] (4*Sin[a + b*x]^8)/b - (32*Sin[a + b*x]^10)/(5*b) + (8*Sin[a + b*x]^12)/(3*
b)
```

Rubi [A] time = 0.0687566, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4288, 2564, 266, 43}

$$\frac{8\sin^{12}(a+bx)}{3b}-\frac{32\sin^{10}(a+bx)}{5b}+\frac{4\sin^8(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (4*Sin[a + b*x]^8)/b - (32*Sin[a + b*x]^10)/(5*b) + (8*Sin[a + b*x]^12)/(3*
b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sin^2(a+bx)\sin^5(2a+2bx)\,dx &= 32\int \cos^5(a+bx)\sin^7(a+bx)\,dx \\
&= \frac{32\,\text{Subst}\left(\int x^7\left(1-x^2\right)^2\,dx, x, \sin(a+bx)\right)}{b} \\
&= \frac{16\,\text{Subst}\left(\int (1-x)^2x^3\,dx, x, \sin^2(a+bx)\right)}{b} \\
&= \frac{16\,\text{Subst}\left(\int \left(x^3-2x^4+x^5\right)\,dx, x, \sin^2(a+bx)\right)}{b} \\
&= \frac{4\sin^8(a+bx)}{b}-\frac{32\sin^{10}(a+bx)}{5b}+\frac{8\sin^{12}(a+bx)}{3b}
\end{aligned}
$$

Mathematica [A] time = 0.34191, size = 68, normalized size = 1.55

$$
\frac{-600\cos(2(a+bx))+75\cos(4(a+bx))+100\cos(6(a+bx))-30\cos(8(a+bx))-12\cos(10(a+bx))+5\cos(12(a+bx))}{3840b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (-600*Cos[2*(a + b*x)] + 75*Cos[4*(a + b*x)] + 100*Cos[6*(a + b*x)] - 30*Co
s[8*(a + b*x)] - 12*Cos[10*(a + b*x)] + 5*Cos[12*(a + b*x)])/(3840*b)
```

Maple [B] time = 0.022, size = 86, normalized size = 2.

$$
-\frac{5\cos(2bx+2a)}{32b}+\frac{5\cos(4bx+4a)}{256b}+\frac{5\cos(6bx+6a)}{192b}-\frac{\cos(8bx+8a)}{128b}-\frac{\cos(10bx+10a)}{320b}+\frac{\cos(12bx+12a)}{768b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a)^5,x)
```

```
[Out] -5/32*cos(2*b*x+2*a)/b+5/256*cos(4*b*x+4*a)/b+5/192*cos(6*b*x+6*a)/b-1/128*
cos(8*b*x+8*a)/b-1/320*cos(10*b*x+10*a)/b+1/768*cos(12*b*x+12*a)/b
```

Maxima [A] time = 1.19228, size = 97, normalized size = 2.2

$$
\frac{5\cos(12bx+12a)-12\cos(10bx+10a)-30\cos(8bx+8a)+100\cos(6bx+6a)+75\cos(4bx+4a)-600\cos(2bx+2a)}{3840b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] 1/3840*(5*cos(12*b*x + 12*a) - 12*cos(10*b*x + 10*a) - 30*cos(8*b*x + 8*a)
+ 100*cos(6*b*x + 6*a) + 75*cos(4*b*x + 4*a) - 600*cos(2*b*x + 2*a))/b
```

Fricas [A] time = 0.504133, size = 122, normalized size = 2.77

$$\frac{4\left(10\cos\left(bx+a\right)^{12}-36\cos\left(bx+a\right)^{10}+45\cos\left(bx+a\right)^{8}-20\cos\left(bx+a\right)^{6}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] 4/15*(10*cos(b*x + a)^12 - 36*cos(b*x + a)^10 + 45*cos(b*x + a)^8 - 20*cos(
b*x + a)^6)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.44942, size = 115, normalized size = 2.61

$$\frac{\cos\left(12\,bx+12\,a\right)}{768\,b}-\frac{\cos\left(10\,bx+10\,a\right)}{320\,b}-\frac{\cos\left(8\,bx+8\,a\right)}{128\,b}+\frac{5\cos\left(6\,bx+6\,a\right)}{192\,b}+\frac{5\cos\left(4\,bx+4\,a\right)}{256\,b}-\frac{5\cos\left(2\,bx+2\,a\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] 1/768*cos(12*b*x + 12*a)/b - 1/320*cos(10*b*x + 10*a)/b - 1/128*cos(8*b*x +
 8*a)/b + 5/192*cos(6*b*x + 6*a)/b + 5/256*cos(4*b*x + 4*a)/b - 5/32*cos(2*
b*x + 2*a)/b
```

3.14 $\int \sin^2(a+bx)\sin^4(2a+2bx)\,dx$

Optimal. Leaf size=76

$$-\frac{\sin^5(2a+2bx)}{20b}-\frac{\sin^3(2a+2bx)\cos(2a+2bx)}{16b}-\frac{3\sin(2a+2bx)\cos(2a+2bx)}{32b}+\frac{3x}{16}$$

```
[Out] (3*x)/16 - (3*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(32*b) - (Cos[2*a + 2*b*x]
*Sin[2*a + 2*b*x]^3)/(16*b) - Sin[2*a + 2*b*x]^5/(20*b)
```

Rubi [A] time = 0.0676803, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4286, 2635, 8, 2564, 30}

$$-\frac{\sin^5(2a+2bx)}{20b}-\frac{\sin^3(2a+2bx)\cos(2a+2bx)}{16b}-\frac{3\sin(2a+2bx)\cos(2a+2bx)}{32b}+\frac{3x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (3*x)/16 - (3*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(32*b) - (Cos[2*a + 2*b*x]
*Sin[2*a + 2*b*x]^3)/(16*b) - Sin[2*a + 2*b*x]^5/(20*b)
```

Rule 4286

```
Int[sin[(a_.) + (b_.)*(x_)]^2*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbo
l] :> Dist[1/2, Int[(g*Sin[c + d*x])^p, x], x] - Dist[1/2, Int[Cos[c + d*x]
*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d,
0] && EqQ[d/b, 2] && IGtQ[p/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \sin^2(a+bx)\sin^4(2a+2bx)\,dx &= \frac{1}{2}\int \sin^4(2a+2bx)\,dx - \frac{1}{2}\int \cos(2a+2bx)\sin^4(2a+2bx)\,dx \\
&= -\frac{\cos(2a+2bx)\sin^3(2a+2bx)}{16b} + \frac{3}{8}\int \sin^2(2a+2bx)\,dx - \frac{\mathrm{Subst}\left(\int x^4\,dx, x, \sin\right.}{4b} \\
&= -\frac{3\cos(2a+2bx)\sin(2a+2bx)}{32b} - \frac{\cos(2a+2bx)\sin^3(2a+2bx)}{16b} - \frac{\sin^5(2a+2bx)}{20b} \\
&= \frac{3x}{16} - \frac{3\cos(2a+2bx)\sin(2a+2bx)}{32b} - \frac{\cos(2a+2bx)\sin^3(2a+2bx)}{16b} - \frac{\sin^5(2a+}{20b}
\end{aligned}
$$

Mathematica [A] time = 0.193295, size = 62, normalized size = 0.82

$$
\frac{-20\sin(2(a+bx)) - 40\sin(4(a+bx)) + 10\sin(6(a+bx)) + 5\sin(8(a+bx)) - 2\sin(10(a+bx)) + 120bx}{640b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (120*b*x - 20*Sin[2*(a + b*x)] - 40*Sin[4*(a + b*x)] + 10*Sin[6*(a + b*x)]
+ 5*Sin[8*(a + b*x)] - 2*Sin[10*(a + b*x)])/(640*b)
```

Maple [A] time = 0.027, size = 75, normalized size = 1.

$$
\frac{3x}{16} - \frac{\sin(2bx+2a)}{32b} - \frac{\sin(4bx+4a)}{16b} + \frac{\sin(6bx+6a)}{64b} + \frac{\sin(8bx+8a)}{128b} - \frac{\sin(10bx+10a)}{320b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a)^4,x)
```

```
[Out] 3/16*x-1/32*sin(2*b*x+2*a)/b-1/16/b*sin(4*b*x+4*a)+1/64/b*sin(6*b*x+6*a)+1/
128/b*sin(8*b*x+8*a)-1/320/b*sin(10*b*x+10*a)
```

Maxima [A] time = 1.12836, size = 88, normalized size = 1.16

$$
\frac{120bx - 2\sin(10bx+10a) + 5\sin(8bx+8a) + 10\sin(6bx+6a) - 40\sin(4bx+4a) - 20\sin(2bx+2a)}{640b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/640*(120*b*x - 2*sin(10*b*x + 10*a) + 5*sin(8*b*x + 8*a) + 10*sin(6*b*x +
 6*a) - 40*sin(4*b*x + 4*a) - 20*sin(2*b*x + 2*a))/b
```

Fricas [A] time = 0.511443, size = 180, normalized size = 2.37

$$
\frac{15bx - \left(128\cos(bx+a)^9 - 336\cos(bx+a)^7 + 248\cos(bx+a)^5 - 10\cos(bx+a)^3 - 15\cos(bx+a)\right)\sin(bx+a)}{80b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] 1/80*(15*b*x - (128*cos(b*x + a)^9 - 336*cos(b*x + a)^7 + 248*cos(b*x + a)^
5 - 10*cos(b*x + a)^3 - 15*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35376, size = 100, normalized size = 1.32

$$\frac{3}{16}\,x - \frac{\sin\left(10\,bx + 10\,a\right)}{320\,b} + \frac{\sin\left(8\,bx + 8\,a\right)}{128\,b} + \frac{\sin\left(6\,bx + 6\,a\right)}{64\,b} - \frac{\sin\left(4\,bx + 4\,a\right)}{16\,b} - \frac{\sin\left(2\,bx + 2\,a\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] 3/16*x - 1/320*sin(10*b*x + 10*a)/b + 1/128*sin(8*b*x + 8*a)/b + 1/64*sin(6
*b*x + 6*a)/b - 1/16*sin(4*b*x + 4*a)/b - 1/32*sin(2*b*x + 2*a)/b
```

3.15 $\int \sin^2(a+bx)\sin^3(2a+2bx)\,dx$

Optimal. Leaf size=29

$$\frac{4\sin^6(a+bx)}{3b}-\frac{\sin^8(a+bx)}{b}$$

```
[Out] (4*Sin[a + b*x]^6)/(3*b) - Sin[a + b*x]^8/b
```

Rubi [A] time = 0.0591716, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4288, 2564, 14}

$$\frac{4\sin^6(a+bx)}{3b}-\frac{\sin^8(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]

[Out] (4*Sin[a + b*x]^6)/(3*b) - Sin[a + b*x]^8/b
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+bx)\sin^3(2a+2bx)\,dx &= 8\int \cos^3(a+bx)\sin^5(a+bx)\,dx\\ &= \frac{8\,\text{Subst}\left(\int x^5\left(1-x^2\right)dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{8\,\text{Subst}\left(\int \left(x^5-x^7\right)dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{4\sin^6(a+bx)}{3b}-\frac{\sin^8(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.113724, size = 48, normalized size = 1.66

$$\frac{-72\cos(2(a+bx))+12\cos(4(a+bx))+8\cos(6(a+bx))-3\cos(8(a+bx))}{384b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (-72*Cos[2*(a + b*x)] + 12*Cos[4*(a + b*x)] + 8*Cos[6*(a + b*x)] - 3*Cos[8*
(a + b*x)])/(384*b)
```

Maple [B] time = 0.01, size = 58, normalized size = 2.

$$-\frac{3\cos(2bx+2a)}{16b}+\frac{\cos(4bx+4a)}{32b}+\frac{\cos(6bx+6a)}{48b}-\frac{\cos(8bx+8a)}{128b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a)^3,x)
```

```
[Out] -3/16*cos(2*b*x+2*a)/b+1/32*cos(4*b*x+4*a)/b+1/48*cos(6*b*x+6*a)/b-1/128*co
s(8*b*x+8*a)/b
```

Maxima [A] time = 1.0841, size = 68, normalized size = 2.34

$$-\frac{3\cos(8bx+8a)-8\cos(6bx+6a)-12\cos(4bx+4a)+72\cos(2bx+2a)}{384b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] -1/384*(3*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) - 12*cos(4*b*x + 4*a) + 72*
cos(2*b*x + 2*a))/b
```

Fricas [A] time = 0.483444, size = 88, normalized size = 3.03

$$-\frac{3\cos(bx+a)^8-8\cos(bx+a)^6+6\cos(bx+a)^4}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*cos(b*x + a)^8 - 8*cos(b*x + a)^6 + 6*cos(b*x + a)^4)/b
```

Sympy [A] time = 63.2081, size = 362, normalized size = 12.48

$$\begin{cases}\frac{3x\sin^2(a+bx)\sin^3(2a+2bx)}{16}+\frac{3x\sin^2(a+bx)\sin(2a+2bx)\cos^2(2a+2bx)}{16}+\frac{3x\sin(a+bx)\sin^2(2a+2bx)\cos(a+bx)\cos(2a+2bx)}{8}+\frac{3x\sin(a+bx)\cos(a+b}{8}\\ x\sin^2(a)\sin^3(2a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**3,x)
```

```
[Out] Piecewise((3*x*sin(a + b*x)**2*sin(2*a + 2*b*x)**3/16 + 3*x*sin(a + b*x)**2
*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**2/16 + 3*x*sin(a + b*x)*sin(2*a + 2*b*x
)**2*cos(a + b*x)*cos(2*a + 2*b*x)/8 + 3*x*sin(a + b*x)*cos(a + b*x)*cos(2*
a + 2*b*x)**3/8 - 3*x*sin(2*a + 2*b*x)**3*cos(a + b*x)**2/16 - 3*x*sin(2*a
+ 2*b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/16 + 17*sin(a + b*x)**2*cos(2*
a + 2*b*x)**3/(96*b) - 13*sin(a + b*x)*sin(2*a + 2*b*x)**3*cos(a + b*x)/(16
*b) - 7*sin(a + b*x)*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**2/(8*b
) - sin(2*a + 2*b*x)**2*cos(a + b*x)**2*cos(2*a + 2*b*x)/(2*b) - 49*cos(a +
 b*x)**2*cos(2*a + 2*b*x)**3/(96*b), Ne(b, 0)), (x*sin(a)**2*sin(2*a)**3, T
rue))
```

Giac [B] time = 1.37836, size = 77, normalized size = 2.66

$$-\frac{\cos(8\,bx+8\,a)}{128\,b}+\frac{\cos(6\,bx+6\,a)}{48\,b}+\frac{\cos(4\,bx+4\,a)}{32\,b}-\frac{3\,\cos(2\,bx+2\,a)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] -1/128*cos(8*b*x + 8*a)/b + 1/48*cos(6*b*x + 6*a)/b + 1/32*cos(4*b*x + 4*a)
/b - 3/16*cos(2*b*x + 2*a)/b
```

3.16 $\int \sin^2(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=49

$$-\frac{\sin^3(2a+2bx)}{12b}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{8b}+\frac{x}{4}$$

```
[Out] x/4 - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(8*b) - Sin[2*a + 2*b*x]^3/(12*b)
```

Rubi [A] time = 0.0560334, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4286, 2635, 8, 2564, 30}

$$-\frac{\sin^3(2a+2bx)}{12b}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{8b}+\frac{x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] x/4 - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(8*b) - Sin[2*a + 2*b*x]^3/(12*b)
```

Rule 4286

```
Int[sin[(a_.) + (b_.)*(x_)]^2*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbo
l] :> Dist[1/2, Int[(g*Sin[c + d*x])^p, x], x] - Dist[1/2, Int[Cos[c + d*x]
*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d,
0] && EqQ[d/b, 2] && IGtQ[p/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sin^2(a+bx)\sin^2(2a+2bx)\,dx = \frac{1}{2}\int \sin^2(2a+2bx)\,dx - \frac{1}{2}\int \cos(2a+2bx)\sin^2(2a+2bx)\,dx$$
$$= -\frac{\cos(2a+2bx)\sin(2a+2bx)}{8b} + \frac{\int 1\,dx}{4} - \frac{\text{Subst}\left(\int x^2\,dx, x, \sin(2a+2bx)\right)}{4b}$$
$$= \frac{x}{4} - \frac{\cos(2a+2bx)\sin(2a+2bx)}{8b} - \frac{\sin^3(2a+2bx)}{12b}$$

Mathematica [A] time = 0.0718913, size = 40, normalized size = 0.82

$$\frac{-3\sin(2(a+bx)) - 3\sin(4(a+bx)) + \sin(6(a+bx)) + 12bx}{48b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (12*b*x - 3*Sin[2*(a + b*x)] - 3*Sin[4*(a + b*x)] + Sin[6*(a + b*x)])/(48*b
)
```

Maple [A] time = 0.009, size = 47, normalized size = 1.

$$\frac{x}{4} - \frac{\sin(2bx+2a)}{16b} - \frac{\sin(4bx+4a)}{16b} + \frac{\sin(6bx+6a)}{48b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a)^2,x)
```

```
[Out] 1/4*x-1/16*sin(2*b*x+2*a)/b-1/16/b*sin(4*b*x+4*a)+1/48/b*sin(6*b*x+6*a)
```

Maxima [A] time = 1.11582, size = 55, normalized size = 1.12

$$\frac{12\,bx + \sin(6\,bx+6\,a) - 3\,\sin(4\,bx+4\,a) - 3\,\sin(2\,bx+2\,a)}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(12*b*x + sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))/
b
```

Fricas [A] time = 0.488772, size = 117, normalized size = 2.39

$$\frac{3\,bx + \left(8\,\cos(bx+a)^5 - 14\,\cos(bx+a)^3 + 3\,\cos(bx+a)\right)\sin(bx+a)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*x + (8*cos(b*x + a)^5 - 14*cos(b*x + a)^3 + 3*cos(b*x + a))*sin(b
*x + a))/b
```

Sympy [A] time = 17.4472, size = 231, normalized size = 4.71

$$\begin{cases} \frac{x\sin^2(a+bx)\sin^2(2a+2bx)}{4} + \frac{x\sin^2(a+bx)\cos^2(2a+2bx)}{4} + \frac{x\sin^2(2a+2bx)\cos^2(a+bx)}{4} + \frac{x\cos^2(a+bx)\cos^2(2a+2bx)}{4} - \frac{7\sin^2(a+bx)\sin(2a+2bx)\cos}{24b} \\ x\sin^2(a)\sin^2(2a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**2,x)
```

```
[Out] Piecewise((x*sin(a + b*x)**2*sin(2*a + 2*b*x)**2/4 + x*sin(a + b*x)**2*cos(
2*a + 2*b*x)**2/4 + x*sin(2*a + 2*b*x)**2*cos(a + b*x)**2/4 + x*cos(a + b*x
)**2*cos(2*a + 2*b*x)**2/4 - 7*sin(a + b*x)**2*sin(2*a + 2*b*x)*cos(2*a + 2
*b*x)/(24*b) - sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(a + b*x)/(6*b) - sin(a
+ b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**2/(3*b) + sin(2*a + 2*b*x)*cos(a + b*
x)**2*cos(2*a + 2*b*x)/(24*b), Ne(b, 0)), (x*sin(a)**2*sin(2*a)**2, True))
```

Giac [A] time = 1.34635, size = 62, normalized size = 1.27

$$\frac{1}{4}x + \frac{\sin(6bx + 6a)}{48b} - \frac{\sin(4bx + 4a)}{16b} - \frac{\sin(2bx + 2a)}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] 1/4*x + 1/48*sin(6*b*x + 6*a)/b - 1/16*sin(4*b*x + 4*a)/b - 1/16*sin(2*b*x
+ 2*a)/b
```

3.17 $\int \sin^2(a+bx)\sin(2a+2bx)\,dx$

Optimal. Leaf size=15

$$\frac{\sin^4(a+bx)}{2b}$$

```
[Out] Sin[a + b*x]^4/(2*b)
```

Rubi [A] time = 0.0332715, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2564, 30}

$$\frac{\sin^4(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x],x]

[Out] Sin[a + b*x]^4/(2*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+bx)\sin(2a+2bx)\,dx &= 2\int \cos(a+bx)\sin^3(a+bx)\,dx\\ &= \frac{2\,\text{Subst}\left(\int x^3\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\sin^4(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0045543, size = 15, normalized size = 1.

$$\frac{\sin^4(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x],x]
```

```
[Out] Sin[a + b*x]^4/(2*b)
```

Maple [B] time = 0.009, size = 30, normalized size = 2.

$$-\frac{\cos(2bx+2a)}{4b}+\frac{\cos(4bx+4a)}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a),x)
```

```
[Out] -1/4*cos(2*b*x+2*a)/b+1/16*cos(4*b*x+4*a)/b
```

Maxima [A] time = 1.19698, size = 35, normalized size = 2.33

$$\frac{\cos(4bx+4a)-4\cos(2bx+2a)}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] 1/16*(cos(4*b*x + 4*a) - 4*cos(2*b*x + 2*a))/b
```

Fricas [A] time = 0.474385, size = 58, normalized size = 3.87

$$\frac{\cos(bx+a)^4-2\cos(bx+a)^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 1/2*(cos(b*x + a)^4 - 2*cos(b*x + a)^2)/b
```

Sympy [A] time = 4.49128, size = 133, normalized size = 8.87

$$\begin{cases}\frac{x\sin^2(a+bx)\sin(2a+2bx)}{4}+\frac{x\sin(a+bx)\cos(a+bx)\cos(2a+2bx)}{2}-\frac{x\sin(2a+2bx)\cos^2(a+bx)}{4}-\frac{3\sin(a+bx)\sin(2a+2bx)\cos(a+bx)}{4b}-\frac{\cos^2(a+bx)\cos}{2b}\\ x\sin^2(a)\sin(2a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a),x)
```

```
[Out] Piecewise((x*sin(a + b*x)**2*sin(2*a + 2*b*x)/4 + x*sin(a + b*x)*cos(a + b*
x)*cos(2*a + 2*b*x)/2 - x*sin(2*a + 2*b*x)*cos(a + b*x)**2/4 - 3*sin(a + b*
x)*sin(2*a + 2*b*x)*cos(a + b*x)/(4*b) - cos(a + b*x)**2*cos(2*a + 2*b*x)/(
2*b), Ne(b, 0)), (x*sin(a)**2*sin(2*a), True))
```

Giac [B] time = 1.40023, size = 39, normalized size = 2.6

$$\frac{\cos(4\,bx + 4\,a)}{16\,b} - \frac{\cos(2\,bx + 2\,a)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] 1/16*cos(4*b*x + 4*a)/b - 1/4*cos(2*b*x + 2*a)/b
```

3.18 $\int \csc(2a+2bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=14

$$-\frac{\log(\cos(a+bx))}{2b}$$

```
[Out] -Log[Cos[a + b*x]]/(2*b)
```

Rubi [A] time = 0.026394, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4288, 3475}

$$-\frac{\log(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]*Sin[a + b*x]^2,x]

[Out] -Log[Cos[a + b*x]]/(2*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc(2a+2bx)\sin^2(a+bx)\,dx &= \frac{1}{2}\int \tan(a+bx)\,dx\\ &= -\frac{\log(\cos(a+bx))}{2b}\end{aligned}$$

Mathematica [A] time = 0.011125, size = 14, normalized size = 1.

$$-\frac{\log(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]*Sin[a + b*x]^2,x]

[Out] -Log[Cos[a + b*x]]/(2*b)
```

Maple [A] time = 0.022, size = 13, normalized size = 0.9

$$-\frac{\ln\left(\cos\left(bx+a\right)\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)*sin(b*x+a)^2,x)
```

```
[Out] -1/2*ln(cos(b*x+a))/b
```

Maxima [B] time = 1.12982, size = 74, normalized size = 5.29

$$-\frac{\log\left(\cos\left(2\,bx\right)^{2}+2\,\cos\left(2\,bx\right)\cos\left(2\,a\right)+\cos\left(2\,a\right)^{2}+\sin\left(2\,bx\right)^{2}-2\,\sin\left(2\,bx\right)\sin\left(2\,a\right)+\sin\left(2\,a\right)^{2}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 -
 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2)/b
```

Fricas [A] time = 0.497735, size = 36, normalized size = 2.57

$$-\frac{\log\left(-\cos\left(bx+a\right)\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*log(-cos(b*x + a))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.50726, size = 128, normalized size = 9.14

$$\frac{\log\left(\tan\left(bx+4\,a\right)^{2}+1\right)-2\,\log\left(\left|6\,\tan\left(bx+4\,a\right)\tan\left(\frac{1}{2}\,a\right)^{5}-\tan\left(\frac{1}{2}\,a\right)^{6}-20\,\tan\left(bx+4\,a\right)\tan\left(\frac{1}{2}\,a\right)^{3}+15\,\tan\left(\frac{1}{2}\right.\right.\right.}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(log(tan(b*x + 4*a)^2 + 1) - 2*log(abs(6*tan(b*x + 4*a)*tan(1/2*a)^5 -
tan(1/2*a)^6 - 20*tan(b*x + 4*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 + 6*tan(b*x
 + 4*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)))/b
```

3.19 $\int \csc^2(2a+2bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=13

$$\frac{\tan(a+bx)}{4b}$$

```
[Out] Tan[a + b*x]/(4*b)
```

Rubi [A] time = 0.034776, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4288, 3767, 8}

$$\frac{\tan(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] Tan[a + b*x]/(4*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \csc^2(2a+2bx)\sin^2(a+bx)\,dx &= \frac{1}{4}\int \sec^2(a+bx)\,dx\\ &= -\frac{\operatorname{Subst}(\int 1\,dx, x, -\tan(a+bx))}{4b}\\ &= \frac{\tan(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0075078, size = 13, normalized size = 1.

$$\frac{\tan(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] Tan[a + b*x]/(4*b)
```

Maple [A] time = 0.091, size = 12, normalized size = 0.9

$$\frac{\tan(bx+a)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^2*sin(b*x+a)^2,x)
```

```
[Out] 1/4*tan(b*x+a)/b
```

Maxima [B] time = 1.11319, size = 72, normalized size = 5.54

$$\frac{\sin(2bx+2a)}{2\left(b\cos(2bx+2a)^2+b\sin(2bx+2a)^2+2b\cos(2bx+2a)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*sin(2*b*x + 2*a)/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 + 2*b*cos
(2*b*x + 2*a) + b)
```

Fricas [A] time = 0.453275, size = 47, normalized size = 3.62

$$\frac{\sin(bx+a)}{4b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*sin(b*x + a)/(b*cos(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**2*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

<u>Giac</u> [B] time = 1.47716, size = 205, normalized size = 15.77

$$-\frac{\tan\left(\frac{1}{2}a\right)^{12} + 6\tan\left(\frac{1}{2}a\right)^{10} + 15\tan\left(\frac{1}{2}a\right)^{8} + 20\tan\left(\frac{1}{2}a\right)^{6} + 15\tan\left(\right.}{8\left(6\tan\left(bx+4a\right)\tan\left(\frac{1}{2}a\right)^{5} - \tan\left(\frac{1}{2}a\right)^{6} - 20\tan\left(bx+4a\right)\tan\left(\frac{1}{2}a\right)^{3} + 15\tan\left(\frac{1}{2}a\right)^{4} + 6\tan\left(bx+4a\right)\tan\left(\frac{1}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/8*(tan(1/2*a)^12 + 6*tan(1/2*a)^10 + 15*tan(1/2*a)^8 + 20*tan(1/2*a)^6 +
 15*tan(1/2*a)^4 + 6*tan(1/2*a)^2 + 1)/((6*tan(b*x + 4*a)*tan(1/2*a)^5 - ta
n(1/2*a)^6 - 20*tan(b*x + 4*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 + 6*tan(b*x +
 4*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)*(3*tan(1/2*a)^5 - 10*tan(1/2*a)^3 +
 3*tan(1/2*a))*b)
```

3.20 $\int \csc^3(2a+2bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=30

$$\frac{\tan^2(a+bx)}{16b}+\frac{\log(\tan(a+bx))}{8b}$$

```
[Out] Log[Tan[a + b*x]]/(8*b) + Tan[a + b*x]^2/(16*b)
```

Rubi [A] time = 0.0478498, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4288, 2620, 14}

$$\frac{\tan^2(a+bx)}{16b}+\frac{\log(\tan(a+bx))}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] Log[Tan[a + b*x]]/(8*b) + Tan[a + b*x]^2/(16*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \csc^3(2a+2bx)\sin^2(a+bx)\,dx &= \frac{1}{8}\int \csc(a+bx)\sec^3(a+bx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{1+x^2}{x}\,dx,x,\tan(a+bx)\right)}{8b}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+x\right)\,dx,x,\tan(a+bx)\right)}{8b}\\
&= \frac{\log(\tan(a+bx))}{8b}+\frac{\tan^2(a+bx)}{16b}
\end{aligned}$$

Mathematica [A] time = 0.0355133, size = 36, normalized size = 1.2

$$-\frac{-\sec^2(a+bx)-2\log(\sin(a+bx))+2\log(\cos(a+bx))}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] -(2*Log[Cos[a + b*x]] - 2*Log[Sin[a + b*x]] - Sec[a + b*x]^2)/(16*b)
```

Maple [A] time = 0.036, size = 27, normalized size = 0.9

$$\frac{1}{16\,b\,(\cos(bx+a))^2}+\frac{\ln(\tan(bx+a))}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^3*sin(b*x+a)^2,x)
```

```
[Out] 1/16/b/cos(b*x+a)^2+1/8*ln(tan(b*x+a))/b
```

Maxima [B] time = 1.12181, size = 865, normalized size = 28.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + 8*cos(2*b*x + 2*a)^2 - (2*(2*co
s(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2
*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*
b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(
2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2) + (2
*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b
*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*
sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(b*x)^2 + 2*cos(b*x)*co
s(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + (2*(2*cos(2*
b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^
2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x
+ 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos
(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*sin(4*b*x + 4*a)*sin
(2*b*x + 2*a) + 8*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a))/(b*cos(4*b*x + 4
*a)^2 + 4*b*cos(2*b*x + 2*a)^2 + b*sin(4*b*x + 4*a)^2 + 4*b*sin(4*b*x + 4*a
)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 + 2*(2*b*cos(2*b*x + 2*a) + b)*
cos(4*b*x + 4*a) + 4*b*cos(2*b*x + 2*a) + b)
```

Fricas [B] time = 0.498528, size = 155, normalized size = 5.17

$$-\frac{\cos(bx+a)^2\log\left(\cos(bx+a)^2\right)-\cos(bx+a)^2\log\left(-\frac{1}{4}\,\cos(bx+a)^2+\frac{1}{4}\right)-1}{16\,b\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(cos(b*x + a)^2*log(cos(b*x + a)^2) - cos(b*x + a)^2*log(-1/4*cos(b*x
 + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**3*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.63354, size = 991, normalized size = 33.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/64*((12*tan(b*x + 4*a)*tan(1/2*a)^23 - tan(1/2*a)^24 + 3888*tan(b*x + 4*a
)^2*tan(1/2*a)^20 - 1444*tan(b*x + 4*a)*tan(1/2*a)^21 + 168*tan(1/2*a)^22 -
 51840*tan(b*x + 4*a)^2*tan(1/2*a)^18 + 32052*tan(b*x + 4*a)*tan(1/2*a)^19
- 4074*tan(1/2*a)^20 + 274752*tan(b*x + 4*a)^2*tan(1/2*a)^16 - 260028*tan(b
*x + 4*a)*tan(1/2*a)^17 + 49480*tan(1/2*a)^18 - 731520*tan(b*x + 4*a)^2*tan
(1/2*a)^14 + 979064*tan(b*x + 4*a)*tan(1/2*a)^15 - 276687*tan(1/2*a)^16 + 1
021728*tan(b*x + 4*a)^2*tan(1/2*a)^12 - 1873128*tan(b*x + 4*a)*tan(1/2*a)^1
3 + 737808*tan(1/2*a)^14 - 731520*tan(b*x + 4*a)^2*tan(1/2*a)^10 + 1873128*
tan(b*x + 4*a)*tan(1/2*a)^11 - 1009292*tan(1/2*a)^12 + 274752*tan(b*x + 4*a
)^2*tan(1/2*a)^8 - 979064*tan(b*x + 4*a)*tan(1/2*a)^9 + 737808*tan(1/2*a)^1
0 - 51840*tan(b*x + 4*a)^2*tan(1/2*a)^6 + 260028*tan(b*x + 4*a)*tan(1/2*a)^
7 - 276687*tan(1/2*a)^8 + 3888*tan(b*x + 4*a)^2*tan(1/2*a)^4 - 32052*tan(b*
x + 4*a)*tan(1/2*a)^5 + 49480*tan(1/2*a)^6 + 1444*tan(b*x + 4*a)*tan(1/2*a)
^3 - 4074*tan(1/2*a)^4 - 12*tan(b*x + 4*a)*tan(1/2*a) + 168*tan(1/2*a)^2 -
1)/((9*tan(1/2*a)^10 - 60*tan(1/2*a)^8 + 118*tan(1/2*a)^6 - 60*tan(1/2*a)^4
 + 9*tan(1/2*a)^2)*(6*tan(b*x + 4*a)*tan(1/2*a)^5 - tan(1/2*a)^6 - 20*tan(b
*x + 4*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 + 6*tan(b*x + 4*a)*tan(1/2*a) - 15
*tan(1/2*a)^2 + 1)^2) + 8*log(abs(tan(b*x + 4*a)*tan(1/2*a)^6 - 15*tan(b*x
+ 4*a)*tan(1/2*a)^4 + 6*tan(1/2*a)^5 + 15*tan(b*x + 4*a)*tan(1/2*a)^2 - 20*
tan(1/2*a)^3 - tan(b*x + 4*a) + 6*tan(1/2*a))) - 8*log(abs(6*tan(b*x + 4*a)
*tan(1/2*a)^5 - tan(1/2*a)^6 - 20*tan(b*x + 4*a)*tan(1/2*a)^3 + 15*tan(1/2*
a)^4 + 6*tan(b*x + 4*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)))/b
```

3.21 $\int \csc^4(2a+2bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=42

$$\frac{\tan^3(a+bx)}{48b}+\frac{\tan(a+bx)}{8b}-\frac{\cot(a+bx)}{16b}$$

```
[Out] -Cot[a + b*x]/(16*b) + Tan[a + b*x]/(8*b) + Tan[a + b*x]^3/(48*b)
```

Rubi [A] time = 0.062722, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4288, 2620, 270}

$$\frac{\tan^3(a+bx)}{48b}+\frac{\tan(a+bx)}{8b}-\frac{\cot(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^4*Sin[a + b*x]^2,x]
```

```
[Out] -Cot[a + b*x]/(16*b) + Tan[a + b*x]/(8*b) + Tan[a + b*x]^3/(48*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \csc^4(2a+2bx)\sin^2(a+bx)\,dx &= \frac{1}{16}\int \csc^2(a+bx)\sec^4(a+bx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^2}\,dx, x, \tan(a+bx)\right)}{16b}\\
&= \frac{\text{Subst}\left(\int \left(2+\frac{1}{x^2}+x^2\right)dx, x, \tan(a+bx)\right)}{16b}\\
&= -\frac{\cot(a+bx)}{16b}+\frac{\tan(a+bx)}{8b}+\frac{\tan^3(a+bx)}{48b}
\end{aligned}$$

Mathematica [A] time = 0.0538419, size = 48, normalized size = 1.14

$$\frac{5\tan(a+bx)}{48b}-\frac{\cot(a+bx)}{16b}+\frac{\tan(a+bx)\sec^2(a+bx)}{48b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^4*Sin[a + b*x]^2,x]
```

```
[Out] -Cot[a + b*x]/(16*b) + (5*Tan[a + b*x])/(48*b) + (Sec[a + b*x]^2*Tan[a + b*
x])/(48*b)
```

Maple [A] time = 0.057, size = 51, normalized size = 1.2

$$\frac{1}{16\,b}\left(\frac{1}{3\,\sin(bx+a)\,(\cos(bx+a))^3}+\frac{4}{3\,\sin(bx+a)\cos(bx+a)}-\frac{8\,\cot(bx+a)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^4*sin(b*x+a)^2,x)
```

```
[Out] 1/16/b*(1/3/sin(b*x+a)/cos(b*x+a)^3+4/3/sin(b*x+a)/cos(b*x+a)-8/3*cot(b*x+a
))
```

Maxima [B] time = 1.07467, size = 416, normalized size = 9.9

$$-\frac{(2\cos(2\,bx+2}{3\left(b\cos(8\,bx+8\,a)^2+4\,b\cos(6\,bx+6\,a)^2+4\,b\cos(2\,bx+2\,a)^2+b\sin(8\,bx+8\,a)^2+4\,b\sin(6\,bx+6\,a)^2-8\,b\sin(6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*((2*cos(2*b*x + 2*a) + 1)*sin(8*b*x + 8*a) + 2*(2*cos(2*b*x + 2*a) + 1
)*sin(6*b*x + 6*a) - 2*cos(8*b*x + 8*a)*sin(2*b*x + 2*a) - 4*cos(6*b*x + 6*
a)*sin(2*b*x + 2*a))/(b*cos(8*b*x + 8*a)^2 + 4*b*cos(6*b*x + 6*a)^2 + 4*b*c
os(2*b*x + 2*a)^2 + b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 - 8*b*sin
(6*b*x + 6*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 + 2*(2*b*cos(6*b*x
+ 6*a) - 2*b*cos(2*b*x + 2*a) - b)*cos(8*b*x + 8*a) - 4*(2*b*cos(2*b*x + 2*
a) + b)*cos(6*b*x + 6*a) + 4*b*cos(2*b*x + 2*a) + 4*(b*sin(6*b*x + 6*a) - b
*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + b)
```

Fricas [A] time = 0.469919, size = 109, normalized size = 2.6

$$-\frac{8\,\cos(bx+a)^4-4\,\cos(bx+a)^2-1}{48\,b\cos(bx+a)^3\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/48*(8*cos(b*x + a)^4 - 4*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^3*sin(b*x +
 a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**4*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.69862, size = 1396, normalized size = 33.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/384*(24*(tan(1/2*a)^12 + 6*tan(1/2*a)^10 + 15*tan(1/2*a)^8 + 20*tan(1/2*
a)^6 + 15*tan(1/2*a)^4 + 6*tan(1/2*a)^2 + 1)/((tan(b*x + 4*a)*tan(1/2*a)^6
- 15*tan(b*x + 4*a)*tan(1/2*a)^4 + 6*tan(1/2*a)^5 + 15*tan(b*x + 4*a)*tan(1
/2*a)^2 - 20*tan(1/2*a)^3 - tan(b*x + 4*a) + 6*tan(1/2*a))*(tan(1/2*a)^6 -
15*tan(1/2*a)^4 + 15*tan(1/2*a)^2 - 1)) + (108*tan(b*x + 4*a)^2*tan(1/2*a)^
34 - 18*tan(b*x + 4*a)*tan(1/2*a)^35 + tan(1/2*a)^36 + 4464*tan(b*x + 4*a)^
2*tan(1/2*a)^32 - 1182*tan(b*x + 4*a)*tan(1/2*a)^33 + 126*tan(1/2*a)^34 - 2
8608*tan(b*x + 4*a)^2*tan(1/2*a)^30 + 26208*tan(b*x + 4*a)*tan(1/2*a)^31 -
3159*tan(1/2*a)^32 + 14544*tan(b*x + 4*a)^2*tan(1/2*a)^28 - 81216*tan(b*x +
 4*a)*tan(1/2*a)^29 + 29232*tan(1/2*a)^30 + 197136*tan(b*x + 4*a)^2*tan(1/2
*a)^26 - 130344*tan(b*x + 4*a)*tan(1/2*a)^27 - 26460*tan(1/2*a)^28 - 230160
*tan(b*x + 4*a)^2*tan(1/2*a)^24 + 657288*tan(b*x + 4*a)*tan(1/2*a)^25 - 228
600*tan(1/2*a)^26 - 753984*tan(b*x + 4*a)^2*tan(1/2*a)^22 + 419328*tan(b*x
+ 4*a)*tan(1/2*a)^23 + 237588*tan(1/2*a)^24 + 604368*tan(b*x + 4*a)^2*tan(1
/2*a)^20 - 2188128*tan(b*x + 4*a)*tan(1/2*a)^21 + 944208*tan(1/2*a)^22 + 19
57128*tan(b*x + 4*a)^2*tan(1/2*a)^18 - 1928412*tan(b*x + 4*a)*tan(1/2*a)^19
 - 142434*tan(1/2*a)^20 + 604368*tan(b*x + 4*a)^2*tan(1/2*a)^16 + 1928412*t
an(b*x + 4*a)*tan(1/2*a)^17 - 1358860*tan(1/2*a)^18 - 753984*tan(b*x + 4*a)
^2*tan(1/2*a)^14 + 2188128*tan(b*x + 4*a)*tan(1/2*a)^15 - 142434*tan(1/2*a)
^16 - 230160*tan(b*x + 4*a)^2*tan(1/2*a)^12 - 419328*tan(b*x + 4*a)*tan(1/2
*a)^13 + 944208*tan(1/2*a)^14 + 197136*tan(b*x + 4*a)^2*tan(1/2*a)^10 - 657
288*tan(b*x + 4*a)*tan(1/2*a)^11 + 237588*tan(1/2*a)^12 + 14544*tan(b*x + 4
*a)^2*tan(1/2*a)^8 + 130344*tan(b*x + 4*a)*tan(1/2*a)^9 - 228600*tan(1/2*a)
^10 - 28608*tan(b*x + 4*a)^2*tan(1/2*a)^6 + 81216*tan(b*x + 4*a)*tan(1/2*a)
^7 - 26460*tan(1/2*a)^8 + 4464*tan(b*x + 4*a)^2*tan(1/2*a)^4 - 26208*tan(b*
x + 4*a)*tan(1/2*a)^5 + 29232*tan(1/2*a)^6 + 108*tan(b*x + 4*a)^2*tan(1/2*a
)^2 + 1182*tan(b*x + 4*a)*tan(1/2*a)^3 - 3159*tan(1/2*a)^4 + 18*tan(b*x + 4
*a)*tan(1/2*a) + 126*tan(1/2*a)^2 + 1)/((27*tan(1/2*a)^15 - 270*tan(1/2*a)^
13 + 981*tan(1/2*a)^11 - 1540*tan(1/2*a)^9 + 981*tan(1/2*a)^7 - 270*tan(1/2
*a)^5 + 27*tan(1/2*a)^3)*(6*tan(b*x + 4*a)*tan(1/2*a)^5 - tan(1/2*a)^6 - 20
*tan(b*x + 4*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 + 6*tan(b*x + 4*a)*tan(1/2*a
) - 15*tan(1/2*a)^2 + 1)^3))/b
```

3.22 $\int \csc^5(2a+2bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=60

$$\frac{\tan^4(a+bx)}{128b}+\frac{3\tan^2(a+bx)}{64b}-\frac{\cot^2(a+bx)}{64b}+\frac{3\log(\tan(a+bx))}{32b}$$

```
[Out] -Cot[a + b*x]^2/(64*b) + (3*Log[Tan[a + b*x]])/(32*b) + (3*Tan[a + b*x]^2)/
(64*b) + Tan[a + b*x]^4/(128*b)
```

Rubi [A] time = 0.0713194, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4288, 2620, 266, 43}

$$\frac{\tan^4(a+bx)}{128b}+\frac{3\tan^2(a+bx)}{64b}-\frac{\cot^2(a+bx)}{64b}+\frac{3\log(\tan(a+bx))}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^5*Sin[a + b*x]^2,x]
```

```
[Out] -Cot[a + b*x]^2/(64*b) + (3*Log[Tan[a + b*x]])/(32*b) + (3*Tan[a + b*x]^2)/
(64*b) + Tan[a + b*x]^4/(128*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \csc^5(2a+2bx)\sin^2(a+bx)\,dx = \frac{1}{32}\int \csc^3(a+bx)\sec^5(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^3}\,dx, x, \tan(a+bx)\right)}{32b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{x^2}\,dx, x, \tan^2(a+bx)\right)}{64b}$$
$$= \frac{\text{Subst}\left(\int \left(3+\frac{1}{x^2}+\frac{3}{x}+x\right)dx, x, \tan^2(a+bx)\right)}{64b}$$
$$= -\frac{\cot^2(a+bx)}{64b} + \frac{3\log(\tan(a+bx))}{32b} + \frac{3\tan^2(a+bx)}{64b} + \frac{\tan^4(a+bx)}{128b}$$

Mathematica [A] time = 0.241152, size = 56, normalized size = 0.93

$$-\frac{2\csc^2(a+bx) - \sec^4(a+bx) - 4\sec^2(a+bx) - 12\log(\sin(a+bx)) + 12\log(\cos(a+bx))}{128b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^5*Sin[a + b*x]^2,x]
```

```
[Out] -(2*Csc[a + b*x]^2 + 12*Log[Cos[a + b*x]] - 12*Log[Sin[a + b*x]] - 4*Sec[a
+ b*x]^2 - Sec[a + b*x]^4)/(128*b)
```

Maple [A] time = 0.036, size = 69, normalized size = 1.2

$$\frac{1}{128\,b\,(\sin(bx+a))^2(\cos(bx+a))^4} + \frac{3}{128\,b\,(\sin(bx+a))^2(\cos(bx+a))^2} - \frac{3}{64\,b\,(\sin(bx+a))^2} + \frac{3\ln(\tan(bx+a))}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^5*sin(b*x+a)^2,x)
```

```
[Out] 1/128/b/sin(b*x+a)^2/cos(b*x+a)^4+3/128/b/sin(b*x+a)^2/cos(b*x+a)^2-3/64/b/
sin(b*x+a)^2+3/32*ln(tan(b*x+a))/b
```

Maxima [B] time = 1.53095, size = 4271, normalized size = 71.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/64*(4*(3*cos(10*b*x + 10*a) + 6*cos(8*b*x + 8*a) - 2*cos(6*b*x + 6*a) + 6
*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a))*cos(12*b*x + 12*a) + 4*(9*cos(8*b*x
 + 8*a) - 16*cos(6*b*x + 6*a) + 9*cos(4*b*x + 4*a) + 12*cos(2*b*x + 2*a) +
3)*cos(10*b*x + 10*a) + 24*cos(10*b*x + 10*a)^2 - 4*(22*cos(6*b*x + 6*a) +
12*cos(4*b*x + 4*a) - 9*cos(2*b*x + 2*a) - 6)*cos(8*b*x + 8*a) - 24*cos(8*b
```

```
*x + 8*a)^2 - 8*(11*cos(4*b*x + 4*a) + 8*cos(2*b*x + 2*a) + 1)*cos(6*b*x +
6*a) + 32*cos(6*b*x + 6*a)^2 + 12*(3*cos(2*b*x + 2*a) + 2)*cos(4*b*x + 4*a)
 - 24*cos(4*b*x + 4*a)^2 + 24*cos(2*b*x + 2*a)^2 - 3*(2*(2*cos(10*b*x + 10*
a) - cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) - cos(4*b*x + 4*a) + 2*cos(2*b*x
 + 2*a) + 1)*cos(12*b*x + 12*a) + cos(12*b*x + 12*a)^2 - 4*(cos(8*b*x + 8*a
) + 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) - 1)*cos(10*
b*x + 10*a) + 4*cos(10*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*a) + cos(4*b*x +
4*a) - 2*cos(2*b*x + 2*a) - 1)*cos(8*b*x + 8*a) + cos(8*b*x + 8*a)^2 + 8*(c
os(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) + 16*cos(6*b*x +
 6*a)^2 - 2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2
+ 4*cos(2*b*x + 2*a)^2 + 2*(2*sin(10*b*x + 10*a) - sin(8*b*x + 8*a) - 4*sin
(6*b*x + 6*a) - sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) +
 sin(12*b*x + 12*a)^2 - 4*(sin(8*b*x + 8*a) + 4*sin(6*b*x + 6*a) + sin(4*b*
x + 4*a) - 2*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + 4*sin(10*b*x + 10*a)^2
+ 2*(4*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(8*b*x
+ 8*a) + sin(8*b*x + 8*a)^2 + 8*(sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin
(6*b*x + 6*a) + 16*sin(6*b*x + 6*a)^2 + sin(4*b*x + 4*a)^2 - 4*sin(4*b*x +
4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(
cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*
b*x)*sin(2*a) + sin(2*a)^2) + 3*(2*(2*cos(10*b*x + 10*a) - cos(8*b*x + 8*a)
 - 4*cos(6*b*x + 6*a) - cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) + 1)*cos(12*b
*x + 12*a) + cos(12*b*x + 12*a)^2 - 4*(cos(8*b*x + 8*a) + 4*cos(6*b*x + 6*a
) + cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) - 1)*cos(10*b*x + 10*a) + 4*cos(1
0*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) - 2*cos(2*b*x +
2*a) - 1)*cos(8*b*x + 8*a) + cos(8*b*x + 8*a)^2 + 8*(cos(4*b*x + 4*a) - 2*c
os(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) + 16*cos(6*b*x + 6*a)^2 - 2*(2*cos(2*
b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^
2 + 2*(2*sin(10*b*x + 10*a) - sin(8*b*x + 8*a) - 4*sin(6*b*x + 6*a) - sin(4
*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) + sin(12*b*x + 12*a)^2
 - 4*(sin(8*b*x + 8*a) + 4*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) - 2*sin(2*b*
x + 2*a))*sin(10*b*x + 10*a) + 4*sin(10*b*x + 10*a)^2 + 2*(4*sin(6*b*x + 6*
a) + sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + sin(8*b*x +
8*a)^2 + 8*(sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + 16*si
n(6*b*x + 6*a)^2 + sin(4*b*x + 4*a)^2 - 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a)
 + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(b*x)^2 + 2*cos(b*
x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + 3*(2*(2
*cos(10*b*x + 10*a) - cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) - cos(4*b*x + 4
*a) + 2*cos(2*b*x + 2*a) + 1)*cos(12*b*x + 12*a) + cos(12*b*x + 12*a)^2 - 4
*(cos(8*b*x + 8*a) + 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) - 2*cos(2*b*x +
2*a) - 1)*cos(10*b*x + 10*a) + 4*cos(10*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*
a) + cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) - 1)*cos(8*b*x + 8*a) + cos(8*b*
x + 8*a)^2 + 8*(cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a)
 + 16*cos(6*b*x + 6*a)^2 - 2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + co
s(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + 2*(2*sin(10*b*x + 10*a) - sin(8*b
*x + 8*a) - 4*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin
(12*b*x + 12*a) + sin(12*b*x + 12*a)^2 - 4*(sin(8*b*x + 8*a) + 4*sin(6*b*x
+ 6*a) + sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + 4*sin(
10*b*x + 10*a)^2 + 2*(4*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) - 2*sin(2*b*x +
 2*a))*sin(8*b*x + 8*a) + sin(8*b*x + 8*a)^2 + 8*(sin(4*b*x + 4*a) - 2*sin(
2*b*x + 2*a))*sin(6*b*x + 6*a) + 16*sin(6*b*x + 6*a)^2 + sin(4*b*x + 4*a)^2
 - 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x
 + 2*a) + 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2
*sin(b*x)*sin(a) + sin(a)^2) + 4*(3*sin(10*b*x + 10*a) + 6*sin(8*b*x + 8*a)
 - 2*sin(6*b*x + 6*a) + 6*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(12*b*x
 + 12*a) + 4*(9*sin(8*b*x + 8*a) - 16*sin(6*b*x + 6*a) + 9*sin(4*b*x + 4*a)
 + 12*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + 24*sin(10*b*x + 10*a)^2 - 4*(2
2*sin(6*b*x + 6*a) + 12*sin(4*b*x + 4*a) - 9*sin(2*b*x + 2*a))*sin(8*b*x +
8*a) - 24*sin(8*b*x + 8*a)^2 - 8*(11*sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))
*sin(6*b*x + 6*a) + 32*sin(6*b*x + 6*a)^2 - 24*sin(4*b*x + 4*a)^2 + 36*sin(
```

```
4*b*x + 4*a)*sin(2*b*x + 2*a) + 24*sin(2*b*x + 2*a)^2 + 12*cos(2*b*x + 2*a)
)/(b*cos(12*b*x + 12*a)^2 + 4*b*cos(10*b*x + 10*a)^2 + b*cos(8*b*x + 8*a)^2
 + 16*b*cos(6*b*x + 6*a)^2 + b*cos(4*b*x + 4*a)^2 + 4*b*cos(2*b*x + 2*a)^2
+ b*sin(12*b*x + 12*a)^2 + 4*b*sin(10*b*x + 10*a)^2 + b*sin(8*b*x + 8*a)^2
+ 16*b*sin(6*b*x + 6*a)^2 + b*sin(4*b*x + 4*a)^2 - 4*b*sin(4*b*x + 4*a)*sin
(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 + 2*(2*b*cos(10*b*x + 10*a) - b*cos(
8*b*x + 8*a) - 4*b*cos(6*b*x + 6*a) - b*cos(4*b*x + 4*a) + 2*b*cos(2*b*x +
2*a) + b)*cos(12*b*x + 12*a) - 4*(b*cos(8*b*x + 8*a) + 4*b*cos(6*b*x + 6*a)
 + b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) - b)*cos(10*b*x + 10*a) + 2*(4
*b*cos(6*b*x + 6*a) + b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) - b)*cos(8*
b*x + 8*a) + 8*(b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) - b)*cos(6*b*x +
6*a) - 2*(2*b*cos(2*b*x + 2*a) + b)*cos(4*b*x + 4*a) + 4*b*cos(2*b*x + 2*a)
 + 2*(2*b*sin(10*b*x + 10*a) - b*sin(8*b*x + 8*a) - 4*b*sin(6*b*x + 6*a) -
b*sin(4*b*x + 4*a) + 2*b*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - 4*(b*sin(8*
b*x + 8*a) + 4*b*sin(6*b*x + 6*a) + b*sin(4*b*x + 4*a) - 2*b*sin(2*b*x + 2*
a))*sin(10*b*x + 10*a) + 2*(4*b*sin(6*b*x + 6*a) + b*sin(4*b*x + 4*a) - 2*b
*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + 8*(b*sin(4*b*x + 4*a) - 2*b*sin(2*b*x
 + 2*a))*sin(6*b*x + 6*a) + b)
```

Fricas [B] time = 0.517188, size = 289, normalized size = 4.82

$$\frac{6 \cos(bx+a)^4 - 3 \cos(bx+a)^2 - 6\left(\cos(bx+a)^6 - \cos(bx+a)^4\right)\log\left(\cos(bx+a)^2\right) + 6\left(\cos(bx+a)^6 - \cos(bx+\right.}{128\left(b\cos(bx+a)^6 - b\cos(bx+a)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/128*(6*cos(b*x + a)^4 - 3*cos(b*x + a)^2 - 6*(cos(b*x + a)^6 - cos(b*x +
a)^4)*log(cos(b*x + a)^2) + 6*(cos(b*x + a)^6 - cos(b*x + a)^4)*log(-1/4*co
s(b*x + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^6 - b*cos(b*x + a)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**5*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.57972, size = 3587, normalized size = 59.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2048*(32*(9*tan(b*x + 4*a)^2*tan(1/2*a)^24 - 540*tan(b*x + 4*a)^2*tan(1/
2*a)^22 + 96*tan(b*x + 4*a)*tan(1/2*a)^23 + tan(1/2*a)^24 + 12690*tan(b*x +
```

```
 4*a)^2*tan(1/2*a)^20 - 5072*tan(b*x + 4*a)*tan(1/2*a)^21 + 264*tan(1/2*a)^
22 - 145836*tan(b*x + 4*a)^2*tan(1/2*a)^18 + 94416*tan(b*x + 4*a)*tan(1/2*a
)^19 - 11766*tan(1/2*a)^20 + 833895*tan(b*x + 4*a)^2*tan(1/2*a)^16 - 775584
*tan(b*x + 4*a)*tan(1/2*a)^17 + 152744*tan(1/2*a)^18 - 2212920*tan(b*x + 4*
a)^2*tan(1/2*a)^14 + 2952832*tan(b*x + 4*a)*tan(1/2*a)^15 - 825777*tan(1/2*
a)^16 + 3025404*tan(b*x + 4*a)^2*tan(1/2*a)^12 - 5609184*tan(b*x + 4*a)*tan
(1/2*a)^13 + 2205264*tan(1/2*a)^14 - 2212920*tan(b*x + 4*a)^2*tan(1/2*a)^10
 + 5609184*tan(b*x + 4*a)*tan(1/2*a)^11 - 3045556*tan(1/2*a)^12 + 833895*ta
n(b*x + 4*a)^2*tan(1/2*a)^8 - 2952832*tan(b*x + 4*a)*tan(1/2*a)^9 + 2205264
*tan(1/2*a)^10 - 145836*tan(b*x + 4*a)^2*tan(1/2*a)^6 + 775584*tan(b*x + 4*
a)*tan(1/2*a)^7 - 825777*tan(1/2*a)^8 + 12690*tan(b*x + 4*a)^2*tan(1/2*a)^4
 - 94416*tan(b*x + 4*a)*tan(1/2*a)^5 + 152744*tan(1/2*a)^6 - 540*tan(b*x +
4*a)^2*tan(1/2*a)^2 + 5072*tan(b*x + 4*a)*tan(1/2*a)^3 - 11766*tan(1/2*a)^4
 + 9*tan(b*x + 4*a)^2 - 96*tan(b*x + 4*a)*tan(1/2*a) + 264*tan(1/2*a)^2 + 1
)/((tan(1/2*a)^12 - 30*tan(1/2*a)^10 + 255*tan(1/2*a)^8 - 452*tan(1/2*a)^6
+ 255*tan(1/2*a)^4 - 30*tan(1/2*a)^2 + 1)*(tan(b*x + 4*a)*tan(1/2*a)^6 - 15
*tan(b*x + 4*a)*tan(1/2*a)^4 + 6*tan(1/2*a)^5 + 15*tan(b*x + 4*a)*tan(1/2*a
)^2 - 20*tan(1/2*a)^3 - tan(b*x + 4*a) + 6*tan(1/2*a))^2) - (864*tan(b*x +
4*a)^3*tan(1/2*a)^45 - 216*tan(b*x + 4*a)^2*tan(1/2*a)^46 + 24*tan(b*x + 4*
a)*tan(1/2*a)^47 - tan(1/2*a)^48 + 50976*tan(b*x + 4*a)^3*tan(1/2*a)^43 - 1
8000*tan(b*x + 4*a)^2*tan(1/2*a)^44 + 2584*tan(b*x + 4*a)*tan(1/2*a)^45 - 9
6*tan(1/2*a)^46 + 41990400*tan(b*x + 4*a)^4*tan(1/2*a)^40 - 29618496*tan(b*
x + 4*a)^3*tan(1/2*a)^41 + 8128920*tan(b*x + 4*a)^2*tan(1/2*a)^42 - 1000584
*tan(b*x + 4*a)*tan(1/2*a)^43 + 46860*tan(1/2*a)^44 - 1119744000*tan(b*x +
4*a)^4*tan(1/2*a)^38 + 1086111040*tan(b*x + 4*a)^3*tan(1/2*a)^39 - 36523900
8*tan(b*x + 4*a)^2*tan(1/2*a)^40 + 52636536*tan(b*x + 4*a)*tan(1/2*a)^41 -
2775328*tan(1/2*a)^42 + 13399603200*tan(b*x + 4*a)^4*tan(1/2*a)^36 - 169820
62752*tan(b*x + 4*a)^3*tan(1/2*a)^37 + 7267900536*tan(b*x + 4*a)^2*tan(1/2*
a)^38 - 1271292760*tan(b*x + 4*a)*tan(1/2*a)^39 + 78743742*tan(1/2*a)^40 -
94929408000*tan(b*x + 4*a)^4*tan(1/2*a)^34 + 150936080928*tan(b*x + 4*a)^3*
tan(1/2*a)^35 - 81088683024*tan(b*x + 4*a)^2*tan(1/2*a)^36 + 17523815592*ta
n(b*x + 4*a)*tan(1/2*a)^37 - 1302867360*tan(1/2*a)^38 + 442437811200*tan(b*
x + 4*a)^4*tan(1/2*a)^32 - 858638750464*tan(b*x + 4*a)^3*tan(1/2*a)^33 + 56
7089250120*tan(b*x + 4*a)^2*tan(1/2*a)^34 - 150754238328*tan(b*x + 4*a)*tan
(1/2*a)^35 + 13639996380*tan(1/2*a)^36 - 1426650624000*tan(b*x + 4*a)^4*tan
(1/2*a)^30 + 3319597849344*tan(b*x + 4*a)^3*tan(1/2*a)^31 - 2647648010880*t
an(b*x + 4*a)^2*tan(1/2*a)^32 + 854992222856*tan(b*x + 4*a)*tan(1/2*a)^33 -
 94087910880*tan(1/2*a)^34 + 3262626662400*tan(b*x + 4*a)^4*tan(1/2*a)^28 -
 9004992609600*tan(b*x + 4*a)^3*tan(1/2*a)^29 + 8563235603472*tan(b*x + 4*a
)^2*tan(1/2*a)^30 - 3316579144464*tan(b*x + 4*a)*tan(1/2*a)^31 + 4400725427
37*tan(1/2*a)^32 - 5349143040000*tan(b*x + 4*a)^4*tan(1/2*a)^26 + 174203278
27008*tan(b*x + 4*a)^3*tan(1/2*a)^27 - 19596982709664*tan(b*x + 4*a)^2*tan(
1/2*a)^28 + 9014744094960*tan(b*x + 4*a)*tan(1/2*a)^29 - 1427870886848*tan(
1/2*a)^30 + 6307092928000*tan(b*x + 4*a)^4*tan(1/2*a)^24 - 24202990429056*t
an(b*x + 4*a)^3*tan(1/2*a)^25 + 32094110324592*tan(b*x + 4*a)^2*tan(1/2*a)^
26 - 17430945047632*tan(b*x + 4*a)*tan(1/2*a)^27 + 3269665274136*tan(1/2*a)
^28 - 5349143040000*tan(b*x + 4*a)^4*tan(1/2*a)^22 + 24202990429056*tan(b*x
 + 4*a)^3*tan(1/2*a)^23 - 37811152430400*tan(b*x + 4*a)^2*tan(1/2*a)^24 + 2
4188717892528*tan(b*x + 4*a)*tan(1/2*a)^25 - 5348679038784*tan(1/2*a)^26 +
3262626662400*tan(b*x + 4*a)^4*tan(1/2*a)^20 - 17420327827008*tan(b*x + 4*a
)^3*tan(1/2*a)^21 + 32094110324592*tan(b*x + 4*a)^2*tan(1/2*a)^22 - 2418871
7892528*tan(b*x + 4*a)*tan(1/2*a)^23 + 6296990528100*tan(1/2*a)^24 - 142665
0624000*tan(b*x + 4*a)^4*tan(1/2*a)^18 + 9004992609600*tan(b*x + 4*a)^3*tan
(1/2*a)^19 - 19596982709664*tan(b*x + 4*a)^2*tan(1/2*a)^20 + 17430945047632
*tan(b*x + 4*a)*tan(1/2*a)^21 - 5348679038784*tan(1/2*a)^22 + 442437811200*
tan(b*x + 4*a)^4*tan(1/2*a)^16 - 3319597849344*tan(b*x + 4*a)^3*tan(1/2*a)^
17 + 8563235603472*tan(b*x + 4*a)^2*tan(1/2*a)^18 - 9014744094960*tan(b*x +
 4*a)*tan(1/2*a)^19 + 3269665274136*tan(1/2*a)^20 - 94929408000*tan(b*x + 4
*a)^4*tan(1/2*a)^14 + 858638750464*tan(b*x + 4*a)^3*tan(1/2*a)^15 - 2647648
```

```
010880*tan(b*x + 4*a)^2*tan(1/2*a)^16 + 3316579144464*tan(b*x + 4*a)*tan(1/
2*a)^17 - 1427870886848*tan(1/2*a)^18 + 13399603200*tan(b*x + 4*a)^4*tan(1/
2*a)^12 - 150936080928*tan(b*x + 4*a)^3*tan(1/2*a)^13 + 567089250120*tan(b*
x + 4*a)^2*tan(1/2*a)^14 - 854992222856*tan(b*x + 4*a)*tan(1/2*a)^15 + 4400
72542737*tan(1/2*a)^16 - 1119744000*tan(b*x + 4*a)^4*tan(1/2*a)^10 + 169820
62752*tan(b*x + 4*a)^3*tan(1/2*a)^11 - 81088683024*tan(b*x + 4*a)^2*tan(1/2
*a)^12 + 150754238328*tan(b*x + 4*a)*tan(1/2*a)^13 - 94087910880*tan(1/2*a)
^14 + 41990400*tan(b*x + 4*a)^4*tan(1/2*a)^8 - 1086111040*tan(b*x + 4*a)^3*
tan(1/2*a)^9 + 7267900536*tan(b*x + 4*a)^2*tan(1/2*a)^10 - 17523815592*tan(
b*x + 4*a)*tan(1/2*a)^11 + 13639996380*tan(1/2*a)^12 + 29618496*tan(b*x + 4
*a)^3*tan(1/2*a)^7 - 365239008*tan(b*x + 4*a)^2*tan(1/2*a)^8 + 1271292760*t
an(b*x + 4*a)*tan(1/2*a)^9 - 1302867360*tan(1/2*a)^10 - 50976*tan(b*x + 4*a
)^3*tan(1/2*a)^5 + 8128920*tan(b*x + 4*a)^2*tan(1/2*a)^6 - 52636536*tan(b*x
 + 4*a)*tan(1/2*a)^7 + 78743742*tan(1/2*a)^8 - 864*tan(b*x + 4*a)^3*tan(1/2
*a)^3 - 18000*tan(b*x + 4*a)^2*tan(1/2*a)^4 + 1000584*tan(b*x + 4*a)*tan(1/
2*a)^5 - 2775328*tan(1/2*a)^6 - 216*tan(b*x + 4*a)^2*tan(1/2*a)^2 - 2584*ta
n(b*x + 4*a)*tan(1/2*a)^3 + 46860*tan(1/2*a)^4 - 24*tan(b*x + 4*a)*tan(1/2*
a) - 96*tan(1/2*a)^2 - 1)/((81*tan(1/2*a)^20 - 1080*tan(1/2*a)^18 + 5724*ta
n(1/2*a)^16 - 15240*tan(1/2*a)^14 + 21286*tan(1/2*a)^12 - 15240*tan(1/2*a)^
10 + 5724*tan(1/2*a)^8 - 1080*tan(1/2*a)^6 + 81*tan(1/2*a)^4)*(6*tan(b*x +
4*a)*tan(1/2*a)^5 - tan(1/2*a)^6 - 20*tan(b*x + 4*a)*tan(1/2*a)^3 + 15*tan(
1/2*a)^4 + 6*tan(b*x + 4*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)^4) - 192*log(
abs(tan(b*x + 4*a)*tan(1/2*a)^6 - 15*tan(b*x + 4*a)*tan(1/2*a)^4 + 6*tan(1/
2*a)^5 + 15*tan(b*x + 4*a)*tan(1/2*a)^2 - 20*tan(1/2*a)^3 - tan(b*x + 4*a)
+ 6*tan(1/2*a))) + 192*log(abs(6*tan(b*x + 4*a)*tan(1/2*a)^5 - tan(1/2*a)^6
 - 20*tan(b*x + 4*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 + 6*tan(b*x + 4*a)*tan(
1/2*a) - 15*tan(1/2*a)^2 + 1)))/b
```

3.23 $\int \sin^3(a+bx)\sin^5(2a+2bx)\,dx$

Optimal. Leaf size=46

$$\frac{32\sin^{13}(a+bx)}{13b}-\frac{64\sin^{11}(a+bx)}{11b}+\frac{32\sin^9(a+bx)}{9b}$$

```
[Out] (32*Sin[a + b*x]^9)/(9*b) - (64*Sin[a + b*x]^11)/(11*b) + (32*Sin[a + b*x]^
13)/(13*b)
```

Rubi [A] time = 0.0631673, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4288, 2564, 270}

$$\frac{32\sin^{13}(a+bx)}{13b}-\frac{64\sin^{11}(a+bx)}{11b}+\frac{32\sin^9(a+bx)}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (32*Sin[a + b*x]^9)/(9*b) - (64*Sin[a + b*x]^11)/(11*b) + (32*Sin[a + b*x]^
13)/(13*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(a+bx)\sin^5(2a+2bx)\,dx &= 32\int \cos^5(a+bx)\sin^8(a+bx)\,dx\\ &= \frac{32\,\text{Subst}\left(\int x^8\left(1-x^2\right)^2\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{32\,\text{Subst}\left(\int \left(x^8-2x^{10}+x^{12}\right)dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{32\sin^9(a+bx)}{9b}-\frac{64\sin^{11}(a+bx)}{11b}+\frac{32\sin^{13}(a+bx)}{13b}\end{aligned}$$

Mathematica [A] time = 0.373848, size = 37, normalized size = 0.8

$$\frac{4\sin^9(a+bx)(540\cos(2(a+bx))+99\cos(4(a+bx))+505)}{1287b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (4*(505 + 540*Cos[2*(a + b*x)] + 99*Cos[4*(a + b*x)])*Sin[a + b*x]^9)/(1287
*b)
```

Maple [B] time = 0.012, size = 97, normalized size = 2.1

$$\frac{5\sin(bx+a)}{32b}-\frac{25\sin(3bx+3a)}{384b}-\frac{\sin(5bx+5a)}{128b}+\frac{\sin(7bx+7a)}{64b}-\frac{\sin(9bx+9a)}{576b}-\frac{3\sin(11bx+11a)}{1408b}+\frac{\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(2*b*x+2*a)^5,x)
```

```
[Out] 5/32*sin(b*x+a)/b-25/384*sin(3*b*x+3*a)/b-1/128/b*sin(5*b*x+5*a)+1/64/b*sin
(7*b*x+7*a)-1/576/b*sin(9*b*x+9*a)-3/1408/b*sin(11*b*x+11*a)+1/1664/b*sin(1
3*b*x+13*a)
```

Maxima [A] time = 1.06181, size = 108, normalized size = 2.35

$$\frac{99\sin(13bx+13a)-351\sin(11bx+11a)-286\sin(9bx+9a)+2574\sin(7bx+7a)-1287\sin(5bx+5a)-1}{164736b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] 1/164736*(99*sin(13*b*x + 13*a) - 351*sin(11*b*x + 11*a) - 286*sin(9*b*x +
9*a) + 2574*sin(7*b*x + 7*a) - 1287*sin(5*b*x + 5*a) - 10725*sin(3*b*x + 3*
a) + 25740*sin(b*x + a))/b
```

Fricas [A] time = 0.515469, size = 204, normalized size = 4.43

$$\frac{32\left(99\cos(bx+a)^{12}-360\cos(bx+a)^{10}+458\cos(bx+a)^{8}-212\cos(bx+a)^{6}+3\cos(bx+a)^{4}+4\cos(bx+a)^{2}\right.}{1287b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] 32/1287*(99*cos(b*x + a)^12 - 360*cos(b*x + a)^10 + 458*cos(b*x + a)^8 - 21
2*cos(b*x + a)^6 + 3*cos(b*x + a)^4 + 4*cos(b*x + a)^2 + 8)*sin(b*x + a)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.59182, size = 130, normalized size = 2.83

$$\frac{\sin(13\,bx+13\,a)}{1664\,b}-\frac{3\,\sin(11\,bx+11\,a)}{1408\,b}-\frac{\sin(9\,bx+9\,a)}{576\,b}+\frac{\sin(7\,bx+7\,a)}{64\,b}-\frac{\sin(5\,bx+5\,a)}{128\,b}-\frac{25\,\sin(3\,bx+3\,a)}{384\,b}+\frac{5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] 1/1664*sin(13*b*x + 13*a)/b - 3/1408*sin(11*b*x + 11*a)/b - 1/576*sin(9*b*x
 + 9*a)/b + 1/64*sin(7*b*x + 7*a)/b - 1/128*sin(5*b*x + 5*a)/b - 25/384*sin
(3*b*x + 3*a)/b + 5/32*sin(b*x + a)/b
```

3.24 $\int \sin^3(a+bx)\sin^4(2a+2bx)\,dx$

Optimal. Leaf size=61

$$\frac{16\cos^{11}(a+bx)}{11b}-\frac{16\cos^9(a+bx)}{3b}+\frac{48\cos^7(a+bx)}{7b}-\frac{16\cos^5(a+bx)}{5b}$$

```
[Out] (-16*Cos[a + b*x]^5)/(5*b) + (48*Cos[a + b*x]^7)/(7*b) - (16*Cos[a + b*x]^9
)/(3*b) + (16*Cos[a + b*x]^11)/(11*b)
```

Rubi [A] time = 0.0685345, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4288, 2565, 270}

$$\frac{16\cos^{11}(a+bx)}{11b}-\frac{16\cos^9(a+bx)}{3b}+\frac{48\cos^7(a+bx)}{7b}-\frac{16\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (-16*Cos[a + b*x]^5)/(5*b) + (48*Cos[a + b*x]^7)/(7*b) - (16*Cos[a + b*x]^9
)/(3*b) + (16*Cos[a + b*x]^11)/(11*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(a+bx)\sin^4(2a+2bx)\,dx &= 16\int \cos^4(a+bx)\sin^7(a+bx)\,dx\\ &= -\frac{16\,\text{Subst}\left(\int x^4\left(1-x^2\right)^3\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{16\,\text{Subst}\left(\int \left(x^4-3x^6+3x^8-x^{10}\right)dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{16\cos^5(a+bx)}{5b}+\frac{48\cos^7(a+bx)}{7b}-\frac{16\cos^9(a+bx)}{3b}+\frac{16\cos^{11}(a+bx)}{11b}\end{aligned}$$

Mathematica [A] time = 0.225861, size = 47, normalized size = 0.77

$$\frac{\cos^5(a+bx)(3335\cos(2(a+bx))-910\cos(4(a+bx))+105\cos(6(a+bx))-3042)}{2310b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (Cos[a + b*x]^5*(-3042 + 3335*Cos[2*(a + b*x)] - 910*Cos[4*(a + b*x)] + 105
*Cos[6*(a + b*x)]))/(2310*b)
```

Maple [A] time = 0.011, size = 83, normalized size = 1.4

$$-\frac{7\cos(bx+a)}{32b}-\frac{\cos(3bx+3a)}{32b}+\frac{11\cos(5bx+5a)}{320b}-\frac{\cos(7bx+7a)}{448b}-\frac{\cos(9bx+9a)}{192b}+\frac{\cos(11bx+11a)}{704b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(2*b*x+2*a)^4,x)
```

```
[Out] -7/32*cos(b*x+a)/b-1/32*cos(3*b*x+3*a)/b+11/320*cos(5*b*x+5*a)/b-1/448*cos(
7*b*x+7*a)/b-1/192*cos(9*b*x+9*a)/b+1/704*cos(11*b*x+11*a)/b
```

Maxima [A] time = 1.21816, size = 93, normalized size = 1.52

$$\frac{105\cos(11bx+11a)-385\cos(9bx+9a)-165\cos(7bx+7a)+2541\cos(5bx+5a)-2310\cos(3bx+3a)-1617}{73920b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/73920*(105*cos(11*b*x + 11*a) - 385*cos(9*b*x + 9*a) - 165*cos(7*b*x + 7*
a) + 2541*cos(5*b*x + 5*a) - 2310*cos(3*b*x + 3*a) - 16170*cos(b*x + a))/b
```

Fricas [A] time = 0.503269, size = 130, normalized size = 2.13

$$\frac{16\left(105\cos(bx+a)^{11}-385\cos(bx+a)^9+495\cos(bx+a)^7-231\cos(bx+a)^5\right)}{1155b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] 16/1155*(105*cos(b*x + a)^11 - 385*cos(b*x + a)^9 + 495*cos(b*x + a)^7 - 23
1*cos(b*x + a)^5)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.42921, size = 111, normalized size = 1.82

$$\frac{\cos\left(11\,bx+11\,a\right)}{704\,b}-\frac{\cos\left(9\,bx+9\,a\right)}{192\,b}-\frac{\cos\left(7\,bx+7\,a\right)}{448\,b}+\frac{11\,\cos\left(5\,bx+5\,a\right)}{320\,b}-\frac{\cos\left(3\,bx+3\,a\right)}{32\,b}-\frac{7\,\cos\left(bx+a\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] 1/704*cos(11*b*x + 11*a)/b - 1/192*cos(9*b*x + 9*a)/b - 1/448*cos(7*b*x + 7
*a)/b + 11/320*cos(5*b*x + 5*a)/b - 1/32*cos(3*b*x + 3*a)/b - 7/32*cos(b*x
+ a)/b
```

3.25 $\int \sin^3(a+bx)\sin^3(2a+2bx)\,dx$

Optimal. Leaf size=31

$$\frac{8\sin^7(a+bx)}{7b}-\frac{8\sin^9(a+bx)}{9b}$$

```
[Out] (8*Sin[a + b*x]^7)/(7*b) - (8*Sin[a + b*x]^9)/(9*b)
```

Rubi [A] time = 0.059593, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4288, 2564, 14}

$$\frac{8\sin^7(a+bx)}{7b}-\frac{8\sin^9(a+bx)}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]

[Out] (8*Sin[a + b*x]^7)/(7*b) - (8*Sin[a + b*x]^9)/(9*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \sin^3(a+bx)\sin^3(2a+2bx)\,dx &= 8\int \cos^3(a+bx)\sin^6(a+bx)\,dx\\
&= \frac{8\,\text{Subst}\left(\int x^6\left(1-x^2\right)\,dx,x,\sin(a+bx)\right)}{b}\\
&= \frac{8\,\text{Subst}\left(\int \left(x^6-x^8\right)\,dx,x,\sin(a+bx)\right)}{b}\\
&= \frac{8\sin^7(a+bx)}{7b}-\frac{8\sin^9(a+bx)}{9b}
\end{aligned}$$

Mathematica [A] time = 0.141587, size = 27, normalized size = 0.87

$$\frac{4\sin^7(a+bx)(7\cos(2(a+bx))+11)}{63b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (4*(11 + 7*Cos[2*(a + b*x)])*Sin[a + b*x]^7)/(63*b)
```

Maple [A] time = 0.01, size = 55, normalized size = 1.8

$$\frac{3\sin(bx+a)}{16\,b}-\frac{\sin(3\,bx+3\,a)}{12\,b}+\frac{3\sin(7\,bx+7\,a)}{224\,b}-\frac{\sin(9\,bx+9\,a)}{288\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(2*b*x+2*a)^3,x)
```

```
[Out] 3/16*sin(b*x+a)/b-1/12*sin(3*b*x+3*a)/b+3/224/b*sin(7*b*x+7*a)-1/288/b*sin(
9*b*x+9*a)
```

Maxima [A] time = 1.176, size = 63, normalized size = 2.03

$$-\frac{7\sin(9\,bx+9\,a)-27\sin(7\,bx+7\,a)+168\sin(3\,bx+3\,a)-378\sin(bx+a)}{2016\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] -1/2016*(7*sin(9*b*x + 9*a) - 27*sin(7*b*x + 7*a) + 168*sin(3*b*x + 3*a) -
378*sin(b*x + a))/b
```

Fricas [A] time = 0.488846, size = 138, normalized size = 4.45

$$-\frac{8\left(7\cos(bx+a)^8-19\cos(bx+a)^6+15\cos(bx+a)^4-\cos(bx+a)^2-2\right)\sin(bx+a)}{63\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] -8/63*(7*cos(b*x + a)^8 - 19*cos(b*x + a)^6 + 15*cos(b*x + a)^4 - cos(b*x +
 a)^2 - 2)*sin(b*x + a)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28407, size = 73, normalized size = 2.35

$$-\frac{\sin\left(9\,bx + 9\,a\right)}{288\,b} + \frac{3\,\sin\left(7\,bx + 7\,a\right)}{224\,b} - \frac{\sin\left(3\,bx + 3\,a\right)}{12\,b} + \frac{3\,\sin\left(bx + a\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] -1/288*sin(9*b*x + 9*a)/b + 3/224*sin(7*b*x + 7*a)/b - 1/12*sin(3*b*x + 3*a
)/b + 3/16*sin(b*x + a)/b
```

3.26 $\int \sin^3(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=46

$$-\frac{4\cos^7(a+bx)}{7b}+\frac{8\cos^5(a+bx)}{5b}-\frac{4\cos^3(a+bx)}{3b}$$

```
[Out] (-4*Cos[a + b*x]^3)/(3*b) + (8*Cos[a + b*x]^5)/(5*b) - (4*Cos[a + b*x]^7)/(
7*b)
```

Rubi [A] time = 0.0635506, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4288, 2565, 270}

$$-\frac{4\cos^7(a+bx)}{7b}+\frac{8\cos^5(a+bx)}{5b}-\frac{4\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (-4*Cos[a + b*x]^3)/(3*b) + (8*Cos[a + b*x]^5)/(5*b) - (4*Cos[a + b*x]^7)/(
7*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \sin^3(a+bx)\sin^2(2a+2bx)\,dx &= 4\int \cos^2(a+bx)\sin^5(a+bx)\,dx\\
&= -\frac{4\,\text{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{4\,\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{4\cos^3(a+bx)}{3b}+\frac{8\cos^5(a+bx)}{5b}-\frac{4\cos^7(a+bx)}{7b}
\end{aligned}$$

Mathematica [A] time = 0.109167, size = 37, normalized size = 0.8

$$\frac{\cos^3(a+bx)(108\cos(2(a+bx))-15\cos(4(a+bx))-157)}{210b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (Cos[a + b*x]^3*(-157 + 108*Cos[2*(a + b*x)] - 15*Cos[4*(a + b*x)]))/(210*b
)
```

Maple [A] time = 0.01, size = 55, normalized size = 1.2

$$-\frac{5\cos(bx+a)}{16b}-\frac{\cos(3bx+3a)}{48b}+\frac{3\cos(5bx+5a)}{80b}-\frac{\cos(7bx+7a)}{112b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(2*b*x+2*a)^2,x)
```

```
[Out] -5/16*cos(b*x+a)/b-1/48*cos(3*b*x+3*a)/b+3/80*cos(5*b*x+5*a)/b-1/112*cos(7*
b*x+7*a)/b
```

Maxima [A] time = 1.14872, size = 63, normalized size = 1.37

$$-\frac{15\cos(7bx+7a)-63\cos(5bx+5a)+35\cos(3bx+3a)+525\cos(bx+a)}{1680b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] -1/1680*(15*cos(7*b*x + 7*a) - 63*cos(5*b*x + 5*a) + 35*cos(3*b*x + 3*a) +
525*cos(b*x + a))/b
```

Fricas [A] time = 0.485046, size = 95, normalized size = 2.07

$$-\frac{4\left(15\cos(bx+a)^7-42\cos(bx+a)^5+35\cos(bx+a)^3\right)}{105b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] -4/105*(15*cos(b*x + a)^7 - 42*cos(b*x + a)^5 + 35*cos(b*x + a)^3)/b
```

Sympy [A] time = 73.1723, size = 202, normalized size = 4.39

$$\begin{cases} -\frac{12\sin^3(a+bx)\sin(2a+2bx)\cos(2a+2bx)}{35b}-\frac{11\sin^2(a+bx)\sin^2(2a+2bx)\cos(a+bx)}{35b}-\frac{24\sin^2(a+bx)\cos(a+bx)\cos^2(2a+2bx)}{35b}+\frac{8\sin(a+bx)\sin(2a+2b \\ x\sin^3(a)\sin^2(2a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**2,x)
```

```
[Out] Piecewise((-12*sin(a + b*x)**3*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)/(35*b) - 1
1*sin(a + b*x)**2*sin(2*a + 2*b*x)**2*cos(a + b*x)/(35*b) - 24*sin(a + b*x)
**2*cos(a + b*x)*cos(2*a + 2*b*x)**2/(35*b) + 8*sin(a + b*x)*sin(2*a + 2*b*
x)*cos(a + b*x)**2*cos(2*a + 2*b*x)/(35*b) - 38*sin(2*a + 2*b*x)**2*cos(a +
 b*x)**3/(105*b) - 32*cos(a + b*x)**3*cos(2*a + 2*b*x)**2/(105*b), Ne(b, 0)
), (x*sin(a)**3*sin(2*a)**2, True))
```

Giac [A] time = 1.30693, size = 73, normalized size = 1.59

$$-\frac{\cos(7\,bx+7\,a)}{112\,b}+\frac{3\,\cos(5\,bx+5\,a)}{80\,b}-\frac{\cos(3\,bx+3\,a)}{48\,b}-\frac{5\,\cos(bx+a)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] -1/112*cos(7*b*x + 7*a)/b + 3/80*cos(5*b*x + 5*a)/b - 1/48*cos(3*b*x + 3*a)
/b - 5/16*cos(b*x + a)/b
```

3.27 $\int \sin^3(a+bx)\sin(2a+2bx)\,dx$

Optimal. Leaf size=15

$$\frac{2\sin^5(a+bx)}{5b}$$

```
[Out] (2*Sin[a + b*x]^5)/(5*b)
```

Rubi [A] time = 0.0330478, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2564, 30}

$$\frac{2\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x],x]

[Out] (2*Sin[a + b*x]^5)/(5*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sin^3(a+bx)\sin(2a+2bx)\,dx &= 2\int \cos(a+bx)\sin^4(a+bx)\,dx\\ &= \frac{2\,\text{Subst}\left(\int x^4\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{2\sin^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.0073833, size = 15, normalized size = 1.

$$\frac{2\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x],x]
```

```
[Out] (2*Sin[a + b*x]^5)/(5*b)
```

Maple [B] time = 0.009, size = 41, normalized size = 2.7

$$\frac{\sin \left(bx+a\right)}{4\,b}-\frac{\sin \left(3\,bx+3\,a\right)}{8\,b}+\frac{\sin \left(5\,bx+5\,a\right)}{40\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(2*b*x+2*a),x)
```

```
[Out] 1/4*sin(b*x+a)/b-1/8*sin(3*b*x+3*a)/b+1/40/b*sin(5*b*x+5*a)
```

Maxima [B] time = 1.07632, size = 46, normalized size = 3.07

$$\frac{\sin \left(5\,bx+5\,a\right)-5\,\sin \left(3\,bx+3\,a\right)+10\,\sin \left(bx+a\right)}{40\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] 1/40*(sin(5*b*x + 5*a) - 5*sin(3*b*x + 3*a) + 10*sin(b*x + a))/b
```

Fricas [B] time = 0.470941, size = 81, normalized size = 5.4

$$\frac{2\left(\cos \left(bx+a\right)^{4}-2\,\cos \left(bx+a\right)^{2}+1\right)\sin \left(bx+a\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 2/5*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sin(b*x + a)/b
```

Sympy [A] time = 19.1654, size = 117, normalized size = 7.8

$$\begin{cases} -\frac{2\sin^{3}(a+bx)\cos(2a+2bx)}{5b}-\frac{\sin^{2}(a+bx)\sin(2a+2bx)\cos(a+bx)}{5b}-\frac{4\sin(a+bx)\cos^{2}(a+bx)\cos(2a+2bx)}{5b}+\frac{2\sin(2a+2bx)\cos^{3}(a+bx)}{5b} & \text{for } b \neq \\ x\sin^{3}(a)\sin(2a) & \text{otherw} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(2*b*x+2*a),x)
```

```
[Out] Piecewise((-2*sin(a + b*x)**3*cos(2*a + 2*b*x)/(5*b) - sin(a + b*x)**2*sin(
2*a + 2*b*x)*cos(a + b*x)/(5*b) - 4*sin(a + b*x)*cos(a + b*x)**2*cos(2*a +
2*b*x)/(5*b) + 2*sin(2*a + 2*b*x)*cos(a + b*x)**3/(5*b), Ne(b, 0)), (x*sin(
a)**3*sin(2*a), True))
```

Giac [B] time = 1.2782, size = 54, normalized size = 3.6

$$\frac{\sin(5\,bx+5\,a)}{40\,b}-\frac{\sin(3\,bx+3\,a)}{8\,b}+\frac{\sin(bx+a)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] 1/40*sin(5*b*x + 5*a)/b - 1/8*sin(3*b*x + 3*a)/b + 1/4*sin(b*x + a)/b
```

3.28 $\int \csc(2a+2bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=28

$$\frac{\tanh^{-1}(\sin(a+bx))}{2b}-\frac{\sin(a+bx)}{2b}$$

```
[Out] ArcTanh[Sin[a + b*x]]/(2*b) - Sin[a + b*x]/(2*b)
```

Rubi [A] time = 0.0372884, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4288, 2592, 321, 206}

$$\frac{\tanh^{-1}(\sin(a+bx))}{2b}-\frac{\sin(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]*Sin[a + b*x]^3,x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(2*b) - Sin[a + b*x]/(2*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc(2a+2bx)\sin^3(a+bx)\,dx = \frac{1}{2}\int \sin(a+bx)\tan(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(a+bx)\right)}{2b}$$
$$= -\frac{\sin(a+bx)}{2b} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(a+bx)\right)}{2b}$$
$$= \frac{\tanh^{-1}(\sin(a+bx))}{2b} - \frac{\sin(a+bx)}{2b}$$

Mathematica [A] time = 0.0120244, size = 27, normalized size = 0.96

$$\frac{1}{2}\left(\frac{\tanh^{-1}(\sin(a+bx))}{b} - \frac{\sin(a+bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (ArcTanh[Sin[a + b*x]]/b - Sin[a + b*x]/b)/2
```

Maple [A] time = 0.026, size = 32, normalized size = 1.1

$$-\frac{\sin(bx+a)}{2b} + \frac{\ln(\sec(bx+a) + \tan(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)*sin(b*x+a)^3,x)
```

```
[Out] -1/2*sin(b*x+a)/b+1/2/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [B] time = 1.75747, size = 167, normalized size = 5.96

$$-\frac{\log\left(\frac{\cos(bx+2a)^2+\cos(a)^2-2\cos(a)\sin(bx+2a)+\sin(bx+2a)^2+2\cos(bx+2a)\sin(a)+\sin(a)^2}{\cos(bx+2a)^2+\cos(a)^2+2\cos(a)\sin(bx+2a)+\sin(bx+2a)^2-2\cos(bx+2a)\sin(a)+\sin(a)^2}\right) + 2\sin(bx+a)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x
 + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2
 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + s
in(a)^2)) + 2*sin(b*x + a))/b
```

Fricas [A] time = 0.502638, size = 99, normalized size = 3.54

$$\frac{\log(\sin(bx+a)+1) - \log(-\sin(bx+a)+1) - 2\sin(bx+a)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1) - 2*sin(b*x + a))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.61057, size = 834, normalized size = 29.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*((tan(1/2*a)^15 + 3*tan(1/2*a)^14 + 3*tan(1/2*a)^13 + 17*tan(1/2*a)^12
- 3*tan(1/2*a)^11 + 39*tan(1/2*a)^10 - 25*tan(1/2*a)^9 + 45*tan(1/2*a)^8 -
45*tan(1/2*a)^7 + 25*tan(1/2*a)^6 - 39*tan(1/2*a)^5 + 3*tan(1/2*a)^4 - 17*t
an(1/2*a)^3 - 3*tan(1/2*a)^2 - 3*tan(1/2*a) - 1)*log(abs(tan(1/2*b*x + 2*a)
*tan(1/2*a)^3 + 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - tan(1/2*a)^3 - 3*tan(1/
2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 - tan(1/2*b*x + 2*a) + 3*tan(1/2*a
) - 1))/(tan(1/2*a)^3 + 3*tan(1/2*a)^2 - 3*tan(1/2*a) - 1) - (tan(1/2*a)^15
 - 3*tan(1/2*a)^14 + 3*tan(1/2*a)^13 - 17*tan(1/2*a)^12 - 3*tan(1/2*a)^11 -
 39*tan(1/2*a)^10 - 25*tan(1/2*a)^9 - 45*tan(1/2*a)^8 - 45*tan(1/2*a)^7 - 2
5*tan(1/2*a)^6 - 39*tan(1/2*a)^5 - 3*tan(1/2*a)^4 - 17*tan(1/2*a)^3 + 3*tan
(1/2*a)^2 - 3*tan(1/2*a) + 1)*log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 3*t
an(1/2*b*x + 2*a)*tan(1/2*a)^2 + tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/
2*a) + 3*tan(1/2*a)^2 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a) - 1))/(tan(1/2*a)
^3 - 3*tan(1/2*a)^2 - 3*tan(1/2*a) + 1) + 2*(tan(1/2*b*x + 2*a)*tan(1/2*a)^
12 - 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^10 + 6*tan(1/2*a)^11 - 27*tan(1/2*b*x
 + 2*a)*tan(1/2*a)^8 - 2*tan(1/2*a)^9 - 36*tan(1/2*a)^7 + 27*tan(1/2*b*x +
2*a)*tan(1/2*a)^4 - 36*tan(1/2*a)^5 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 -
2*tan(1/2*a)^3 - tan(1/2*b*x + 2*a) + 6*tan(1/2*a))/(tan(1/2*b*x + 2*a)^2 +
 1))/b
```

3.29 $\int \csc^2(2a+2bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=13

$$\frac{\sec(a+bx)}{4b}$$

[Out] Sec[a + b*x]/(4*b)

Rubi [A] time = 0.0353868, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4288, 2606, 8}

$$\frac{\sec(a+bx)}{4b}$$

Antiderivative was successfully verified.

[In] Int[Csc[2*a + 2*b*x]^2*Sin[a + b*x]^3,x]

[Out] Sec[a + b*x]/(4*b)

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \csc^2(2a+2bx)\sin^3(a+bx)\,dx &= \frac{1}{4}\int \sec(a+bx)\tan(a+bx)\,dx\\ &= \frac{\text{Subst}\left(\int 1\,dx, x, \sec(a+bx)\right)}{4b}\\ &= \frac{\sec(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0094908, size = 13, normalized size = 1.

$$\frac{\sec(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] Sec[a + b*x]/(4*b)
```

Maple [A] time = 0.02, size = 14, normalized size = 1.1

$$\frac{1}{4\,b\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^2*sin(b*x+a)^3,x)
```

```
[Out] 1/4/b/cos(b*x+a)
```

Maxima [B] time = 1.14944, size = 112, normalized size = 8.62

$$\frac{\cos\left(2\,bx+2\,a\right)\cos\left(bx+a\right)+\sin\left(2\,bx+2\,a\right)\sin\left(bx+a\right)+\cos\left(bx+a\right)}{2\left(b\cos\left(2\,bx+2\,a\right)^{2}+b\sin\left(2\,bx+2\,a\right)^{2}+2\,b\cos\left(2\,bx+2\,a\right)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(cos(2*b*x + 2*a)*cos(b*x + a) + sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*
x + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 + 2*b*cos(2*b*x + 2*a)
 + b)
```

Fricas [A] time = 0.465013, size = 30, normalized size = 2.31

$$\frac{1}{4\,b\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4/(b*cos(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**2*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.61941, size = 431, normalized size = 33.15

$$-\frac{6\tan\left(\frac{1}{2}bx+2a\right)\tan\left(\frac{1}{2}a\right)^{11}-\tan\left(\frac{1}{2}a\right)^{12}-2\tan\left(\frac{1}{2}bx+2a\right)\tan\left(\frac{1}{2}a\right)^{9}+12\tan\left(\frac{1}{2}a\right)^{10}-36}{2\left(\tan\left(\frac{1}{2}bx+2a\right)^{2}\tan\left(\frac{1}{2}a\right)^{6}-15\tan\left(\frac{1}{2}bx+2a\right)^{2}\tan\left(\frac{1}{2}a\right)^{4}+12\tan\left(\frac{1}{2}bx+2a\right)\tan\left(\frac{1}{2}a\right)^{5}-\tan\left(\frac{1}{2}a\right)^{6}+15\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(6*tan(1/2*b*x + 2*a)*tan(1/2*a)^11 - tan(1/2*a)^12 - 2*tan(1/2*b*x +
2*a)*tan(1/2*a)^9 + 12*tan(1/2*a)^10 - 36*tan(1/2*b*x + 2*a)*tan(1/2*a)^7 +
 27*tan(1/2*a)^8 - 36*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - 2*tan(1/2*b*x + 2*a
)*tan(1/2*a)^3 - 27*tan(1/2*a)^4 + 6*tan(1/2*b*x + 2*a)*tan(1/2*a) - 12*tan
(1/2*a)^2 + 1)/((tan(1/2*b*x + 2*a)^2*tan(1/2*a)^6 - 15*tan(1/2*b*x + 2*a)^
2*tan(1/2*a)^4 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - tan(1/2*a)^6 + 15*tan
(1/2*b*x + 2*a)^2*tan(1/2*a)^2 - 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 15*ta
n(1/2*a)^4 - tan(1/2*b*x + 2*a)^2 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a) - 15*t
an(1/2*a)^2 + 1)*(tan(1/2*a)^6 - 15*tan(1/2*a)^4 + 15*tan(1/2*a)^2 - 1)*b)
```

3.30 $\int \csc^3(2a+2bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=34

$$\frac{\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\tan(a+bx)\sec(a+bx)}{16b}$$

```
[Out] ArcTanh[Sin[a + b*x]]/(16*b) + (Sec[a + b*x]*Tan[a + b*x])/(16*b)
```

Rubi [A] time = 0.0413876, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4288, 3768, 3770}

$$\frac{\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\tan(a+bx)\sec(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(16*b) + (Sec[a + b*x]*Tan[a + b*x])/(16*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^3(2a+2bx)\sin^3(a+bx)\,dx &= \frac{1}{8}\int \sec^3(a+bx)\,dx\\ &= \frac{\sec(a+bx)\tan(a+bx)}{16b}+\frac{1}{16}\int \sec(a+bx)\,dx\\ &= \frac{\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\sec(a+bx)\tan(a+bx)}{16b}\end{aligned}$$

Mathematica [A] time = 0.0107836, size = 38, normalized size = 1.12

$$\frac{1}{8}\left(\frac{\tanh^{-1}(\sin(a+bx))}{2b}+\frac{\tan(a+bx)\sec(a+bx)}{2b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (ArcTanh[Sin[a + b*x]]/(2*b) + (Sec[a + b*x]*Tan[a + b*x])/(2*b))/8
```

Maple [A] time = 0.066, size = 38, normalized size = 1.1

$$\frac{\sec(bx+a)\tan(bx+a)}{16b}+\frac{\ln(\sec(bx+a)+\tan(bx+a))}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^3*sin(b*x+a)^3,x)
```

```
[Out] 1/16*sec(b*x+a)*tan(b*x+a)/b+1/16/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [B] time = 1.72041, size = 648, normalized size = 19.06

$$\frac{4(\sin(3bx+3a)-\sin(bx+a))\cos(4bx+4a)-\left(2(2\cos(2bx+2a)+1)\cos(4bx+4a)+\cos(4bx+4a)^2+4\cos(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(4*(sin(3*b*x + 3*a) - sin(b*x + a))*cos(4*b*x + 4*a) - (2*(2*cos(2*b*
x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2
+ sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x +
2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a
)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(
cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 -
2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(cos(3*b*x + 3*a) - cos(b*x + a))*
sin(4*b*x + 4*a) + 4*(2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a) - 8*cos(3*b*
x + 3*a)*sin(2*b*x + 2*a) + 8*cos(b*x + a)*sin(2*b*x + 2*a) - 8*cos(2*b*x +
 2*a)*sin(b*x + a) - 4*sin(b*x + a))/(b*cos(4*b*x + 4*a)^2 + 4*b*cos(2*b*x
+ 2*a)^2 + b*sin(4*b*x + 4*a)^2 + 4*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4
*b*sin(2*b*x + 2*a)^2 + 2*(2*b*cos(2*b*x + 2*a) + b)*cos(4*b*x + 4*a) + 4*b
*cos(2*b*x + 2*a) + b)
```

Fricas [B] time = 0.492381, size = 163, normalized size = 4.79

$$\frac{\cos(bx+a)^2\log(\sin(bx+a)+1)-\cos(bx+a)^2\log(-\sin(bx+a)+1)+2\sin(bx+a)}{32b\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(cos(b*x + a)^2*log(sin(b*x + a) + 1) - cos(b*x + a)^2*log(-sin(b*x +
a) + 1) + 2*sin(b*x + a))/(b*cos(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**3*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.18763, size = 1500, normalized size = 44.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/16*(2*(tan(1/2*b*x + 2*a)^3*tan(1/2*a)^24 + 30*tan(1/2*b*x + 2*a)^3*tan(
1/2*a)^22 - 6*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^23 + tan(1/2*b*x + 2*a)*tan(1
/2*a)^24 - 756*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^20 + 614*tan(1/2*b*x + 2*a)^
2*tan(1/2*a)^21 - 114*tan(1/2*b*x + 2*a)*tan(1/2*a)^22 + 6*tan(1/2*a)^23 +
2058*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^18 - 4578*tan(1/2*b*x + 2*a)^2*tan(1/2
*a)^19 + 1932*tan(1/2*b*x + 2*a)*tan(1/2*a)^20 - 182*tan(1/2*a)^21 - 27*tan
(1/2*b*x + 2*a)^3*tan(1/2*a)^16 + 6210*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^17 -
 7462*tan(1/2*b*x + 2*a)*tan(1/2*a)^18 + 1554*tan(1/2*a)^19 - 9396*tan(1/2*
b*x + 2*a)^3*tan(1/2*a)^14 + 15588*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^15 - 233
1*tan(1/2*b*x + 2*a)*tan(1/2*a)^16 - 2178*tan(1/2*a)^17 - 21924*tan(1/2*b*x
 + 2*a)^2*tan(1/2*a)^13 + 26028*tan(1/2*b*x + 2*a)*tan(1/2*a)^14 - 5668*tan
(1/2*a)^15 + 9396*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^10 - 21924*tan(1/2*b*x +
2*a)^2*tan(1/2*a)^11 + 6468*tan(1/2*a)^13 + 27*tan(1/2*b*x + 2*a)^3*tan(1/2
*a)^8 + 15588*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^9 - 26028*tan(1/2*b*x + 2*a)*
tan(1/2*a)^10 + 6468*tan(1/2*a)^11 - 2058*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^6
 + 6210*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^7 + 2331*tan(1/2*b*x + 2*a)*tan(1/2
*a)^8 - 5668*tan(1/2*a)^9 + 756*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^4 - 4578*ta
n(1/2*b*x + 2*a)^2*tan(1/2*a)^5 + 7462*tan(1/2*b*x + 2*a)*tan(1/2*a)^6 - 21
78*tan(1/2*a)^7 - 30*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^2 + 614*tan(1/2*b*x +
2*a)^2*tan(1/2*a)^3 - 1932*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 1554*tan(1/2*a
)^5 - tan(1/2*b*x + 2*a)^3 - 6*tan(1/2*b*x + 2*a)^2*tan(1/2*a) + 114*tan(1/
2*b*x + 2*a)*tan(1/2*a)^2 - 182*tan(1/2*a)^3 - tan(1/2*b*x + 2*a) + 6*tan(1
/2*a))/((tan(1/2*a)^12 - 30*tan(1/2*a)^10 + 255*tan(1/2*a)^8 - 452*tan(1/2*
a)^6 + 255*tan(1/2*a)^4 - 30*tan(1/2*a)^2 + 1)*(tan(1/2*b*x + 2*a)^2*tan(1/
2*a)^6 - 15*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^4 + 12*tan(1/2*b*x + 2*a)*tan(1
/2*a)^5 - tan(1/2*a)^6 + 15*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2 - 40*tan(1/2*
b*x + 2*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 - tan(1/2*b*x + 2*a)^2 + 12*tan(1
/2*b*x + 2*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)^2) - log(abs(tan(1/2*b*x +
2*a)*tan(1/2*a)^3 + 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - tan(1/2*a)^3 - 3*ta
n(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 - tan(1/2*b*x + 2*a) + 3*tan(1
/2*a) - 1)) + log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a
)*tan(1/2*a)^2 + tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2
*a)^2 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a) - 1)))/b
```

3.31 $\int \csc^4(2a+2bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=43

$$\frac{\sec^3(a+bx)}{48b}+\frac{\sec(a+bx)}{16b}-\frac{\tanh^{-1}(\cos(a+bx))}{16b}$$

```
[Out] -ArcTanh[Cos[a + b*x]]/(16*b) + Sec[a + b*x]/(16*b) + Sec[a + b*x]^3/(48*b)
```

Rubi [A] time = 0.0529464, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4288, 2622, 302, 207}

$$\frac{\sec^3(a+bx)}{48b}+\frac{\sec(a+bx)}{16b}-\frac{\tanh^{-1}(\cos(a+bx))}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^4*Sin[a + b*x]^3,x]
```

```
[Out] -ArcTanh[Cos[a + b*x]]/(16*b) + Sec[a + b*x]/(16*b) + Sec[a + b*x]^3/(48*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^4(2a+2bx)\sin^3(a+bx)\,dx = \frac{1}{16}\int \csc(a+bx)\sec^4(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{16b}$$
$$= \frac{\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \sec(a+bx)\right)}{16b}$$
$$= \frac{\sec(a+bx)}{16b} + \frac{\sec^3(a+bx)}{48b} + \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{16b}$$
$$= -\frac{\tanh^{-1}(\cos(a+bx))}{16b} + \frac{\sec(a+bx)}{16b} + \frac{\sec^3(a+bx)}{48b}$$

Mathematica [A] time = 0.0263777, size = 61, normalized size = 1.42

$$\frac{1}{16}\left(\frac{\sec^3(a+bx)}{3b} + \frac{\sec(a+bx)}{b} + \frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b} - \frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^4*Sin[a + b*x]^3,x]
```

```
[Out] (-(Log[Cos[(a + b*x)/2]]/b) + Log[Sin[(a + b*x)/2]]/b + Sec[a + b*x]/b + Se
c[a + b*x]^3/(3*b))/16
```

Maple [A] time = 0.035, size = 49, normalized size = 1.1

$$\frac{1}{48\,b\,(\cos(bx+a))^3} + \frac{1}{16\,b\cos(bx+a)} + \frac{\ln(\csc(bx+a) - \cot(bx+a))}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^4*sin(b*x+a)^3,x)
```

```
[Out] 1/48/b/cos(b*x+a)^3+1/16/b/cos(b*x+a)+1/16/b*ln(csc(b*x+a)-cot(b*x+a))
```

Maxima [B] time = 1.29424, size = 1332, normalized size = 30.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/96*(4*(3*cos(5*b*x + 5*a) + 10*cos(3*b*x + 3*a) + 3*cos(b*x + a))*cos(6*b
*x + 6*a) + 12*(3*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) + 1)*cos(5*b*x + 5*
a) + 12*(10*cos(3*b*x + 3*a) + 3*cos(b*x + a))*cos(4*b*x + 4*a) + 40*(3*cos
(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a) + 36*cos(2*b*x + 2*a)*cos(b*x + a) - 3*
(2*(3*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) + cos(6*b
```

```
*x + 6*a)^2 + 6*(3*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 9*cos(4*b*x + 4
*a)^2 + 9*cos(2*b*x + 2*a)^2 + 6*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(
6*b*x + 6*a) + sin(6*b*x + 6*a)^2 + 9*sin(4*b*x + 4*a)^2 + 18*sin(4*b*x + 4
*a)*sin(2*b*x + 2*a) + 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2*a) + 1)*log(c
os(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) +
 sin(a)^2) + 3*(2*(3*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x +
 6*a) + cos(6*b*x + 6*a)^2 + 6*(3*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) +
9*cos(4*b*x + 4*a)^2 + 9*cos(2*b*x + 2*a)^2 + 6*(sin(4*b*x + 4*a) + sin(2*b
*x + 2*a))*sin(6*b*x + 6*a) + sin(6*b*x + 6*a)^2 + 9*sin(4*b*x + 4*a)^2 + 1
8*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x +
2*a) + 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*si
n(b*x)*sin(a) + sin(a)^2) + 4*(3*sin(5*b*x + 5*a) + 10*sin(3*b*x + 3*a) + 3
*sin(b*x + a))*sin(6*b*x + 6*a) + 36*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*
sin(5*b*x + 5*a) + 12*(10*sin(3*b*x + 3*a) + 3*sin(b*x + a))*sin(4*b*x + 4*
a) + 120*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) + 36*sin(2*b*x + 2*a)*sin(b*x +
a) + 12*cos(b*x + a))/(b*cos(6*b*x + 6*a)^2 + 9*b*cos(4*b*x + 4*a)^2 + 9*b*
cos(2*b*x + 2*a)^2 + b*sin(6*b*x + 6*a)^2 + 9*b*sin(4*b*x + 4*a)^2 + 18*b*s
in(4*b*x + 4*a)*sin(2*b*x + 2*a) + 9*b*sin(2*b*x + 2*a)^2 + 2*(3*b*cos(4*b*
x + 4*a) + 3*b*cos(2*b*x + 2*a) + b)*cos(6*b*x + 6*a) + 6*(3*b*cos(2*b*x +
2*a) + b)*cos(4*b*x + 4*a) + 6*b*cos(2*b*x + 2*a) + 6*(b*sin(4*b*x + 4*a) +
 b*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + b)
```

Fricas [A] time = 0.501539, size = 194, normalized size = 4.51

$$-\frac{3 \cos\left(bx+a\right)^{3} \log\left(\frac{1}{2} \cos\left(bx+a\right)+\frac{1}{2}\right)-3 \cos\left(bx+a\right)^{3} \log\left(-\frac{1}{2} \cos\left(bx+a\right)+\frac{1}{2}\right)-6 \cos\left(bx+a\right)^{2}-2}{96\, b \cos\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/96*(3*cos(b*x + a)^3*log(1/2*cos(b*x + a) + 1/2) - 3*cos(b*x + a)^3*log(
-1/2*cos(b*x + a) + 1/2) - 6*cos(b*x + a)^2 - 2)/(b*cos(b*x + a)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**4*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.82518, size = 2854, normalized size = 66.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/48*(4*(18*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^35 - 3*tan(1/2*b*x + 2*a)^4*ta
n(1/2*a)^36 - 762*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^33 + 486*tan(1/2*b*x + 2*
a)^4*tan(1/2*a)^34 - 72*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^35 + 3*tan(1/2*b*x
+ 2*a)^2*tan(1/2*a)^36 + 13644*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^31 - 15561*t
an(1/2*b*x + 2*a)^4*tan(1/2*a)^32 + 5424*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^33
 - 756*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^34 + 54*tan(1/2*b*x + 2*a)*tan(1/2*a
)^35 - 2*tan(1/2*a)^36 - 140076*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^29 + 233916
*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^30 - 126936*tan(1/2*b*x + 2*a)^3*tan(1/2*a
)^31 + 28701*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^32 - 2934*tan(1/2*b*x + 2*a)*t
an(1/2*a)^33 + 126*tan(1/2*a)^34 + 811140*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^2
7 - 1893744*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^28 + 1474776*tan(1/2*b*x + 2*a)
^3*tan(1/2*a)^29 - 471816*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^30 + 64908*tan(1/
2*b*x + 2*a)*tan(1/2*a)^31 - 3330*tan(1/2*a)^32 - 1739556*tan(1/2*b*x + 2*a
)^5*tan(1/2*a)^25 + 6839316*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^26 - 8095576*ta
n(1/2*b*x + 2*a)^3*tan(1/2*a)^27 + 3832164*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^
28 - 727596*tan(1/2*b*x + 2*a)*tan(1/2*a)^29 + 47148*tan(1/2*a)^30 - 461700
*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^23 - 6247800*tan(1/2*b*x + 2*a)^4*tan(1/2*
a)^24 + 16873272*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^25 - 13541976*tan(1/2*b*x
+ 2*a)^2*tan(1/2*a)^26 + 4049556*tan(1/2*b*x + 2*a)*tan(1/2*a)^27 - 368892*
tan(1/2*a)^28 + 5103972*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^21 - 14529780*tan(1
/2*b*x + 2*a)^4*tan(1/2*a)^22 + 3596040*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^23
+ 12515100*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^24 - 8506836*tan(1/2*b*x + 2*a)*
tan(1/2*a)^25 + 1396836*tan(1/2*a)^26 - 3586680*tan(1/2*b*x + 2*a)^5*tan(1/
2*a)^19 + 28026162*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^20 - 51234312*tan(1/2*b*
x + 2*a)^3*tan(1/2*a)^21 + 28649880*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^22 - 19
42020*tan(1/2*b*x + 2*a)*tan(1/2*a)^23 - 1226940*tan(1/2*a)^24 - 3586680*ta
n(1/2*b*x + 2*a)^5*tan(1/2*a)^17 + 37245240*tan(1/2*b*x + 2*a)^3*tan(1/2*a)
^19 - 56612142*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^20 + 25560228*tan(1/2*b*x +
2*a)*tan(1/2*a)^21 - 2935620*tan(1/2*a)^22 + 5103972*tan(1/2*b*x + 2*a)^5*t
an(1/2*a)^15 - 28026162*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^16 + 37245240*tan(1
/2*b*x + 2*a)^3*tan(1/2*a)^17 - 18495360*tan(1/2*b*x + 2*a)*tan(1/2*a)^19 +
 5548608*tan(1/2*a)^20 - 461700*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^13 + 145297
80*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^14 - 51234312*tan(1/2*b*x + 2*a)^3*tan(1
/2*a)^15 + 56612142*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^16 - 18495360*tan(1/2*b
*x + 2*a)*tan(1/2*a)^17 - 1739556*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^11 + 6247
800*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^12 + 3596040*tan(1/2*b*x + 2*a)^3*tan(1
/2*a)^13 - 28649880*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^14 + 25560228*tan(1/2*b
*x + 2*a)*tan(1/2*a)^15 - 5548608*tan(1/2*a)^16 + 811140*tan(1/2*b*x + 2*a)
^5*tan(1/2*a)^9 - 6839316*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^10 + 16873272*tan
(1/2*b*x + 2*a)^3*tan(1/2*a)^11 - 12515100*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^
12 - 1942020*tan(1/2*b*x + 2*a)*tan(1/2*a)^13 + 2935620*tan(1/2*a)^14 - 140
076*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^7 + 1893744*tan(1/2*b*x + 2*a)^4*tan(1/
2*a)^8 - 8095576*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^9 + 13541976*tan(1/2*b*x +
 2*a)^2*tan(1/2*a)^10 - 8506836*tan(1/2*b*x + 2*a)*tan(1/2*a)^11 + 1226940*
tan(1/2*a)^12 + 13644*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^5 - 233916*tan(1/2*b*
x + 2*a)^4*tan(1/2*a)^6 + 1474776*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^7 - 38321
64*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^8 + 4049556*tan(1/2*b*x + 2*a)*tan(1/2*a
)^9 - 1396836*tan(1/2*a)^10 - 762*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^3 + 15561
*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^4 - 126936*tan(1/2*b*x + 2*a)^3*tan(1/2*a)
^5 + 471816*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^6 - 727596*tan(1/2*b*x + 2*a)*t
an(1/2*a)^7 + 368892*tan(1/2*a)^8 + 18*tan(1/2*b*x + 2*a)^5*tan(1/2*a) - 48
6*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^2 + 5424*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^
3 - 28701*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^4 + 64908*tan(1/2*b*x + 2*a)*tan(
1/2*a)^5 - 47148*tan(1/2*a)^6 + 3*tan(1/2*b*x + 2*a)^4 - 72*tan(1/2*b*x + 2
*a)^3*tan(1/2*a) + 756*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2 - 2934*tan(1/2*b*x
 + 2*a)*tan(1/2*a)^3 + 3330*tan(1/2*a)^4 - 3*tan(1/2*b*x + 2*a)^2 + 54*tan(
1/2*b*x + 2*a)*tan(1/2*a) - 126*tan(1/2*a)^2 + 2)/((tan(1/2*a)^18 - 45*tan(
1/2*a)^16 + 720*tan(1/2*a)^14 - 4728*tan(1/2*a)^12 + 10890*tan(1/2*a)^10 -
10890*tan(1/2*a)^8 + 4728*tan(1/2*a)^6 - 720*tan(1/2*a)^4 + 45*tan(1/2*a)^2
```

```
 - 1)*(tan(1/2*b*x + 2*a)^2*tan(1/2*a)^6 - 15*tan(1/2*b*x + 2*a)^2*tan(1/2*
a)^4 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - tan(1/2*a)^6 + 15*tan(1/2*b*x +
 2*a)^2*tan(1/2*a)^2 - 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4
 - tan(1/2*b*x + 2*a)^2 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a) - 15*tan(1/2*a)^
2 + 1)^3) + 3*log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a
)*tan(1/2*a) + 3*tan(1/2*a)^2 - 1)) - 3*log(abs(3*tan(1/2*b*x + 2*a)*tan(1/
2*a)^2 - tan(1/2*a)^3 - tan(1/2*b*x + 2*a) + 3*tan(1/2*a))))/b
```

3.32 $\int \csc^5(2a+2bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=70

$$-\frac{15\csc(a+bx)}{256b}+\frac{15\tanh^{-1}(\sin(a+bx))}{256b}+\frac{\csc(a+bx)\sec^4(a+bx)}{128b}+\frac{5\csc(a+bx)\sec^2(a+bx)}{256b}$$

```
[Out] (15*ArcTanh[Sin[a + b*x]])/(256*b) - (15*Csc[a + b*x])/(256*b) + (5*Csc[a +
 b*x]*Sec[a + b*x]^2)/(256*b) + (Csc[a + b*x]*Sec[a + b*x]^4)/(128*b)
```

Rubi [A] time = 0.0721454, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4288, 2621, 288, 321, 207}

$$-\frac{15\csc(a+bx)}{256b}+\frac{15\tanh^{-1}(\sin(a+bx))}{256b}+\frac{\csc(a+bx)\sec^4(a+bx)}{128b}+\frac{5\csc(a+bx)\sec^2(a+bx)}{256b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^5*Sin[a + b*x]^3,x]
```

```
[Out] (15*ArcTanh[Sin[a + b*x]])/(256*b) - (15*Csc[a + b*x])/(256*b) + (5*Csc[a +
 b*x]*Sec[a + b*x]^2)/(256*b) + (Csc[a + b*x]*Sec[a + b*x]^4)/(128*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^5(2a+2bx)\sin^3(a+bx)\,dx = \frac{1}{32}\int \csc^2(a+bx)\sec^5(a+bx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^3}\,dx, x, \csc(a+bx)\right)}{32b}$$
$$= \frac{\csc(a+bx)\sec^4(a+bx)}{128b} - \frac{5\,\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \csc(a+bx)\right)}{128b}$$
$$= \frac{5\csc(a+bx)\sec^2(a+bx)}{256b} + \frac{\csc(a+bx)\sec^4(a+bx)}{128b} - \frac{15\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc\right.}{256b}$$
$$= -\frac{15\csc(a+bx)}{256b} + \frac{5\csc(a+bx)\sec^2(a+bx)}{256b} + \frac{\csc(a+bx)\sec^4(a+bx)}{128b} - \frac{15\,\text{Subst}}{}$$
$$= \frac{15\tanh^{-1}(\sin(a+bx))}{256b} - \frac{15\csc(a+bx)}{256b} + \frac{5\csc(a+bx)\sec^2(a+bx)}{256b} + \frac{\csc(a+bx)}{12}$$

Mathematica [C] time = 0.0345482, size = 29, normalized size = 0.41

$$-\frac{\csc(a+bx)\text{Hypergeometric2F1}\left(-\frac{1}{2},3,\frac{1}{2},\sin^2(a+bx)\right)}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^5*Sin[a + b*x]^3,x]

[Out] -(Csc[a + b*x]*Hypergeometric2F1[-1/2, 3, 1/2, Sin[a + b*x]^2])/(32*b)
```

Maple [A] time = 0.036, size = 76, normalized size = 1.1

$$\frac{1}{128\,b\sin(bx+a)(\cos(bx+a))^4} + \frac{5}{256\,b\sin(bx+a)(\cos(bx+a))^2} - \frac{15}{256\,b\sin(bx+a)} + \frac{15\ln(\sec(bx+a)+\tan(bx}{256\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^5*sin(b*x+a)^3,x)

[Out] 1/128/b/sin(b*x+a)/cos(b*x+a)^4+5/256/b/sin(b*x+a)/cos(b*x+a)^2-15/256/b/si
n(b*x+a)+15/256/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [B] time = 2.06677, size = 2437, normalized size = 34.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/512*(4*(15*sin(9*b*x + 9*a) + 40*sin(7*b*x + 7*a) + 18*sin(5*b*x + 5*a) +
 40*sin(3*b*x + 3*a) + 15*sin(b*x + a))*cos(10*b*x + 10*a) - 60*(3*sin(8*b*
x + 8*a) + 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*co
s(9*b*x + 9*a) + 12*(40*sin(7*b*x + 7*a) + 18*sin(5*b*x + 5*a) + 40*sin(3*b
*x + 3*a) + 15*sin(b*x + a))*cos(8*b*x + 8*a) - 160*(2*sin(6*b*x + 6*a) - 2
*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*cos(7*b*x + 7*a) + 8*(18*sin(5*b*x
+ 5*a) + 40*sin(3*b*x + 3*a) + 15*sin(b*x + a))*cos(6*b*x + 6*a) + 72*(2*si
n(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*cos(5*b*x + 5*a) - 40*(8*sin(3*b*x + 3
*a) + 3*sin(b*x + a))*cos(4*b*x + 4*a) - 15*(2*(3*cos(8*b*x + 8*a) + 2*cos(
6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) - 1)*cos(10*b*x + 10
*a) + cos(10*b*x + 10*a)^2 + 6*(2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - 3
*cos(2*b*x + 2*a) - 1)*cos(8*b*x + 8*a) + 9*cos(8*b*x + 8*a)^2 - 4*(2*cos(4
*b*x + 4*a) + 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) + 4*cos(6*b*x + 6*a)
^2 + 4*(3*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*cos(4*b*x + 4*a)^2 + 9
*cos(2*b*x + 2*a)^2 + 2*(3*sin(8*b*x + 8*a) + 2*sin(6*b*x + 6*a) - 2*sin(4*
b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + sin(10*b*x + 10*a)^2
+ 6*(2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(8*b*
x + 8*a) + 9*sin(8*b*x + 8*a)^2 - 4*(2*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a
))*sin(6*b*x + 6*a) + 4*sin(6*b*x + 6*a)^2 + 4*sin(4*b*x + 4*a)^2 + 12*sin(
4*b*x + 4*a)*sin(2*b*x + 2*a) + 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2*a) +
 1)*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x +
2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 +
 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin
(a)^2)) - 4*(15*cos(9*b*x + 9*a) + 40*cos(7*b*x + 7*a) + 18*cos(5*b*x + 5*a
) + 40*cos(3*b*x + 3*a) + 15*cos(b*x + a))*sin(10*b*x + 10*a) + 60*(3*cos(8
*b*x + 8*a) + 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a)
- 1)*sin(9*b*x + 9*a) - 12*(40*cos(7*b*x + 7*a) + 18*cos(5*b*x + 5*a) + 40*
cos(3*b*x + 3*a) + 15*cos(b*x + a))*sin(8*b*x + 8*a) + 160*(2*cos(6*b*x + 6
*a) - 2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) - 1)*sin(7*b*x + 7*a) - 8*(18
*cos(5*b*x + 5*a) + 40*cos(3*b*x + 3*a) + 15*cos(b*x + a))*sin(6*b*x + 6*a)
 - 72*(2*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) + 1)*sin(5*b*x + 5*a) + 40*(
8*cos(3*b*x + 3*a) + 3*cos(b*x + a))*sin(4*b*x + 4*a) - 160*(3*cos(2*b*x +
2*a) + 1)*sin(3*b*x + 3*a) + 480*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) + 180*co
s(b*x + a)*sin(2*b*x + 2*a) - 180*cos(2*b*x + 2*a)*sin(b*x + a) - 60*sin(b*
x + a))/(b*cos(10*b*x + 10*a)^2 + 9*b*cos(8*b*x + 8*a)^2 + 4*b*cos(6*b*x +
6*a)^2 + 4*b*cos(4*b*x + 4*a)^2 + 9*b*cos(2*b*x + 2*a)^2 + b*sin(10*b*x + 1
0*a)^2 + 9*b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 + 4*b*sin(4*b*x +
4*a)^2 + 12*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 9*b*sin(2*b*x + 2*a)^2 +
2*(3*b*cos(8*b*x + 8*a) + 2*b*cos(6*b*x + 6*a) - 2*b*cos(4*b*x + 4*a) - 3*b
*cos(2*b*x + 2*a) - b)*cos(10*b*x + 10*a) + 6*(2*b*cos(6*b*x + 6*a) - 2*b*c
os(4*b*x + 4*a) - 3*b*cos(2*b*x + 2*a) - b)*cos(8*b*x + 8*a) - 4*(2*b*cos(4
*b*x + 4*a) + 3*b*cos(2*b*x + 2*a) + b)*cos(6*b*x + 6*a) + 4*(3*b*cos(2*b*x
 + 2*a) + b)*cos(4*b*x + 4*a) + 6*b*cos(2*b*x + 2*a) + 2*(3*b*sin(8*b*x + 8
*a) + 2*b*sin(6*b*x + 6*a) - 2*b*sin(4*b*x + 4*a) - 3*b*sin(2*b*x + 2*a))*s
in(10*b*x + 10*a) + 6*(2*b*sin(6*b*x + 6*a) - 2*b*sin(4*b*x + 4*a) - 3*b*si
n(2*b*x + 2*a))*sin(8*b*x + 8*a) - 4*(2*b*sin(4*b*x + 4*a) + 3*b*sin(2*b*x
+ 2*a))*sin(6*b*x + 6*a) + b)
```

Fricas [A] time = 0.509329, size = 262, normalized size = 3.74

$$\frac{15\cos(bx+a)^4\log(\sin(bx+a)+1)\sin(bx+a)-15\cos(bx+a)^4\log(-\sin(bx+a)+1)\sin(bx+a)-30\cos(bx}{512\,b\cos(bx+a)^4\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/512*(15*cos(b*x + a)^4*log(sin(b*x + a) + 1)*sin(b*x + a) - 15*cos(b*x +
a)^4*log(-sin(b*x + a) + 1)*sin(b*x + a) - 30*cos(b*x + a)^4 + 10*cos(b*x +
 a)^2 + 4)/(b*cos(b*x + a)^4*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**5*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 5.78996, size = 5443, normalized size = 77.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/256*(4*(tan(1/2*b*x + 2*a)*tan(1/2*a)^12 - 12*tan(1/2*b*x + 2*a)*tan(1/2
*a)^10 + 6*tan(1/2*a)^11 - 27*tan(1/2*b*x + 2*a)*tan(1/2*a)^8 - 2*tan(1/2*a
)^9 - 36*tan(1/2*a)^7 + 27*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 - 36*tan(1/2*a)^
5 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - 2*tan(1/2*a)^3 - tan(1/2*b*x + 2*a
) + 6*tan(1/2*a))/((3*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 - tan(1/2*b*x + 2*a
)*tan(1/2*a)^6 - 10*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 + 15*tan(1/2*b*x + 2*
a)*tan(1/2*a)^4 - 3*tan(1/2*a)^5 + 3*tan(1/2*b*x + 2*a)^2*tan(1/2*a) - 15*t
an(1/2*b*x + 2*a)*tan(1/2*a)^2 + 10*tan(1/2*a)^3 + tan(1/2*b*x + 2*a) - 3*t
an(1/2*a))*(3*tan(1/2*a)^5 - 10*tan(1/2*a)^3 + 3*tan(1/2*a))) + 2*(9*tan(1/
2*b*x + 2*a)^7*tan(1/2*a)^48 - 54*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^46 + 90*t
an(1/2*b*x + 2*a)^6*tan(1/2*a)^47 - tan(1/2*b*x + 2*a)^5*tan(1/2*a)^48 - 16
938*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^44 + 10878*tan(1/2*b*x + 2*a)^6*tan(1/2
*a)^45 - 2058*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^46 + 162*tan(1/2*b*x + 2*a)^4
*tan(1/2*a)^47 - tan(1/2*b*x + 2*a)^3*tan(1/2*a)^48 + 648690*tan(1/2*b*x +
2*a)^7*tan(1/2*a)^42 - 772902*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^43 + 320922*t
an(1/2*b*x + 2*a)^5*tan(1/2*a)^44 - 59994*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^4
5 + 5718*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^46 - 306*tan(1/2*b*x + 2*a)^2*tan(
1/2*a)^47 + 9*tan(1/2*b*x + 2*a)*tan(1/2*a)^48 - 11649780*tan(1/2*b*x + 2*a
)^7*tan(1/2*a)^40 + 20073870*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^41 - 12606642*
tan(1/2*b*x + 2*a)^5*tan(1/2*a)^42 + 3790962*tan(1/2*b*x + 2*a)^4*tan(1/2*a
)^43 - 601830*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^44 + 53562*tan(1/2*b*x + 2*a)
^2*tan(1/2*a)^45 - 2646*tan(1/2*b*x + 2*a)*tan(1/2*a)^46 + 54*tan(1/2*a)^47
 + 122301894*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^38 - 281501690*tan(1/2*b*x + 2
*a)^6*tan(1/2*a)^39 + 240373332*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^40 - 992325
06*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^41 + 21805614*tan(1/2*b*x + 2*a)^3*tan(1
/2*a)^42 - 2617650*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^43 + 166230*tan(1/2*b*x
+ 2*a)*tan(1/2*a)^44 - 4446*tan(1/2*a)^45 - 762446542*tan(1/2*b*x + 2*a)^7*
tan(1/2*a)^36 + 2299471746*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^37 - 2589365766*
tan(1/2*b*x + 2*a)^5*tan(1/2*a)^38 + 1410179550*tan(1/2*b*x + 2*a)^4*tan(1/
2*a)^39 - 404516268*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^40 + 62438634*tan(1/2*b
```

*x + 2*a)^2*tan(1/2*a)^41 - 4933038*tan(1/2*b*x + 2*a)*tan(1/2*a)^42 + 1594
62*tan(1/2*a)^43 + 2624819022*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^34 - 10699256
970*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^35 + 16002520222*tan(1/2*b*x + 2*a)^5*t
an(1/2*a)^36 - 11500345350*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^37 + 4307714970*
tan(1/2*b*x + 2*a)^3*tan(1/2*a)^38 - 849637038*tan(1/2*b*x + 2*a)^2*tan(1/2
*a)^39 + 83542284*tan(1/2*b*x + 2*a)*tan(1/2*a)^40 - 3248766*tan(1/2*a)^41
- 4363726131*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^32 + 26510577474*tan(1/2*b*x +
2*a)^6*tan(1/2*a)^33 - 55167038478*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^34 + 53
489761470*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^35 - 26616485090*tan(1/2*b*x + 2*
a)^3*tan(1/2*a)^36 + 6857208774*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^37 - 856321
050*tan(1/2*b*x + 2*a)*tan(1/2*a)^38 + 40894922*tan(1/2*a)^39 + 148294628*t
an(1/2*b*x + 2*a)^7*tan(1/2*a)^30 - 23718089916*tan(1/2*b*x + 2*a)^6*tan(1/
2*a)^31 + 91329113691*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^32 - 132573864918*tan
(1/2*b*x + 2*a)^4*tan(1/2*a)^33 + 92023852050*tan(1/2*b*x + 2*a)^3*tan(1/2*
a)^34 - 32055103710*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^35 + 5284344754*tan(1/2
*b*x + 2*a)*tan(1/2*a)^36 - 322093122*tan(1/2*a)^37 + 11061475644*tan(1/2*b
*x + 2*a)^7*tan(1/2*a)^28 - 36130112340*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^29
- 3422040164*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^30 + 118445882004*tan(1/2*b*x
+ 2*a)^4*tan(1/2*a)^31 - 152452905381*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^32 +
79837550214*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^33 - 18462316050*tan(1/2*b*x +
2*a)*tan(1/2*a)^34 + 1522201770*tan(1/2*a)^35 - 14430929868*tan(1/2*b*x + 2
*a)^7*tan(1/2*a)^26 + 104533629188*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^27 - 231
729549276*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^28 + 180365650620*tan(1/2*b*x + 2
*a)^4*tan(1/2*a)^29 + 5258310492*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^30 - 71047
415988*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^31 + 30651378381*tan(1/2*b*x + 2*a)*
tan(1/2*a)^32 - 3833122290*tan(1/2*a)^33 - 62534029428*tan(1/2*b*x + 2*a)^6
*tan(1/2*a)^25 + 304139029068*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^26 - 52273462
2092*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^27 + 386477003556*tan(1/2*b*x + 2*a)^3
*tan(1/2*a)^28 - 109057608828*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^29 - 75017558
0*tan(1/2*b*x + 2*a)*tan(1/2*a)^30 + 3377366748*tan(1/2*a)^31 + 14430929868
*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^22 - 62534029428*tan(1/2*b*x + 2*a)^6*tan(
1/2*a)^23 + 313058642364*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^25 - 506043183348*
tan(1/2*b*x + 2*a)^3*tan(1/2*a)^26 + 313110921612*tan(1/2*b*x + 2*a)^2*tan(
1/2*a)^27 - 77490791364*tan(1/2*b*x + 2*a)*tan(1/2*a)^28 + 5273542932*tan(1
/2*a)^29 - 11061475644*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^20 + 104533629188*ta
n(1/2*b*x + 2*a)^6*tan(1/2*a)^21 - 304139029068*tan(1/2*b*x + 2*a)^5*tan(1/
2*a)^22 + 313058642364*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^23 - 186855789276*ta
n(1/2*b*x + 2*a)^2*tan(1/2*a)^25 + 100600780788*tan(1/2*b*x + 2*a)*tan(1/2*
a)^26 - 14857714436*tan(1/2*a)^27 - 148294628*tan(1/2*b*x + 2*a)^7*tan(1/2*
a)^18 - 36130112340*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^19 + 231729549276*tan(1
/2*b*x + 2*a)^5*tan(1/2*a)^20 - 522734622092*tan(1/2*b*x + 2*a)^4*tan(1/2*a
)^21 + 506043183348*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^22 - 186855789276*tan(1
/2*b*x + 2*a)^2*tan(1/2*a)^23 + 8802017172*tan(1/2*a)^25 + 4363726131*tan(1
/2*b*x + 2*a)^7*tan(1/2*a)^16 - 23718089916*tan(1/2*b*x + 2*a)^6*tan(1/2*a)
^17 + 3422040164*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^18 + 180365650620*tan(1/2*
b*x + 2*a)^4*tan(1/2*a)^19 - 386477003556*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^2
0 + 313110921612*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^21 - 100600780788*tan(1/2*
b*x + 2*a)*tan(1/2*a)^22 + 8802017172*tan(1/2*a)^23 - 2624819022*tan(1/2*b*
x + 2*a)^7*tan(1/2*a)^14 + 26510577474*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^15 -
91329113691*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^16 + 118445882004*tan(1/2*b*x
+ 2*a)^4*tan(1/2*a)^17 - 5258310492*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^18 - 10
9057608828*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^19 + 77490791364*tan(1/2*b*x + 2
*a)*tan(1/2*a)^20 - 14857714436*tan(1/2*a)^21 + 762446542*tan(1/2*b*x + 2*a
)^7*tan(1/2*a)^12 - 10699256970*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^13 + 551670
38478*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^14 - 132573864918*tan(1/2*b*x + 2*a)^
4*tan(1/2*a)^15 + 152452905381*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^16 - 7104741
5988*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^17 + 750175580*tan(1/2*b*x + 2*a)*tan(
1/2*a)^18 + 5273542932*tan(1/2*a)^19 - 122301894*tan(1/2*b*x + 2*a)^7*tan(1
/2*a)^10 + 2299471746*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^11 - 16002520222*tan(

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1/2*b*x + 2*a)^5*tan(1/2*a)^12 + 53489761470*tan(1/2*b*x + 2*a)^4*tan(1/2*a
)^13 - 92023852050*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^14 + 79837550214*tan(1/2
*b*x + 2*a)^2*tan(1/2*a)^15 - 30651378381*tan(1/2*b*x + 2*a)*tan(1/2*a)^16
+ 3377366748*tan(1/2*a)^17 + 11649780*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^8 - 2
81501690*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^9 + 2589365766*tan(1/2*b*x + 2*a)^
5*tan(1/2*a)^10 - 11500345350*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^11 + 26616485
090*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^12 - 32055103710*tan(1/2*b*x + 2*a)^2*t
an(1/2*a)^13 + 18462316050*tan(1/2*b*x + 2*a)*tan(1/2*a)^14 - 3833122290*ta
n(1/2*a)^15 - 648690*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^6 + 20073870*tan(1/2*b
*x + 2*a)^6*tan(1/2*a)^7 - 240373332*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^8 + 14
10179550*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^9 - 4307714970*tan(1/2*b*x + 2*a)^
3*tan(1/2*a)^10 + 6857208774*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^11 - 528434475
4*tan(1/2*b*x + 2*a)*tan(1/2*a)^12 + 1522201770*tan(1/2*a)^13 + 16938*tan(1
/2*b*x + 2*a)^7*tan(1/2*a)^4 - 772902*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^5 + 1
2606642*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^6 - 99232506*tan(1/2*b*x + 2*a)^4*t
an(1/2*a)^7 + 404516268*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^8 - 849637038*tan(1
/2*b*x + 2*a)^2*tan(1/2*a)^9 + 856321050*tan(1/2*b*x + 2*a)*tan(1/2*a)^10 -
 322093122*tan(1/2*a)^11 + 54*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^2 + 10878*tan
(1/2*b*x + 2*a)^6*tan(1/2*a)^3 - 320922*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^4 +
 3790962*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^5 - 21805614*tan(1/2*b*x + 2*a)^3*
tan(1/2*a)^6 + 62438634*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^7 - 83542284*tan(1/
2*b*x + 2*a)*tan(1/2*a)^8 + 40894922*tan(1/2*a)^9 - 9*tan(1/2*b*x + 2*a)^7
+ 90*tan(1/2*b*x + 2*a)^6*tan(1/2*a) + 2058*tan(1/2*b*x + 2*a)^5*tan(1/2*a)
^2 - 59994*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^3 + 601830*tan(1/2*b*x + 2*a)^3*
tan(1/2*a)^4 - 2617650*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 + 4933038*tan(1/2*
b*x + 2*a)*tan(1/2*a)^6 - 3248766*tan(1/2*a)^7 + tan(1/2*b*x + 2*a)^5 + 162
*tan(1/2*b*x + 2*a)^4*tan(1/2*a) - 5718*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^2 +
 53562*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 - 166230*tan(1/2*b*x + 2*a)*tan(1/
2*a)^4 + 159462*tan(1/2*a)^5 + tan(1/2*b*x + 2*a)^3 - 306*tan(1/2*b*x + 2*a
)^2*tan(1/2*a) + 2646*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - 4446*tan(1/2*a)^3 -
 9*tan(1/2*b*x + 2*a) + 54*tan(1/2*a))/((tan(1/2*a)^24 - 60*tan(1/2*a)^22 +
 1410*tan(1/2*a)^20 - 16204*tan(1/2*a)^18 + 92655*tan(1/2*a)^16 - 245880*ta
n(1/2*a)^14 + 336156*tan(1/2*a)^12 - 245880*tan(1/2*a)^10 + 92655*tan(1/2*a
)^8 - 16204*tan(1/2*a)^6 + 1410*tan(1/2*a)^4 - 60*tan(1/2*a)^2 + 1)*(tan(1/
2*b*x + 2*a)^2*tan(1/2*a)^6 - 15*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^4 + 12*tan
(1/2*b*x + 2*a)*tan(1/2*a)^5 - tan(1/2*a)^6 + 15*tan(1/2*b*x + 2*a)^2*tan(1
/2*a)^2 - 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 - tan(1/2*b*
x + 2*a)^2 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)^4) - 1
5*log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 3*tan(1/2*b*x + 2*a)*tan(1/2*a)
^2 - tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 - tan(
1/2*b*x + 2*a) + 3*tan(1/2*a) - 1)) + 15*log(abs(tan(1/2*b*x + 2*a)*tan(1/2
*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 + tan(1/2*a)^3 - 3*tan(1/2*b*x +
2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a) - 1)))
/b
```

3.33 $\int \csc(a+bx)\sin^8(2a+2bx)\,dx$

Optimal. Leaf size=61

$$\frac{256\cos^{15}(a+bx)}{15b}-\frac{768\cos^{13}(a+bx)}{13b}+\frac{768\cos^{11}(a+bx)}{11b}-\frac{256\cos^9(a+bx)}{9b}$$

```
[Out] (-256*Cos[a + b*x]^9)/(9*b) + (768*Cos[a + b*x]^11)/(11*b) - (768*Cos[a + b
*x]^13)/(13*b) + (256*Cos[a + b*x]^15)/(15*b)
```

Rubi [A] time = 0.0613116, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2565, 270}

$$\frac{256\cos^{15}(a+bx)}{15b}-\frac{768\cos^{13}(a+bx)}{13b}+\frac{768\cos^{11}(a+bx)}{11b}-\frac{256\cos^9(a+bx)}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^8,x]
```

```
[Out] (-256*Cos[a + b*x]^9)/(9*b) + (768*Cos[a + b*x]^11)/(11*b) - (768*Cos[a + b
*x]^13)/(13*b) + (256*Cos[a + b*x]^15)/(15*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \csc(a+bx)\sin^8(2a+2bx)\,dx &= 256\int \cos^8(a+bx)\sin^7(a+bx)\,dx\\ &= -\frac{256\,\text{Subst}\left(\int x^8\left(1-x^2\right)^3\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{256\,\text{Subst}\left(\int \left(x^8-3x^{10}+3x^{12}-x^{14}\right)dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{256\cos^9(a+bx)}{9b}+\frac{768\cos^{11}(a+bx)}{11b}-\frac{768\cos^{13}(a+bx)}{13b}+\frac{256\cos^{15}(a+bx)}{15b}\end{aligned}$$

Mathematica [A] time = 0.0936457, size = 119, normalized size = 1.95

$$-\frac{35\cos(a+bx)}{64b}-\frac{35\cos(3(a+bx))}{192b}+\frac{21\cos(5(a+bx))}{320b}+\frac{3\cos(7(a+bx))}{64b}-\frac{7\cos(9(a+bx))}{576b}-\frac{7\cos(11(a+bx))}{704b}+\frac{\text{c}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^8,x]
```

```
[Out] (-35*Cos[a + b*x])/(64*b) - (35*Cos[3*(a + b*x)])/(192*b) + (21*Cos[5*(a +
b*x)])/(320*b) + (3*Cos[7*(a + b*x)])/(64*b) - (7*Cos[9*(a + b*x)])/(576*b)
 - (7*Cos[11*(a + b*x)])/(704*b) + Cos[13*(a + b*x)]/(832*b) + Cos[15*(a +
b*x)]/(960*b)
```

Maple [A] time = 0.036, size = 71, normalized size = 1.2

$$256\,\frac{1}{b}\left(-1/15\,(\sin(bx+a))^{6}(\cos(bx+a))^{9}-\frac{2\,(\sin(bx+a))^{4}(\cos(bx+a))^{9}}{65}-\frac{8\,(\sin(bx+a))^{2}(\cos(bx+a))^{9}}{715}-\frac{16\,(\text{c}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^8,x)
```

```
[Out] 256/b*(-1/15*sin(b*x+a)^6*cos(b*x+a)^9-2/65*sin(b*x+a)^4*cos(b*x+a)^9-8/715
*sin(b*x+a)^2*cos(b*x+a)^9-16/6435*cos(b*x+a)^9)
```

Maxima [A] time = 1.06837, size = 123, normalized size = 2.02

$$\frac{429\,\cos(15\,bx+15\,a)+495\,\cos(13\,bx+13\,a)-4095\,\cos(11\,bx+11\,a)-5005\,\cos(9\,bx+9\,a)+19305\,\cos(7\,bx+7\,a}{411840\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^8,x, algorithm="maxima")
```

```
[Out] 1/411840*(429*cos(15*b*x + 15*a) + 495*cos(13*b*x + 13*a) - 4095*cos(11*b*x
 + 11*a) - 5005*cos(9*b*x + 9*a) + 19305*cos(7*b*x + 7*a) + 27027*cos(5*b*x
 + 5*a) - 75075*cos(3*b*x + 3*a) - 225225*cos(b*x + a))/b
```

Fricas [A] time = 0.538362, size = 136, normalized size = 2.23

$$\frac{256\left(429\,\cos(bx+a)^{15}-1485\,\cos(bx+a)^{13}+1755\,\cos(bx+a)^{11}-715\,\cos(bx+a)^{9}\right)}{6435\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^8,x, algorithm="fricas")
```

```
[Out] 256/6435*(429*cos(b*x + a)^15 - 1485*cos(b*x + a)^13 + 1755*cos(b*x + a)^11
 - 715*cos(b*x + a)^9)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.93562, size = 365, normalized size = 5.98

$$-\frac{8192\left(\frac{15\,(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{105\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{455\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{5070\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+\frac{30030\,(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5}+\frac{70070\,(\cos(bx+a)-1)^6}{(\cos(bx+a)+1)^6}\cdots}{6435\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^8,x, algorithm="giac")
```

```
[Out] -8192/6435*(15*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 105*(cos(b*x + a) -
1)^2/(cos(b*x + a) + 1)^2 + 455*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 +
 5070*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 30030*(cos(b*x + a) - 1)^
5/(cos(b*x + a) + 1)^5 + 70070*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 +
115830*(cos(b*x + a) - 1)^7/(cos(b*x + a) + 1)^7 + 109395*(cos(b*x + a) - 1
)^8/(cos(b*x + a) + 1)^8 + 75075*(cos(b*x + a) - 1)^9/(cos(b*x + a) + 1)^9
+ 27027*(cos(b*x + a) - 1)^10/(cos(b*x + a) + 1)^10 + 6435*(cos(b*x + a) -
1)^11/(cos(b*x + a) + 1)^11 - 1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1)
- 1)^15)
```

3.34 $\int \csc(a+bx)\sin^7(2a+2bx)\,dx$

Optimal. Leaf size=61

$$-\frac{128\sin^{13}(a+bx)}{13b}+\frac{384\sin^{11}(a+bx)}{11b}-\frac{128\sin^{9}(a+bx)}{3b}+\frac{128\sin^{7}(a+bx)}{7b}$$

```
[Out] (128*Sin[a + b*x]^7)/(7*b) - (128*Sin[a + b*x]^9)/(3*b) + (384*Sin[a + b*x]
^11)/(11*b) - (128*Sin[a + b*x]^13)/(13*b)
```

Rubi [A] time = 0.0606673, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2564, 270}

$$-\frac{128\sin^{13}(a+bx)}{13b}+\frac{384\sin^{11}(a+bx)}{11b}-\frac{128\sin^{9}(a+bx)}{3b}+\frac{128\sin^{7}(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (128*Sin[a + b*x]^7)/(7*b) - (128*Sin[a + b*x]^9)/(3*b) + (384*Sin[a + b*x]
^11)/(11*b) - (128*Sin[a + b*x]^13)/(13*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \csc(a+bx)\sin^7(2a+2bx)\,dx &= 128\int \cos^7(a+bx)\sin^6(a+bx)\,dx\\
&= \frac{128\,\text{Subst}\left(\int x^6\left(1-x^2\right)^3\,dx,x,\sin(a+bx)\right)}{b}\\
&= \frac{128\,\text{Subst}\left(\int \left(x^6-3x^8+3x^{10}-x^{12}\right)dx,x,\sin(a+bx)\right)}{b}\\
&= \frac{128\sin^7(a+bx)}{7b}-\frac{128\sin^9(a+bx)}{3b}+\frac{384\sin^{11}(a+bx)}{11b}-\frac{128\sin^{13}(a+bx)}{13b}
\end{aligned}$$

Mathematica [A] time = 0.213718, size = 48, normalized size = 0.79

$$\frac{128\left(-231\sin^{13}(a+bx)+819\sin^{11}(a+bx)-1001\sin^{9}(a+bx)+429\sin^{7}(a+bx)\right)}{3003b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (128*(429*Sin[a + b*x]^7 - 1001*Sin[a + b*x]^9 + 819*Sin[a + b*x]^11 - 231*
Sin[a + b*x]^13))/(3003*b)
```

Maple [A] time = 0.06, size = 97, normalized size = 1.6

$$128\,\frac{1}{b}\left(-1/13\,(\sin(bx+a))^{5}(\cos(bx+a))^{8}-\frac{5\,(\sin(bx+a))^{3}(\cos(bx+a))^{8}}{143}-\frac{5\,\sin(bx+a)(\cos(bx+a))^{8}}{429}+\frac{5\,\sin\ldots}{3\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^7,x)
```

```
[Out] 128/b*(-1/13*sin(b*x+a)^5*cos(b*x+a)^8-5/143*sin(b*x+a)^3*cos(b*x+a)^8-5/42
9*sin(b*x+a)*cos(b*x+a)^8+5/3003*(16/5+cos(b*x+a)^6+6/5*cos(b*x+a)^4+8/5*co
s(b*x+a)^2)*sin(b*x+a))
```

Maxima [A] time = 1.2241, size = 108, normalized size = 1.77

$$-\frac{231\,\sin(13\,bx+13\,a)+273\,\sin(11\,bx+11\,a)-2002\,\sin(9\,bx+9\,a)-2574\,\sin(7\,bx+7\,a)+9009\,\sin(5\,bx+5\,a\ldots}{96096\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="maxima")
```

```
[Out] -1/96096*(231*sin(13*b*x + 13*a) + 273*sin(11*b*x + 11*a) - 2002*sin(9*b*x
+ 9*a) - 2574*sin(7*b*x + 7*a) + 9009*sin(5*b*x + 5*a) + 15015*sin(3*b*x +
3*a) - 60060*sin(b*x + a))/b
```

Fricas [A] time = 0.520104, size = 207, normalized size = 3.39

$$-\frac{128\left(231\,\cos(bx+a)^{12}-567\,\cos(bx+a)^{10}+371\,\cos(bx+a)^{8}-5\,\cos(bx+a)^{6}-6\,\cos(bx+a)^{4}-8\,\cos(bx+a\ldots\right.}{3003\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="fricas")
```

```
[Out] -128/3003*(231*cos(b*x + a)^12 - 567*cos(b*x + a)^10 + 371*cos(b*x + a)^8 -
 5*cos(b*x + a)^6 - 6*cos(b*x + a)^4 - 8*cos(b*x + a)^2 - 16)*sin(b*x + a)/
b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**7,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.82603, size = 62, normalized size = 1.02

$$-\frac{128\left(231\,\sin\left(bx+a\right)^{13}-819\,\sin\left(bx+a\right)^{11}+1001\,\sin\left(bx+a\right)^{9}-429\,\sin\left(bx+a\right)^{7}\right)}{3003\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="giac")
```

```
[Out] -128/3003*(231*sin(b*x + a)^13 - 819*sin(b*x + a)^11 + 1001*sin(b*x + a)^9
- 429*sin(b*x + a)^7)/b
```

3.35 $\int \csc(a+bx)\sin^6(2a+2bx)\,dx$

Optimal. Leaf size=46

$$-\frac{64\cos^{11}(a+bx)}{11b}+\frac{128\cos^9(a+bx)}{9b}-\frac{64\cos^7(a+bx)}{7b}$$

```
[Out] (-64*Cos[a + b*x]^7)/(7*b) + (128*Cos[a + b*x]^9)/(9*b) - (64*Cos[a + b*x]^
11)/(11*b)
```

Rubi [A] time = 0.0559452, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2565, 270}

$$-\frac{64\cos^{11}(a+bx)}{11b}+\frac{128\cos^9(a+bx)}{9b}-\frac{64\cos^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

```
[Out] (-64*Cos[a + b*x]^7)/(7*b) + (128*Cos[a + b*x]^9)/(9*b) - (64*Cos[a + b*x]^
11)/(11*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \csc(a+bx)\sin^6(2a+2bx)\,dx &= 64\int \cos^6(a+bx)\sin^5(a+bx)\,dx\\
&= -\frac{64\,\text{Subst}\left(\int x^6\left(1-x^2\right)^2\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{64\,\text{Subst}\left(\int \left(x^6-2x^8+x^{10}\right)\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{64\cos^7(a+bx)}{7b}+\frac{128\cos^9(a+bx)}{9b}-\frac{64\cos^{11}(a+bx)}{11b}
\end{aligned}$$

Mathematica [A] time = 0.0579333, size = 89, normalized size = 1.93

$$-\frac{5\cos(a+bx)}{8b}-\frac{5\cos(3(a+bx))}{24b}+\frac{\cos(5(a+bx))}{16b}+\frac{5\cos(7(a+bx))}{112b}-\frac{\cos(9(a+bx))}{144b}-\frac{\cos(11(a+bx))}{176b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

```
[Out] (-5*Cos[a + b*x])/(8*b) - (5*Cos[3*(a + b*x)])/(24*b) + Cos[5*(a + b*x)]/(1
6*b) + (5*Cos[7*(a + b*x)])/(112*b) - Cos[9*(a + b*x)]/(144*b) - Cos[11*(a
+ b*x)]/(176*b)
```

Maple [A] time = 0.029, size = 53, normalized size = 1.2

$$64\,\frac{1}{b}\left(-1/11\,(\sin(bx+a))^4(\cos(bx+a))^7-\frac{4\,(\sin(bx+a))^2(\cos(bx+a))^7}{99}-\frac{8\,(\cos(bx+a))^7}{693}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^6,x)
```

```
[Out] 64/b*(-1/11*sin(b*x+a)^4*cos(b*x+a)^7-4/99*sin(b*x+a)^2*cos(b*x+a)^7-8/693*
cos(b*x+a)^7)
```

Maxima [A] time = 1.1505, size = 93, normalized size = 2.02

$$-\frac{63\,\cos(11\,bx+11\,a)+77\,\cos(9\,bx+9\,a)-495\,\cos(7\,bx+7\,a)-693\,\cos(5\,bx+5\,a)+2310\,\cos(3\,bx+3\,a)+6930}{11088\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="maxima")
```

```
[Out] -1/11088*(63*cos(11*b*x + 11*a) + 77*cos(9*b*x + 9*a) - 495*cos(7*b*x + 7*a
) - 693*cos(5*b*x + 5*a) + 2310*cos(3*b*x + 3*a) + 6930*cos(b*x + a))/b
```

Fricas [A] time = 0.503029, size = 99, normalized size = 2.15

$$-\frac{64\left(63\,\cos(bx+a)^{11}-154\,\cos(bx+a)^9+99\,\cos(bx+a)^7\right)}{693\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="fricas")
```

```
[Out] -64/693*(63*cos(b*x + a)^11 - 154*cos(b*x + a)^9 + 99*cos(b*x + a)^7)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.59592, size = 275, normalized size = 5.98

$$-\frac{1024\left(\frac{11\,(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{55\,(\cos(bx+a)-1)^{2}}{(\cos(bx+a)+1)^{2}}-\frac{297\,(\cos(bx+a)-1)^{3}}{(\cos(bx+a)+1)^{3}}-\frac{1485\,(\cos(bx+a)-1)^{4}}{(\cos(bx+a)+1)^{4}}-\frac{2079\,(\cos(bx+a)-1)^{5}}{(\cos(bx+a)+1)^{5}}-\frac{2541\,(\cos(bx+a)-1)^{6}}{(\cos(bx+a)+1)^{6}}-11}{693\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="giac")
```

```
[Out] -1024/693*(11*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 55*(cos(b*x + a) - 1)
^2/(cos(b*x + a) + 1)^2 - 297*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 - 1
485*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 2079*(cos(b*x + a) - 1)^5/(
cos(b*x + a) + 1)^5 - 2541*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 - 1155
*(cos(b*x + a) - 1)^7/(cos(b*x + a) + 1)^7 - 462*(cos(b*x + a) - 1)^8/(cos(
b*x + a) + 1)^8 - 1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^11)
```

3.36 $\int \csc(a+bx)\sin^5(2a+2bx)\,dx$

Optimal. Leaf size=46

$$\frac{32\sin^9(a+bx)}{9b}-\frac{64\sin^7(a+bx)}{7b}+\frac{32\sin^5(a+bx)}{5b}$$

```
[Out] (32*Sin[a + b*x]^5)/(5*b) - (64*Sin[a + b*x]^7)/(7*b) + (32*Sin[a + b*x]^9)
/(9*b)
```

Rubi [A] time = 0.0565101, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2564, 270}

$$\frac{32\sin^9(a+bx)}{9b}-\frac{64\sin^7(a+bx)}{7b}+\frac{32\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (32*Sin[a + b*x]^5)/(5*b) - (64*Sin[a + b*x]^7)/(7*b) + (32*Sin[a + b*x]^9)
/(9*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \csc(a+bx)\sin^5(2a+2bx)\,dx &= 32\int \cos^5(a+bx)\sin^4(a+bx)\,dx\\
&= \frac{32\,\text{Subst}\left(\int x^4\left(1-x^2\right)^2\,dx,x,\sin(a+bx)\right)}{b}\\
&= \frac{32\,\text{Subst}\left(\int \left(x^4-2x^6+x^8\right)\,dx,x,\sin(a+bx)\right)}{b}\\
&= \frac{32\sin^5(a+bx)}{5b}-\frac{64\sin^7(a+bx)}{7b}+\frac{32\sin^9(a+bx)}{9b}
\end{aligned}$$

Mathematica [A] time = 0.0981108, size = 38, normalized size = 0.83

$$\frac{32\left(35\sin^9(a+bx)-90\sin^7(a+bx)+63\sin^5(a+bx)\right)}{315b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (32*(63*Sin[a + b*x]^5 - 90*Sin[a + b*x]^7 + 35*Sin[a + b*x]^9))/(315*b)
```

Maple [A] time = 0.056, size = 69, normalized size = 1.5

$$32\,\frac{1}{b}\left(-1/9\,(\sin(bx+a))^3(\cos(bx+a))^6-1/21\,\sin(bx+a)(\cos(bx+a))^6+\frac{\left(8/3+(\cos(bx+a))^4+4/3\,(\cos(bx+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^5,x)
```

```
[Out] 32/b*(-1/9*sin(b*x+a)^3*cos(b*x+a)^6-1/21*sin(b*x+a)*cos(b*x+a)^6+1/105*(8/
3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a))
```

Maxima [A] time = 1.16279, size = 78, normalized size = 1.7

$$\frac{35\,\sin(9\,bx+9\,a)+45\,\sin(7\,bx+7\,a)-252\,\sin(5\,bx+5\,a)-420\,\sin(3\,bx+3\,a)+1890\,\sin(bx+a)}{2520\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] 1/2520*(35*sin(9*b*x + 9*a) + 45*sin(7*b*x + 7*a) - 252*sin(5*b*x + 5*a) -
420*sin(3*b*x + 3*a) + 1890*sin(b*x + a))/b
```

Fricas [A] time = 0.494995, size = 142, normalized size = 3.09

$$\frac{32\left(35\,\cos(bx+a)^8-50\,\cos(bx+a)^6+3\,\cos(bx+a)^4+4\,\cos(bx+a)^2+8\right)\sin(bx+a)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] 32/315*(35*cos(b*x + a)^8 - 50*cos(b*x + a)^6 + 3*cos(b*x + a)^4 + 4*cos(b*
x + a)^2 + 8)*sin(b*x + a)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40332, size = 49, normalized size = 1.07

$$\frac{32\left(35\sin\left(bx+a\right)^{9}-90\sin\left(bx+a\right)^{7}+63\sin\left(bx+a\right)^{5}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] 32/315*(35*sin(b*x + a)^9 - 90*sin(b*x + a)^7 + 63*sin(b*x + a)^5)/b
```

3.37 $\int \csc(a+bx)\sin^4(2a+2bx)\,dx$

Optimal. Leaf size=31

$$\frac{16\cos^7(a+bx)}{7b}-\frac{16\cos^5(a+bx)}{5b}$$

```
[Out] (-16*Cos[a + b*x]^5)/(5*b) + (16*Cos[a + b*x]^7)/(7*b)
```

Rubi [A] time = 0.0519787, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2565, 14}

$$\frac{16\cos^7(a+bx)}{7b}-\frac{16\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (-16*Cos[a + b*x]^5)/(5*b) + (16*Cos[a + b*x]^7)/(7*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \csc(a+bx)\sin^4(2a+2bx)\,dx &= 16\int \cos^4(a+bx)\sin^3(a+bx)\,dx\\
&= -\frac{16\,\text{Subst}\left(\int x^4\left(1-x^2\right)dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{16\,\text{Subst}\left(\int \left(x^4-x^6\right)dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{16\cos^5(a+bx)}{5b}+\frac{16\cos^7(a+bx)}{7b}
\end{aligned}$$

Mathematica [A] time = 0.0380123, size = 59, normalized size = 1.9

$$-\frac{3\cos(a+bx)}{4b}-\frac{\cos(3(a+bx))}{4b}+\frac{\cos(5(a+bx))}{20b}+\frac{\cos(7(a+bx))}{28b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (-3*Cos[a + b*x])/(4*b) - Cos[3*(a + b*x)]/(4*b) + Cos[5*(a + b*x)]/(20*b)
+ Cos[7*(a + b*x)]/(28*b)
```

Maple [A] time = 0.029, size = 35, normalized size = 1.1

$$16\,\frac{1}{b}\left(-1/7\,\left(\sin\left(bx+a\right)\right)^{2}\left(\cos\left(bx+a\right)\right)^{5}-\frac{2\,\left(\cos\left(bx+a\right)\right)^{5}}{35}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^4,x)
```

```
[Out] 16/b*(-1/7*sin(b*x+a)^2*cos(b*x+a)^5-2/35*cos(b*x+a)^5)
```

Maxima [A] time = 1.14041, size = 63, normalized size = 2.03

$$\frac{5\,\cos\left(7\,bx+7\,a\right)+7\,\cos\left(5\,bx+5\,a\right)-35\,\cos\left(3\,bx+3\,a\right)-105\,\cos\left(bx+a\right)}{140\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/140*(5*cos(7*b*x + 7*a) + 7*cos(5*b*x + 5*a) - 35*cos(3*b*x + 3*a) - 105*
cos(b*x + a))/b
```

Fricas [A] time = 0.482037, size = 63, normalized size = 2.03

$$\frac{16\left(5\,\cos\left(bx+a\right)^{7}-7\,\cos\left(bx+a\right)^{5}\right)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] 16/35*(5*cos(b*x + a)^7 - 7*cos(b*x + a)^5)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.40043, size = 186, normalized size = 6.

$$-\frac{64\left(\frac{7(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{14(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{70(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{35(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+\frac{35(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5}-1\right)}{35\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] -64/35*(7*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 14*(cos(b*x + a) - 1)^2/(
cos(b*x + a) + 1)^2 + 70*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 35*(co
s(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 35*(cos(b*x + a) - 1)^5/(cos(b*x +
 a) + 1)^5 - 1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^7)
```

3.38 $\int \csc(a+bx)\sin^3(2a+2bx)\,dx$

Optimal. Leaf size=31

$$\frac{8\sin^3(a+bx)}{3b}-\frac{8\sin^5(a+bx)}{5b}$$

```
[Out] (8*Sin[a + b*x]^3)/(3*b) - (8*Sin[a + b*x]^5)/(5*b)
```

Rubi [A] time = 0.0513627, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2564, 14}

$$\frac{8\sin^3(a+bx)}{3b}-\frac{8\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (8*Sin[a + b*x]^3)/(3*b) - (8*Sin[a + b*x]^5)/(5*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \csc(a+bx)\sin^3(2a+2bx)\,dx &= 8\int \cos^3(a+bx)\sin^2(a+bx)\,dx\\ &= \frac{8\,\text{Subst}\left(\int x^2\left(1-x^2\right)dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{8\,\text{Subst}\left(\int \left(x^2-x^4\right)dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{8\sin^3(a+bx)}{3b}-\frac{8\sin^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.0475814, size = 28, normalized size = 0.9

$$\frac{8\left(5\sin^3(a+bx)-3\sin^5(a+bx)\right)}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (8*(5*Sin[a + b*x]^3 - 3*Sin[a + b*x]^5))/(15*b)
```

Maple [A] time = 0.047, size = 41, normalized size = 1.3

$$8\,\frac{-1/5\,\sin(bx+a)\,(\cos(bx+a))^4+1/15\,\left(2+(\cos(bx+a))^2\right)\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^3,x)
```

```
[Out] 8/b*(-1/5*sin(b*x+a)*cos(b*x+a)^4+1/15*(2+cos(b*x+a)^2)*sin(b*x+a))
```

Maxima [A] time = 1.20096, size = 49, normalized size = 1.58

$$-\frac{3\,\sin(5\,bx+5\,a)+5\,\sin(3\,bx+3\,a)-30\,\sin(bx+a)}{30\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] -1/30*(3*sin(5*b*x + 5*a) + 5*sin(3*b*x + 3*a) - 30*sin(b*x + a))/b
```

Fricas [A] time = 0.477091, size = 84, normalized size = 2.71

$$-\frac{8\left(3\,\cos(bx+a)^4-\cos(bx+a)^2-2\right)\sin(bx+a)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] -8/15*(3*cos(b*x + a)^4 - cos(b*x + a)^2 - 2)*sin(b*x + a)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34838, size = 35, normalized size = 1.13

$$-\frac{8\left(3\,\sin\left(bx+a\right)^{5}-5\,\sin\left(bx+a\right)^{3}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] -8/15*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)/b
```

3.39 $\int \csc(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=15

$$-\frac{4\cos^3(a+bx)}{3b}$$

```
[Out] (-4*Cos[a + b*x]^3)/(3*b)
```

Rubi [A] time = 0.0372745, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2565, 30}

$$-\frac{4\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (-4*Cos[a + b*x]^3)/(3*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \csc(a+bx)\sin^2(2a+2bx)\,dx &= 4\int \cos^2(a+bx)\sin(a+bx)\,dx\\
&= -\frac{4\,\text{Subst}\left(\int x^2\,dx, x, \cos(a+bx)\right)}{b}\\
&= -\frac{4\cos^3(a+bx)}{3b}
\end{aligned}$$

Mathematica [A] time = 0.0068125, size = 15, normalized size = 1.

$$-\frac{4\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (-4*Cos[a + b*x]^3)/(3*b)
```

Maple [A] time = 0.019, size = 14, normalized size = 0.9

$$-\frac{4 (\cos(bx+a))^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^2,x)
```

```
[Out] -4/3*cos(b*x+a)^3/b
```

Maxima [A] time = 1.17229, size = 31, normalized size = 2.07

$$-\frac{\cos(3\,bx+3\,a)+3\,\cos(bx+a)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(cos(3*b*x + 3*a) + 3*cos(b*x + a))/b
```

Fricas [A] time = 0.471947, size = 31, normalized size = 2.07

$$-\frac{4\,\cos(bx+a)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] -4/3*cos(b*x + a)^3/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26339, size = 70, normalized size = 4.67

$$\frac{8\left(\frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+1\right)}{3b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] 8/3*(3*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 + 1)/(b*((cos(b*x + a) - 1
)/(cos(b*x + a) + 1) - 1)^3)
```

3.40 $\int \csc(a+bx)\sin(2a+2bx)\,dx$

Optimal. Leaf size=11

$$\frac{2\sin(a+bx)}{b}$$

```
[Out] (2*Sin[a + b*x])/b
```

Rubi [A] time = 0.0165524, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4288, 2637}

$$\frac{2\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x],x]
```

```
[Out] (2*Sin[a + b*x])/b
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc(a+bx)\sin(2a+2bx)\,dx &= 2\int \cos(a+bx)\,dx\\ &= \frac{2\sin(a+bx)}{b}\end{aligned}$$

Mathematica [B] time = 0.0081312, size = 23, normalized size = 2.09

$$2\left(\frac{\sin(a)\cos(bx)}{b}+\frac{\cos(a)\sin(bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x],x]
```

```
[Out] 2*((Cos[b*x]*Sin[a])/b + (Cos[a]*Sin[b*x])/b)
```

Maple [A] time = 0.018, size = 12, normalized size = 1.1

$$2\,\frac{\sin\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a),x)
```

```
[Out] 2*sin(b*x+a)/b
```

Maxima [A] time = 1.07669, size = 15, normalized size = 1.36

$$\frac{2\,\sin\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] 2*sin(b*x + a)/b
```

Fricas [A] time = 0.46841, size = 24, normalized size = 2.18

$$\frac{2\,\sin\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 2*sin(b*x + a)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30253, size = 15, normalized size = 1.36

$$\frac{2\,\sin\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] 2*sin(b*x + a)/b
```

3.41 $\int \csc(a+bx)\csc(2a+2bx)\,dx$

Optimal. Leaf size=28

$$\frac{\tanh^{-1}(\sin(a+bx))}{2b}-\frac{\csc(a+bx)}{2b}$$

```
[Out] ArcTanh[Sin[a + b*x]]/(2*b) - Csc[a + b*x]/(2*b)
```

Rubi [A] time = 0.0413485, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4288, 2621, 321, 207}

$$\frac{\tanh^{-1}(\sin(a+bx))}{2b}-\frac{\csc(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Csc[2*a + 2*b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(2*b) - Csc[a + b*x]/(2*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc(a+bx)\csc(2a+2bx)\,dx = \frac{1}{2}\int \csc^2(a+bx)\sec(a+bx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{2b}$$
$$= -\frac{\csc(a+bx)}{2b} - \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{2b}$$
$$= \frac{\tanh^{-1}(\sin(a+bx))}{2b} - \frac{\csc(a+bx)}{2b}$$

Mathematica [C] time = 0.0168705, size = 29, normalized size = 1.04

$$-\frac{\csc(a+bx)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\sin^2(a+bx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Csc[2*a + 2*b*x],x]
```

```
[Out] -(Csc[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[a + b*x]^2])/(2*b)
```

Maple [A] time = 0.03, size = 34, normalized size = 1.2

$$-\frac{1}{2\,b\sin(bx+a)} + \frac{\ln(\sec(bx+a)+\tan(bx+a))}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*csc(2*b*x+2*a),x)
```

```
[Out] -1/2/b/sin(b*x+a)+1/2/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [B] time = 1.80033, size = 315, normalized size = 11.25

$$-\frac{\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2-2\,\cos(2\,bx+2\,a)+1\right)\log\left(\frac{\cos(bx+2\,a)^2+\cos(a)^2-2\,\cos(a)\sin(bx+2\,a)+\sin(bx+2\,a)^2+2\,\cos(}{\cos(bx+2\,a)^2+\cos(a)^2+2\,\cos(a)\sin(bx+2\,a)+\sin(bx+2\,a)^2-2\,\cos(}\right.}{4\left(b\cos(2\,bx+2\,a)^2+b\sin(2\,bx+2\,a)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -1/4*((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*lo
g((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2
 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos
(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)
) + 4*cos(b*x + a)*sin(2*b*x + 2*a) - 4*cos(2*b*x + 2*a)*sin(b*x + a) + 4*s
in(b*x + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 - 2*b*cos(2*b*x +
 2*a) + b)
```

Fricas [B] time = 0.490565, size = 136, normalized size = 4.86

$$\frac{\log\left(\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-\log\left(-\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-2}{4\,b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 1/4*(log(sin(b*x + a) + 1)*sin(b*x + a) - log(-sin(b*x + a) + 1)*sin(b*x +
a) - 2)/(b*sin(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\csc\left(a+bx\right)\csc\left(2a+2bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a),x)
```

```
[Out] Integral(csc(a + b*x)*csc(2*a + 2*b*x), x)
```

Giac [B] time = 1.60129, size = 653, normalized size = 23.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] -1/4*((tan(1/2*b*x + 2*a)*tan(1/2*a)^12 - 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^
10 + 6*tan(1/2*a)^11 - 27*tan(1/2*b*x + 2*a)*tan(1/2*a)^8 - 2*tan(1/2*a)^9
- 36*tan(1/2*a)^7 + 27*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 - 36*tan(1/2*a)^5 +
12*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - 2*tan(1/2*a)^3 - tan(1/2*b*x + 2*a) +
6*tan(1/2*a))/((3*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 - tan(1/2*b*x + 2*a)*ta
n(1/2*a)^6 - 10*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 + 15*tan(1/2*b*x + 2*a)*t
an(1/2*a)^4 - 3*tan(1/2*a)^5 + 3*tan(1/2*b*x + 2*a)^2*tan(1/2*a) - 15*tan(1
/2*b*x + 2*a)*tan(1/2*a)^2 + 10*tan(1/2*a)^3 + tan(1/2*b*x + 2*a) - 3*tan(1
/2*a))*(3*tan(1/2*a)^5 - 10*tan(1/2*a)^3 + 3*tan(1/2*a))) - 2*log(abs(tan(1
/2*b*x + 2*a)*tan(1/2*a)^3 + 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - tan(1/2*a)
^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 - tan(1/2*b*x + 2*a)
+ 3*tan(1/2*a) - 1)) + 2*log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 3*tan(1/
2*b*x + 2*a)*tan(1/2*a)^2 + tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a)
+ 3*tan(1/2*a)^2 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a) - 1)))/b
```

3.42 $\int \csc(a+bx)\csc^2(2a+2bx)\,dx$

Optimal. Leaf size=49

$$\frac{3\sec(a+bx)}{8b}-\frac{3\tanh^{-1}(\cos(a+bx))}{8b}-\frac{\csc^2(a+bx)\sec(a+bx)}{8b}$$

```
[Out] (-3*ArcTanh[Cos[a + b*x]])/(8*b) + (3*Sec[a + b*x])/(8*b) - (Csc[a + b*x]^2
*Sec[a + b*x])/(8*b)
```

Rubi [A] time = 0.058755, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {4288, 2622, 288, 321, 207}

$$\frac{3\sec(a+bx)}{8b}-\frac{3\tanh^{-1}(\cos(a+bx))}{8b}-\frac{\csc^2(a+bx)\sec(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] (-3*ArcTanh[Cos[a + b*x]])/(8*b) + (3*Sec[a + b*x])/(8*b) - (Csc[a + b*x]^2
*Sec[a + b*x])/(8*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \csc(a+bx)\csc^2(2a+2bx)\,dx &= \frac{1}{4}\int \csc^3(a+bx)\sec^2(a+bx)\,dx\\ &= \frac{\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, \sec(a+bx)\right)}{4b}\\ &= -\frac{\csc^2(a+bx)\sec(a+bx)}{8b}+\frac{3\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{8b}\\ &= \frac{3\sec(a+bx)}{8b}-\frac{\csc^2(a+bx)\sec(a+bx)}{8b}+\frac{3\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{8b}\\ &= -\frac{3\tanh^{-1}(\cos(a+bx))}{8b}+\frac{3\sec(a+bx)}{8b}-\frac{\csc^2(a+bx)\sec(a+bx)}{8b}\end{aligned}$$

Mathematica [B] time = 0.258844, size = 143, normalized size = 2.92

$$\frac{\csc^4(a+bx)\left(-6\cos(2(a+bx))+2\cos(3(a+bx))+3\cos(3(a+bx))\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)-3\cos(3(a+bx))\log\left(\sin\left(\frac{1}{2}\right.\right.\right.}{8b\left(\csc^2\left(\frac{1}{2}(a+bx)\right)-\sec^2\left(\frac{1}{2}(a+bx\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] (Csc[a + b*x]^4*(2 - 6*Cos[2*(a + b*x)] + 2*Cos[3*(a + b*x)] + 3*Cos[3*(a +
 b*x)]*Log[Cos[(a + b*x)/2]] - 3*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] + C
os[a + b*x]*(-2 - 3*Log[Cos[(a + b*x)/2]] + 3*Log[Sin[(a + b*x)/2]])))/(8*b
*(Csc[(a + b*x)/2]^2 - Sec[(a + b*x)/2]^2))
```

Maple [A] time = 0.032, size = 57, normalized size = 1.2

$$-\frac{1}{8\,b\left(\sin(bx+a)\right)^2\cos(bx+a)}+\frac{3}{8\,b\cos(bx+a)}+\frac{3\,\ln\left(\csc(bx+a)-\cot(bx+a)\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*csc(2*b*x+2*a)^2,x)
```

```
[Out] -1/8/b/sin(b*x+a)^2/cos(b*x+a)+3/8/b/cos(b*x+a)+3/8/b*ln(csc(b*x+a)-cot(b*x
+a))
```

Maxima [B] time = 1.0884, size = 1315, normalized size = 26.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*(3*cos(5*b*x + 5*a) - 2*cos(3*b*x + 3*a) + 3*cos(b*x + a))*cos(6*b*
x + 6*a) - 12*(cos(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(5*b*x + 5*a) +
4*(2*cos(3*b*x + 3*a) - 3*cos(b*x + a))*cos(4*b*x + 4*a) + 8*(cos(2*b*x + 2
*a) - 1)*cos(3*b*x + 3*a) - 12*cos(2*b*x + 2*a)*cos(b*x + a) + 3*(2*(cos(4*
b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - cos(6*b*x + 6*a)^2 -
2*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - cos(2*b*x
+ 2*a)^2 + 2*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - sin(6
*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) -
sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*co
s(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 3*(2*(cos(4*
b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - cos(6*b*x + 6*a)^2 -
2*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - cos(2*b*x
+ 2*a)^2 + 2*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - sin(6
*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) -
sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*co
s(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*(3*sin(5*b
*x + 5*a) - 2*sin(3*b*x + 3*a) + 3*sin(b*x + a))*sin(6*b*x + 6*a) - 12*(sin
(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(5*b*x + 5*a) + 4*(2*sin(3*b*x + 3*a)
- 3*sin(b*x + a))*sin(4*b*x + 4*a) + 8*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) -
12*sin(2*b*x + 2*a)*sin(b*x + a) + 12*cos(b*x + a))/(b*cos(6*b*x + 6*a)^2 +
 b*cos(4*b*x + 4*a)^2 + b*cos(2*b*x + 2*a)^2 + b*sin(6*b*x + 6*a)^2 + b*sin
(4*b*x + 4*a)^2 + 2*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b*sin(2*b*x + 2*a
)^2 - 2*(b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a) - b)*cos(6*b*x + 6*a) + 2*
(b*cos(2*b*x + 2*a) - b)*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) - 2*(b*sin
(4*b*x + 4*a) + b*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + b)
```

Fricas [B] time = 0.500152, size = 262, normalized size = 5.35

$$\frac{6\cos(bx+a)^2 - 3\left(\cos(bx+a)^3 - \cos(bx+a)\right)\log\left(\frac{1}{2}\cos(bx+a) + \frac{1}{2}\right) + 3\left(\cos(bx+a)^3 - \cos(bx+a)\right)\log\left(-\frac{1}{2}\,c\right.}{16\left(b\cos(bx+a)^3 - b\cos(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(6*cos(b*x + a)^2 - 3*(cos(b*x + a)^3 - cos(b*x + a))*log(1/2*cos(b*x
+ a) + 1/2) + 3*(cos(b*x + a)^3 - cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2
) - 4)/(b*cos(b*x + a)^3 - b*cos(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(a+bx)\csc^2(2a+2bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)**2,x)
```

```
[Out] Integral(csc(a + b*x)*csc(2*a + 2*b*x)**2, x)
```

Giac [B] time = 1.92672, size = 1791, normalized size = 36.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] -1/32*(16*(6*tan(1/2*b*x + 2*a)*tan(1/2*a)^11 - tan(1/2*a)^12 - 2*tan(1/2*b
*x + 2*a)*tan(1/2*a)^9 + 12*tan(1/2*a)^10 - 36*tan(1/2*b*x + 2*a)*tan(1/2*a
)^7 + 27*tan(1/2*a)^8 - 36*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - 2*tan(1/2*b*x
+ 2*a)*tan(1/2*a)^3 - 27*tan(1/2*a)^4 + 6*tan(1/2*b*x + 2*a)*tan(1/2*a) - 1
2*tan(1/2*a)^2 + 1)/((tan(1/2*b*x + 2*a)^2*tan(1/2*a)^6 - 15*tan(1/2*b*x +
2*a)^2*tan(1/2*a)^4 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - tan(1/2*a)^6 + 1
5*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2 - 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 +
15*tan(1/2*a)^4 - tan(1/2*b*x + 2*a)^2 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a) -
 15*tan(1/2*a)^2 + 1)*(tan(1/2*a)^6 - 15*tan(1/2*a)^4 + 15*tan(1/2*a)^2 - 1
)) + (6*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^23 - tan(1/2*b*x + 2*a)^2*tan(1/2*a
)^24 - 74*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^21 + 60*tan(1/2*b*x + 2*a)^2*tan(
1/2*a)^22 - 6*tan(1/2*b*x + 2*a)*tan(1/2*a)^23 + 798*tan(1/2*b*x + 2*a)^3*t
an(1/2*a)^19 - 924*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^20 + 290*tan(1/2*b*x + 2
*a)*tan(1/2*a)^21 - 18*tan(1/2*a)^22 - 1170*tan(1/2*b*x + 2*a)^3*tan(1/2*a)
^17 + 3892*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^18 - 2310*tan(1/2*b*x + 2*a)*tan
(1/2*a)^19 + 336*tan(1/2*a)^20 - 3188*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^15 +
1467*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^16 + 3186*tan(1/2*b*x + 2*a)*tan(1/2*a
)^17 - 1190*tan(1/2*a)^18 + 2604*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^13 - 12744
*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^14 + 8148*tan(1/2*b*x + 2*a)*tan(1/2*a)^15
 - 288*tan(1/2*a)^16 + 2604*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^11 - 10332*tan(
1/2*b*x + 2*a)*tan(1/2*a)^13 + 4428*tan(1/2*a)^14 - 3188*tan(1/2*b*x + 2*a)
^3*tan(1/2*a)^9 + 12744*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^10 - 10332*tan(1/2*
b*x + 2*a)*tan(1/2*a)^11 - 1170*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^7 - 1467*ta
n(1/2*b*x + 2*a)^2*tan(1/2*a)^8 + 8148*tan(1/2*b*x + 2*a)*tan(1/2*a)^9 - 44
28*tan(1/2*a)^10 + 798*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^5 - 3892*tan(1/2*b*x
 + 2*a)^2*tan(1/2*a)^6 + 3186*tan(1/2*b*x + 2*a)*tan(1/2*a)^7 + 288*tan(1/2
*a)^8 - 74*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^3 + 924*tan(1/2*b*x + 2*a)^2*tan
(1/2*a)^4 - 2310*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 + 1190*tan(1/2*a)^6 + 6*ta
n(1/2*b*x + 2*a)^3*tan(1/2*a) - 60*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2 + 290*
tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 336*tan(1/2*a)^4 + tan(1/2*b*x + 2*a)^2 -
 6*tan(1/2*b*x + 2*a)*tan(1/2*a) + 18*tan(1/2*a)^2)/((9*tan(1/2*a)^10 - 60*
tan(1/2*a)^8 + 118*tan(1/2*a)^6 - 60*tan(1/2*a)^4 + 9*tan(1/2*a)^2)*(3*tan(
1/2*b*x + 2*a)^2*tan(1/2*a)^5 - tan(1/2*b*x + 2*a)*tan(1/2*a)^6 - 10*tan(1/
2*b*x + 2*a)^2*tan(1/2*a)^3 + 15*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 - 3*tan(1/
2*a)^5 + 3*tan(1/2*b*x + 2*a)^2*tan(1/2*a) - 15*tan(1/2*b*x + 2*a)*tan(1/2*
a)^2 + 10*tan(1/2*a)^3 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a))^2) + 12*log(abs
(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(
1/2*a)^2 - 1)) - 12*log(abs(3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - tan(1/2*a)^
3 - tan(1/2*b*x + 2*a) + 3*tan(1/2*a))))/b
```

3.43 $\int \csc(a+bx)\csc^3(2a+2bx)\,dx$

Optimal. Leaf size=66

$$-\frac{5\csc^3(a+bx)}{48b}-\frac{5\csc(a+bx)}{16b}+\frac{5\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\csc^3(a+bx)\sec^2(a+bx)}{16b}$$

```
[Out] (5*ArcTanh[Sin[a + b*x]])/(16*b) - (5*Csc[a + b*x])/(16*b) - (5*Csc[a + b*x
]^3)/(48*b) + (Csc[a + b*x]^3*Sec[a + b*x]^2)/(16*b)
```

Rubi [A] time = 0.0627575, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {4288, 2621, 288, 302, 207}

$$-\frac{5\csc^3(a+bx)}{48b}-\frac{5\csc(a+bx)}{16b}+\frac{5\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\csc^3(a+bx)\sec^2(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] (5*ArcTanh[Sin[a + b*x]])/(16*b) - (5*Csc[a + b*x])/(16*b) - (5*Csc[a + b*x
]^3)/(48*b) + (Csc[a + b*x]^3*Sec[a + b*x]^2)/(16*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc(a+bx)\csc^3(2a+2bx)\,dx = \frac{1}{8}\int \csc^4(a+bx)\sec^3(a+bx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, \csc(a+bx)\right)}{8b}$$
$$= \frac{\csc^3(a+bx)\sec^2(a+bx)}{16b} - \frac{5\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{16b}$$
$$= \frac{\csc^3(a+bx)\sec^2(a+bx)}{16b} - \frac{5\,\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \csc(a+bx)\right)}{16b}$$
$$= -\frac{5\csc(a+bx)}{16b} - \frac{5\csc^3(a+bx)}{48b} + \frac{\csc^3(a+bx)\sec^2(a+bx)}{16b} - \frac{5\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x\right.}{16b}$$
$$= \frac{5\tanh^{-1}(\sin(a+bx))}{16b} - \frac{5\csc(a+bx)}{16b} - \frac{5\csc^3(a+bx)}{48b} + \frac{\csc^3(a+bx)\sec^2(a+bx)}{16b}$$

Mathematica [C] time = 0.0208439, size = 31, normalized size = 0.47

$$-\frac{\csc^3(a+bx)\text{Hypergeometric2F1}\left(-\frac{3}{2},2,-\frac{1}{2},\sin^2(a+bx)\right)}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] -(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 2, -1/2, Sin[a + b*x]^2])/(24*b)
```

Maple [A] time = 0.035, size = 76, normalized size = 1.2

$$-\frac{1}{24\,b\left(\sin\left(bx+a\right)\right)^3\left(\cos\left(bx+a\right)\right)^2} + \frac{5}{48\,b\sin\left(bx+a\right)\left(\cos\left(bx+a\right)\right)^2} - \frac{5}{16\,b\sin\left(bx+a\right)} + \frac{5\,\ln\left(\sec\left(bx+a\right)+\tan\left(bx\right.\right.}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*csc(2*b*x+2*a)^3,x)
```

```
[Out] -1/24/b/sin(b*x+a)^3/cos(b*x+a)^2+5/48/b/sin(b*x+a)/cos(b*x+a)^2-5/16/b/sin
(b*x+a)+5/16/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [B] time = 1.95001, size = 2403, normalized size = 36.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 1/96*(4*(15*sin(9*b*x + 9*a) - 20*sin(7*b*x + 7*a) - 22*sin(5*b*x + 5*a) -
20*sin(3*b*x + 3*a) + 15*sin(b*x + a))*cos(10*b*x + 10*a) + 60*(sin(8*b*x +
 8*a) + 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(9*b
*x + 9*a) + 4*(20*sin(7*b*x + 7*a) + 22*sin(5*b*x + 5*a) + 20*sin(3*b*x + 3
*a) - 15*sin(b*x + a))*cos(8*b*x + 8*a) - 80*(2*sin(6*b*x + 6*a) - 2*sin(4*
b*x + 4*a) - sin(2*b*x + 2*a))*cos(7*b*x + 7*a) + 8*(22*sin(5*b*x + 5*a) +
20*sin(3*b*x + 3*a) - 15*sin(b*x + a))*cos(6*b*x + 6*a) + 88*(2*sin(4*b*x +
 4*a) + sin(2*b*x + 2*a))*cos(5*b*x + 5*a) - 40*(4*sin(3*b*x + 3*a) - 3*sin
(b*x + a))*cos(4*b*x + 4*a) + 15*(2*(cos(8*b*x + 8*a) + 2*cos(6*b*x + 6*a)
- 2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(10*b*x + 10*a) - cos(10*b*
x + 10*a)^2 - 2*(2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a)
 + 1)*cos(8*b*x + 8*a) - cos(8*b*x + 8*a)^2 + 4*(2*cos(4*b*x + 4*a) + cos(2
*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 4*cos(6*b*x + 6*a)^2 - 4*(cos(2*b*x + 2
*a) - 1)*cos(4*b*x + 4*a) - 4*cos(4*b*x + 4*a)^2 - cos(2*b*x + 2*a)^2 + 2*(
sin(8*b*x + 8*a) + 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) - sin(2*b*x + 2*
a))*sin(10*b*x + 10*a) - sin(10*b*x + 10*a)^2 - 2*(2*sin(6*b*x + 6*a) - 2*s
in(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - sin(8*b*x + 8*a)^2 +
 4*(2*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 4*sin(6*b*x +
 6*a)^2 - 4*sin(4*b*x + 4*a)^2 - 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - sin(
2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log((cos(b*x + 2*a)^2 + cos(a)^2 -
 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin
(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2
*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(15*cos(9*b*x + 9*a) - 20*
cos(7*b*x + 7*a) - 22*cos(5*b*x + 5*a) - 20*cos(3*b*x + 3*a) + 15*cos(b*x +
 a))*sin(10*b*x + 10*a) - 60*(cos(8*b*x + 8*a) + 2*cos(6*b*x + 6*a) - 2*cos
(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*sin(9*b*x + 9*a) - 4*(20*cos(7*b*x +
7*a) + 22*cos(5*b*x + 5*a) + 20*cos(3*b*x + 3*a) - 15*cos(b*x + a))*sin(8*b
*x + 8*a) + 80*(2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a)
+ 1)*sin(7*b*x + 7*a) - 8*(22*cos(5*b*x + 5*a) + 20*cos(3*b*x + 3*a) - 15*c
os(b*x + a))*sin(6*b*x + 6*a) - 88*(2*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) -
 1)*sin(5*b*x + 5*a) + 40*(4*cos(3*b*x + 3*a) - 3*cos(b*x + a))*sin(4*b*x +
 4*a) - 80*(cos(2*b*x + 2*a) - 1)*sin(3*b*x + 3*a) + 80*cos(3*b*x + 3*a)*si
n(2*b*x + 2*a) - 60*cos(b*x + a)*sin(2*b*x + 2*a) + 60*cos(2*b*x + 2*a)*sin
(b*x + a) - 60*sin(b*x + a))/(b*cos(10*b*x + 10*a)^2 + b*cos(8*b*x + 8*a)^2
 + 4*b*cos(6*b*x + 6*a)^2 + 4*b*cos(4*b*x + 4*a)^2 + b*cos(2*b*x + 2*a)^2 +
 b*sin(10*b*x + 10*a)^2 + b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 + 4
*b*sin(4*b*x + 4*a)^2 + 4*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b*sin(2*b*x
 + 2*a)^2 - 2*(b*cos(8*b*x + 8*a) + 2*b*cos(6*b*x + 6*a) - 2*b*cos(4*b*x +
4*a) - b*cos(2*b*x + 2*a) + b)*cos(10*b*x + 10*a) + 2*(2*b*cos(6*b*x + 6*a)
 - 2*b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) - 4*(2*b
*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a) - b)*cos(6*b*x + 6*a) + 4*(b*cos(2*b
*x + 2*a) - b)*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) - 2*(b*sin(8*b*x + 8
*a) + 2*b*sin(6*b*x + 6*a) - 2*b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*sin
(10*b*x + 10*a) + 2*(2*b*sin(6*b*x + 6*a) - 2*b*sin(4*b*x + 4*a) - b*sin(2*
b*x + 2*a))*sin(8*b*x + 8*a) - 4*(2*b*sin(4*b*x + 4*a) + b*sin(2*b*x + 2*a)
)*sin(6*b*x + 6*a) + b)
```

Fricas [B] time = 0.505113, size = 342, normalized size = 5.18

$$-\frac{30\,\cos\left(bx+a\right)^{4}-15\left(\cos\left(bx+a\right)^{4}-\cos\left(bx+a\right)^{2}\right)\log\left(\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)+15\left(\cos\left(bx+a\right)^{4}-\cos\left(bx\cdot\right.\right.}{96\left(b\cos\left(bx+a\right)^{4}-b\cos\left(bx+a\right)^{2}\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] -1/96*(30*cos(b*x + a)^4 - 15*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(sin(b*x
```

```
 + a) + 1)*sin(b*x + a) + 15*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(-sin(b*x
 + a) + 1)*sin(b*x + a) - 40*cos(b*x + a)^2 + 6)/((b*cos(b*x + a)^4 - b*cos
(b*x + a)^2)*sin(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(a+bx)\csc^3(2a+2bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)**3,x)
```

```
[Out] Integral(csc(a + b*x)*csc(2*a + 2*b*x)**3, x)
```

Giac [B] time = 4.39821, size = 4095, normalized size = 62.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] -1/192*(24*(tan(1/2*b*x + 2*a)^3*tan(1/2*a)^24 + 30*tan(1/2*b*x + 2*a)^3*ta
n(1/2*a)^22 - 6*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^23 + tan(1/2*b*x + 2*a)*tan
(1/2*a)^24 - 756*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^20 + 614*tan(1/2*b*x + 2*a
)^2*tan(1/2*a)^21 - 114*tan(1/2*b*x + 2*a)*tan(1/2*a)^22 + 6*tan(1/2*a)^23
+ 2058*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^18 - 4578*tan(1/2*b*x + 2*a)^2*tan(1
/2*a)^19 + 1932*tan(1/2*b*x + 2*a)*tan(1/2*a)^20 - 182*tan(1/2*a)^21 - 27*t
an(1/2*b*x + 2*a)^3*tan(1/2*a)^16 + 6210*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^17
 - 7462*tan(1/2*b*x + 2*a)*tan(1/2*a)^18 + 1554*tan(1/2*a)^19 - 9396*tan(1/
2*b*x + 2*a)^3*tan(1/2*a)^14 + 15588*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^15 - 2
331*tan(1/2*b*x + 2*a)*tan(1/2*a)^16 - 2178*tan(1/2*a)^17 - 21924*tan(1/2*b
*x + 2*a)^2*tan(1/2*a)^13 + 26028*tan(1/2*b*x + 2*a)*tan(1/2*a)^14 - 5668*t
an(1/2*a)^15 + 9396*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^10 - 21924*tan(1/2*b*x
+ 2*a)^2*tan(1/2*a)^11 + 6468*tan(1/2*a)^13 + 27*tan(1/2*b*x + 2*a)^3*tan(1
/2*a)^8 + 15588*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^9 - 26028*tan(1/2*b*x + 2*a
)*tan(1/2*a)^10 + 6468*tan(1/2*a)^11 - 2058*tan(1/2*b*x + 2*a)^3*tan(1/2*a)
^6 + 6210*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^7 + 2331*tan(1/2*b*x + 2*a)*tan(1
/2*a)^8 - 5668*tan(1/2*a)^9 + 756*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^4 - 4578*
tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 + 7462*tan(1/2*b*x + 2*a)*tan(1/2*a)^6 -
2178*tan(1/2*a)^7 - 30*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^2 + 614*tan(1/2*b*x
+ 2*a)^2*tan(1/2*a)^3 - 1932*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 1554*tan(1/2
*a)^5 - tan(1/2*b*x + 2*a)^3 - 6*tan(1/2*b*x + 2*a)^2*tan(1/2*a) + 114*tan(
1/2*b*x + 2*a)*tan(1/2*a)^2 - 182*tan(1/2*a)^3 - tan(1/2*b*x + 2*a) + 6*tan
(1/2*a))/((tan(1/2*a)^12 - 30*tan(1/2*a)^10 + 255*tan(1/2*a)^8 - 452*tan(1/
2*a)^6 + 255*tan(1/2*a)^4 - 30*tan(1/2*a)^2 + 1)*(tan(1/2*b*x + 2*a)^2*tan(
1/2*a)^6 - 15*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^4 + 12*tan(1/2*b*x + 2*a)*tan
(1/2*a)^5 - tan(1/2*a)^6 + 15*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2 - 40*tan(1/
2*b*x + 2*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 - tan(1/2*b*x + 2*a)^2 + 12*tan
(1/2*b*x + 2*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)^2) + (27*tan(1/2*b*x + 2*
a)^5*tan(1/2*a)^34 - 9*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^35 + tan(1/2*b*x + 2
*a)^3*tan(1/2*a)^36 + 1602*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^32 - 915*tan(1/2
*b*x + 2*a)^4*tan(1/2*a)^33 + 126*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^34 + 9*ta
n(1/2*b*x + 2*a)^2*tan(1/2*a)^35 - 50082*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^30
```

+ 58626*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^31 - 21141*tan(1/2*b*x + 2*a)^3*ta
n(1/2*a)^32 + 2373*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^33 + 27*tan(1/2*b*x + 2*
a)*tan(1/2*a)^34 + 400050*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^28 - 783810*tan(1
/2*b*x + 2*a)^4*tan(1/2*a)^29 + 487692*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^30 -
118404*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^31 + 9378*tan(1/2*b*x + 2*a)*tan(1/
2*a)^32 + 54*tan(1/2*a)^33 - 1301382*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^26 + 4
035870*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^27 - 3943944*tan(1/2*b*x + 2*a)^3*ta
n(1/2*a)^28 + 1555092*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^29 - 247074*tan(1/2*b
*x + 2*a)*tan(1/2*a)^30 + 11610*tan(1/2*a)^31 + 1238250*tan(1/2*b*x + 2*a)^
5*tan(1/2*a)^24 - 8115822*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^25 + 13194468*tan
(1/2*b*x + 2*a)^3*tan(1/2*a)^26 - 8075232*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2
7 + 1983762*tan(1/2*b*x + 2*a)*tan(1/2*a)^28 - 160362*tan(1/2*a)^29 + 26423
10*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^22 - 1212390*tan(1/2*b*x + 2*a)^4*tan(1/
2*a)^23 - 12476880*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^24 + 16317288*tan(1/2*b*
x + 2*a)^2*tan(1/2*a)^25 - 6559110*tan(1/2*b*x + 2*a)*tan(1/2*a)^26 + 80578
6*tan(1/2*a)^27 - 5947398*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^20 + 25548198*tan
(1/2*b*x + 2*a)^4*tan(1/2*a)^21 - 27336420*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^
22 + 2575260*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^23 + 6211530*tan(1/2*b*x + 2*a
)*tan(1/2*a)^24 - 1599858*tan(1/2*a)^25 - 19824660*tan(1/2*b*x + 2*a)^4*tan
(1/2*a)^19 + 58330530*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^20 - 51141420*tan(1/2
*b*x + 2*a)^2*tan(1/2*a)^21 + 13453830*tan(1/2*b*x + 2*a)*tan(1/2*a)^22 - 2
09790*tan(1/2*a)^23 + 5947398*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^16 - 19824660
*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^17 + 39278250*tan(1/2*b*x + 2*a)^2*tan(1/2
*a)^19 - 29430054*tan(1/2*b*x + 2*a)*tan(1/2*a)^20 + 5059854*tan(1/2*a)^21
- 2642310*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^14 + 25548198*tan(1/2*b*x + 2*a)^
4*tan(1/2*a)^15 - 58330530*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^16 + 39278250*ta
n(1/2*b*x + 2*a)^2*tan(1/2*a)^17 - 4136670*tan(1/2*a)^19 - 1238250*tan(1/2*
b*x + 2*a)^5*tan(1/2*a)^12 - 1212390*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^13 + 2
7336420*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^14 - 51141420*tan(1/2*b*x + 2*a)^2*
tan(1/2*a)^15 + 29430054*tan(1/2*b*x + 2*a)*tan(1/2*a)^16 - 4136670*tan(1/2
*a)^17 + 1301382*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^10 - 8115822*tan(1/2*b*x +
2*a)^4*tan(1/2*a)^11 + 12476880*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^12 + 25752
60*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^13 - 13453830*tan(1/2*b*x + 2*a)*tan(1/2
*a)^14 + 5059854*tan(1/2*a)^15 - 400050*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^8 +
4035870*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^9 - 13194468*tan(1/2*b*x + 2*a)^3*
tan(1/2*a)^10 + 16317288*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^11 - 6211530*tan(1
/2*b*x + 2*a)*tan(1/2*a)^12 - 209790*tan(1/2*a)^13 + 50082*tan(1/2*b*x + 2*
a)^5*tan(1/2*a)^6 - 783810*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^7 + 3943944*tan(
1/2*b*x + 2*a)^3*tan(1/2*a)^8 - 8075232*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^9 +
6559110*tan(1/2*b*x + 2*a)*tan(1/2*a)^10 - 1599858*tan(1/2*a)^11 - 1602*ta
n(1/2*b*x + 2*a)^5*tan(1/2*a)^4 + 58626*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^5 -
487692*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^6 + 1555092*tan(1/2*b*x + 2*a)^2*ta
n(1/2*a)^7 - 1983762*tan(1/2*b*x + 2*a)*tan(1/2*a)^8 + 805786*tan(1/2*a)^9
- 27*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^2 - 915*tan(1/2*b*x + 2*a)^4*tan(1/2*a
)^3 + 21141*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^4 - 118404*tan(1/2*b*x + 2*a)^2
*tan(1/2*a)^5 + 247074*tan(1/2*b*x + 2*a)*tan(1/2*a)^6 - 160362*tan(1/2*a)^
7 - 9*tan(1/2*b*x + 2*a)^4*tan(1/2*a) - 126*tan(1/2*b*x + 2*a)^3*tan(1/2*a)
^2 + 2373*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 - 9378*tan(1/2*b*x + 2*a)*tan(1
/2*a)^4 + 11610*tan(1/2*a)^5 - tan(1/2*b*x + 2*a)^3 + 9*tan(1/2*b*x + 2*a)^
2*tan(1/2*a) - 27*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 + 54*tan(1/2*a)^3)/((27*t
an(1/2*a)^15 - 270*tan(1/2*a)^13 + 981*tan(1/2*a)^11 - 1540*tan(1/2*a)^9 +
981*tan(1/2*a)^7 - 270*tan(1/2*a)^5 + 27*tan(1/2*a)^3)*(3*tan(1/2*b*x + 2*a
)^2*tan(1/2*a)^5 - tan(1/2*b*x + 2*a)*tan(1/2*a)^6 - 10*tan(1/2*b*x + 2*a)^
2*tan(1/2*a)^3 + 15*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 - 3*tan(1/2*a)^5 + 3*ta
n(1/2*b*x + 2*a)^2*tan(1/2*a) - 15*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 + 10*tan
(1/2*a)^3 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a))^3) - 60*log(abs(tan(1/2*b*x
+ 2*a)*tan(1/2*a)^3 + 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - tan(1/2*a)^3 - 3*
tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 - tan(1/2*b*x + 2*a) + 3*tan
(1/2*a) - 1)) + 60*log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 3*tan(1/2*b*x

```
+ 2*a)*tan(1/2*a)^2 + tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*ta
n(1/2*a)^2 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a) - 1)))/b
```

3.44 $\int \csc(a+bx)\csc^4(2a+2bx)\,dx$

Optimal. Leaf size=89

$$\frac{35\sec^3(a+bx)}{384b}+\frac{35\sec(a+bx)}{128b}-\frac{35\tanh^{-1}(\cos(a+bx))}{128b}-\frac{\csc^4(a+bx)\sec^3(a+bx)}{64b}-\frac{7\csc^2(a+bx)\sec^3(a+bx)}{128b}$$

```
[Out] (-35*ArcTanh[Cos[a + b*x]])/(128*b) + (35*Sec[a + b*x])/(128*b) + (35*Sec[a
 + b*x]^3)/(384*b) - (7*Csc[a + b*x]^2*Sec[a + b*x]^3)/(128*b) - (Csc[a + b
*x]^4*Sec[a + b*x]^3)/(64*b)
```

Rubi [A] time = 0.07168, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {4288, 2622, 288, 302, 207}

$$\frac{35\sec^3(a+bx)}{384b}+\frac{35\sec(a+bx)}{128b}-\frac{35\tanh^{-1}(\cos(a+bx))}{128b}-\frac{\csc^4(a+bx)\sec^3(a+bx)}{64b}-\frac{7\csc^2(a+bx)\sec^3(a+bx)}{128b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] (-35*ArcTanh[Cos[a + b*x]])/(128*b) + (35*Sec[a + b*x])/(128*b) + (35*Sec[a
 + b*x]^3)/(384*b) - (7*Csc[a + b*x]^2*Sec[a + b*x]^3)/(128*b) - (Csc[a + b
*x]^4*Sec[a + b*x]^3)/(64*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc(a+bx)\csc^4(2a+2bx)\,dx = \frac{1}{16}\int \csc^5(a+bx)\sec^4(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^8}{(-1+x^2)^3}\,dx, x, \sec(a+bx)\right)}{16b}$$
$$= -\frac{\csc^4(a+bx)\sec^3(a+bx)}{64b} + \frac{7\,\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, \sec(a+bx)\right)}{64b}$$
$$= -\frac{7\csc^2(a+bx)\sec^3(a+bx)}{128b} - \frac{\csc^4(a+bx)\sec^3(a+bx)}{64b} + \frac{35\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \text{s}\right.}{128b}$$
$$= -\frac{7\csc^2(a+bx)\sec^3(a+bx)}{128b} - \frac{\csc^4(a+bx)\sec^3(a+bx)}{64b} + \frac{35\,\text{Subst}\left(\int \left(1+x^2+\frac{}{-1}\right.\right.}{128}$$
$$= \frac{35\sec(a+bx)}{128b} + \frac{35\sec^3(a+bx)}{384b} - \frac{7\csc^2(a+bx)\sec^3(a+bx)}{128b} - \frac{\csc^4(a+bx)\sec^3(a}{64b}$$
$$= -\frac{35\tanh^{-1}(\cos(a+bx))}{128b} + \frac{35\sec(a+bx)}{128b} + \frac{35\sec^3(a+bx)}{384b} - \frac{7\csc^2(a+bx)\sec^3(a}{128b}$$

Mathematica [B] time = 0.511508, size = 268, normalized size = 3.01

$$-\frac{\csc^{10}(a+bx)\left(658\cos(2(a+bx)) - 228\cos(3(a+bx)) + 140\cos(4(a+bx)) - 76\cos(5(a+bx)) - 210\cos(6(a+bx)) + \right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] -(Csc[a + b*x]^10*(-204 + 658*Cos[2*(a + b*x)] - 228*Cos[3*(a + b*x)] + 140
*Cos[4*(a + b*x)] - 76*Cos[5*(a + b*x)] - 210*Cos[6*(a + b*x)] + 76*Cos[7*(
a + b*x)] - 315*Cos[3*(a + b*x)]*Log[Cos[(a + b*x)/2]] - 105*Cos[5*(a + b*x
)]*Log[Cos[(a + b*x)/2]] + 105*Cos[7*(a + b*x)]*Log[Cos[(a + b*x)/2]] + 3*C
os[a + b*x]*(76 + 105*Log[Cos[(a + b*x)/2]] - 105*Log[Sin[(a + b*x)/2]]) +
315*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] + 105*Cos[5*(a + b*x)]*Log[Sin[(
a + b*x)/2]] - 105*Cos[7*(a + b*x)]*Log[Sin[(a + b*x)/2]]))/(384*b*(Csc[(a
+ b*x)/2]^2 - Sec[(a + b*x)/2]^2)^3)
```

Maple [A] time = 0.037, size = 99, normalized size = 1.1

$$-\frac{1}{64\,b\,(\sin(bx+a))^4(\cos(bx+a))^3} + \frac{7}{192\,b\,(\sin(bx+a))^2(\cos(bx+a))^3} - \frac{35}{384\,b\,(\sin(bx+a))^2\cos(bx+a)} + \frac{3}{128\,b\,\text{co}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*csc(2*b*x+2*a)^4,x)
```

```
[Out] -1/64/b/sin(b*x+a)^4/cos(b*x+a)^3+7/192/b/sin(b*x+a)^2/cos(b*x+a)^3-35/384/
b/sin(b*x+a)^2/cos(b*x+a)+35/128/b/cos(b*x+a)+35/128/b*ln(csc(b*x+a)-cot(b*
x+a))
```

Maxima [B] time = 1.78768, size = 5192, normalized size = 58.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/768*(4*(105*cos(13*b*x + 13*a) - 70*cos(11*b*x + 11*a) - 329*cos(9*b*x +
9*a) + 204*cos(7*b*x + 7*a) - 329*cos(5*b*x + 5*a) - 70*cos(3*b*x + 3*a) +
105*cos(b*x + a))*cos(14*b*x + 14*a) - 420*(cos(12*b*x + 12*a) + 3*cos(10*b
*x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) +
 cos(2*b*x + 2*a) - 1)*cos(13*b*x + 13*a) + 4*(70*cos(11*b*x + 11*a) + 329*
cos(9*b*x + 9*a) - 204*cos(7*b*x + 7*a) + 329*cos(5*b*x + 5*a) + 70*cos(3*b
*x + 3*a) - 105*cos(b*x + a))*cos(12*b*x + 12*a) + 280*(3*cos(10*b*x + 10*a
) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) + cos(2*b*
x + 2*a) - 1)*cos(11*b*x + 11*a) + 12*(329*cos(9*b*x + 9*a) - 204*cos(7*b*x
 + 7*a) + 329*cos(5*b*x + 5*a) + 70*cos(3*b*x + 3*a) - 105*cos(b*x + a))*co
s(10*b*x + 10*a) - 1316*(3*cos(8*b*x + 8*a) + 3*cos(6*b*x + 6*a) - 3*cos(4*
b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(9*b*x + 9*a) + 12*(204*cos(7*b*x + 7
*a) - 329*cos(5*b*x + 5*a) - 70*cos(3*b*x + 3*a) + 105*cos(b*x + a))*cos(8*
b*x + 8*a) + 816*(3*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a
) + 1)*cos(7*b*x + 7*a) - 84*(47*cos(5*b*x + 5*a) + 10*cos(3*b*x + 3*a) - 1
5*cos(b*x + a))*cos(6*b*x + 6*a) + 1316*(3*cos(4*b*x + 4*a) + cos(2*b*x + 2
*a) - 1)*cos(5*b*x + 5*a) + 420*(2*cos(3*b*x + 3*a) - 3*cos(b*x + a))*cos(4
*b*x + 4*a) + 280*(cos(2*b*x + 2*a) - 1)*cos(3*b*x + 3*a) - 420*cos(2*b*x +
 2*a)*cos(b*x + a) + 105*(2*(cos(12*b*x + 12*a) + 3*cos(10*b*x + 10*a) - 3*
cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) + cos(2*b*x + 2*
a) - 1)*cos(14*b*x + 14*a) - cos(14*b*x + 14*a)^2 - 2*(3*cos(10*b*x + 10*a)
 - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) + cos(2*b*x
 + 2*a) - 1)*cos(12*b*x + 12*a) - cos(12*b*x + 12*a)^2 + 6*(3*cos(8*b*x + 8
*a) + 3*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(1
0*b*x + 10*a) - 9*cos(10*b*x + 10*a)^2 - 6*(3*cos(6*b*x + 6*a) - 3*cos(4*b*
x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - 9*cos(8*b*x + 8*a)^2 +
6*(3*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 9*cos(6*b*
x + 6*a)^2 - 6*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 9*cos(4*b*x + 4*a)
^2 - cos(2*b*x + 2*a)^2 + 2*(sin(12*b*x + 12*a) + 3*sin(10*b*x + 10*a) - 3*
sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + sin(2*b*x + 2*
a))*sin(14*b*x + 14*a) - sin(14*b*x + 14*a)^2 - 2*(3*sin(10*b*x + 10*a) - 3
*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + sin(2*b*x + 2
*a))*sin(12*b*x + 12*a) - sin(12*b*x + 12*a)^2 + 6*(3*sin(8*b*x + 8*a) + 3*
sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(10*b*x + 10*a
) - 9*sin(10*b*x + 10*a)^2 - 6*(3*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - s
in(2*b*x + 2*a))*sin(8*b*x + 8*a) - 9*sin(8*b*x + 8*a)^2 + 6*(3*sin(4*b*x +
 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 9*sin(6*b*x + 6*a)^2 - 9*sin(4
*b*x + 4*a)^2 - 6*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - sin(2*b*x + 2*a)^2 +
2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin
(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 105*(2*(cos(12*b*x + 12*a) + 3*co
s(10*b*x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x +
4*a) + cos(2*b*x + 2*a) - 1)*cos(14*b*x + 14*a) - cos(14*b*x + 14*a)^2 - 2*
(3*cos(10*b*x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b
*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(12*b*x + 12*a) - cos(12*b*x + 12*a)^2
 + 6*(3*cos(8*b*x + 8*a) + 3*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - cos(2*
```

```
b*x + 2*a) + 1)*cos(10*b*x + 10*a) - 9*cos(10*b*x + 10*a)^2 - 6*(3*cos(6*b*
x + 6*a) - 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - 9*
cos(8*b*x + 8*a)^2 + 6*(3*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*
x + 6*a) - 9*cos(6*b*x + 6*a)^2 - 6*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a)
 - 9*cos(4*b*x + 4*a)^2 - cos(2*b*x + 2*a)^2 + 2*(sin(12*b*x + 12*a) + 3*si
n(10*b*x + 10*a) - 3*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x +
4*a) + sin(2*b*x + 2*a))*sin(14*b*x + 14*a) - sin(14*b*x + 14*a)^2 - 2*(3*s
in(10*b*x + 10*a) - 3*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x +
 4*a) + sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - sin(12*b*x + 12*a)^2 + 6*(3*
sin(8*b*x + 8*a) + 3*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - sin(2*b*x + 2*
a))*sin(10*b*x + 10*a) - 9*sin(10*b*x + 10*a)^2 - 6*(3*sin(6*b*x + 6*a) - 3
*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 9*sin(8*b*x + 8*a)
^2 + 6*(3*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 9*sin(6*b
*x + 6*a)^2 - 9*sin(4*b*x + 4*a)^2 - 6*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) -
sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*co
s(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*(105*sin(1
3*b*x + 13*a) - 70*sin(11*b*x + 11*a) - 329*sin(9*b*x + 9*a) + 204*sin(7*b*
x + 7*a) - 329*sin(5*b*x + 5*a) - 70*sin(3*b*x + 3*a) + 105*sin(b*x + a))*s
in(14*b*x + 14*a) - 420*(sin(12*b*x + 12*a) + 3*sin(10*b*x + 10*a) - 3*sin(
8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*
sin(13*b*x + 13*a) + 4*(70*sin(11*b*x + 11*a) + 329*sin(9*b*x + 9*a) - 204*
sin(7*b*x + 7*a) + 329*sin(5*b*x + 5*a) + 70*sin(3*b*x + 3*a) - 105*sin(b*x
 + a))*sin(12*b*x + 12*a) + 280*(3*sin(10*b*x + 10*a) - 3*sin(8*b*x + 8*a)
- 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(11*b*x +
11*a) + 12*(329*sin(9*b*x + 9*a) - 204*sin(7*b*x + 7*a) + 329*sin(5*b*x + 5
*a) + 70*sin(3*b*x + 3*a) - 105*sin(b*x + a))*sin(10*b*x + 10*a) - 1316*(3*
sin(8*b*x + 8*a) + 3*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - sin(2*b*x + 2*
a))*sin(9*b*x + 9*a) + 12*(204*sin(7*b*x + 7*a) - 329*sin(5*b*x + 5*a) - 70
*sin(3*b*x + 3*a) + 105*sin(b*x + a))*sin(8*b*x + 8*a) + 816*(3*sin(6*b*x +
 6*a) - 3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(7*b*x + 7*a) - 84*(47*si
n(5*b*x + 5*a) + 10*sin(3*b*x + 3*a) - 15*sin(b*x + a))*sin(6*b*x + 6*a) +
1316*(3*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(5*b*x + 5*a) + 420*(2*sin(
3*b*x + 3*a) - 3*sin(b*x + a))*sin(4*b*x + 4*a) + 280*sin(3*b*x + 3*a)*sin(
2*b*x + 2*a) - 420*sin(2*b*x + 2*a)*sin(b*x + a) + 420*cos(b*x + a))/(b*cos
(14*b*x + 14*a)^2 + b*cos(12*b*x + 12*a)^2 + 9*b*cos(10*b*x + 10*a)^2 + 9*b
*cos(8*b*x + 8*a)^2 + 9*b*cos(6*b*x + 6*a)^2 + 9*b*cos(4*b*x + 4*a)^2 + b*c
os(2*b*x + 2*a)^2 + b*sin(14*b*x + 14*a)^2 + b*sin(12*b*x + 12*a)^2 + 9*b*s
in(10*b*x + 10*a)^2 + 9*b*sin(8*b*x + 8*a)^2 + 9*b*sin(6*b*x + 6*a)^2 + 9*b
*sin(4*b*x + 4*a)^2 + 6*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b*sin(2*b*x +
 2*a)^2 - 2*(b*cos(12*b*x + 12*a) + 3*b*cos(10*b*x + 10*a) - 3*b*cos(8*b*x
+ 8*a) - 3*b*cos(6*b*x + 6*a) + 3*b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a) -
 b)*cos(14*b*x + 14*a) + 2*(3*b*cos(10*b*x + 10*a) - 3*b*cos(8*b*x + 8*a) -
 3*b*cos(6*b*x + 6*a) + 3*b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a) - b)*cos(
12*b*x + 12*a) - 6*(3*b*cos(8*b*x + 8*a) + 3*b*cos(6*b*x + 6*a) - 3*b*cos(4
*b*x + 4*a) - b*cos(2*b*x + 2*a) + b)*cos(10*b*x + 10*a) + 6*(3*b*cos(6*b*x
 + 6*a) - 3*b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) -
 6*(3*b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a) - b)*cos(6*b*x + 6*a) + 6*(b*
cos(2*b*x + 2*a) - b)*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) - 2*(b*sin(12
*b*x + 12*a) + 3*b*sin(10*b*x + 10*a) - 3*b*sin(8*b*x + 8*a) - 3*b*sin(6*b*
x + 6*a) + 3*b*sin(4*b*x + 4*a) + b*sin(2*b*x + 2*a))*sin(14*b*x + 14*a) +
2*(3*b*sin(10*b*x + 10*a) - 3*b*sin(8*b*x + 8*a) - 3*b*sin(6*b*x + 6*a) + 3
*b*sin(4*b*x + 4*a) + b*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - 6*(3*b*sin(8
*b*x + 8*a) + 3*b*sin(6*b*x + 6*a) - 3*b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2
*a))*sin(10*b*x + 10*a) + 6*(3*b*sin(6*b*x + 6*a) - 3*b*sin(4*b*x + 4*a) -
b*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 6*(3*b*sin(4*b*x + 4*a) + b*sin(2*b*
x + 2*a))*sin(6*b*x + 6*a) + b)
```

Fricas [A] time = 0.518579, size = 417, normalized size = 4.69

$$\frac{210 \cos(bx+a)^6 - 350 \cos(bx+a)^4 + 112 \cos(bx+a)^2 - 105\left(\cos(bx+a)^7 - 2\cos(bx+a)^5 + \cos(bx+a)^3\right)\log\left(\right.}{768\left(b\cos(bx+a)^7 - 2b\cos(bx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] 1/768*(210*cos(b*x + a)^6 - 350*cos(b*x + a)^4 + 112*cos(b*x + a)^2 - 105*(
cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3)*log(1/2*cos(b*x + a) +
1/2) + 105*(cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3)*log(-1/2*co
s(b*x + a) + 1/2) + 16)/(b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*cos(b*x
+ a)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(a+bx)\csc^4(2a+2bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)**4,x)
```

```
[Out] Integral(csc(a + b*x)*csc(2*a + 2*b*x)**4, x)
```

Giac [B] time = 12.2181, size = 7393, normalized size = 83.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] -1/3072*(256*(36*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^35 - 6*tan(1/2*b*x + 2*a)^
4*tan(1/2*a)^36 - 1848*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^33 + 1134*tan(1/2*b*
x + 2*a)^4*tan(1/2*a)^34 - 180*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^35 + 9*tan(1
/2*b*x + 2*a)^2*tan(1/2*a)^36 + 39276*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^31 -
42894*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^32 + 14532*tan(1/2*b*x + 2*a)^3*tan(1
/2*a)^33 - 2052*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^34 + 144*tan(1/2*b*x + 2*a)
*tan(1/2*a)^35 - 5*tan(1/2*a)^36 - 433836*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^2
9 + 709068*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^30 - 376632*tan(1/2*b*x + 2*a)^3
*tan(1/2*a)^31 + 83367*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^32 - 8364*tan(1/2*b*
x + 2*a)*tan(1/2*a)^33 + 342*tan(1/2*a)^34 + 2430348*tan(1/2*b*x + 2*a)^5*t
an(1/2*a)^27 - 5742396*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^28 + 4466808*tan(1/2
*b*x + 2*a)^3*tan(1/2*a)^29 - 1422120*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^30 +
193068*tan(1/2*b*x + 2*a)*tan(1/2*a)^31 - 9639*tan(1/2*a)^32 - 5123196*tan(
1/2*b*x + 2*a)^5*tan(1/2*a)^25 + 20329092*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^2
6 - 24275368*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^27 + 11529468*tan(1/2*b*x + 2*
a)^2*tan(1/2*a)^28 - 2196396*tan(1/2*b*x + 2*a)*tan(1/2*a)^29 + 141660*tan(
1/2*a)^30 - 1201860*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^23 - 18742620*tan(1/2*b
*x + 2*a)^4*tan(1/2*a)^24 + 50327784*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^25 - 4
0521528*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^26 + 12145596*tan(1/2*b*x + 2*a)*ta
```

n(1/2*a)^27 - 1116072*tan(1/2*a)^28 + 15332100*tan(1/2*b*x + 2*a)^5*tan(1/2
*a)^21 - 42937380*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^22 + 10264680*tan(1/2*b*x
 + 2*a)^3*tan(1/2*a)^23 + 37504740*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^24 - 254
25036*tan(1/2*b*x + 2*a)*tan(1/2*a)^25 + 4162356*tan(1/2*a)^26 - 11041020*t
an(1/2*b*x + 2*a)^5*tan(1/2*a)^19 + 84945240*tan(1/2*b*x + 2*a)^4*tan(1/2*a
)^20 - 153596328*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^21 + 85465080*tan(1/2*b*x
+ 2*a)^2*tan(1/2*a)^22 - 5642820*tan(1/2*b*x + 2*a)*tan(1/2*a)^23 - 3694080
*tan(1/2*a)^24 - 11041020*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^17 + 112912560*ta
n(1/2*b*x + 2*a)^3*tan(1/2*a)^19 - 170450298*tan(1/2*b*x + 2*a)^2*tan(1/2*a
)^20 + 76700868*tan(1/2*b*x + 2*a)*tan(1/2*a)^21 - 8751060*tan(1/2*a)^22 +
15332100*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^15 - 84945240*tan(1/2*b*x + 2*a)^4
*tan(1/2*a)^16 + 112912560*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^17 - 55767060*ta
n(1/2*b*x + 2*a)*tan(1/2*a)^19 + 16730118*tan(1/2*a)^20 - 1201860*tan(1/2*b
*x + 2*a)^5*tan(1/2*a)^13 + 42937380*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^14 - 1
53596328*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^15 + 170450298*tan(1/2*b*x + 2*a)^
2*tan(1/2*a)^16 - 55767060*tan(1/2*b*x + 2*a)*tan(1/2*a)^17 - 5123196*tan(1
/2*b*x + 2*a)^5*tan(1/2*a)^11 + 18742620*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^12
 + 10264680*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^13 - 85465080*tan(1/2*b*x + 2*a
)^2*tan(1/2*a)^14 + 76700868*tan(1/2*b*x + 2*a)*tan(1/2*a)^15 - 16730118*ta
n(1/2*a)^16 + 2430348*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^9 - 20329092*tan(1/2*
b*x + 2*a)^4*tan(1/2*a)^10 + 50327784*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^11 -
37504740*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^12 - 5642820*tan(1/2*b*x + 2*a)*ta
n(1/2*a)^13 + 8751060*tan(1/2*a)^14 - 433836*tan(1/2*b*x + 2*a)^5*tan(1/2*a
)^7 + 5742396*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^8 - 24275368*tan(1/2*b*x + 2*
a)^3*tan(1/2*a)^9 + 40521528*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^10 - 25425036*
tan(1/2*b*x + 2*a)*tan(1/2*a)^11 + 3694080*tan(1/2*a)^12 + 39276*tan(1/2*b*
x + 2*a)^5*tan(1/2*a)^5 - 709068*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^6 + 446680
8*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^7 - 11529468*tan(1/2*b*x + 2*a)^2*tan(1/2
*a)^8 + 12145596*tan(1/2*b*x + 2*a)*tan(1/2*a)^9 - 4162356*tan(1/2*a)^10 -
1848*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^3 + 42894*tan(1/2*b*x + 2*a)^4*tan(1/2
*a)^4 - 376632*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^5 + 1422120*tan(1/2*b*x + 2*
a)^2*tan(1/2*a)^6 - 2196396*tan(1/2*b*x + 2*a)*tan(1/2*a)^7 + 1116072*tan(1
/2*a)^8 + 36*tan(1/2*b*x + 2*a)^5*tan(1/2*a) - 1134*tan(1/2*b*x + 2*a)^4*ta
n(1/2*a)^2 + 14532*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^3 - 83367*tan(1/2*b*x +
2*a)^2*tan(1/2*a)^4 + 193068*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - 141660*tan(1
/2*a)^6 + 6*tan(1/2*b*x + 2*a)^4 - 180*tan(1/2*b*x + 2*a)^3*tan(1/2*a) + 20
52*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2 - 8364*tan(1/2*b*x + 2*a)*tan(1/2*a)^3
 + 9639*tan(1/2*a)^4 - 9*tan(1/2*b*x + 2*a)^2 + 144*tan(1/2*b*x + 2*a)*tan(
1/2*a) - 342*tan(1/2*a)^2 + 5)/((tan(1/2*a)^18 - 45*tan(1/2*a)^16 + 720*tan
(1/2*a)^14 - 4728*tan(1/2*a)^12 + 10890*tan(1/2*a)^10 - 10890*tan(1/2*a)^8
+ 4728*tan(1/2*a)^6 - 720*tan(1/2*a)^4 + 45*tan(1/2*a)^2 - 1)*(tan(1/2*b*x
+ 2*a)^2*tan(1/2*a)^6 - 15*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^4 + 12*tan(1/2*b
*x + 2*a)*tan(1/2*a)^5 - tan(1/2*a)^6 + 15*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^
2 - 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 - tan(1/2*b*x + 2*
a)^2 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)^3) + 3*(108*
tan(1/2*b*x + 2*a)^7*tan(1/2*a)^45 - 54*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^46
+ 12*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^47 - tan(1/2*b*x + 2*a)^4*tan(1/2*a)^4
8 + 8316*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^43 - 6120*tan(1/2*b*x + 2*a)^6*tan
(1/2*a)^44 + 1400*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^45 - 24*tan(1/2*b*x + 2*a
)^4*tan(1/2*a)^46 - 12*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^47 - 253008*tan(1/2*
b*x + 2*a)^7*tan(1/2*a)^41 + 354774*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^42 - 16
5384*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^43 + 30180*tan(1/2*b*x + 2*a)^4*tan(1/
2*a)^44 - 1400*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^45 - 54*tan(1/2*b*x + 2*a)^2
*tan(1/2*a)^46 + 3918128*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^39 - 7606080*tan(1
/2*b*x + 2*a)^6*tan(1/2*a)^40 + 5365548*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^41
- 1698920*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^42 + 235368*tan(1/2*b*x + 2*a)^3*
tan(1/2*a)^43 - 10008*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^44 - 108*tan(1/2*b*x
+ 2*a)*tan(1/2*a)^45 - 32664372*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^37 + 853525
14*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^38 - 82649988*tan(1/2*b*x + 2*a)^5*tan(1

/2*a)^39 + 37937604*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^40 - 8596476*tan(1/2*b*
x + 2*a)^3*tan(1/2*a)^41 + 879006*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^42 - 2775
6*tan(1/2*b*x + 2*a)*tan(1/2*a)^43 - 162*tan(1/2*a)^44 + 150470268*tan(1/2*
b*x + 2*a)^7*tan(1/2*a)^35 - 527449208*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^36 +
684424728*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^37 - 427365432*tan(1/2*b*x + 2*a
)^4*tan(1/2*a)^38 + 137583540*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^39 - 22235328
*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^40 + 1556784*tan(1/2*b*x + 2*a)*tan(1/2*a)
^41 - 28944*tan(1/2*a)^42 - 369285568*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^33 +
1800957150*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^34 - 3157829352*tan(1/2*b*x + 2*
a)^5*tan(1/2*a)^35 + 2638274028*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^36 - 114501
5352*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^37 + 257818938*tan(1/2*b*x + 2*a)^2*ta
n(1/2*a)^38 - 27438800*tan(1/2*b*x + 2*a)*tan(1/2*a)^39 + 1015632*tan(1/2*a
)^40 + 332678976*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^31 - 2973661056*tan(1/2*b*
x + 2*a)^6*tan(1/2*a)^32 + 7767810012*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^33 -
9003276360*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^34 + 5266355688*tan(1/2*b*x + 2*
a)^3*tan(1/2*a)^35 - 1589641352*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^36 + 231030
852*tan(1/2*b*x + 2*a)*tan(1/2*a)^37 - 12400752*tan(1/2*a)^38 + 498198168*t
an(1/2*b*x + 2*a)^7*tan(1/2*a)^29 + 141213844*tan(1/2*b*x + 2*a)^6*tan(1/2*
a)^30 - 6978686184*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^31 + 14860730619*tan(1/2
*b*x + 2*a)^4*tan(1/2*a)^32 - 12919461276*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^3
3 + 5392538622*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^34 - 1054828476*tan(1/2*b*x
+ 2*a)*tan(1/2*a)^35 + 76246378*tan(1/2*a)^36 - 1473564360*tan(1/2*b*x + 2*
a)^7*tan(1/2*a)^27 + 7565433072*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^28 - 104925
95088*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^29 - 707654192*tan(1/2*b*x + 2*a)^4*t
an(1/2*a)^30 + 11624633640*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^31 - 8882796672*
tan(1/2*b*x + 2*a)^2*tan(1/2*a)^32 + 2571218880*tan(1/2*b*x + 2*a)*tan(1/2*
a)^33 - 256007232*tan(1/2*a)^34 + 883677600*tan(1/2*b*x + 2*a)^7*tan(1/2*a)
^25 - 10018470420*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^26 + 30880710896*tan(1/2*
b*x + 2*a)^5*tan(1/2*a)^27 - 37780516920*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^28
+ 17392225680*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^29 + 478350836*tan(1/2*b*x +
2*a)^2*tan(1/2*a)^30 - 2321948736*tan(1/2*b*x + 2*a)*tan(1/2*a)^31 + 42019
1712*tan(1/2*a)^32 + 883677600*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^23 - 1862376
5160*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^25 + 50161528368*tan(1/2*b*x + 2*a)^4*
tan(1/2*a)^26 - 51515325744*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^27 + 2260797672
0*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^28 - 3433884120*tan(1/2*b*x + 2*a)*tan(1/
2*a)^29 - 26975424*tan(1/2*a)^30 - 1473564360*tan(1/2*b*x + 2*a)^7*tan(1/2*
a)^21 + 10018470420*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^22 - 18623765160*tan(1/
2*b*x + 2*a)^5*tan(1/2*a)^23 + 31169987784*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^
25 - 30241452708*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^26 + 10324445736*tan(1/2*b
*x + 2*a)*tan(1/2*a)^27 - 1070628372*tan(1/2*a)^28 + 498198168*tan(1/2*b*x
+ 2*a)^7*tan(1/2*a)^19 - 7565433072*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^20 + 30
880710896*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^21 - 50161528368*tan(1/2*b*x + 2*
a)^4*tan(1/2*a)^22 + 31169987784*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^23 - 62969
40000*tan(1/2*b*x + 2*a)*tan(1/2*a)^25 + 1453140000*tan(1/2*a)^26 + 3326789
76*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^17 - 141213844*tan(1/2*b*x + 2*a)^6*tan(
1/2*a)^18 - 10492595088*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^19 + 37780516920*ta
n(1/2*b*x + 2*a)^4*tan(1/2*a)^20 - 51515325744*tan(1/2*b*x + 2*a)^3*tan(1/2
*a)^21 + 30241452708*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^22 - 6296940000*tan(1/
2*b*x + 2*a)*tan(1/2*a)^23 - 369285568*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^15 +
2973661056*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^16 - 6978686184*tan(1/2*b*x + 2
*a)^5*tan(1/2*a)^17 + 707654192*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^18 + 173922
25680*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^19 - 22607976720*tan(1/2*b*x + 2*a)^2
*tan(1/2*a)^20 + 10324445736*tan(1/2*b*x + 2*a)*tan(1/2*a)^21 - 1453140000*
tan(1/2*a)^22 + 150470268*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^13 - 1800957150*t
an(1/2*b*x + 2*a)^6*tan(1/2*a)^14 + 7767810012*tan(1/2*b*x + 2*a)^5*tan(1/2
*a)^15 - 14860730619*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^16 + 11624633640*tan(1
/2*b*x + 2*a)^3*tan(1/2*a)^17 - 478350836*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^1
8 - 3433884120*tan(1/2*b*x + 2*a)*tan(1/2*a)^19 + 1070628372*tan(1/2*a)^20
- 32664372*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^11 + 527449208*tan(1/2*b*x + 2*a

```
)^6*tan(1/2*a)^12 - 3157829352*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^13 + 9003276
360*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^14 - 12919461276*tan(1/2*b*x + 2*a)^3*t
an(1/2*a)^15 + 8882796672*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^16 - 2321948736*t
an(1/2*b*x + 2*a)*tan(1/2*a)^17 + 26975424*tan(1/2*a)^18 + 3918128*tan(1/2*
b*x + 2*a)^7*tan(1/2*a)^9 - 85352514*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^10 + 6
84424728*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^11 - 2638274028*tan(1/2*b*x + 2*a)
^4*tan(1/2*a)^12 + 5266355688*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^13 - 53925386
22*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^14 + 2571218880*tan(1/2*b*x + 2*a)*tan(1
/2*a)^15 - 420191712*tan(1/2*a)^16 - 253008*tan(1/2*b*x + 2*a)^7*tan(1/2*a)
^7 + 7606080*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^8 - 82649988*tan(1/2*b*x + 2*a
)^5*tan(1/2*a)^9 + 427365432*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^10 - 114501535
2*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^11 + 1589641352*tan(1/2*b*x + 2*a)^2*tan(
1/2*a)^12 - 1054828476*tan(1/2*b*x + 2*a)*tan(1/2*a)^13 + 256007232*tan(1/2
*a)^14 + 8316*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^5 - 354774*tan(1/2*b*x + 2*a)
^6*tan(1/2*a)^6 + 5365548*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^7 - 37937604*tan(
1/2*b*x + 2*a)^4*tan(1/2*a)^8 + 137583540*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^9
 - 257818938*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^10 + 231030852*tan(1/2*b*x + 2
*a)*tan(1/2*a)^11 - 76246378*tan(1/2*a)^12 + 108*tan(1/2*b*x + 2*a)^7*tan(1
/2*a)^3 + 6120*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^4 - 165384*tan(1/2*b*x + 2*a
)^5*tan(1/2*a)^5 + 1698920*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^6 - 8596476*tan(
1/2*b*x + 2*a)^3*tan(1/2*a)^7 + 22235328*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^8
- 27438800*tan(1/2*b*x + 2*a)*tan(1/2*a)^9 + 12400752*tan(1/2*a)^10 + 54*ta
n(1/2*b*x + 2*a)^6*tan(1/2*a)^2 + 1400*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^3 -
30180*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^4 + 235368*tan(1/2*b*x + 2*a)^3*tan(1
/2*a)^5 - 879006*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^6 + 1556784*tan(1/2*b*x +
2*a)*tan(1/2*a)^7 - 1015632*tan(1/2*a)^8 + 12*tan(1/2*b*x + 2*a)^5*tan(1/2*
a) + 24*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^2 - 1400*tan(1/2*b*x + 2*a)^3*tan(1
/2*a)^3 + 10008*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^4 - 27756*tan(1/2*b*x + 2*a
)*tan(1/2*a)^5 + 28944*tan(1/2*a)^6 + tan(1/2*b*x + 2*a)^4 - 12*tan(1/2*b*x
 + 2*a)^3*tan(1/2*a) + 54*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2 - 108*tan(1/2*b
*x + 2*a)*tan(1/2*a)^3 + 162*tan(1/2*a)^4)/((81*tan(1/2*a)^20 - 1080*tan(1/
2*a)^18 + 5724*tan(1/2*a)^16 - 15240*tan(1/2*a)^14 + 21286*tan(1/2*a)^12 -
15240*tan(1/2*a)^10 + 5724*tan(1/2*a)^8 - 1080*tan(1/2*a)^6 + 81*tan(1/2*a)
^4)*(3*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 - tan(1/2*b*x + 2*a)*tan(1/2*a)^6
- 10*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 + 15*tan(1/2*b*x + 2*a)*tan(1/2*a)^4
 - 3*tan(1/2*a)^5 + 3*tan(1/2*b*x + 2*a)^2*tan(1/2*a) - 15*tan(1/2*b*x + 2*
a)*tan(1/2*a)^2 + 10*tan(1/2*a)^3 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a))^4) +
 840*log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2
*a) + 3*tan(1/2*a)^2 - 1)) - 840*log(abs(3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2
- tan(1/2*a)^3 - tan(1/2*b*x + 2*a) + 3*tan(1/2*a))))/b
```

3.45 $\int \csc^2(a+bx)\sin^8(2a+2bx)\,dx$

Optimal. Leaf size=155

$$-\frac{128\sin^5(a+bx)\cos^9(a+bx)}{7b}-\frac{160\sin^3(a+bx)\cos^9(a+bx)}{21b}-\frac{16\sin(a+bx)\cos^9(a+bx)}{7b}+\frac{2\sin(a+bx)\cos^7(a\cdot}{7b}$$

```
[Out] (5*x)/8 + (5*Cos[a + b*x]*Sin[a + b*x])/(8*b) + (5*Cos[a + b*x]^3*Sin[a + b
*x])/(12*b) + (Cos[a + b*x]^5*Sin[a + b*x])/(3*b) + (2*Cos[a + b*x]^7*Sin[a
 + b*x])/(7*b) - (16*Cos[a + b*x]^9*Sin[a + b*x])/(7*b) - (160*Cos[a + b*x]
^9*Sin[a + b*x]^3)/(21*b) - (128*Cos[a + b*x]^9*Sin[a + b*x]^5)/(7*b)
```

Rubi [A] time = 0.171443, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4288, 2568, 2635, 8}

$$-\frac{128\sin^5(a+bx)\cos^9(a+bx)}{7b}-\frac{160\sin^3(a+bx)\cos^9(a+bx)}{21b}-\frac{16\sin(a+bx)\cos^9(a+bx)}{7b}+\frac{2\sin(a+bx)\cos^7(a\cdot}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^8,x]
```

```
[Out] (5*x)/8 + (5*Cos[a + b*x]*Sin[a + b*x])/(8*b) + (5*Cos[a + b*x]^3*Sin[a + b
*x])/(12*b) + (Cos[a + b*x]^5*Sin[a + b*x])/(3*b) + (2*Cos[a + b*x]^7*Sin[a
 + b*x])/(7*b) - (16*Cos[a + b*x]^9*Sin[a + b*x])/(7*b) - (160*Cos[a + b*x]
^9*Sin[a + b*x]^3)/(21*b) - (128*Cos[a + b*x]^9*Sin[a + b*x]^5)/(7*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \csc^2(a+bx)\sin^8(2a+2bx)\,dx = 256\int \cos^8(a+bx)\sin^6(a+bx)\,dx$$
$$= -\frac{128\cos^9(a+bx)\sin^5(a+bx)}{7b} + \frac{640}{7}\int \cos^8(a+bx)\sin^4(a+bx)\,dx$$
$$= -\frac{160\cos^9(a+bx)\sin^3(a+bx)}{21b} - \frac{128\cos^9(a+bx)\sin^5(a+bx)}{7b} + \frac{160}{7}\int \cos^8(a+b$$
$$= -\frac{16\cos^9(a+bx)\sin(a+bx)}{7b} - \frac{160\cos^9(a+bx)\sin^3(a+bx)}{21b} - \frac{128\cos^9(a+bx)\sin}{7b}$$
$$= \frac{2\cos^7(a+bx)\sin(a+bx)}{7b} - \frac{16\cos^9(a+bx)\sin(a+bx)}{7b} - \frac{160\cos^9(a+bx)\sin^3(a+}{21b}$$
$$= \frac{\cos^5(a+bx)\sin(a+bx)}{3b} + \frac{2\cos^7(a+bx)\sin(a+bx)}{7b} - \frac{16\cos^9(a+bx)\sin(a+bx)}{7b}$$
$$= \frac{5\cos^3(a+bx)\sin(a+bx)}{12b} + \frac{\cos^5(a+bx)\sin(a+bx)}{3b} + \frac{2\cos^7(a+bx)\sin(a+bx)}{7b} -$$
$$= \frac{5\cos(a+bx)\sin(a+bx)}{8b} + \frac{5\cos^3(a+bx)\sin(a+bx)}{12b} + \frac{\cos^5(a+bx)\sin(a+bx)}{3b} +$$
$$= \frac{5x}{8} + \frac{5\cos(a+bx)\sin(a+bx)}{8b} + \frac{5\cos^3(a+bx)\sin(a+bx)}{12b} + \frac{\cos^5(a+bx)\sin(a+}{3b}$$

Mathematica [A] time = 0.248023, size = 85, normalized size = 0.55

$$\frac{105\sin(2(a+bx)) - 315\sin(4(a+bx)) - 63\sin(6(a+bx)) + 63\sin(8(a+bx)) + 21\sin(10(a+bx)) - 7\sin(12(a+bx)) -}{1344b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^8,x]
```

```
[Out] (840*a + 840*b*x + 105*Sin[2*(a + b*x)] - 315*Sin[4*(a + b*x)] - 63*Sin[6*(
a + b*x)] + 63*Sin[8*(a + b*x)] + 21*Sin[10*(a + b*x)] - 7*Sin[12*(a + b*x)
] - 3*Sin[14*(a + b*x)])/(1344*b)
```

Maple [A] time = 0.06, size = 111, normalized size = 0.7

$$256\,\frac{1}{b}\left(-1/14\,(\sin(bx+a))^5(\cos(bx+a))^9 - \frac{5\,(\sin(bx+a))^3(\cos(bx+a))^9}{168} - \frac{\sin(bx+a)(\cos(bx+a))^9}{112} + \frac{\sin(bx+a}{896}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^8,x)
```

```
[Out] 256/b*(-1/14*sin(b*x+a)^5*cos(b*x+a)^9-5/168*sin(b*x+a)^3*cos(b*x+a)^9-1/11
2*sin(b*x+a)*cos(b*x+a)^9+1/896*(cos(b*x+a)^7+7/6*cos(b*x+a)^5+35/24*cos(b*
x+a)^3+35/16*cos(b*x+a))*sin(b*x+a)+5/2048*b*x+5/2048*a)
```

Maxima [A] time = 1.11489, size = 117, normalized size = 0.75

$$\frac{840\,bx - 3\,\sin(14\,bx+14\,a) - 7\,\sin(12\,bx+12\,a) + 21\,\sin(10\,bx+10\,a) + 63\,\sin(8\,bx+8\,a) - 63\,\sin(6\,bx+6\,a) - 31}{1344\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^8,x, algorithm="maxima")
```

```
[Out] 1/1344*(840*b*x - 3*sin(14*b*x + 14*a) - 7*sin(12*b*x + 12*a) + 21*sin(10*b
*x + 10*a) + 63*sin(8*b*x + 8*a) - 63*sin(6*b*x + 6*a) - 315*sin(4*b*x + 4*
a) + 105*sin(2*b*x + 2*a))/b
```

Fricas [A] time = 0.545882, size = 244, normalized size = 1.57

$$\frac{105\,bx - \left(3072\,\cos\left(bx+a\right)^{13} - 7424\,\cos\left(bx+a\right)^{11} + 4736\,\cos\left(bx+a\right)^{9} - 48\,\cos\left(bx+a\right)^{7} - 56\,\cos\left(bx+a\right)^{5} - 70}{168\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^8,x, algorithm="fricas")
```

```
[Out] 1/168*(105*b*x - (3072*cos(b*x + a)^13 - 7424*cos(b*x + a)^11 + 4736*cos(b*
x + a)^9 - 48*cos(b*x + a)^7 - 56*cos(b*x + a)^5 - 70*cos(b*x + a)^3 - 105*
cos(b*x + a))*sin(b*x + a))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.89029, size = 128, normalized size = 0.83

$$\frac{105\,bx + 105\,a + \frac{105\,\tan(bx+a)^{13} + 700\,\tan(bx+a)^{11} + 1981\,\tan(bx+a)^{9} + 3072\,\tan(bx+a)^{7} - 1981\,\tan(bx+a)^{5} - 700\,\tan(bx+a)^{3} - 105\,\tan(bx+a)}{\left(\tan(bx+a)^{2}+1\right)^{7}}}{168\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^8,x, algorithm="giac")
```

```
[Out] 1/168*(105*b*x + 105*a + (105*tan(b*x + a)^13 + 700*tan(b*x + a)^11 + 1981*
tan(b*x + a)^9 + 3072*tan(b*x + a)^7 - 1981*tan(b*x + a)^5 - 700*tan(b*x +
a)^3 - 105*tan(b*x + a))/(tan(b*x + a)^2 + 1)^7)/b
```

3.46 $\int \csc^2(a+bx)\sin^7(2a+2bx)\,dx$

Optimal. Leaf size=44

$$-\frac{32\cos^{12}(a+bx)}{3b}+\frac{128\cos^{10}(a+bx)}{5b}-\frac{16\cos^8(a+bx)}{b}$$

```
[Out] (-16*Cos[a + b*x]^8)/b + (128*Cos[a + b*x]^10)/(5*b) - (32*Cos[a + b*x]^12)
/(3*b)
```

Rubi [A] time = 0.0653625, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4288, 2565, 266, 43}

$$-\frac{32\cos^{12}(a+bx)}{3b}+\frac{128\cos^{10}(a+bx)}{5b}-\frac{16\cos^8(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (-16*Cos[a + b*x]^8)/b + (128*Cos[a + b*x]^10)/(5*b) - (32*Cos[a + b*x]^12)
/(3*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \csc^2(a+bx)\sin^7(2a+2bx)\,dx = 128\int \cos^7(a+bx)\sin^5(a+bx)\,dx$$
$$= -\frac{128\,\text{Subst}\left(\int x^7\left(1-x^2\right)^2\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{64\,\text{Subst}\left(\int (1-x)^2x^3\,dx, x, \cos^2(a+bx)\right)}{b}$$
$$= -\frac{64\,\text{Subst}\left(\int \left(x^3-2x^4+x^5\right)\,dx, x, \cos^2(a+bx)\right)}{b}$$
$$= -\frac{16\cos^8(a+bx)}{b}+\frac{128\cos^{10}(a+bx)}{5b}-\frac{32\cos^{12}(a+bx)}{3b}$$

Mathematica [A] time = 0.172471, size = 48, normalized size = 1.09

$$\frac{16\left(-10\sin^{12}(a+bx)+36\sin^{10}(a+bx)-45\sin^8(a+bx)+20\sin^6(a+bx)\right)}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (16*(20*Sin[a + b*x]^6 - 45*Sin[a + b*x]^8 + 36*Sin[a + b*x]^10 - 10*Sin[a
+ b*x]^12))/(15*b)
```

Maple [A] time = 0.027, size = 53, normalized size = 1.2

$$128\,\frac{1}{b}\left(-1/12\,\left(\sin\left(bx+a\right)\right)^4\left(\cos\left(bx+a\right)\right)^8-1/30\,\left(\sin\left(bx+a\right)\right)^2\left(\cos\left(bx+a\right)\right)^8-\frac{\left(\cos\left(bx+a\right)\right)^8}{120}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^7,x)
```

```
[Out] 128/b*(-1/12*sin(b*x+a)^4*cos(b*x+a)^8-1/30*sin(b*x+a)^2*cos(b*x+a)^8-1/120
*cos(b*x+a)^8)
```

Maxima [A] time = 1.08059, size = 97, normalized size = 2.2

$$-\frac{5\,\cos\left(12\,bx+12\,a\right)+12\,\cos\left(10\,bx+10\,a\right)-30\,\cos\left(8\,bx+8\,a\right)-100\,\cos\left(6\,bx+6\,a\right)+75\,\cos\left(4\,bx+4\,a\right)+600\,c}{960\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^7,x, algorithm="maxima")
```

```
[Out] -1/960*(5*cos(12*b*x + 12*a) + 12*cos(10*b*x + 10*a) - 30*cos(8*b*x + 8*a)
- 100*cos(6*b*x + 6*a) + 75*cos(4*b*x + 4*a) + 600*cos(2*b*x + 2*a))/b
```

Fricas [A] time = 0.510969, size = 97, normalized size = 2.2

$$-\frac{16\left(10\cos\left(bx+a\right)^{12}-24\cos\left(bx+a\right)^{10}+15\cos\left(bx+a\right)^{8}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^7,x, algorithm="fricas")
```

```
[Out] -16/15*(10*cos(b*x + a)^12 - 24*cos(b*x + a)^10 + 15*cos(b*x + a)^8)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**7,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.6791, size = 247, normalized size = 5.61

$$-\frac{4096\left(\frac{5\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{15\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+\frac{39\,(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5}+\frac{42\,(\cos(bx+a)-1)^6}{(\cos(bx+a)+1)^6}+\frac{39\,(\cos(bx+a)-1)^7}{(\cos(bx+a)+1)^7}+\frac{15\,(\cos(bx+a)-1)^8}{(\cos(bx+a)+1)^8}+\frac{5\,(\cos(bx+a)-}{(\cos(bx+a)+}\right.}{15\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^7,x, algorithm="giac")
```

```
[Out] -4096/15*(5*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 15*(cos(b*x + a) -
1)^4/(cos(b*x + a) + 1)^4 + 39*(cos(b*x + a) - 1)^5/(cos(b*x + a) + 1)^5 +
42*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 + 39*(cos(b*x + a) - 1)^7/(cos
(b*x + a) + 1)^7 + 15*(cos(b*x + a) - 1)^8/(cos(b*x + a) + 1)^8 + 5*(cos(b*
x + a) - 1)^9/(cos(b*x + a) + 1)^9)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) +
1) - 1)^12)
```

3.47 $\int \csc^2(a+bx)\sin^6(2a+2bx)\,dx$

Optimal. Leaf size=111

$$-\frac{32\sin^3(a+bx)\cos^7(a+bx)}{5b}-\frac{12\sin(a+bx)\cos^7(a+bx)}{5b}+\frac{2\sin(a+bx)\cos^5(a+bx)}{5b}+\frac{\sin(a+bx)\cos^3(a+bx)}{2b}$$

```
[Out] (3*x)/4 + (3*Cos[a + b*x]*Sin[a + b*x])/(4*b) + (Cos[a + b*x]^3*Sin[a + b*x
])/(2*b) + (2*Cos[a + b*x]^5*Sin[a + b*x])/(5*b) - (12*Cos[a + b*x]^7*Sin[a
 + b*x])/(5*b) - (32*Cos[a + b*x]^7*Sin[a + b*x]^3)/(5*b)
```

Rubi [A] time = 0.120024, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4288, 2568, 2635, 8}

$$-\frac{32\sin^3(a+bx)\cos^7(a+bx)}{5b}-\frac{12\sin(a+bx)\cos^7(a+bx)}{5b}+\frac{2\sin(a+bx)\cos^5(a+bx)}{5b}+\frac{\sin(a+bx)\cos^3(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^6,x]
```

```
[Out] (3*x)/4 + (3*Cos[a + b*x]*Sin[a + b*x])/(4*b) + (Cos[a + b*x]^3*Sin[a + b*x
])/(2*b) + (2*Cos[a + b*x]^5*Sin[a + b*x])/(5*b) - (12*Cos[a + b*x]^7*Sin[a
 + b*x])/(5*b) - (32*Cos[a + b*x]^7*Sin[a + b*x]^3)/(5*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \csc^2(a+bx)\sin^6(2a+2bx)\,dx &= 64\int \cos^6(a+bx)\sin^4(a+bx)\,dx\\ &= -\frac{32\cos^7(a+bx)\sin^3(a+bx)}{5b}+\frac{96}{5}\int \cos^6(a+bx)\sin^2(a+bx)\,dx\\ &= -\frac{12\cos^7(a+bx)\sin(a+bx)}{5b}-\frac{32\cos^7(a+bx)\sin^3(a+bx)}{5b}+\frac{12}{5}\int \cos^6(a+bx)\,dx\\ &= \frac{2\cos^5(a+bx)\sin(a+bx)}{5b}-\frac{12\cos^7(a+bx)\sin(a+bx)}{5b}-\frac{32\cos^7(a+bx)\sin^3(a+\\ &= \frac{\cos^3(a+bx)\sin(a+bx)}{2b}+\frac{2\cos^5(a+bx)\sin(a+bx)}{5b}-\frac{12\cos^7(a+bx)\sin(a+bx)}{5b}\\ &= \frac{3\cos(a+bx)\sin(a+bx)}{4b}+\frac{\cos^3(a+bx)\sin(a+bx)}{2b}+\frac{2\cos^5(a+bx)\sin(a+bx)}{5b}-\\ &= \frac{3x}{4}+\frac{3\cos(a+bx)\sin(a+bx)}{4b}+\frac{\cos^3(a+bx)\sin(a+bx)}{2b}+\frac{2\cos^5(a+bx)\sin(a+\\ &\;5b\end{aligned}$$

Mathematica [A] time = 0.197873, size = 62, normalized size = 0.56

$$\frac{20\sin(2(a+bx))-40\sin(4(a+bx))-10\sin(6(a+bx))+5\sin(8(a+bx))+2\sin(10(a+bx))+120bx}{160b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^6,x]
```

```
[Out] (120*b*x + 20*Sin[2*(a + b*x)] - 40*Sin[4*(a + b*x)] - 10*Sin[6*(a + b*x)]
 + 5*Sin[8*(a + b*x)] + 2*Sin[10*(a + b*x)])/(160*b)
```

Maple [A] time = 0.052, size = 83, normalized size = 0.8

$$64\,\frac{1}{b}\left(-1/10\,(\sin(bx+a))^3(\cos(bx+a))^7-\frac{3\,\sin(bx+a)\,(\cos(bx+a))^7}{80}+\frac{\sin(bx+a)}{160}\left((\cos(bx+a))^5+5/4\,(\cos(bx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^6,x)
```

```
[Out] 64/b*(-1/10*sin(b*x+a)^3*cos(b*x+a)^7-3/80*sin(b*x+a)*cos(b*x+a)^7+1/160*(c
os(b*x+a)^5+5/4*cos(b*x+a)^3+15/8*cos(b*x+a))*sin(b*x+a)+3/256*b*x+3/256*a)
```

Maxima [A] time = 1.19874, size = 88, normalized size = 0.79

$$\frac{120\,bx+2\,\sin(10\,bx+10\,a)+5\,\sin(8\,bx+8\,a)-10\,\sin(6\,bx+6\,a)-40\,\sin(4\,bx+4\,a)+20\,\sin(2\,bx+2\,a)}{160\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^6,x, algorithm="maxima")
```

```
[Out] 1/160*(120*b*x + 2*sin(10*b*x + 10*a) + 5*sin(8*b*x + 8*a) - 10*sin(6*b*x +
 6*a) - 40*sin(4*b*x + 4*a) + 20*sin(2*b*x + 2*a))/b
```

Fricas [A] time = 0.512515, size = 177, normalized size = 1.59

$$\frac{15\,bx + \left(128\,\cos\left(bx+a\right)^{9} - 176\,\cos\left(bx+a\right)^{7} + 8\,\cos\left(bx+a\right)^{5} + 10\,\cos\left(bx+a\right)^{3} + 15\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{20\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^6,x, algorithm="fricas")
```

```
[Out] 1/20*(15*b*x + (128*cos(b*x + a)^9 - 176*cos(b*x + a)^7 + 8*cos(b*x + a)^5
+ 10*cos(b*x + a)^3 + 15*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.55702, size = 101, normalized size = 0.91

$$\frac{15\,bx + 15\,a + \frac{15\,\tan\left(bx+a\right)^{9} + 70\,\tan\left(bx+a\right)^{7} + 128\,\tan\left(bx+a\right)^{5} - 70\,\tan\left(bx+a\right)^{3} - 15\,\tan\left(bx+a\right)}{\left(\tan\left(bx+a\right)^{2}+1\right)^{5}}}{20\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^6,x, algorithm="giac")
```

```
[Out] 1/20*(15*b*x + 15*a + (15*tan(b*x + a)^9 + 70*tan(b*x + a)^7 + 128*tan(b*x
+ a)^5 - 70*tan(b*x + a)^3 - 15*tan(b*x + a))/(tan(b*x + a)^2 + 1)^5)/b
```

3.48 $\int \csc^2(a+bx)\sin^5(2a+2bx)\,dx$

Optimal. Leaf size=29

$$\frac{4\cos^8(a+bx)}{b}-\frac{16\cos^6(a+bx)}{3b}$$

```
[Out] (-16*Cos[a + b*x]^6)/(3*b) + (4*Cos[a + b*x]^8)/b
```

Rubi [A] time = 0.0567422, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4288, 2565, 14}

$$\frac{4\cos^8(a+bx)}{b}-\frac{16\cos^6(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (-16*Cos[a + b*x]^6)/(3*b) + (4*Cos[a + b*x]^8)/b
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \csc^2(a+bx)\sin^5(2a+2bx)\,dx &= 32\int \cos^5(a+bx)\sin^3(a+bx)\,dx\\
&= -\frac{32\,\text{Subst}\left(\int x^5\left(1-x^2\right)\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{32\,\text{Subst}\left(\int \left(x^5-x^7\right)\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{16\cos^6(a+bx)}{3b}+\frac{4\cos^8(a+bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.124245, size = 48, normalized size = 1.66

$$\frac{-72\cos(2(a+bx))-12\cos(4(a+bx))+8\cos(6(a+bx))+3\cos(8(a+bx))}{96b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (-72*Cos[2*(a + b*x)] - 12*Cos[4*(a + b*x)] + 8*Cos[6*(a + b*x)] + 3*Cos[8*
(a + b*x)])/(96*b)
```

Maple [A] time = 0.025, size = 35, normalized size = 1.2

$$32\,\frac{-1/8\,(\sin(bx+a))^2(\cos(bx+a))^6-1/24\,(\cos(bx+a))^6}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^5,x)
```

```
[Out] 32/b*(-1/8*sin(b*x+a)^2*cos(b*x+a)^6-1/24*cos(b*x+a)^6)
```

Maxima [A] time = 1.12087, size = 68, normalized size = 2.34

$$\frac{3\,\cos(8\,bx+8\,a)+8\,\cos(6\,bx+6\,a)-12\,\cos(4\,bx+4\,a)-72\,\cos(2\,bx+2\,a)}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] 1/96*(3*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 12*cos(4*b*x + 4*a) - 72*co
s(2*b*x + 2*a))/b
```

Fricas [A] time = 0.489855, size = 61, normalized size = 2.1

$$\frac{4\left(3\,\cos(bx+a)^8-4\,\cos(bx+a)^6\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] 4/3*(3*cos(b*x + a)^8 - 4*cos(b*x + a)^6)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.45094, size = 188, normalized size = 6.48

$$\frac{128\left(\frac{3\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{4\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{10\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+\frac{4\,(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5}+\frac{3\,(\cos(bx+a)-1)^6}{(\cos(bx+a)+1)^6}\right)}{3\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] 128/3*(3*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 + 4*(cos(b*x + a) - 1)^3
/(cos(b*x + a) + 1)^3 + 10*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 4*(c
os(b*x + a) - 1)^5/(cos(b*x + a) + 1)^5 + 3*(cos(b*x + a) - 1)^6/(cos(b*x +
 a) + 1)^6)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^8)
```

3.49 $\int \csc^2(a+bx)\sin^4(2a+2bx)\,dx$

Optimal. Leaf size=60

$$-\frac{8\sin(a+bx)\cos^5(a+bx)}{3b}+\frac{2\sin(a+bx)\cos^3(a+bx)}{3b}+\frac{\sin(a+bx)\cos(a+bx)}{b}+x$$

```
[Out] x + (Cos[a + b*x]*Sin[a + b*x])/b + (2*Cos[a + b*x]^3*Sin[a + b*x])/(3*b) -
 (8*Cos[a + b*x]^5*Sin[a + b*x])/(3*b)
```

Rubi [A] time = 0.0743127, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4288, 2568, 2635, 8}

$$-\frac{8\sin(a+bx)\cos^5(a+bx)}{3b}+\frac{2\sin(a+bx)\cos^3(a+bx)}{3b}+\frac{\sin(a+bx)\cos(a+bx)}{b}+x$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] x + (Cos[a + b*x]*Sin[a + b*x])/b + (2*Cos[a + b*x]^3*Sin[a + b*x])/(3*b) -
 (8*Cos[a + b*x]^5*Sin[a + b*x])/(3*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \csc^2(a+bx)\sin^4(2a+2bx)\,dx = 16\int \cos^4(a+bx)\sin^2(a+bx)\,dx$$
$$= -\frac{8\cos^5(a+bx)\sin(a+bx)}{3b} + \frac{8}{3}\int \cos^4(a+bx)\,dx$$
$$= \frac{2\cos^3(a+bx)\sin(a+bx)}{3b} - \frac{8\cos^5(a+bx)\sin(a+bx)}{3b} + 2\int \cos^2(a+bx)\,dx$$
$$= \frac{\cos(a+bx)\sin(a+bx)}{b} + \frac{2\cos^3(a+bx)\sin(a+bx)}{3b} - \frac{8\cos^5(a+bx)\sin(a+bx)}{3b} +$$
$$= x + \frac{\cos(a+bx)\sin(a+bx)}{b} + \frac{2\cos^3(a+bx)\sin(a+bx)}{3b} - \frac{8\cos^5(a+bx)\sin(a+bx)}{3b}$$

Mathematica [A] time = 0.099401, size = 40, normalized size = 0.67

$$-\frac{-3\sin(2(a+bx)) + 3\sin(4(a+bx)) + \sin(6(a+bx)) - 12bx}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] -(-12*b*x - 3*Sin[2*(a + b*x)] + 3*Sin[4*(a + b*x)] + Sin[6*(a + b*x)])/(12
*b)
```

Maple [A] time = 0.05, size = 55, normalized size = 0.9

$$16\,\frac{-1/6\,\sin(bx+a)(\cos(bx+a))^5 + 1/24\left((\cos(bx+a))^3 + 3/2\,\cos(bx+a)\right)\sin(bx+a) + 1/16\,bx + a/16}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^4,x)
```

```
[Out] 16/b*(-1/6*sin(b*x+a)*cos(b*x+a)^5+1/24*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b
*x+a)+1/16*b*x+1/16*a)
```

Maxima [A] time = 1.19902, size = 58, normalized size = 0.97

$$\frac{12\,bx - \sin(6\,bx + 6\,a) - 3\,\sin(4\,bx + 4\,a) + 3\,\sin(2\,bx + 2\,a)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/12*(12*b*x - sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))/
b
```

Fricas [A] time = 0.485046, size = 115, normalized size = 1.92

$$\frac{3\,bx - \left(8\,\cos(bx+a)^5 - 2\,\cos(bx+a)^3 - 3\,\cos(bx+a)\right)\sin(bx+a)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b*x - (8*cos(b*x + a)^5 - 2*cos(b*x + a)^3 - 3*cos(b*x + a))*sin(b*x
 + a))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38985, size = 74, normalized size = 1.23

$$\frac{3\,bx + 3\,a + \frac{3\,\tan(bx+a)^5 + 8\,\tan(bx+a)^3 - 3\,\tan(bx+a)}{\left(\tan(bx+a)^2+1\right)^3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*b*x + 3*a + (3*tan(b*x + a)^5 + 8*tan(b*x + a)^3 - 3*tan(b*x + a))/(
tan(b*x + a)^2 + 1)^3)/b
```

3.50 $\int \csc^2(a+bx)\sin^3(2a+2bx)\,dx$

Optimal. Leaf size=13

$$-\frac{2\cos^4(a+bx)}{b}$$

```
[Out] (-2*Cos[a + b*x]^4)/b
```

Rubi [A] time = 0.0414177, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4288, 2565, 30}

$$-\frac{2\cos^4(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (-2*Cos[a + b*x]^4)/b
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \csc^2(a+bx)\sin^3(2a+2bx)\,dx &= 8\int \cos^3(a+bx)\sin(a+bx)\,dx\\ &= -\frac{8\,\text{Subst}\left(\int x^3\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{2\cos^4(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0055429, size = 13, normalized size = 1.

$$-\frac{2\cos^4(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (-2*Cos[a + b*x]^4)/b
```

Maple [A] time = 0.02, size = 14, normalized size = 1.1

$$-2\,{\frac {\left(\cos\left(bx+a\right)\right)^{4}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^3,x)
```

```
[Out] -2*cos(b*x+a)^4/b
```

Maxima [A] time = 1.09913, size = 35, normalized size = 2.69

$$-\frac{\cos\left(4\,bx+4\,a\right)+4\,\cos\left(2\,bx+2\,a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(cos(4*b*x + 4*a) + 4*cos(2*b*x + 2*a))/b
```

Fricas [A] time = 0.461666, size = 28, normalized size = 2.15

$$-\frac{2\,\cos\left(bx+a\right)^{4}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] -2*cos(b*x + a)^4/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.33947, size = 93, normalized size = 7.15

$$-\frac{16\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+\frac{(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}\right)}{b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] -16*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + (cos(b*x + a) - 1)^3/(cos(b*x
+ a) + 1)^3)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^4)
```

3.51 $\int \csc^2(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=21

$$\frac{2\sin(a+bx)\cos(a+bx)}{b}+2x$$

```
[Out] 2*x + (2*Cos[a + b*x]*Sin[a + b*x])/b
```

Rubi [A] time = 0.0343239, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4288, 2635, 8}

$$\frac{2\sin(a+bx)\cos(a+bx)}{b}+2x$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] 2*x + (2*Cos[a + b*x]*Sin[a + b*x])/b
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \csc^2(a+bx)\sin^2(2a+2bx)\,dx &= 4\int \cos^2(a+bx)\,dx\\ &= \frac{2\cos(a+bx)\sin(a+bx)}{b}+2\int 1\,dx\\ &= 2x+\frac{2\cos(a+bx)\sin(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0243541, size = 20, normalized size = 0.95

$$\frac{2(a+bx)+\sin(2(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (2*(a + b*x) + Sin[2*(a + b*x)])/b
```

Maple [A] time = 0.025, size = 28, normalized size = 1.3

$$4\,\frac{1/2\,\cos(bx+a)\sin(bx+a)+1/2\,bx+a/2}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^2,x)
```

```
[Out] 4/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)
```

Maxima [A] time = 1.12614, size = 24, normalized size = 1.14

$$\frac{2\,bx+\sin(2\,bx+2\,a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] (2*b*x + sin(2*b*x + 2*a))/b
```

Fricas [A] time = 0.476171, size = 53, normalized size = 2.52

$$\frac{2\,(bx+\cos(bx+a)\sin(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] 2*(b*x + cos(b*x + a)*sin(b*x + a))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29708, size = 39, normalized size = 1.86

$$\frac{2\left(bx+a+\frac{\tan(bx+a)}{\tan(bx+a)^2+1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] 2*(b*x + a + tan(b*x + a)/(tan(b*x + a)^2 + 1))/b
```

3.52 $\int \csc^2(a+bx)\sin(2a+2bx)\,dx$

Optimal. Leaf size=12

$$\frac{2\log(\sin(a+bx))}{b}$$

```
[Out] (2*Log[Sin[a + b*x]])/b
```

Rubi [A] time = 0.0201196, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4288, 3475}

$$\frac{2\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x],x]
```

```
[Out] (2*Log[Sin[a + b*x]])/b
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^2(a+bx)\sin(2a+2bx)\,dx &= 2\int \cot(a+bx)\,dx\\ &= \frac{2\log(\sin(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.0151928, size = 20, normalized size = 1.67

$$\frac{2(\log(\tan(a+bx))+\log(\cos(a+bx)))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x],x]
```

```
[Out] (2*(Log[Cos[a + b*x]] + Log[Tan[a + b*x]]))/b
```

Maple [A] time = 0.02, size = 13, normalized size = 1.1

$$2\,\frac{\ln\left(\sin\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a),x)
```

```
[Out] 2*ln(sin(b*x+a))/b
```

Maxima [B] time = 1.2166, size = 109, normalized size = 9.08

$$\frac{\log\left(\cos\left(bx\right)^{2}+2\,\cos\left(bx\right)\cos\left(a\right)+\cos\left(a\right)^{2}+\sin\left(bx\right)^{2}-2\,\sin\left(bx\right)\sin\left(a\right)+\sin\left(a\right)^{2}\right)+\log\left(\cos\left(bx\right)^{2}-2\,\cos\left(bx\right)\cos\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] (log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*si
n(a) + sin(a)^2) + log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)
^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b
```

Fricas [A] time = 0.483796, size = 36, normalized size = 3.

$$\frac{2\,\log\left(\frac{1}{2}\,\sin\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 2*log(1/2*sin(b*x + a))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30846, size = 74, normalized size = 6.17

$$\frac{\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)-2\,\log\left(\left|-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] (log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) - 2*log(abs(-(cos(b*x +
a) - 1)/(cos(b*x + a) + 1) + 1)))/b
```

3.53 $\int \csc^2(a+bx)\csc(2a+2bx)\,dx$

Optimal. Leaf size=30

$$\frac{\log(\tan(a+bx))}{2b}-\frac{\cot^2(a+bx)}{4b}$$

```
[Out] -Cot[a + b*x]^2/(4*b) + Log[Tan[a + b*x]]/(2*b)
```

Rubi [A] time = 0.0420328, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2620, 14}

$$\frac{\log(\tan(a+bx))}{2b}-\frac{\cot^2(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x],x]
```

```
[Out] -Cot[a + b*x]^2/(4*b) + Log[Tan[a + b*x]]/(2*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \csc^2(a+bx)\csc(2a+2bx)\,dx &= \frac{1}{2}\int \csc^3(a+bx)\sec(a+bx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{1+x^2}{x^3}\,dx, x, \tan(a+bx)\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{1}{x}\right)dx, x, \tan(a+bx)\right)}{2b}\\
&= -\frac{\cot^2(a+bx)}{4b}+\frac{\log(\tan(a+bx))}{2b}
\end{aligned}$$

Mathematica [A] time = 0.051582, size = 34, normalized size = 1.13

$$-\frac{\csc^2(a+bx)-2\log(\sin(a+bx))+2\log(\cos(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x],x]
```

```
[Out] -(Csc[a + b*x]^2 + 2*Log[Cos[a + b*x]] - 2*Log[Sin[a + b*x]])/(4*b)
```

Maple [A] time = 0.03, size = 27, normalized size = 0.9

$$-\frac{1}{4\,b\,(\sin(bx+a))^2}+\frac{\ln(\tan(bx+a))}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*csc(2*b*x+2*a),x)
```

```
[Out] -1/4/b/sin(b*x+a)^2+1/2*ln(tan(b*x+a))/b
```

Maxima [B] time = 1.22505, size = 886, normalized size = 29.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] 1/4*(4*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) - 8*cos(2*b*x + 2*a)^2 + (2*(2*cos
(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*
a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b
*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2
*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2) - (2*
(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*
x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*s
in(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos
(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (2*(2*cos(2*b
*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2
 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x +
 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(
a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*sin(4*b*x + 4*a)*sin(
2*b*x + 2*a) - 8*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a))/(b*cos(4*b*x + 4*
a)^2 + 4*b*cos(2*b*x + 2*a)^2 + b*sin(4*b*x + 4*a)^2 - 4*b*sin(4*b*x + 4*a)
*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(2*b*x + 2*a) - b)*c
os(4*b*x + 4*a) - 4*b*cos(2*b*x + 2*a) + b)
```

Fricas [B] time = 0.499717, size = 176, normalized size = 5.87

$$-\frac{\left(\cos(bx+a)^2-1\right)\log\left(\cos(bx+a)^2\right)-\left(\cos(bx+a)^2-1\right)\log\left(-\frac{1}{4}\cos(bx+a)^2+\frac{1}{4}\right)-1}{4\left(b\cos(bx+a)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] -1/4*((cos(b*x + a)^2 - 1)*log(cos(b*x + a)^2) - (cos(b*x + a)^2 - 1)*log(-
1/4*cos(b*x + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^2 - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \csc^2(a+bx)\csc(2a+2bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*csc(2*b*x+2*a),x)
```

```
[Out] Integral(csc(a + b*x)**2*csc(2*a + 2*b*x), x)
```

Giac [B] time = 1.47037, size = 1098, normalized size = 36.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] -1/4*((3*tan(b*x + 4*a)^2*tan(1/2*a)^24 - 180*tan(b*x + 4*a)^2*tan(1/2*a)^2
2 + 24*tan(b*x + 4*a)*tan(1/2*a)^23 + tan(1/2*a)^24 + 4230*tan(b*x + 4*a)^2
*tan(1/2*a)^20 - 1592*tan(b*x + 4*a)*tan(1/2*a)^21 + 48*tan(1/2*a)^22 - 486
12*tan(b*x + 4*a)^2*tan(1/2*a)^18 + 31704*tan(b*x + 4*a)*tan(1/2*a)^19 - 38
46*tan(1/2*a)^20 + 277965*tan(b*x + 4*a)^2*tan(1/2*a)^16 - 259128*tan(b*x +
 4*a)*tan(1/2*a)^17 + 51632*tan(1/2*a)^18 - 737640*tan(b*x + 4*a)^2*tan(1/2
*a)^14 + 982192*tan(b*x + 4*a)*tan(1/2*a)^15 - 274545*tan(1/2*a)^16 + 10084
68*tan(b*x + 4*a)^2*tan(1/2*a)^12 - 1871088*tan(b*x + 4*a)*tan(1/2*a)^13 +
733728*tan(1/2*a)^14 - 737640*tan(b*x + 4*a)^2*tan(1/2*a)^10 + 1871088*tan(
b*x + 4*a)*tan(1/2*a)^11 - 1018132*tan(1/2*a)^12 + 277965*tan(b*x + 4*a)^2*
tan(1/2*a)^8 - 982192*tan(b*x + 4*a)*tan(1/2*a)^9 + 733728*tan(1/2*a)^10 -
48612*tan(b*x + 4*a)^2*tan(1/2*a)^6 + 259128*tan(b*x + 4*a)*tan(1/2*a)^7 -
274545*tan(1/2*a)^8 + 4230*tan(b*x + 4*a)^2*tan(1/2*a)^4 - 31704*tan(b*x +
4*a)*tan(1/2*a)^5 + 51632*tan(1/2*a)^6 - 180*tan(b*x + 4*a)^2*tan(1/2*a)^2
+ 1592*tan(b*x + 4*a)*tan(1/2*a)^3 - 3846*tan(1/2*a)^4 + 3*tan(b*x + 4*a)^2
 - 24*tan(b*x + 4*a)*tan(1/2*a) + 48*tan(1/2*a)^2 + 1)/((tan(1/2*a)^12 - 30
*tan(1/2*a)^10 + 255*tan(1/2*a)^8 - 452*tan(1/2*a)^6 + 255*tan(1/2*a)^4 - 3
0*tan(1/2*a)^2 + 1)*(tan(b*x + 4*a)*tan(1/2*a)^6 - 15*tan(b*x + 4*a)*tan(1/
2*a)^4 + 6*tan(1/2*a)^5 + 15*tan(b*x + 4*a)*tan(1/2*a)^2 - 20*tan(1/2*a)^3
- tan(b*x + 4*a) + 6*tan(1/2*a))^2) - 2*log(abs(tan(b*x + 4*a)*tan(1/2*a)^6
 - 15*tan(b*x + 4*a)*tan(1/2*a)^4 + 6*tan(1/2*a)^5 + 15*tan(b*x + 4*a)*tan(
1/2*a)^2 - 20*tan(1/2*a)^3 - tan(b*x + 4*a) + 6*tan(1/2*a))) + 2*log(abs(6*
tan(b*x + 4*a)*tan(1/2*a)^5 - tan(1/2*a)^6 - 20*tan(b*x + 4*a)*tan(1/2*a)^3
 + 15*tan(1/2*a)^4 + 6*tan(b*x + 4*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)))/b
```

3.54 $\int \csc^2(a+bx)\csc^2(2a+2bx)\,dx$

Optimal. Leaf size=42

$$\frac{\tan(a+bx)}{4b}-\frac{\cot^3(a+bx)}{12b}-\frac{\cot(a+bx)}{2b}$$

```
[Out] -Cot[a + b*x]/(2*b) - Cot[a + b*x]^3/(12*b) + Tan[a + b*x]/(4*b)
```

Rubi [A] time = 0.0590592, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4288, 2620, 270}

$$\frac{\tan(a+bx)}{4b}-\frac{\cot^3(a+bx)}{12b}-\frac{\cot(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] -Cot[a + b*x]/(2*b) - Cot[a + b*x]^3/(12*b) + Tan[a + b*x]/(4*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^2(a+bx)\csc^2(2a+2bx)\,dx &= \frac{1}{4}\int \csc^4(a+bx)\sec^2(a+bx)\,dx\\ &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^4}\,dx,x,\tan(a+bx)\right)}{4b}\\ &= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}+\frac{2}{x^2}\right)dx,x,\tan(a+bx)\right)}{4b}\\ &= -\frac{\cot(a+bx)}{2b}-\frac{\cot^3(a+bx)}{12b}+\frac{\tan(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0795716, size = 48, normalized size = 1.14

$$\frac{\tan(a+bx)}{4b}-\frac{5\cot(a+bx)}{12b}-\frac{\cot(a+bx)\csc^2(a+bx)}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] (-5*Cot[a + b*x])/(12*b) - (Cot[a + b*x]*Csc[a + b*x]^2)/(12*b) + Tan[a + b
*x]/(4*b)
```

Maple [A] time = 0.032, size = 51, normalized size = 1.2

$$\frac{1}{4b}\left(-\frac{1}{3(\sin(bx+a))^3\cos(bx+a)}+\frac{4}{3\cos(bx+a)\sin(bx+a)}-\frac{8\cot(bx+a)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*csc(2*b*x+2*a)^2,x)
```

```
[Out] 1/4/b*(-1/3/sin(b*x+a)^3/cos(b*x+a)+4/3/sin(b*x+a)/cos(b*x+a)-8/3*cot(b*x+a
))
```

Maxima [B] time = 1.20777, size = 416, normalized size = 9.9

$$\frac{4((2\cos(2bx+}{3\left(b\cos(8bx+8a)^2+4b\cos(6bx+6a)^2+4b\cos(2bx+2a)^2+b\sin(8bx+8a)^2+4b\sin(6bx+6a)^2-8b\sin(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] 4/3*((2*cos(2*b*x + 2*a) - 1)*sin(8*b*x + 8*a) - 2*(2*cos(2*b*x + 2*a) - 1)
*sin(6*b*x + 6*a) - 2*cos(8*b*x + 8*a)*sin(2*b*x + 2*a) + 4*cos(6*b*x + 6*a
)*sin(2*b*x + 2*a))/(b*cos(8*b*x + 8*a)^2 + 4*b*cos(6*b*x + 6*a)^2 + 4*b*co
s(2*b*x + 2*a)^2 + b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 - 8*b*sin(
6*b*x + 6*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(6*b*x +
 6*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) - 4*(2*b*cos(2*b*x + 2*a
) - b)*cos(6*b*x + 6*a) - 4*b*cos(2*b*x + 2*a) - 4*(b*sin(6*b*x + 6*a) - b*
sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + b)
```

Fricas [A] time = 0.466911, size = 136, normalized size = 3.24

$$-\frac{8\cos(bx+a)^4-12\cos(bx+a)^2+3}{12\left(b\cos(bx+a)^3-b\cos(bx+a)\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] -1/12*(8*cos(b*x + a)^4 - 12*cos(b*x + a)^2 + 3)/((b*cos(b*x + a)^3 - b*cos
(b*x + a))*sin(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \csc^2(a+bx)\csc^2(2a+2bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*csc(2*b*x+2*a)**2,x)
```

```
[Out] Integral(csc(a + b*x)**2*csc(2*a + 2*b*x)**2, x)
```

Giac [B] time = 1.54552, size = 1457, normalized size = 34.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] -1/24*(3*(tan(1/2*a)^12 + 6*tan(1/2*a)^10 + 15*tan(1/2*a)^8 + 20*tan(1/2*a)
^6 + 15*tan(1/2*a)^4 + 6*tan(1/2*a)^2 + 1)/((6*tan(b*x + 4*a)*tan(1/2*a)^5
- tan(1/2*a)^6 - 20*tan(b*x + 4*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 + 6*tan(b
*x + 4*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)*(3*tan(1/2*a)^5 - 10*tan(1/2*a)
^3 + 3*tan(1/2*a))) + 2*(6*tan(b*x + 4*a)^2*tan(1/2*a)^36 - 216*tan(b*x + 4
*a)^2*tan(1/2*a)^34 + 54*tan(b*x + 4*a)*tan(1/2*a)^35 + tan(1/2*a)^36 + 307
8*tan(b*x + 4*a)^2*tan(1/2*a)^32 - 1638*tan(b*x + 4*a)*tan(1/2*a)^33 + 126*
tan(1/2*a)^34 - 23328*tan(b*x + 4*a)^2*tan(1/2*a)^30 + 19872*tan(b*x + 4*a)
*tan(1/2*a)^31 - 3159*tan(1/2*a)^32 + 62856*tan(b*x + 4*a)^2*tan(1/2*a)^28
- 100224*tan(b*x + 4*a)*tan(1/2*a)^29 + 29232*tan(1/2*a)^30 + 328608*tan(b*
x + 4*a)^2*tan(1/2*a)^26 - 127176*tan(b*x + 4*a)*tan(1/2*a)^27 - 26460*tan(
1/2*a)^28 - 96504*tan(b*x + 4*a)^2*tan(1/2*a)^24 + 808488*tan(b*x + 4*a)*ta
n(1/2*a)^25 - 228600*tan(1/2*a)^26 - 879840*tan(b*x + 4*a)^2*tan(1/2*a)^22
+ 833472*tan(b*x + 4*a)*tan(1/2*a)^23 + 237588*tan(1/2*a)^24 + 30564*tan(b*
x + 4*a)^2*tan(1/2*a)^20 - 1653792*tan(b*x + 4*a)*tan(1/2*a)^21 + 944208*ta
n(1/2*a)^22 + 1149552*tan(b*x + 4*a)^2*tan(1/2*a)^18 - 1673388*tan(b*x + 4*
a)*tan(1/2*a)^19 - 142434*tan(1/2*a)^20 + 30564*tan(b*x + 4*a)^2*tan(1/2*a)
^16 + 1673388*tan(b*x + 4*a)*tan(1/2*a)^17 - 1358860*tan(1/2*a)^18 - 879840
*tan(b*x + 4*a)^2*tan(1/2*a)^14 + 1653792*tan(b*x + 4*a)*tan(1/2*a)^15 - 14
2434*tan(1/2*a)^16 - 96504*tan(b*x + 4*a)^2*tan(1/2*a)^12 - 833472*tan(b*x
+ 4*a)*tan(1/2*a)^13 + 944208*tan(1/2*a)^14 + 328608*tan(b*x + 4*a)^2*tan(1
/2*a)^10 - 808488*tan(b*x + 4*a)*tan(1/2*a)^11 + 237588*tan(1/2*a)^12 + 628
56*tan(b*x + 4*a)^2*tan(1/2*a)^8 + 127176*tan(b*x + 4*a)*tan(1/2*a)^9 - 228
600*tan(1/2*a)^10 - 23328*tan(b*x + 4*a)^2*tan(1/2*a)^6 + 100224*tan(b*x +
4*a)*tan(1/2*a)^7 - 26460*tan(1/2*a)^8 + 3078*tan(b*x + 4*a)^2*tan(1/2*a)^4
 - 19872*tan(b*x + 4*a)*tan(1/2*a)^5 + 29232*tan(1/2*a)^6 - 216*tan(b*x + 4
*a)^2*tan(1/2*a)^2 + 1638*tan(b*x + 4*a)*tan(1/2*a)^3 - 3159*tan(1/2*a)^4 +
 6*tan(b*x + 4*a)^2 - 54*tan(b*x + 4*a)*tan(1/2*a) + 126*tan(1/2*a)^2 + 1)/
((tan(1/2*a)^18 - 45*tan(1/2*a)^16 + 720*tan(1/2*a)^14 - 4728*tan(1/2*a)^12
 + 10890*tan(1/2*a)^10 - 10890*tan(1/2*a)^8 + 4728*tan(1/2*a)^6 - 720*tan(1
/2*a)^4 + 45*tan(1/2*a)^2 - 1)*(tan(b*x + 4*a)*tan(1/2*a)^6 - 15*tan(b*x +
4*a)*tan(1/2*a)^4 + 6*tan(1/2*a)^5 + 15*tan(b*x + 4*a)*tan(1/2*a)^2 - 20*ta
n(1/2*a)^3 - tan(b*x + 4*a) + 6*tan(1/2*a))^3))/b
```

3.55 $\int \csc^2(a+bx)\csc^3(2a+2bx)\,dx$

Optimal. Leaf size=60

$$\frac{\tan^2(a+bx)}{16b}-\frac{\cot^4(a+bx)}{32b}-\frac{3\cot^2(a+bx)}{16b}+\frac{3\log(\tan(a+bx))}{8b}$$

```
[Out] (-3*Cot[a + b*x]^2)/(16*b) - Cot[a + b*x]^4/(32*b) + (3*Log[Tan[a + b*x]])/
(8*b) + Tan[a + b*x]^2/(16*b)
```

Rubi [A] time = 0.0671915, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4288, 2620, 266, 43}

$$\frac{\tan^2(a+bx)}{16b}-\frac{\cot^4(a+bx)}{32b}-\frac{3\cot^2(a+bx)}{16b}+\frac{3\log(\tan(a+bx))}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] (-3*Cot[a + b*x]^2)/(16*b) - Cot[a + b*x]^4/(32*b) + (3*Log[Tan[a + b*x]])/
(8*b) + Tan[a + b*x]^2/(16*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \csc^2(a+bx)\csc^3(2a+2bx)\,dx = \frac{1}{8}\int \csc^5(a+bx)\sec^3(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^5}\,dx, x, \tan(a+bx)\right)}{8b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{x^3}\,dx, x, \tan^2(a+bx)\right)}{16b}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^3}+\frac{3}{x^2}+\frac{3}{x}\right)dx, x, \tan^2(a+bx)\right)}{16b}$$
$$= -\frac{3\cot^2(a+bx)}{16b} - \frac{\cot^4(a+bx)}{32b} + \frac{3\log(\tan(a+bx))}{8b} + \frac{\tan^2(a+bx)}{16b}$$

Mathematica [A] time = 0.359673, size = 54, normalized size = 0.9

$$-\frac{\csc^4(a+bx)+4\csc^2(a+bx)-2\sec^2(a+bx)-12\log(\sin(a+bx))+12\log(\cos(a+bx))}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] -(4*Csc[a + b*x]^2 + Csc[a + b*x]^4 + 12*Log[Cos[a + b*x]] - 12*Log[Sin[a +
 b*x]] - 2*Sec[a + b*x]^2)/(32*b)
```

Maple [A] time = 0.035, size = 69, normalized size = 1.2

$$-\frac{1}{32\,b\,(\sin(bx+a))^4\,(\cos(bx+a))^2}+\frac{3}{32\,b\,(\sin(bx+a))^2\,(\cos(bx+a))^2}-\frac{3}{16\,b\,(\sin(bx+a))^2}+\frac{3\,\ln(\tan(bx+a))}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*csc(2*b*x+2*a)^3,x)
```

```
[Out] -1/32/b/sin(b*x+a)^4/cos(b*x+a)^2+3/32/b/sin(b*x+a)^2/cos(b*x+a)^2-3/16/b/s
in(b*x+a)^2+3/8*ln(tan(b*x+a))/b
```

Maxima [B] time = 1.45619, size = 4304, normalized size = 71.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(4*(3*cos(10*b*x + 10*a) - 6*cos(8*b*x + 8*a) - 2*cos(6*b*x + 6*a) - 6
*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a))*cos(12*b*x + 12*a) + 4*(9*cos(8*b*x
 + 8*a) + 16*cos(6*b*x + 6*a) + 9*cos(4*b*x + 4*a) - 12*cos(2*b*x + 2*a) +
3)*cos(10*b*x + 10*a) - 24*cos(10*b*x + 10*a)^2 - 4*(22*cos(6*b*x + 6*a) -
12*cos(4*b*x + 4*a) - 9*cos(2*b*x + 2*a) + 6)*cos(8*b*x + 8*a) + 24*cos(8*b
```

```
*x + 8*a)^2 - 8*(11*cos(4*b*x + 4*a) - 8*cos(2*b*x + 2*a) + 1)*cos(6*b*x +
6*a) - 32*cos(6*b*x + 6*a)^2 + 12*(3*cos(2*b*x + 2*a) - 2)*cos(4*b*x + 4*a)
 + 24*cos(4*b*x + 4*a)^2 - 24*cos(2*b*x + 2*a)^2 + 3*(2*(2*cos(10*b*x + 10*
a) + cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*cos(2*b*x
 + 2*a) - 1)*cos(12*b*x + 12*a) - cos(12*b*x + 12*a)^2 - 4*(cos(8*b*x + 8*a
) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(10*
b*x + 10*a) - 4*cos(10*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*a) - cos(4*b*x +
4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - cos(8*b*x + 8*a)^2 + 8*(c
os(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 16*cos(6*b*x +
 6*a)^2 - 2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2
- 4*cos(2*b*x + 2*a)^2 + 2*(2*sin(10*b*x + 10*a) + sin(8*b*x + 8*a) - 4*sin
(6*b*x + 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) -
 sin(12*b*x + 12*a)^2 - 4*(sin(8*b*x + 8*a) - 4*sin(6*b*x + 6*a) + sin(4*b*
x + 4*a) + 2*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 4*sin(10*b*x + 10*a)^2
+ 2*(4*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(8*b*x
+ 8*a) - sin(8*b*x + 8*a)^2 + 8*(sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin
(6*b*x + 6*a) - 16*sin(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 4*sin(4*b*x +
4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(
cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*
b*x)*sin(2*a) + sin(2*a)^2) - 3*(2*(2*cos(10*b*x + 10*a) + cos(8*b*x + 8*a)
 - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(12*b
*x + 12*a) - cos(12*b*x + 12*a)^2 - 4*(cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a
) + cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(10*b*x + 10*a) - 4*cos(1
0*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*a) - cos(4*b*x + 4*a) - 2*cos(2*b*x +
2*a) + 1)*cos(8*b*x + 8*a) - cos(8*b*x + 8*a)^2 + 8*(cos(4*b*x + 4*a) + 2*c
os(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 16*cos(6*b*x + 6*a)^2 - 2*(2*cos(2*
b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^
2 + 2*(2*sin(10*b*x + 10*a) + sin(8*b*x + 8*a) - 4*sin(6*b*x + 6*a) + sin(4
*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - sin(12*b*x + 12*a)^2
 - 4*(sin(8*b*x + 8*a) - 4*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*
x + 2*a))*sin(10*b*x + 10*a) - 4*sin(10*b*x + 10*a)^2 + 2*(4*sin(6*b*x + 6*
a) - sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - sin(8*b*x +
8*a)^2 + 8*(sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 16*si
n(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a)
 - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*
x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 3*(2*(2
*cos(10*b*x + 10*a) + cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4
*a) + 2*cos(2*b*x + 2*a) - 1)*cos(12*b*x + 12*a) - cos(12*b*x + 12*a)^2 - 4
*(cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*cos(2*b*x +
2*a) - 1)*cos(10*b*x + 10*a) - 4*cos(10*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*
a) - cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - cos(8*b*
x + 8*a)^2 + 8*(cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a)
 - 16*cos(6*b*x + 6*a)^2 - 2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - co
s(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 + 2*(2*sin(10*b*x + 10*a) + sin(8*b
*x + 8*a) - 4*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin
(12*b*x + 12*a) - sin(12*b*x + 12*a)^2 - 4*(sin(8*b*x + 8*a) - 4*sin(6*b*x
+ 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 4*sin(
10*b*x + 10*a)^2 + 2*(4*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - 2*sin(2*b*x +
 2*a))*sin(8*b*x + 8*a) - sin(8*b*x + 8*a)^2 + 8*(sin(4*b*x + 4*a) + 2*sin(
2*b*x + 2*a))*sin(6*b*x + 6*a) - 16*sin(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2
 - 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x
 + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2
*sin(b*x)*sin(a) + sin(a)^2) + 4*(3*sin(10*b*x + 10*a) - 6*sin(8*b*x + 8*a)
 - 2*sin(6*b*x + 6*a) - 6*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(12*b*x
 + 12*a) + 4*(9*sin(8*b*x + 8*a) + 16*sin(6*b*x + 6*a) + 9*sin(4*b*x + 4*a)
 - 12*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 24*sin(10*b*x + 10*a)^2 - 4*(2
2*sin(6*b*x + 6*a) - 12*sin(4*b*x + 4*a) - 9*sin(2*b*x + 2*a))*sin(8*b*x +
8*a) + 24*sin(8*b*x + 8*a)^2 - 8*(11*sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))
*sin(6*b*x + 6*a) - 32*sin(6*b*x + 6*a)^2 + 24*sin(4*b*x + 4*a)^2 + 36*sin(
```

```
4*b*x + 4*a)*sin(2*b*x + 2*a) - 24*sin(2*b*x + 2*a)^2 + 12*cos(2*b*x + 2*a)
)/(b*cos(12*b*x + 12*a)^2 + 4*b*cos(10*b*x + 10*a)^2 + b*cos(8*b*x + 8*a)^2
 + 16*b*cos(6*b*x + 6*a)^2 + b*cos(4*b*x + 4*a)^2 + 4*b*cos(2*b*x + 2*a)^2
+ b*sin(12*b*x + 12*a)^2 + 4*b*sin(10*b*x + 10*a)^2 + b*sin(8*b*x + 8*a)^2
+ 16*b*sin(6*b*x + 6*a)^2 + b*sin(4*b*x + 4*a)^2 + 4*b*sin(4*b*x + 4*a)*sin
(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(10*b*x + 10*a) + b*cos(
8*b*x + 8*a) - 4*b*cos(6*b*x + 6*a) + b*cos(4*b*x + 4*a) + 2*b*cos(2*b*x +
2*a) - b)*cos(12*b*x + 12*a) + 4*(b*cos(8*b*x + 8*a) - 4*b*cos(6*b*x + 6*a)
 + b*cos(4*b*x + 4*a) + 2*b*cos(2*b*x + 2*a) - b)*cos(10*b*x + 10*a) - 2*(4
*b*cos(6*b*x + 6*a) - b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(8*
b*x + 8*a) - 8*(b*cos(4*b*x + 4*a) + 2*b*cos(2*b*x + 2*a) - b)*cos(6*b*x +
6*a) + 2*(2*b*cos(2*b*x + 2*a) - b)*cos(4*b*x + 4*a) - 4*b*cos(2*b*x + 2*a)
 - 2*(2*b*sin(10*b*x + 10*a) + b*sin(8*b*x + 8*a) - 4*b*sin(6*b*x + 6*a) +
b*sin(4*b*x + 4*a) + 2*b*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) + 4*(b*sin(8*
b*x + 8*a) - 4*b*sin(6*b*x + 6*a) + b*sin(4*b*x + 4*a) + 2*b*sin(2*b*x + 2*
a))*sin(10*b*x + 10*a) - 2*(4*b*sin(6*b*x + 6*a) - b*sin(4*b*x + 4*a) - 2*b
*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 8*(b*sin(4*b*x + 4*a) + 2*b*sin(2*b*x
 + 2*a))*sin(6*b*x + 6*a) + b)
```

Fricas [B] time = 0.520598, size = 367, normalized size = 6.12

$$\frac{6\cos(bx+a)^4 - 9\cos(bx+a)^2 - 6\left(\cos(bx+a)^6 - 2\cos(bx+a)^4 + \cos(bx+a)^2\right)\log\left(\cos(bx+a)^2\right) + 6\left(\cos(bx+a\right.}{32\left(b\cos(bx+a)^6 - 2\,b\cos(bx+a)^4 + b\cos(bx+a)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(6*cos(b*x + a)^4 - 9*cos(b*x + a)^2 - 6*(cos(b*x + a)^6 - 2*cos(b*x +
 a)^4 + cos(b*x + a)^2)*log(cos(b*x + a)^2) + 6*(cos(b*x + a)^6 - 2*cos(b*x
 + a)^4 + cos(b*x + a)^2)*log(-1/4*cos(b*x + a)^2 + 1/4) + 2)/(b*cos(b*x +
a)^6 - 2*b*cos(b*x + a)^4 + b*cos(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \csc^2(a+bx)\csc^3(2a+2bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*csc(2*b*x+2*a)**3,x)
```

```
[Out] Integral(csc(a + b*x)**2*csc(2*a + 2*b*x)**3, x)
```

Giac [B] time = 2.23716, size = 3791, normalized size = 63.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] 1/64*((12*tan(b*x + 4*a)*tan(1/2*a)^23 - tan(1/2*a)^24 + 11664*tan(b*x + 4*
a)^2*tan(1/2*a)^20 - 4036*tan(b*x + 4*a)*tan(1/2*a)^21 + 384*tan(1/2*a)^22
- 155520*tan(b*x + 4*a)^2*tan(1/2*a)^18 + 96852*tan(b*x + 4*a)*tan(1/2*a)^1
9 - 11994*tan(1/2*a)^20 + 824256*tan(b*x + 4*a)^2*tan(1/2*a)^16 - 781884*ta
n(b*x + 4*a)*tan(1/2*a)^17 + 150592*tan(1/2*a)^18 - 2194560*tan(b*x + 4*a)^
2*tan(1/2*a)^14 + 2930936*tan(b*x + 4*a)*tan(1/2*a)^15 - 827919*tan(1/2*a)^
16 + 3065184*tan(b*x + 4*a)^2*tan(1/2*a)^12 - 5623464*tan(b*x + 4*a)*tan(1/
2*a)^13 + 2209344*tan(1/2*a)^14 - 2194560*tan(b*x + 4*a)^2*tan(1/2*a)^10 +
5623464*tan(b*x + 4*a)*tan(1/2*a)^11 - 3036716*tan(1/2*a)^12 + 824256*tan(b
*x + 4*a)^2*tan(1/2*a)^8 - 2930936*tan(b*x + 4*a)*tan(1/2*a)^9 + 2209344*ta
n(1/2*a)^10 - 155520*tan(b*x + 4*a)^2*tan(1/2*a)^6 + 781884*tan(b*x + 4*a)*
tan(1/2*a)^7 - 827919*tan(1/2*a)^8 + 11664*tan(b*x + 4*a)^2*tan(1/2*a)^4 -
96852*tan(b*x + 4*a)*tan(1/2*a)^5 + 150592*tan(1/2*a)^6 + 4036*tan(b*x + 4*
a)*tan(1/2*a)^3 - 11994*tan(1/2*a)^4 - 12*tan(b*x + 4*a)*tan(1/2*a) + 384*t
an(1/2*a)^2 - 1)/((9*tan(1/2*a)^10 - 60*tan(1/2*a)^8 + 118*tan(1/2*a)^6 - 6
0*tan(1/2*a)^4 + 9*tan(1/2*a)^2)*(6*tan(b*x + 4*a)*tan(1/2*a)^5 - tan(1/2*a
)^6 - 20*tan(b*x + 4*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 + 6*tan(b*x + 4*a)*t
an(1/2*a) - 15*tan(1/2*a)^2 + 1)^2) - 2*(25*tan(b*x + 4*a)^4*tan(1/2*a)^48
- 3000*tan(b*x + 4*a)^4*tan(1/2*a)^46 + 528*tan(b*x + 4*a)^3*tan(1/2*a)^47
+ 6*tan(b*x + 4*a)^2*tan(1/2*a)^48 + 160500*tan(b*x + 4*a)^4*tan(1/2*a)^44
- 60656*tan(b*x + 4*a)^3*tan(1/2*a)^45 + 4032*tan(b*x + 4*a)^2*tan(1/2*a)^4
6 + 48*tan(b*x + 4*a)*tan(1/2*a)^47 + tan(1/2*a)^48 - 5040200*tan(b*x + 4*a
)^4*tan(1/2*a)^42 + 2998224*tan(b*x + 4*a)^3*tan(1/2*a)^43 - 459720*tan(b*x
 + 4*a)^2*tan(1/2*a)^44 + 13808*tan(b*x + 4*a)*tan(1/2*a)^45 + 96*tan(1/2*a
)^46 + 102947250*tan(b*x + 4*a)^4*tan(1/2*a)^40 - 84754224*tan(b*x + 4*a)^3
*tan(1/2*a)^41 + 20740800*tan(b*x + 4*a)^2*tan(1/2*a)^42 - 1553616*tan(b*x
+ 4*a)*tan(1/2*a)^43 + 17940*tan(1/2*a)^44 - 1432641000*tan(b*x + 4*a)^4*ta
n(1/2*a)^38 + 1525423600*tan(b*x + 4*a)^3*tan(1/2*a)^39 - 518242932*tan(b*x
 + 4*a)^2*tan(1/2*a)^40 + 62627184*tan(b*x + 4*a)*tan(1/2*a)^41 - 1976672*t
an(1/2*a)^42 + 13850865700*tan(b*x + 4*a)^4*tan(1/2*a)^36 - 18347115792*tan
(b*x + 4*a)^3*tan(1/2*a)^37 + 8061782976*tan(b*x + 4*a)^2*tan(1/2*a)^38 - 1
352088880*tan(b*x + 4*a)*tan(1/2*a)^39 + 69043458*tan(1/2*a)^40 - 934243830
00*tan(b*x + 4*a)^4*tan(1/2*a)^34 + 150554954928*tan(b*x + 4*a)^3*tan(1/2*a
)^35 - 82205969064*tan(b*x + 4*a)^2*tan(1/2*a)^36 + 17773339152*tan(b*x + 4
*a)*tan(1/2*a)^37 - 1252335840*tan(1/2*a)^38 + 438276972375*tan(b*x + 4*a)^
4*tan(1/2*a)^32 - 849352828496*tan(b*x + 4*a)^3*tan(1/2*a)^33 + 56333896416
0*tan(b*x + 4*a)^2*tan(1/2*a)^34 - 150682961328*tan(b*x + 4*a)*tan(1/2*a)^3
5 + 13569524420*tan(1/2*a)^36 - 1429067476400*tan(b*x + 4*a)^4*tan(1/2*a)^3
0 + 3312518959776*tan(b*x + 4*a)^3*tan(1/2*a)^31 - 2637116003430*tan(b*x +
4*a)^2*tan(1/2*a)^32 + 853300409104*tan(b*x + 4*a)*tan(1/2*a)^33 - 94325029
920*tan(1/2*a)^34 + 3274543905000*tan(b*x + 4*a)^4*tan(1/2*a)^28 - 90295262
76960*tan(b*x + 4*a)^3*tan(1/2*a)^29 + 8569312295808*tan(b*x + 4*a)^2*tan(1
/2*a)^30 - 3315261850656*tan(b*x + 4*a)*tan(1/2*a)^31 + 440742337263*tan(1/
2*a)^32 - 5348018130000*tan(b*x + 4*a)^4*tan(1/2*a)^26 + 17444688475680*tan
(b*x + 4*a)^3*tan(1/2*a)^27 - 19627171727760*tan(b*x + 4*a)^2*tan(1/2*a)^28
 + 9019269429600*tan(b*x + 4*a)*tan(1/2*a)^29 - 1427485388352*tan(1/2*a)^30
 + 6290345645500*tan(b*x + 4*a)^4*tan(1/2*a)^24 - 24168979539936*tan(b*x +
4*a)^3*tan(1/2*a)^25 + 32090952253824*tan(b*x + 4*a)^2*tan(1/2*a)^26 - 1743
5372647456*tan(b*x + 4*a)*tan(1/2*a)^27 + 3267744813864*tan(1/2*a)^28 - 534
8018130000*tan(b*x + 4*a)^4*tan(1/2*a)^22 + 24168979539936*tan(b*x + 4*a)^3
*tan(1/2*a)^23 - 37769347277400*tan(b*x + 4*a)^2*tan(1/2*a)^24 + 2418250446
8064*tan(b*x + 4*a)*tan(1/2*a)^25 - 5348887137216*tan(1/2*a)^26 + 327454390
5000*tan(b*x + 4*a)^4*tan(1/2*a)^20 - 17444688475680*tan(b*x + 4*a)^3*tan(1
/2*a)^21 + 32090952253824*tan(b*x + 4*a)^2*tan(1/2*a)^22 - 24182504468064*t
an(b*x + 4*a)*tan(1/2*a)^23 + 6299635484700*tan(1/2*a)^24 - 1429067476400*t
an(b*x + 4*a)^4*tan(1/2*a)^18 + 9029526276960*tan(b*x + 4*a)^3*tan(1/2*a)^1
9 - 19627171727760*tan(b*x + 4*a)^2*tan(1/2*a)^20 + 17435372647456*tan(b*x
+ 4*a)*tan(1/2*a)^21 - 5348887137216*tan(1/2*a)^22 + 438276972375*tan(b*x +
 4*a)^4*tan(1/2*a)^16 - 3312518959776*tan(b*x + 4*a)^3*tan(1/2*a)^17 + 8569
```

```
312295808*tan(b*x + 4*a)^2*tan(1/2*a)^18 - 9019269429600*tan(b*x + 4*a)*tan
(1/2*a)^19 + 3267744813864*tan(1/2*a)^20 - 93424383000*tan(b*x + 4*a)^4*tan
(1/2*a)^14 + 849352828496*tan(b*x + 4*a)^3*tan(1/2*a)^15 - 2637116003430*ta
n(b*x + 4*a)^2*tan(1/2*a)^16 + 3315261850656*tan(b*x + 4*a)*tan(1/2*a)^17 -
 1427485388352*tan(1/2*a)^18 + 13850865700*tan(b*x + 4*a)^4*tan(1/2*a)^12 -
 150554954928*tan(b*x + 4*a)^3*tan(1/2*a)^13 + 563338964160*tan(b*x + 4*a)^
2*tan(1/2*a)^14 - 853300409104*tan(b*x + 4*a)*tan(1/2*a)^15 + 440742337263*
tan(1/2*a)^16 - 1432641000*tan(b*x + 4*a)^4*tan(1/2*a)^10 + 18347115792*tan
(b*x + 4*a)^3*tan(1/2*a)^11 - 82205969064*tan(b*x + 4*a)^2*tan(1/2*a)^12 +
150682961328*tan(b*x + 4*a)*tan(1/2*a)^13 - 94325029920*tan(1/2*a)^14 + 102
947250*tan(b*x + 4*a)^4*tan(1/2*a)^8 - 1525423600*tan(b*x + 4*a)^3*tan(1/2*
a)^9 + 8061782976*tan(b*x + 4*a)^2*tan(1/2*a)^10 - 17773339152*tan(b*x + 4*
a)*tan(1/2*a)^11 + 13569524420*tan(1/2*a)^12 - 5040200*tan(b*x + 4*a)^4*tan
(1/2*a)^6 + 84754224*tan(b*x + 4*a)^3*tan(1/2*a)^7 - 518242932*tan(b*x + 4*
a)^2*tan(1/2*a)^8 + 1352088880*tan(b*x + 4*a)*tan(1/2*a)^9 - 1252335840*tan
(1/2*a)^10 + 160500*tan(b*x + 4*a)^4*tan(1/2*a)^4 - 2998224*tan(b*x + 4*a)^
3*tan(1/2*a)^5 + 20740800*tan(b*x + 4*a)^2*tan(1/2*a)^6 - 62627184*tan(b*x
+ 4*a)*tan(1/2*a)^7 + 69043458*tan(1/2*a)^8 - 3000*tan(b*x + 4*a)^4*tan(1/2
*a)^2 + 60656*tan(b*x + 4*a)^3*tan(1/2*a)^3 - 459720*tan(b*x + 4*a)^2*tan(1
/2*a)^4 + 1553616*tan(b*x + 4*a)*tan(1/2*a)^5 - 1976672*tan(1/2*a)^6 + 25*t
an(b*x + 4*a)^4 - 528*tan(b*x + 4*a)^3*tan(1/2*a) + 4032*tan(b*x + 4*a)^2*t
an(1/2*a)^2 - 13808*tan(b*x + 4*a)*tan(1/2*a)^3 + 17940*tan(1/2*a)^4 + 6*ta
n(b*x + 4*a)^2 - 48*tan(b*x + 4*a)*tan(1/2*a) + 96*tan(1/2*a)^2 + 1)/((tan(
1/2*a)^24 - 60*tan(1/2*a)^22 + 1410*tan(1/2*a)^20 - 16204*tan(1/2*a)^18 + 9
2655*tan(1/2*a)^16 - 245880*tan(1/2*a)^14 + 336156*tan(1/2*a)^12 - 245880*t
an(1/2*a)^10 + 92655*tan(1/2*a)^8 - 16204*tan(1/2*a)^6 + 1410*tan(1/2*a)^4
- 60*tan(1/2*a)^2 + 1)*(tan(b*x + 4*a)*tan(1/2*a)^6 - 15*tan(b*x + 4*a)*tan
(1/2*a)^4 + 6*tan(1/2*a)^5 + 15*tan(b*x + 4*a)*tan(1/2*a)^2 - 20*tan(1/2*a)
^3 - tan(b*x + 4*a) + 6*tan(1/2*a))^4) + 24*log(abs(tan(b*x + 4*a)*tan(1/2*
a)^6 - 15*tan(b*x + 4*a)*tan(1/2*a)^4 + 6*tan(1/2*a)^5 + 15*tan(b*x + 4*a)*
tan(1/2*a)^2 - 20*tan(1/2*a)^3 - tan(b*x + 4*a) + 6*tan(1/2*a))) - 24*log(a
bs(6*tan(b*x + 4*a)*tan(1/2*a)^5 - tan(1/2*a)^6 - 20*tan(b*x + 4*a)*tan(1/2
*a)^3 + 15*tan(1/2*a)^4 + 6*tan(b*x + 4*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1
)))/b
```

3.56 $\int \csc^2(a+bx)\csc^4(2a+2bx)\,dx$

Optimal. Leaf size=72

$$\frac{\tan^3(a+bx)}{48b}+\frac{\tan(a+bx)}{4b}-\frac{\cot^5(a+bx)}{80b}-\frac{\cot^3(a+bx)}{12b}-\frac{3\cot(a+bx)}{8b}$$

```
[Out] (-3*Cot[a + b*x])/(8*b) - Cot[a + b*x]^3/(12*b) - Cot[a + b*x]^5/(80*b) + T
an[a + b*x]/(4*b) + Tan[a + b*x]^3/(48*b)
```

Rubi [A] time = 0.070376, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4288, 2620, 270}

$$\frac{\tan^3(a+bx)}{48b}+\frac{\tan(a+bx)}{4b}-\frac{\cot^5(a+bx)}{80b}-\frac{\cot^3(a+bx)}{12b}-\frac{3\cot(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] (-3*Cot[a + b*x])/(8*b) - Cot[a + b*x]^3/(12*b) - Cot[a + b*x]^5/(80*b) + T
an[a + b*x]/(4*b) + Tan[a + b*x]^3/(48*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \csc^2(a+bx)\csc^4(2a+2bx)\,dx &= \frac{1}{16}\int \csc^6(a+bx)\sec^4(a+bx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^4}{x^6}\,dx, x, \tan(a+bx)\right)}{16b}\\
&= \frac{\text{Subst}\left(\int \left(4+\frac{1}{x^6}+\frac{4}{x^4}+\frac{6}{x^2}+x^2\right)dx, x, \tan(a+bx)\right)}{16b}\\
&= -\frac{3\cot(a+bx)}{8b}-\frac{\cot^3(a+bx)}{12b}-\frac{\cot^5(a+bx)}{80b}+\frac{\tan(a+bx)}{4b}+\frac{\tan^3(a+bx)}{48b}
\end{aligned}$$

Mathematica [A] time = 0.0535614, size = 90, normalized size = 1.25

$$\frac{11\tan(a+bx)}{48b}-\frac{73\cot(a+bx)}{240b}-\frac{\cot(a+bx)\csc^4(a+bx)}{80b}-\frac{7\cot(a+bx)\csc^2(a+bx)}{120b}+\frac{\tan(a+bx)\sec^2(a+bx)}{48b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] (-73*Cot[a + b*x])/(240*b) - (7*Cot[a + b*x]*Csc[a + b*x]^2)/(120*b) - (Cot
[a + b*x]*Csc[a + b*x]^4)/(80*b) + (11*Tan[a + b*x])/(48*b) + (Sec[a + b*x]
^2*Tan[a + b*x])/(48*b)
```

Maple [A] time = 0.038, size = 87, normalized size = 1.2

$$\frac{1}{16\,b}\left(-\frac{1}{5\,(\sin(bx+a))^5(\cos(bx+a))^3}+\frac{8}{15\,(\sin(bx+a))^3(\cos(bx+a))^3}-\frac{16}{15\,(\sin(bx+a))^3\cos(bx+a)}+\frac{}{15\cos(b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*csc(2*b*x+2*a)^4,x)
```

```
[Out] 1/16/b*(-1/5/sin(b*x+a)^5/cos(b*x+a)^3+8/15/sin(b*x+a)^3/cos(b*x+a)^3-16/15
/sin(b*x+a)^3/cos(b*x+a)+64/15/sin(b*x+a)/cos(b*x+a)-128/15*cot(b*x+a))
```

Maxima [B] time = 1.25661, size = 1656, normalized size = 23.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 16/15*(2*(3*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(16*
b*x + 16*a) - 4*(3*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*
cos(14*b*x + 14*a) - 4*(3*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - sin(2*b*x +
 2*a))*cos(12*b*x + 12*a) + 12*(3*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - sin
(2*b*x + 2*a))*cos(10*b*x + 10*a) - (6*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a
) - 2*cos(2*b*x + 2*a) + 1)*sin(16*b*x + 16*a) + 2*(6*cos(6*b*x + 6*a) - 2*
cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*sin(14*b*x + 14*a) + 2*(6*cos(6*
b*x + 6*a) - 2*cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*sin(12*b*x + 12*a
) - 6*(6*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*si
n(10*b*x + 10*a))/(b*cos(16*b*x + 16*a)^2 + 4*b*cos(14*b*x + 14*a)^2 + 4*b*
cos(12*b*x + 12*a)^2 + 36*b*cos(10*b*x + 10*a)^2 + 36*b*cos(6*b*x + 6*a)^2
+ 4*b*cos(4*b*x + 4*a)^2 + 4*b*cos(2*b*x + 2*a)^2 + b*sin(16*b*x + 16*a)^2
+ 4*b*sin(14*b*x + 14*a)^2 + 4*b*sin(12*b*x + 12*a)^2 + 36*b*sin(10*b*x + 1
0*a)^2 + 36*b*sin(6*b*x + 6*a)^2 + 4*b*sin(4*b*x + 4*a)^2 + 8*b*sin(4*b*x +
 4*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(14*b*x + 14*a)
 + 2*b*cos(12*b*x + 12*a) - 6*b*cos(10*b*x + 10*a) + 6*b*cos(6*b*x + 6*a) -
 2*b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(16*b*x + 16*a) + 4*(2
*b*cos(12*b*x + 12*a) - 6*b*cos(10*b*x + 10*a) + 6*b*cos(6*b*x + 6*a) - 2*b
*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(14*b*x + 14*a) - 4*(6*b*c
os(10*b*x + 10*a) - 6*b*cos(6*b*x + 6*a) + 2*b*cos(4*b*x + 4*a) + 2*b*cos(2
*b*x + 2*a) - b)*cos(12*b*x + 12*a) - 12*(6*b*cos(6*b*x + 6*a) - 2*b*cos(4*
```

```
b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(10*b*x + 10*a) - 12*(2*b*cos(4*b
*x + 4*a) + 2*b*cos(2*b*x + 2*a) - b)*cos(6*b*x + 6*a) + 4*(2*b*cos(2*b*x +
 2*a) - b)*cos(4*b*x + 4*a) - 4*b*cos(2*b*x + 2*a) - 4*(b*sin(14*b*x + 14*a
) + b*sin(12*b*x + 12*a) - 3*b*sin(10*b*x + 10*a) + 3*b*sin(6*b*x + 6*a) -
b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*sin(16*b*x + 16*a) + 8*(b*sin(12*b
*x + 12*a) - 3*b*sin(10*b*x + 10*a) + 3*b*sin(6*b*x + 6*a) - b*sin(4*b*x +
4*a) - b*sin(2*b*x + 2*a))*sin(14*b*x + 14*a) - 8*(3*b*sin(10*b*x + 10*a) -
 3*b*sin(6*b*x + 6*a) + b*sin(4*b*x + 4*a) + b*sin(2*b*x + 2*a))*sin(12*b*x
 + 12*a) - 24*(3*b*sin(6*b*x + 6*a) - b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*
a))*sin(10*b*x + 10*a) - 24*(b*sin(4*b*x + 4*a) + b*sin(2*b*x + 2*a))*sin(6
*b*x + 6*a) + b)
```

Fricas [A] time = 0.481648, size = 228, normalized size = 3.17

$$-\frac{128\cos(bx+a)^8 - 320\cos(bx+a)^6 + 240\cos(bx+a)^4 - 40\cos(bx+a)^2 - 5}{240\left(b\cos(bx+a)^7 - 2b\cos(bx+a)^5 + b\cos(bx+a)^3\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] -1/240*(128*cos(b*x + a)^8 - 320*cos(b*x + a)^6 + 240*cos(b*x + a)^4 - 40*c
os(b*x + a)^2 - 5)/((b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*cos(b*x + a)
^3)*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*csc(2*b*x+2*a)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.42267, size = 4651, normalized size = 64.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] -1/1920*(5*(108*tan(b*x + 4*a)^2*tan(1/2*a)^34 - 18*tan(b*x + 4*a)*tan(1/2*
a)^35 + tan(1/2*a)^36 + 12240*tan(b*x + 4*a)^2*tan(1/2*a)^32 - 3774*tan(b*x
 + 4*a)*tan(1/2*a)^33 + 342*tan(1/2*a)^34 - 85632*tan(b*x + 4*a)^2*tan(1/2*
a)^30 + 75456*tan(b*x + 4*a)*tan(1/2*a)^31 - 9783*tan(1/2*a)^32 + 58608*tan
(b*x + 4*a)^2*tan(1/2*a)^28 - 253152*tan(b*x + 4*a)*tan(1/2*a)^29 + 86064*t
an(1/2*a)^30 + 631440*tan(b*x + 4*a)^2*tan(1/2*a)^26 - 389448*tan(b*x + 4*a
)*tan(1/2*a)^27 - 85500*tan(1/2*a)^28 - 679344*tan(b*x + 4*a)^2*tan(1/2*a)^
24 + 2047464*tan(b*x + 4*a)*tan(1/2*a)^25 - 702936*tan(1/2*a)^26 - 2420352*
```

tan(b*x + 4*a)^2*tan(1/2*a)^22 + 1465056*tan(b*x + 4*a)*tan(1/2*a)^23 + 675
636*tan(1/2*a)^24 + 1394928*tan(b*x + 4*a)^2*tan(1/2*a)^20 - 6297216*tan(b*
x + 4*a)*tan(1/2*a)^21 + 2768976*tan(1/2*a)^22 + 5321736*tan(b*x + 4*a)^2*t
an(1/2*a)^18 - 5657724*tan(b*x + 4*a)*tan(1/2*a)^19 - 514818*tan(1/2*a)^20
+ 1394928*tan(b*x + 4*a)^2*tan(1/2*a)^16 + 5657724*tan(b*x + 4*a)*tan(1/2*a
)^17 - 4173820*tan(1/2*a)^18 - 2420352*tan(b*x + 4*a)^2*tan(1/2*a)^14 + 629
7216*tan(b*x + 4*a)*tan(1/2*a)^15 - 514818*tan(1/2*a)^16 - 679344*tan(b*x +
 4*a)^2*tan(1/2*a)^12 - 1465056*tan(b*x + 4*a)*tan(1/2*a)^13 + 2768976*tan(
1/2*a)^14 + 631440*tan(b*x + 4*a)^2*tan(1/2*a)^10 - 2047464*tan(b*x + 4*a)*
tan(1/2*a)^11 + 675636*tan(1/2*a)^12 + 58608*tan(b*x + 4*a)^2*tan(1/2*a)^8
+ 389448*tan(b*x + 4*a)*tan(1/2*a)^9 - 702936*tan(1/2*a)^10 - 85632*tan(b*x
 + 4*a)^2*tan(1/2*a)^6 + 253152*tan(b*x + 4*a)*tan(1/2*a)^7 - 85500*tan(1/2
*a)^8 + 12240*tan(b*x + 4*a)^2*tan(1/2*a)^4 - 75456*tan(b*x + 4*a)*tan(1/2*
a)^5 + 86064*tan(1/2*a)^6 + 108*tan(b*x + 4*a)^2*tan(1/2*a)^2 + 3774*tan(b*
x + 4*a)*tan(1/2*a)^3 - 9783*tan(1/2*a)^4 + 18*tan(b*x + 4*a)*tan(1/2*a) +
342*tan(1/2*a)^2 + 1)/((27*tan(1/2*a)^15 - 270*tan(1/2*a)^13 + 981*tan(1/2*
a)^11 - 1540*tan(1/2*a)^9 + 981*tan(1/2*a)^7 - 270*tan(1/2*a)^5 + 27*tan(1/
2*a)^3)*(6*tan(b*x + 4*a)*tan(1/2*a)^5 - tan(1/2*a)^6 - 20*tan(b*x + 4*a)*t
an(1/2*a)^3 + 15*tan(1/2*a)^4 + 6*tan(b*x + 4*a)*tan(1/2*a) - 15*tan(1/2*a)
^2 + 1)^3) + 8*(90*tan(b*x + 4*a)^4*tan(1/2*a)^60 - 8100*tan(b*x + 4*a)^4*t
an(1/2*a)^58 + 1800*tan(b*x + 4*a)^3*tan(1/2*a)^59 + 20*tan(b*x + 4*a)^2*ta
n(1/2*a)^60 + 337950*tan(b*x + 4*a)^4*tan(1/2*a)^56 - 151800*tan(b*x + 4*a)
^3*tan(1/2*a)^57 + 13200*tan(b*x + 4*a)^2*tan(1/2*a)^58 + 150*tan(b*x + 4*a
)*tan(1/2*a)^59 + 3*tan(1/2*a)^60 - 8558760*tan(b*x + 4*a)^4*tan(1/2*a)^54
+ 5856480*tan(b*x + 4*a)^3*tan(1/2*a)^55 - 1068900*tan(b*x + 4*a)^2*tan(1/2
*a)^56 + 44050*tan(b*x + 4*a)*tan(1/2*a)^57 + 270*tan(1/2*a)^58 + 143179650
*tan(b*x + 4*a)^4*tan(1/2*a)^52 - 133967520*tan(b*x + 4*a)^3*tan(1/2*a)^53
+ 37650400*tan(b*x + 4*a)^2*tan(1/2*a)^54 - 3349740*tan(b*x + 4*a)*tan(1/2*
a)^55 + 56265*tan(1/2*a)^56 - 1610662860*tan(b*x + 4*a)^4*tan(1/2*a)^50 + 1
962664200*tan(b*x + 4*a)^3*tan(1/2*a)^51 - 761504220*tan(b*x + 4*a)^2*tan(1
/2*a)^52 + 105669900*tan(b*x + 4*a)*tan(1/2*a)^53 - 3921700*tan(1/2*a)^54 +
 11902135590*tan(b*x + 4*a)^4*tan(1/2*a)^48 - 18613230840*tan(b*x + 4*a)^3*
tan(1/2*a)^49 + 9508215600*tan(b*x + 4*a)^2*tan(1/2*a)^50 - 1840189110*tan(
b*x + 4*a)*tan(1/2*a)^51 + 108499095*tan(1/2*a)^52 - 53242909200*tan(b*x +
4*a)^4*tan(1/2*a)^46 + 109725269760*tan(b*x + 4*a)^3*tan(1/2*a)^47 - 733098
82260*tan(b*x + 4*a)^2*tan(1/2*a)^48 + 18891597150*tan(b*x + 4*a)*tan(1/2*a
)^49 - 1563175302*tan(1/2*a)^50 + 107646377490*tan(b*x + 4*a)^4*tan(1/2*a)^
44 - 347626321920*tan(b*x + 4*a)^3*tan(1/2*a)^45 + 324785269440*tan(b*x + 4
*a)^2*tan(1/2*a)^46 - 112787150040*tan(b*x + 4*a)*tan(1/2*a)^47 + 126029226
05*tan(1/2*a)^48 + 100499354940*tan(b*x + 4*a)^4*tan(1/2*a)^42 + 2439543517
20*tan(b*x + 4*a)^3*tan(1/2*a)^43 - 614500276860*tan(b*x + 4*a)^2*tan(1/2*a
)^44 + 342870428760*tan(b*x + 4*a)*tan(1/2*a)^45 - 54544776360*tan(1/2*a)^4
6 - 720524857050*tan(b*x + 4*a)^4*tan(1/2*a)^40 + 1552622504040*tan(b*x + 4
*a)^3*tan(1/2*a)^41 - 592670717680*tan(b*x + 4*a)^2*tan(1/2*a)^42 - 2218645
73610*tan(b*x + 4*a)*tan(1/2*a)^43 + 98588733975*tan(1/2*a)^44 + 5458434944
40*tan(b*x + 4*a)^4*tan(1/2*a)^38 - 3402462596640*tan(b*x + 4*a)^3*tan(1/2*
a)^39 + 4208031154860*tan(b*x + 4*a)^2*tan(1/2*a)^40 - 1502042255310*tan(b*
x + 4*a)*tan(1/2*a)^41 + 99097279550*tan(1/2*a)^42 + 1394520156090*tan(b*x
+ 4*a)^4*tan(1/2*a)^36 - 1060535913120*tan(b*x + 4*a)^3*tan(1/2*a)^37 - 339
9607142880*tan(b*x + 4*a)^2*tan(1/2*a)^38 + 3367956274060*tan(b*x + 4*a)*ta
n(1/2*a)^39 - 681790695915*tan(1/2*a)^40 - 2195406541260*tan(b*x + 4*a)^4*t
an(1/2*a)^34 + 8940274641000*tan(b*x + 4*a)^3*tan(1/2*a)^35 - 8188018122860
*tan(b*x + 4*a)^2*tan(1/2*a)^36 + 893802691860*tan(b*x + 4*a)*tan(1/2*a)^37
 + 583218519780*tan(1/2*a)^38 - 793687329810*tan(b*x + 4*a)^4*tan(1/2*a)^32
 - 4368381430680*tan(b*x + 4*a)^3*tan(1/2*a)^33 + 13505898965040*tan(b*x +
4*a)^2*tan(1/2*a)^34 - 8980226310150*tan(b*x + 4*a)*tan(1/2*a)^35 + 1327591
075915*tan(1/2*a)^36 + 3207851661600*tan(b*x + 4*a)^4*tan(1/2*a)^30 - 10136
128700160*tan(b*x + 4*a)^3*tan(1/2*a)^31 + 4668559700220*tan(b*x + 4*a)^2*t
an(1/2*a)^32 + 4628264277390*tan(b*x + 4*a)*tan(1/2*a)^33 - 2309792723910*t

```
an(1/2*a)^34 - 793687329810*tan(b*x + 4*a)^4*tan(1/2*a)^28 + 10136128700160
*tan(b*x + 4*a)^3*tan(1/2*a)^29 - 19695904506240*tan(b*x + 4*a)^2*tan(1/2*a
)^30 + 10330657752240*tan(b*x + 4*a)*tan(1/2*a)^31 - 756295285575*tan(1/2*a
)^32 - 2195406541260*tan(b*x + 4*a)^4*tan(1/2*a)^26 + 4368381430680*tan(b*x
 + 4*a)^3*tan(1/2*a)^27 + 4668559700220*tan(b*x + 4*a)^2*tan(1/2*a)^28 - 10
330657752240*tan(b*x + 4*a)*tan(1/2*a)^29 + 3368788208080*tan(1/2*a)^30 + 1
394520156090*tan(b*x + 4*a)^4*tan(1/2*a)^24 - 8940274641000*tan(b*x + 4*a)^
3*tan(1/2*a)^25 + 13505898965040*tan(b*x + 4*a)^2*tan(1/2*a)^26 - 462826427
7390*tan(b*x + 4*a)*tan(1/2*a)^27 - 756295285575*tan(1/2*a)^28 + 5458434944
40*tan(b*x + 4*a)^4*tan(1/2*a)^22 + 1060535913120*tan(b*x + 4*a)^3*tan(1/2*
a)^23 - 8188018122860*tan(b*x + 4*a)^2*tan(1/2*a)^24 + 8980226310150*tan(b*
x + 4*a)*tan(1/2*a)^25 - 2309792723910*tan(1/2*a)^26 - 720524857050*tan(b*x
 + 4*a)^4*tan(1/2*a)^20 + 3402462596640*tan(b*x + 4*a)^3*tan(1/2*a)^21 - 33
99607142880*tan(b*x + 4*a)^2*tan(1/2*a)^22 - 893802691860*tan(b*x + 4*a)*ta
n(1/2*a)^23 + 1327591075915*tan(1/2*a)^24 + 100499354940*tan(b*x + 4*a)^4*t
an(1/2*a)^18 - 1552622504040*tan(b*x + 4*a)^3*tan(1/2*a)^19 + 4208031154860
*tan(b*x + 4*a)^2*tan(1/2*a)^20 - 3367956274060*tan(b*x + 4*a)*tan(1/2*a)^2
1 + 583218519780*tan(1/2*a)^22 + 107646377490*tan(b*x + 4*a)^4*tan(1/2*a)^1
6 - 243954351720*tan(b*x + 4*a)^3*tan(1/2*a)^17 - 592670717680*tan(b*x + 4*
a)^2*tan(1/2*a)^18 + 1502042255310*tan(b*x + 4*a)*tan(1/2*a)^19 - 681790695
915*tan(1/2*a)^20 - 53242909200*tan(b*x + 4*a)^4*tan(1/2*a)^14 + 3476263219
20*tan(b*x + 4*a)^3*tan(1/2*a)^15 - 614500276860*tan(b*x + 4*a)^2*tan(1/2*a
)^16 + 221864573610*tan(b*x + 4*a)*tan(1/2*a)^17 + 99097279550*tan(1/2*a)^1
8 + 11902135590*tan(b*x + 4*a)^4*tan(1/2*a)^12 - 109725269760*tan(b*x + 4*a
)^3*tan(1/2*a)^13 + 324785269440*tan(b*x + 4*a)^2*tan(1/2*a)^14 - 342870428
760*tan(b*x + 4*a)*tan(1/2*a)^15 + 98588733975*tan(1/2*a)^16 - 1610662860*t
an(b*x + 4*a)^4*tan(1/2*a)^10 + 18613230840*tan(b*x + 4*a)^3*tan(1/2*a)^11
 - 73309882260*tan(b*x + 4*a)^2*tan(1/2*a)^12 + 112787150040*tan(b*x + 4*a)*
tan(1/2*a)^13 - 54544776360*tan(1/2*a)^14 + 143179650*tan(b*x + 4*a)^4*tan(
1/2*a)^8 - 1962664200*tan(b*x + 4*a)^3*tan(1/2*a)^9 + 9508215600*tan(b*x +
4*a)^2*tan(1/2*a)^10 - 18891597150*tan(b*x + 4*a)*tan(1/2*a)^11 + 126029226
05*tan(1/2*a)^12 - 8558760*tan(b*x + 4*a)^4*tan(1/2*a)^6 + 133967520*tan(b*
x + 4*a)^3*tan(1/2*a)^7 - 761504220*tan(b*x + 4*a)^2*tan(1/2*a)^8 + 1840189
110*tan(b*x + 4*a)*tan(1/2*a)^9 - 1563175302*tan(1/2*a)^10 + 337950*tan(b*x
 + 4*a)^4*tan(1/2*a)^4 - 5856480*tan(b*x + 4*a)^3*tan(1/2*a)^5 + 37650400*t
an(b*x + 4*a)^2*tan(1/2*a)^6 - 105669900*tan(b*x + 4*a)*tan(1/2*a)^7 + 1084
99095*tan(1/2*a)^8 - 8100*tan(b*x + 4*a)^4*tan(1/2*a)^2 + 151800*tan(b*x +
4*a)^3*tan(1/2*a)^3 - 1068900*tan(b*x + 4*a)^2*tan(1/2*a)^4 + 3349740*tan(b
*x + 4*a)*tan(1/2*a)^5 - 3921700*tan(1/2*a)^6 + 90*tan(b*x + 4*a)^4 - 1800*
tan(b*x + 4*a)^3*tan(1/2*a) + 13200*tan(b*x + 4*a)^2*tan(1/2*a)^2 - 44050*t
an(b*x + 4*a)*tan(1/2*a)^3 + 56265*tan(1/2*a)^4 + 20*tan(b*x + 4*a)^2 - 150
*tan(b*x + 4*a)*tan(1/2*a) + 270*tan(1/2*a)^2 + 3)/((tan(1/2*a)^30 - 75*tan
(1/2*a)^28 + 2325*tan(1/2*a)^26 - 38255*tan(1/2*a)^24 + 356925*tan(1/2*a)^2
2 - 1880175*tan(1/2*a)^20 + 5430385*tan(1/2*a)^18 - 9069075*tan(1/2*a)^16 +
 9069075*tan(1/2*a)^14 - 5430385*tan(1/2*a)^12 + 1880175*tan(1/2*a)^10 - 35
6925*tan(1/2*a)^8 + 38255*tan(1/2*a)^6 - 2325*tan(1/2*a)^4 + 75*tan(1/2*a)^
2 - 1)*(tan(b*x + 4*a)*tan(1/2*a)^6 - 15*tan(b*x + 4*a)*tan(1/2*a)^4 + 6*ta
n(1/2*a)^5 + 15*tan(b*x + 4*a)*tan(1/2*a)^2 - 20*tan(1/2*a)^3 - tan(b*x + 4
*a) + 6*tan(1/2*a))^5))/b
```

3.57 $\int \csc^2(a+bx)\csc^5(2a+2bx)\,dx$

Optimal. Leaf size=90

$$\frac{\tan^4(a+bx)}{128b}+\frac{5\tan^2(a+bx)}{64b}-\frac{\cot^6(a+bx)}{192b}-\frac{5\cot^4(a+bx)}{128b}-\frac{5\cot^2(a+bx)}{32b}+\frac{5\log(\tan(a+bx))}{16b}$$

```
[Out] (-5*Cot[a + b*x]^2)/(32*b) - (5*Cot[a + b*x]^4)/(128*b) - Cot[a + b*x]^6/(1
92*b) + (5*Log[Tan[a + b*x]])/(16*b) + (5*Tan[a + b*x]^2)/(64*b) + Tan[a +
b*x]^4/(128*b)
```

Rubi [A] time = 0.0817974, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4288, 2620, 266, 43}

$$\frac{\tan^4(a+bx)}{128b}+\frac{5\tan^2(a+bx)}{64b}-\frac{\cot^6(a+bx)}{192b}-\frac{5\cot^4(a+bx)}{128b}-\frac{5\cot^2(a+bx)}{32b}+\frac{5\log(\tan(a+bx))}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^5,x]
```


```
[Out] (-5*Cot[a + b*x]^2)/(32*b) - (5*Cot[a + b*x]^4)/(128*b) - Cot[a + b*x]^6/(1
92*b) + (5*Log[Tan[a + b*x]])/(16*b) + (5*Tan[a + b*x]^2)/(64*b) + Tan[a +
b*x]^4/(128*b)
```


Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \csc^2(a+bx)\csc^5(2a+2bx)\,dx = \frac{1}{32}\int \csc^7(a+bx)\sec^5(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x^2)^5}{x^7}\,dx, x, \tan(a+bx)\right)}{32b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x)^5}{x^4}\,dx, x, \tan^2(a+bx)\right)}{64b}$$
$$= \frac{\text{Subst}\left(\int \left(5+\frac{1}{x^4}+\frac{5}{x^3}+\frac{10}{x^2}+\frac{10}{x}+x\right)dx, x, \tan^2(a+bx)\right)}{64b}$$
$$= -\frac{5\cot^2(a+bx)}{32b} - \frac{5\cot^4(a+bx)}{128b} - \frac{\cot^6(a+bx)}{192b} + \frac{5\log(\tan(a+bx))}{16b} + \frac{5\tan^2(}{6}$$

Mathematica [A] time = 0.405514, size = 76, normalized size = 0.84

$$-\frac{2\csc^6(a+bx)+9\csc^4(a+bx)+36\csc^2(a+bx)-3\sec^4(a+bx)-24\sec^2(a+bx)-120\log(\sin(a+bx))+120\log}{384b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^5,x]
```

```
[Out] -(36*Csc[a + b*x]^2 + 9*Csc[a + b*x]^4 + 2*Csc[a + b*x]^6 + 120*Log[Cos[a +
 b*x]] - 120*Log[Sin[a + b*x]] - 24*Sec[a + b*x]^2 - 3*Sec[a + b*x]^4)/(384
*b)
```

Maple [A] time = 0.037, size = 111, normalized size = 1.2

$$-\frac{1}{192\,b\,(\sin(bx+a))^6(\cos(bx+a))^4} + \frac{5}{384\,b\,(\sin(bx+a))^4(\cos(bx+a))^4} - \frac{5}{192\,b\,(\sin(bx+a))^4(\cos(bx+a))^2} + \frac{}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*csc(2*b*x+2*a)^5,x)
```

```
[Out] -1/192/b/sin(b*x+a)^6/cos(b*x+a)^4+5/384/b/sin(b*x+a)^4/cos(b*x+a)^4-5/192/
b/sin(b*x+a)^4/cos(b*x+a)^2+5/64/b/sin(b*x+a)^2/cos(b*x+a)^2-5/32/b/sin(b*x
+a)^2+5/16*ln(tan(b*x+a))/b
```

Maxima [B] time = 2.62064, size = 10328, normalized size = 114.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] 1/96*(4*(15*cos(18*b*x + 18*a) - 30*cos(16*b*x + 16*a) - 40*cos(14*b*x + 14
*a) + 110*cos(12*b*x + 12*a) + 18*cos(10*b*x + 10*a) + 110*cos(8*b*x + 8*a)
 - 40*cos(6*b*x + 6*a) - 30*cos(4*b*x + 4*a) + 15*cos(2*b*x + 2*a))*cos(20*
```

```
b*x + 20*a) + 4*(15*cos(16*b*x + 16*a) + 200*cos(14*b*x + 14*a) - 190*cos(1
2*b*x + 12*a) - 216*cos(10*b*x + 10*a) - 190*cos(8*b*x + 8*a) + 200*cos(6*b
*x + 6*a) + 15*cos(4*b*x + 4*a) - 60*cos(2*b*x + 2*a) + 15)*cos(18*b*x + 18
*a) - 120*cos(18*b*x + 18*a)^2 - 12*(40*cos(14*b*x + 14*a) + 130*cos(12*b*x
 + 12*a) - 102*cos(10*b*x + 10*a) + 130*cos(8*b*x + 8*a) + 40*cos(6*b*x + 6
*a) - 60*cos(4*b*x + 4*a) - 5*cos(2*b*x + 2*a) + 10)*cos(16*b*x + 16*a) + 3
60*cos(16*b*x + 16*a)^2 + 32*(100*cos(12*b*x + 12*a) + 78*cos(10*b*x + 10*a
) + 100*cos(8*b*x + 8*a) - 80*cos(6*b*x + 6*a) - 15*cos(4*b*x + 4*a) + 25*c
os(2*b*x + 2*a) - 5)*cos(14*b*x + 14*a) - 1280*cos(14*b*x + 14*a)^2 - 8*(64
2*cos(10*b*x + 10*a) - 220*cos(8*b*x + 8*a) - 400*cos(6*b*x + 6*a) + 195*co
s(4*b*x + 4*a) + 95*cos(2*b*x + 2*a) - 55)*cos(12*b*x + 12*a) + 880*cos(12*
b*x + 12*a)^2 - 24*(214*cos(8*b*x + 8*a) - 104*cos(6*b*x + 6*a) - 51*cos(4*
b*x + 4*a) + 36*cos(2*b*x + 2*a) - 3)*cos(10*b*x + 10*a) - 864*cos(10*b*x +
 10*a)^2 + 40*(80*cos(6*b*x + 6*a) - 39*cos(4*b*x + 4*a) - 19*cos(2*b*x + 2
*a) + 11)*cos(8*b*x + 8*a) + 880*cos(8*b*x + 8*a)^2 - 160*(3*cos(4*b*x + 4*
a) - 5*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) - 1280*cos(6*b*x + 6*a)^2 + 6
0*(cos(2*b*x + 2*a) - 2)*cos(4*b*x + 4*a) + 360*cos(4*b*x + 4*a)^2 - 120*co
s(2*b*x + 2*a)^2 + 15*(2*(2*cos(18*b*x + 18*a) + 3*cos(16*b*x + 16*a) - 8*c
os(14*b*x + 14*a) - 2*cos(12*b*x + 12*a) + 12*cos(10*b*x + 10*a) - 2*cos(8*
b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) -
 1)*cos(20*b*x + 20*a) - cos(20*b*x + 20*a)^2 - 4*(3*cos(16*b*x + 16*a) - 8
*cos(14*b*x + 14*a) - 2*cos(12*b*x + 12*a) + 12*cos(10*b*x + 10*a) - 2*cos(
8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a)
 - 1)*cos(18*b*x + 18*a) - 4*cos(18*b*x + 18*a)^2 + 6*(8*cos(14*b*x + 14*a)
 + 2*cos(12*b*x + 12*a) - 12*cos(10*b*x + 10*a) + 2*cos(8*b*x + 8*a) + 8*co
s(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(16*b*x +
16*a) - 9*cos(16*b*x + 16*a)^2 - 16*(2*cos(12*b*x + 12*a) - 12*cos(10*b*x +
 10*a) + 2*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - 2*c
os(2*b*x + 2*a) + 1)*cos(14*b*x + 14*a) - 64*cos(14*b*x + 14*a)^2 + 4*(12*c
os(10*b*x + 10*a) - 2*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(4*b*x +
 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(12*b*x + 12*a) - 4*cos(12*b*x + 12*a)^2
 + 24*(2*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - 2*cos
(2*b*x + 2*a) + 1)*cos(10*b*x + 10*a) - 144*cos(10*b*x + 10*a)^2 - 4*(8*cos
(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*
a) - 4*cos(8*b*x + 8*a)^2 + 16*(3*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1
)*cos(6*b*x + 6*a) - 64*cos(6*b*x + 6*a)^2 - 6*(2*cos(2*b*x + 2*a) - 1)*cos
(4*b*x + 4*a) - 9*cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 + 2*(2*sin(18*b
*x + 18*a) + 3*sin(16*b*x + 16*a) - 8*sin(14*b*x + 14*a) - 2*sin(12*b*x + 1
2*a) + 12*sin(10*b*x + 10*a) - 2*sin(8*b*x + 8*a) - 8*sin(6*b*x + 6*a) + 3*
sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(20*b*x + 20*a) - sin(20*b*x + 20
*a)^2 - 4*(3*sin(16*b*x + 16*a) - 8*sin(14*b*x + 14*a) - 2*sin(12*b*x + 12*
a) + 12*sin(10*b*x + 10*a) - 2*sin(8*b*x + 8*a) - 8*sin(6*b*x + 6*a) + 3*si
n(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(18*b*x + 18*a) - 4*sin(18*b*x + 18
*a)^2 + 6*(8*sin(14*b*x + 14*a) + 2*sin(12*b*x + 12*a) - 12*sin(10*b*x + 10
*a) + 2*sin(8*b*x + 8*a) + 8*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - 2*sin(
2*b*x + 2*a))*sin(16*b*x + 16*a) - 9*sin(16*b*x + 16*a)^2 - 16*(2*sin(12*b*
x + 12*a) - 12*sin(10*b*x + 10*a) + 2*sin(8*b*x + 8*a) + 8*sin(6*b*x + 6*a)
 - 3*sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(14*b*x + 14*a) - 64*sin(14*
b*x + 14*a)^2 + 4*(12*sin(10*b*x + 10*a) - 2*sin(8*b*x + 8*a) - 8*sin(6*b*x
 + 6*a) + 3*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - 4*s
in(12*b*x + 12*a)^2 + 24*(2*sin(8*b*x + 8*a) + 8*sin(6*b*x + 6*a) - 3*sin(4
*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 144*sin(10*b*x + 10*
a)^2 - 4*(8*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin
(8*b*x + 8*a) - 4*sin(8*b*x + 8*a)^2 + 16*(3*sin(4*b*x + 4*a) + 2*sin(2*b*x
 + 2*a))*sin(6*b*x + 6*a) - 64*sin(6*b*x + 6*a)^2 - 9*sin(4*b*x + 4*a)^2 -
12*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x +
 2*a) - 1)*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(2*a)^2 + sin(2*b*
x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2) - 15*(2*(2*cos(18*b*x + 18*a) +
3*cos(16*b*x + 16*a) - 8*cos(14*b*x + 14*a) - 2*cos(12*b*x + 12*a) + 12*cos
```

```
(10*b*x + 10*a) - 2*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4
*a) + 2*cos(2*b*x + 2*a) - 1)*cos(20*b*x + 20*a) - cos(20*b*x + 20*a)^2 - 4
*(3*cos(16*b*x + 16*a) - 8*cos(14*b*x + 14*a) - 2*cos(12*b*x + 12*a) + 12*c
os(10*b*x + 10*a) - 2*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(4*b*x +
 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(18*b*x + 18*a) - 4*cos(18*b*x + 18*a)^2
 + 6*(8*cos(14*b*x + 14*a) + 2*cos(12*b*x + 12*a) - 12*cos(10*b*x + 10*a) +
 2*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - 2*cos(2*b*x
 + 2*a) + 1)*cos(16*b*x + 16*a) - 9*cos(16*b*x + 16*a)^2 - 16*(2*cos(12*b*x
 + 12*a) - 12*cos(10*b*x + 10*a) + 2*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a)
- 3*cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(14*b*x + 14*a) - 64*cos(
14*b*x + 14*a)^2 + 4*(12*cos(10*b*x + 10*a) - 2*cos(8*b*x + 8*a) - 8*cos(6*
b*x + 6*a) + 3*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(12*b*x + 12*a
) - 4*cos(12*b*x + 12*a)^2 + 24*(2*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) -
3*cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(10*b*x + 10*a) - 144*cos(1
0*b*x + 10*a)^2 - 4*(8*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - 2*cos(2*b*x
+ 2*a) + 1)*cos(8*b*x + 8*a) - 4*cos(8*b*x + 8*a)^2 + 16*(3*cos(4*b*x + 4*a
) + 2*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 64*cos(6*b*x + 6*a)^2 - 6*(2
*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 9*cos(4*b*x + 4*a)^2 - 4*cos(2*b*
x + 2*a)^2 + 2*(2*sin(18*b*x + 18*a) + 3*sin(16*b*x + 16*a) - 8*sin(14*b*x
+ 14*a) - 2*sin(12*b*x + 12*a) + 12*sin(10*b*x + 10*a) - 2*sin(8*b*x + 8*a)
 - 8*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(20*b*x
 + 20*a) - sin(20*b*x + 20*a)^2 - 4*(3*sin(16*b*x + 16*a) - 8*sin(14*b*x +
14*a) - 2*sin(12*b*x + 12*a) + 12*sin(10*b*x + 10*a) - 2*sin(8*b*x + 8*a) -
 8*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(18*b*x +
 18*a) - 4*sin(18*b*x + 18*a)^2 + 6*(8*sin(14*b*x + 14*a) + 2*sin(12*b*x +
12*a) - 12*sin(10*b*x + 10*a) + 2*sin(8*b*x + 8*a) + 8*sin(6*b*x + 6*a) - 3
*sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(16*b*x + 16*a) - 9*sin(16*b*x +
 16*a)^2 - 16*(2*sin(12*b*x + 12*a) - 12*sin(10*b*x + 10*a) + 2*sin(8*b*x +
 8*a) + 8*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(1
4*b*x + 14*a) - 64*sin(14*b*x + 14*a)^2 + 4*(12*sin(10*b*x + 10*a) - 2*sin(
8*b*x + 8*a) - 8*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a)
)*sin(12*b*x + 12*a) - 4*sin(12*b*x + 12*a)^2 + 24*(2*sin(8*b*x + 8*a) + 8*
sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(10*b*x + 10
*a) - 144*sin(10*b*x + 10*a)^2 - 4*(8*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a)
 - 2*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 4*sin(8*b*x + 8*a)^2 + 16*(3*sin(
4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 64*sin(6*b*x + 6*a)^2
 - 9*sin(4*b*x + 4*a)^2 - 12*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*
x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + c
os(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 15*(2*(2*cos(18*b*x
+ 18*a) + 3*cos(16*b*x + 16*a) - 8*cos(14*b*x + 14*a) - 2*cos(12*b*x + 12*a
) + 12*cos(10*b*x + 10*a) - 2*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos
(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(20*b*x + 20*a) - cos(20*b*x + 2
0*a)^2 - 4*(3*cos(16*b*x + 16*a) - 8*cos(14*b*x + 14*a) - 2*cos(12*b*x + 12
*a) + 12*cos(10*b*x + 10*a) - 2*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*c
os(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(18*b*x + 18*a) - 4*cos(18*b*x
 + 18*a)^2 + 6*(8*cos(14*b*x + 14*a) + 2*cos(12*b*x + 12*a) - 12*cos(10*b*x
 + 10*a) + 2*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - 2
*cos(2*b*x + 2*a) + 1)*cos(16*b*x + 16*a) - 9*cos(16*b*x + 16*a)^2 - 16*(2*
cos(12*b*x + 12*a) - 12*cos(10*b*x + 10*a) + 2*cos(8*b*x + 8*a) + 8*cos(6*b
*x + 6*a) - 3*cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(14*b*x + 14*a)
 - 64*cos(14*b*x + 14*a)^2 + 4*(12*cos(10*b*x + 10*a) - 2*cos(8*b*x + 8*a)
- 8*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(12*
b*x + 12*a) - 4*cos(12*b*x + 12*a)^2 + 24*(2*cos(8*b*x + 8*a) + 8*cos(6*b*x
 + 6*a) - 3*cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(10*b*x + 10*a) -
 144*cos(10*b*x + 10*a)^2 - 4*(8*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - 2*
cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - 4*cos(8*b*x + 8*a)^2 + 16*(3*cos(4
*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 64*cos(6*b*x + 6*a
)^2 - 6*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 9*cos(4*b*x + 4*a)^2 -
4*cos(2*b*x + 2*a)^2 + 2*(2*sin(18*b*x + 18*a) + 3*sin(16*b*x + 16*a) - 8*s
```

```
in(14*b*x + 14*a) - 2*sin(12*b*x + 12*a) + 12*sin(10*b*x + 10*a) - 2*sin(8*
b*x + 8*a) - 8*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*
sin(20*b*x + 20*a) - sin(20*b*x + 20*a)^2 - 4*(3*sin(16*b*x + 16*a) - 8*sin
(14*b*x + 14*a) - 2*sin(12*b*x + 12*a) + 12*sin(10*b*x + 10*a) - 2*sin(8*b*
x + 8*a) - 8*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*si
n(18*b*x + 18*a) - 4*sin(18*b*x + 18*a)^2 + 6*(8*sin(14*b*x + 14*a) + 2*sin
(12*b*x + 12*a) - 12*sin(10*b*x + 10*a) + 2*sin(8*b*x + 8*a) + 8*sin(6*b*x
+ 6*a) - 3*sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(16*b*x + 16*a) - 9*si
n(16*b*x + 16*a)^2 - 16*(2*sin(12*b*x + 12*a) - 12*sin(10*b*x + 10*a) + 2*s
in(8*b*x + 8*a) + 8*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - 2*sin(2*b*x + 2
*a))*sin(14*b*x + 14*a) - 64*sin(14*b*x + 14*a)^2 + 4*(12*sin(10*b*x + 10*a
) - 2*sin(8*b*x + 8*a) - 8*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + 2*sin(2*
b*x + 2*a))*sin(12*b*x + 12*a) - 4*sin(12*b*x + 12*a)^2 + 24*(2*sin(8*b*x +
 8*a) + 8*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(1
0*b*x + 10*a) - 144*sin(10*b*x + 10*a)^2 - 4*(8*sin(6*b*x + 6*a) - 3*sin(4*
b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 4*sin(8*b*x + 8*a)^2 +
16*(3*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 64*sin(6*b*
x + 6*a)^2 - 9*sin(4*b*x + 4*a)^2 - 12*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) -
4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*
cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*(15*sin(
18*b*x + 18*a) - 30*sin(16*b*x + 16*a) - 40*sin(14*b*x + 14*a) + 110*sin(12
*b*x + 12*a) + 18*sin(10*b*x + 10*a) + 110*sin(8*b*x + 8*a) - 40*sin(6*b*x
+ 6*a) - 30*sin(4*b*x + 4*a) + 15*sin(2*b*x + 2*a))*sin(20*b*x + 20*a) + 4*
(15*sin(16*b*x + 16*a) + 200*sin(14*b*x + 14*a) - 190*sin(12*b*x + 12*a) -
216*sin(10*b*x + 10*a) - 190*sin(8*b*x + 8*a) + 200*sin(6*b*x + 6*a) + 15*s
in(4*b*x + 4*a) - 60*sin(2*b*x + 2*a))*sin(18*b*x + 18*a) - 120*sin(18*b*x
+ 18*a)^2 - 12*(40*sin(14*b*x + 14*a) + 130*sin(12*b*x + 12*a) - 102*sin(10
*b*x + 10*a) + 130*sin(8*b*x + 8*a) + 40*sin(6*b*x + 6*a) - 60*sin(4*b*x +
4*a) - 5*sin(2*b*x + 2*a))*sin(16*b*x + 16*a) + 360*sin(16*b*x + 16*a)^2 +
32*(100*sin(12*b*x + 12*a) + 78*sin(10*b*x + 10*a) + 100*sin(8*b*x + 8*a) -
 80*sin(6*b*x + 6*a) - 15*sin(4*b*x + 4*a) + 25*sin(2*b*x + 2*a))*sin(14*b*
x + 14*a) - 1280*sin(14*b*x + 14*a)^2 - 8*(642*sin(10*b*x + 10*a) - 220*sin
(8*b*x + 8*a) - 400*sin(6*b*x + 6*a) + 195*sin(4*b*x + 4*a) + 95*sin(2*b*x
+ 2*a))*sin(12*b*x + 12*a) + 880*sin(12*b*x + 12*a)^2 - 24*(214*sin(8*b*x +
 8*a) - 104*sin(6*b*x + 6*a) - 51*sin(4*b*x + 4*a) + 36*sin(2*b*x + 2*a))*s
in(10*b*x + 10*a) - 864*sin(10*b*x + 10*a)^2 + 40*(80*sin(6*b*x + 6*a) - 39
*sin(4*b*x + 4*a) - 19*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + 880*sin(8*b*x +
 8*a)^2 - 160*(3*sin(4*b*x + 4*a) - 5*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) -
1280*sin(6*b*x + 6*a)^2 + 360*sin(4*b*x + 4*a)^2 + 60*sin(4*b*x + 4*a)*sin(
2*b*x + 2*a) - 120*sin(2*b*x + 2*a)^2 + 60*cos(2*b*x + 2*a))/(b*cos(20*b*x
+ 20*a)^2 + 4*b*cos(18*b*x + 18*a)^2 + 9*b*cos(16*b*x + 16*a)^2 + 64*b*cos(
14*b*x + 14*a)^2 + 4*b*cos(12*b*x + 12*a)^2 + 144*b*cos(10*b*x + 10*a)^2 +
4*b*cos(8*b*x + 8*a)^2 + 64*b*cos(6*b*x + 6*a)^2 + 9*b*cos(4*b*x + 4*a)^2 +
 4*b*cos(2*b*x + 2*a)^2 + b*sin(20*b*x + 20*a)^2 + 4*b*sin(18*b*x + 18*a)^2
 + 9*b*sin(16*b*x + 16*a)^2 + 64*b*sin(14*b*x + 14*a)^2 + 4*b*sin(12*b*x +
12*a)^2 + 144*b*sin(10*b*x + 10*a)^2 + 4*b*sin(8*b*x + 8*a)^2 + 64*b*sin(6*
b*x + 6*a)^2 + 9*b*sin(4*b*x + 4*a)^2 + 12*b*sin(4*b*x + 4*a)*sin(2*b*x + 2
*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(18*b*x + 18*a) + 3*b*cos(16*b*x +
 16*a) - 8*b*cos(14*b*x + 14*a) - 2*b*cos(12*b*x + 12*a) + 12*b*cos(10*b*x
+ 10*a) - 2*b*cos(8*b*x + 8*a) - 8*b*cos(6*b*x + 6*a) + 3*b*cos(4*b*x + 4*a
) + 2*b*cos(2*b*x + 2*a) - b)*cos(20*b*x + 20*a) + 4*(3*b*cos(16*b*x + 16*a
) - 8*b*cos(14*b*x + 14*a) - 2*b*cos(12*b*x + 12*a) + 12*b*cos(10*b*x + 10*
a) - 2*b*cos(8*b*x + 8*a) - 8*b*cos(6*b*x + 6*a) + 3*b*cos(4*b*x + 4*a) + 2
*b*cos(2*b*x + 2*a) - b)*cos(18*b*x + 18*a) - 6*(8*b*cos(14*b*x + 14*a) + 2
*b*cos(12*b*x + 12*a) - 12*b*cos(10*b*x + 10*a) + 2*b*cos(8*b*x + 8*a) + 8*
b*cos(6*b*x + 6*a) - 3*b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(1
6*b*x + 16*a) + 16*(2*b*cos(12*b*x + 12*a) - 12*b*cos(10*b*x + 10*a) + 2*b*
cos(8*b*x + 8*a) + 8*b*cos(6*b*x + 6*a) - 3*b*cos(4*b*x + 4*a) - 2*b*cos(2*
b*x + 2*a) + b)*cos(14*b*x + 14*a) - 4*(12*b*cos(10*b*x + 10*a) - 2*b*cos(8
```

```
*b*x + 8*a) - 8*b*cos(6*b*x + 6*a) + 3*b*cos(4*b*x + 4*a) + 2*b*cos(2*b*x +
 2*a) - b)*cos(12*b*x + 12*a) - 24*(2*b*cos(8*b*x + 8*a) + 8*b*cos(6*b*x +
6*a) - 3*b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(10*b*x + 10*a)
+ 4*(8*b*cos(6*b*x + 6*a) - 3*b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b
)*cos(8*b*x + 8*a) - 16*(3*b*cos(4*b*x + 4*a) + 2*b*cos(2*b*x + 2*a) - b)*c
os(6*b*x + 6*a) + 6*(2*b*cos(2*b*x + 2*a) - b)*cos(4*b*x + 4*a) - 4*b*cos(2
*b*x + 2*a) - 2*(2*b*sin(18*b*x + 18*a) + 3*b*sin(16*b*x + 16*a) - 8*b*sin(
14*b*x + 14*a) - 2*b*sin(12*b*x + 12*a) + 12*b*sin(10*b*x + 10*a) - 2*b*sin
(8*b*x + 8*a) - 8*b*sin(6*b*x + 6*a) + 3*b*sin(4*b*x + 4*a) + 2*b*sin(2*b*x
 + 2*a))*sin(20*b*x + 20*a) + 4*(3*b*sin(16*b*x + 16*a) - 8*b*sin(14*b*x +
14*a) - 2*b*sin(12*b*x + 12*a) + 12*b*sin(10*b*x + 10*a) - 2*b*sin(8*b*x +
8*a) - 8*b*sin(6*b*x + 6*a) + 3*b*sin(4*b*x + 4*a) + 2*b*sin(2*b*x + 2*a))*
sin(18*b*x + 18*a) - 6*(8*b*sin(14*b*x + 14*a) + 2*b*sin(12*b*x + 12*a) - 1
2*b*sin(10*b*x + 10*a) + 2*b*sin(8*b*x + 8*a) + 8*b*sin(6*b*x + 6*a) - 3*b*
sin(4*b*x + 4*a) - 2*b*sin(2*b*x + 2*a))*sin(16*b*x + 16*a) + 16*(2*b*sin(1
2*b*x + 12*a) - 12*b*sin(10*b*x + 10*a) + 2*b*sin(8*b*x + 8*a) + 8*b*sin(6*
b*x + 6*a) - 3*b*sin(4*b*x + 4*a) - 2*b*sin(2*b*x + 2*a))*sin(14*b*x + 14*a
) - 4*(12*b*sin(10*b*x + 10*a) - 2*b*sin(8*b*x + 8*a) - 8*b*sin(6*b*x + 6*a
) + 3*b*sin(4*b*x + 4*a) + 2*b*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - 24*(2
*b*sin(8*b*x + 8*a) + 8*b*sin(6*b*x + 6*a) - 3*b*sin(4*b*x + 4*a) - 2*b*sin
(2*b*x + 2*a))*sin(10*b*x + 10*a) + 4*(8*b*sin(6*b*x + 6*a) - 3*b*sin(4*b*x
 + 4*a) - 2*b*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 16*(3*b*sin(4*b*x + 4*a)
 + 2*b*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + b)
```

Fricas [B] time = 0.534645, size = 514, normalized size = 5.71

$$\frac{60 \cos(bx+a)^8 - 150 \cos(bx+a)^6 + 110 \cos(bx+a)^4 - 15 \cos(bx+a)^2 - 60\left(\cos(bx+a)^{10} - 3 \cos(bx+a)^8 + 3\right.}{384\left(b\cos(bx+a)^{10}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] 1/384*(60*cos(b*x + a)^8 - 150*cos(b*x + a)^6 + 110*cos(b*x + a)^4 - 15*cos
(b*x + a)^2 - 60*(cos(b*x + a)^10 - 3*cos(b*x + a)^8 + 3*cos(b*x + a)^6 - c
os(b*x + a)^4)*log(cos(b*x + a)^2) + 60*(cos(b*x + a)^10 - 3*cos(b*x + a)^8
 + 3*cos(b*x + a)^6 - cos(b*x + a)^4)*log(-1/4*cos(b*x + a)^2 + 1/4) - 3)/(
b*cos(b*x + a)^10 - 3*b*cos(b*x + a)^8 + 3*b*cos(b*x + a)^6 - b*cos(b*x + a
)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*csc(2*b*x+2*a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 7.27584, size = 8605, normalized size = 95.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] 1/6144*((2592*tan(b*x + 4*a)^3*tan(1/2*a)^45 - 648*tan(b*x + 4*a)^2*tan(1/2
*a)^46 + 72*tan(b*x + 4*a)*tan(1/2*a)^47 - 3*tan(1/2*a)^48 + 339552*tan(b*x
 + 4*a)^3*tan(1/2*a)^43 - 131760*tan(b*x + 4*a)^2*tan(1/2*a)^44 + 18120*tan
(b*x + 4*a)*tan(1/2*a)^45 - 720*tan(1/2*a)^46 + 419904000*tan(b*x + 4*a)^4*
tan(1/2*a)^40 - 289600704*tan(b*x + 4*a)^3*tan(1/2*a)^41 + 76838472*tan(b*x
 + 4*a)^2*tan(1/2*a)^42 - 9139608*tan(b*x + 4*a)*tan(1/2*a)^43 + 409140*tan
(1/2*a)^44 - 11197440000*tan(b*x + 4*a)^4*tan(1/2*a)^38 + 10808414400*tan(b
*x + 4*a)^3*tan(1/2*a)^39 - 3590371872*tan(b*x + 4*a)^2*tan(1/2*a)^40 + 507
604968*tan(b*x + 4*a)*tan(1/2*a)^41 - 26159856*tan(1/2*a)^42 + 133996032000
*tan(b*x + 4*a)^4*tan(1/2*a)^36 - 169676010720*tan(b*x + 4*a)^3*tan(1/2*a)^
37 + 72375177960*tan(b*x + 4*a)^2*tan(1/2*a)^38 - 12560378760*tan(b*x + 4*a
)*tan(1/2*a)^39 + 768063258*tan(1/2*a)^40 - 949294080000*tan(b*x + 4*a)^4*t
an(1/2*a)^34 + 1509418412640*tan(b*x + 4*a)^3*tan(1/2*a)^35 - 810503332080*
tan(b*x + 4*a)^2*tan(1/2*a)^36 + 174786910200*tan(b*x + 4*a)*tan(1/2*a)^37
- 12927376560*tan(1/2*a)^38 + 4424378112000*tan(b*x + 4*a)^4*tan(1/2*a)^32
- 8587351903488*tan(b*x + 4*a)^3*tan(1/2*a)^33 + 5672266009560*tan(b*x + 4*
a)^2*tan(1/2*a)^34 - 1507689425640*tan(b*x + 4*a)*tan(1/2*a)^35 + 136259884
900*tan(1/2*a)^36 - 14266506240000*tan(b*x + 4*a)^4*tan(1/2*a)^30 + 3319683
7729536*tan(b*x + 4*a)^3*tan(1/2*a)^31 - 26480349456768*tan(b*x + 4*a)^2*ta
n(1/2*a)^32 + 8552959192472*tan(b*x + 4*a)*tan(1/2*a)^33 - 941351504400*tan
(1/2*a)^34 + 32626266624000*tan(b*x + 4*a)^4*tan(1/2*a)^28 - 90047346555840
*tan(b*x + 4*a)^3*tan(1/2*a)^29 + 85630491456048*tan(b*x + 4*a)^2*tan(1/2*a
)^30 - 33168296242992*tan(b*x + 4*a)*tan(1/2*a)^31 + 4402067046579*tan(1/2*
a)^32 - 53491430400000*tan(b*x + 4*a)^4*tan(1/2*a)^26 + 174200110235840*tan
(b*x + 4*a)^3*tan(1/2*a)^27 - 195958049148000*tan(b*x + 4*a)^2*tan(1/2*a)^2
8 + 90139253610960*tan(b*x + 4*a)*tan(1/2*a)^29 - 14277938470432*tan(1/2*a)
^30 + 63070929280000*tan(b*x + 4*a)^4*tan(1/2*a)^24 - 242033974687872*tan(b
*x + 4*a)^3*tan(1/2*a)^25 + 320941776763728*tan(b*x + 4*a)^2*tan(1/2*a)^26
- 174300870723312*tan(b*x + 4*a)*tan(1/2*a)^27 + 32692804970088*tan(1/2*a)^
28 - 53491430400000*tan(b*x + 4*a)^4*tan(1/2*a)^22 + 242033974687872*tan(b*
x + 4*a)^3*tan(1/2*a)^23 - 378128484292800*tan(b*x + 4*a)^2*tan(1/2*a)^24 +
 241898880641040*tan(b*x + 4*a)*tan(1/2*a)^25 - 53487220641120*tan(1/2*a)^2
6 + 32626266624000*tan(b*x + 4*a)^4*tan(1/2*a)^20 - 174200110235840*tan(b*x
 + 4*a)^3*tan(1/2*a)^21 + 320941776763728*tan(b*x + 4*a)^2*tan(1/2*a)^22 -
241898880641040*tan(b*x + 4*a)*tan(1/2*a)^23 + 62975177889900*tan(1/2*a)^24
 - 14266506240000*tan(b*x + 4*a)^4*tan(1/2*a)^18 + 90047346555840*tan(b*x +
 4*a)^3*tan(1/2*a)^19 - 195958049148000*tan(b*x + 4*a)^2*tan(1/2*a)^20 + 17
4300870723312*tan(b*x + 4*a)*tan(1/2*a)^21 - 53487220641120*tan(1/2*a)^22 +
 4424378112000*tan(b*x + 4*a)^4*tan(1/2*a)^16 - 33196837729536*tan(b*x + 4*
a)^3*tan(1/2*a)^17 + 85630491456048*tan(b*x + 4*a)^2*tan(1/2*a)^18 - 901392
53610960*tan(b*x + 4*a)*tan(1/2*a)^19 + 32692804970088*tan(1/2*a)^20 - 9492
94080000*tan(b*x + 4*a)^4*tan(1/2*a)^14 + 8587351903488*tan(b*x + 4*a)^3*ta
n(1/2*a)^15 - 26480349456768*tan(b*x + 4*a)^2*tan(1/2*a)^16 + 3316829624299
2*tan(b*x + 4*a)*tan(1/2*a)^17 - 14277938470432*tan(1/2*a)^18 + 13399603200
0*tan(b*x + 4*a)^4*tan(1/2*a)^12 - 1509418412640*tan(b*x + 4*a)^3*tan(1/2*a
)^13 + 5672266009560*tan(b*x + 4*a)^2*tan(1/2*a)^14 - 8552959192472*tan(b*x
 + 4*a)*tan(1/2*a)^15 + 4402067046579*tan(1/2*a)^16 - 11197440000*tan(b*x +
 4*a)^4*tan(1/2*a)^10 + 169676010720*tan(b*x + 4*a)^3*tan(1/2*a)^11 - 81050
3332080*tan(b*x + 4*a)^2*tan(1/2*a)^12 + 1507689425640*tan(b*x + 4*a)*tan(1
/2*a)^13 - 941351504400*tan(1/2*a)^14 + 419904000*tan(b*x + 4*a)^4*tan(1/2*
a)^8 - 10808414400*tan(b*x + 4*a)^3*tan(1/2*a)^9 + 72375177960*tan(b*x + 4*
a)^2*tan(1/2*a)^10 - 174786910200*tan(b*x + 4*a)*tan(1/2*a)^11 + 1362598849
00*tan(1/2*a)^12 + 289600704*tan(b*x + 4*a)^3*tan(1/2*a)^7 - 3590371872*tan
(b*x + 4*a)^2*tan(1/2*a)^8 + 12560378760*tan(b*x + 4*a)*tan(1/2*a)^9 - 1292
7376560*tan(1/2*a)^10 - 339552*tan(b*x + 4*a)^3*tan(1/2*a)^5 + 76838472*tan
```

(b*x + 4*a)^2*tan(1/2*a)^6 - 507604968*tan(b*x + 4*a)*tan(1/2*a)^7 + 768063
258*tan(1/2*a)^8 - 2592*tan(b*x + 4*a)^3*tan(1/2*a)^3 - 131760*tan(b*x + 4*
a)^2*tan(1/2*a)^4 + 9139608*tan(b*x + 4*a)*tan(1/2*a)^5 - 26159856*tan(1/2*
a)^6 - 648*tan(b*x + 4*a)^2*tan(1/2*a)^2 - 18120*tan(b*x + 4*a)*tan(1/2*a)^
3 + 409140*tan(1/2*a)^4 - 72*tan(b*x + 4*a)*tan(1/2*a) - 720*tan(1/2*a)^2 -
 3)/((81*tan(1/2*a)^20 - 1080*tan(1/2*a)^18 + 5724*tan(1/2*a)^16 - 15240*ta
n(1/2*a)^14 + 21286*tan(1/2*a)^12 - 15240*tan(1/2*a)^10 + 5724*tan(1/2*a)^8
 - 1080*tan(1/2*a)^6 + 81*tan(1/2*a)^4)*(6*tan(b*x + 4*a)*tan(1/2*a)^5 - ta
n(1/2*a)^6 - 20*tan(b*x + 4*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 + 6*tan(b*x +
 4*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)^4) - 16*(294*tan(b*x + 4*a)^6*tan(1
/2*a)^72 - 52920*tan(b*x + 4*a)^6*tan(1/2*a)^70 + 9864*tan(b*x + 4*a)^5*tan
(1/2*a)^71 + 60*tan(b*x + 4*a)^4*tan(1/2*a)^72 + 4418820*tan(b*x + 4*a)^6*t
an(1/2*a)^68 - 1699320*tan(b*x + 4*a)^5*tan(1/2*a)^69 + 133920*tan(b*x + 4*
a)^4*tan(1/2*a)^70 + 1080*tan(b*x + 4*a)^3*tan(1/2*a)^71 + 15*tan(b*x + 4*a
)^2*tan(1/2*a)^72 - 227030328*tan(b*x + 4*a)^6*tan(1/2*a)^66 + 134207352*ta
n(b*x + 4*a)^5*tan(1/2*a)^67 - 22587480*tan(b*x + 4*a)^4*tan(1/2*a)^68 + 96
3000*tan(b*x + 4*a)^3*tan(1/2*a)^69 + 7020*tan(b*x + 4*a)^2*tan(1/2*a)^70 +
 108*tan(b*x + 4*a)*tan(1/2*a)^71 + 2*tan(1/2*a)^72 + 8027289270*tan(b*x +
4*a)^6*tan(1/2*a)^64 - 6460519176*tan(b*x + 4*a)^5*tan(1/2*a)^65 + 16863489
60*tan(b*x + 4*a)^4*tan(1/2*a)^66 - 159789240*tan(b*x + 4*a)^3*tan(1/2*a)^6
7 + 3912570*tan(b*x + 4*a)^2*tan(1/2*a)^68 + 20700*tan(b*x + 4*a)*tan(1/2*a
)^69 + 180*tan(1/2*a)^70 - 206987901120*tan(b*x + 4*a)^6*tan(1/2*a)^62 + 21
2284461120*tan(b*x + 4*a)^5*tan(1/2*a)^63 - 75682808100*tan(b*x + 4*a)^4*ta
n(1/2*a)^64 + 11222034120*tan(b*x + 4*a)^3*tan(1/2*a)^65 - 635709780*tan(b*
x + 4*a)^2*tan(1/2*a)^66 + 8559972*tan(b*x + 4*a)*tan(1/2*a)^67 + 23220*tan
(1/2*a)^68 + 4024901399904*tan(b*x + 4*a)^6*tan(1/2*a)^60 - 5048602582464*t
an(b*x + 4*a)^5*tan(1/2*a)^61 + 2296406448000*tan(b*x + 4*a)^4*tan(1/2*a)^6
2 - 466347413440*tan(b*x + 4*a)^3*tan(1/2*a)^63 + 41675854455*tan(b*x + 4*a
)^2*tan(1/2*a)^64 - 1349403948*tan(b*x + 4*a)*tan(1/2*a)^65 + 7900020*tan(1
/2*a)^66 - 60212793009600*tan(b*x + 4*a)^6*tan(1/2*a)^58 + 89880504373440*t
an(b*x + 4*a)^5*tan(1/2*a)^59 - 50018473204800*tan(b*x + 4*a)^4*tan(1/2*a)^
60 + 12967578538560*tan(b*x + 4*a)^3*tan(1/2*a)^61 - 1590087843360*tan(b*x
+ 4*a)^2*tan(1/2*a)^62 + 81753694560*tan(b*x + 4*a)*tan(1/2*a)^63 - 1193590
350*tan(1/2*a)^64 + 701137434828600*tan(b*x + 4*a)^6*tan(1/2*a)^56 - 122215
5524626752*tan(b*x + 4*a)^5*tan(1/2*a)^57 + 809309015383680*tan(b*x + 4*a)^
4*tan(1/2*a)^58 - 256626846135360*tan(b*x + 4*a)^3*tan(1/2*a)^59 + 40162301
142000*tan(b*x + 4*a)^2*tan(1/2*a)^60 - 2834816578848*tan(b*x + 4*a)*tan(1/
2*a)^61 + 66006729504*tan(1/2*a)^62 - 6393695038795680*tan(b*x + 4*a)^6*tan
(1/2*a)^54 + 12843489105103968*tan(b*x + 4*a)^5*tan(1/2*a)^55 - 99287277370
10640*tan(b*x + 4*a)^4*tan(1/2*a)^56 + 3743548943593920*tan(b*x + 4*a)^3*ta
n(1/2*a)^57 - 715723706535840*tan(b*x + 4*a)^2*tan(1/2*a)^58 + 643798023336
00*tan(b*x + 4*a)*tan(1/2*a)^59 - 2055305635104*tan(1/2*a)^60 + 45773543210
194800*tan(b*x + 4*a)^6*tan(1/2*a)^52 - 105001341586308000*tan(b*x + 4*a)^5
*tan(1/2*a)^53 + 93508763139878400*tan(b*x + 4*a)^4*tan(1/2*a)^54 - 4111996
0482804960*tan(b*x + 4*a)^3*tan(1/2*a)^55 + 9332822656724940*tan(b*x + 4*a)
^2*tan(1/2*a)^56 - 1023101781887712*tan(b*x + 4*a)*tan(1/2*a)^57 + 41487214
960800*tan(1/2*a)^58 - 257401775291824800*tan(b*x + 4*a)^6*tan(1/2*a)^50 +
670176117430046880*tan(b*x + 4*a)^5*tan(1/2*a)^51 - 681255358380496800*tan(
b*x + 4*a)^4*tan(1/2*a)^52 + 344752124474645280*tan(b*x + 4*a)^3*tan(1/2*a)
^53 - 91096096904718960*tan(b*x + 4*a)^2*tan(1/2*a)^54 + 11822570313904656*
tan(b*x + 4*a)*tan(1/2*a)^55 - 581936260957848*tan(1/2*a)^56 + 113717503913
0480280*tan(b*x + 4*a)^6*tan(1/2*a)^48 - 3346975359162280800*tan(b*x + 4*a)
^5*tan(1/2*a)^49 + 3859521230041977600*tan(b*x + 4*a)^4*tan(1/2*a)^50 - 222
7011325606075680*tan(b*x + 4*a)^3*tan(1/2*a)^51 + 675970919512787160*tan(b*
x + 4*a)^2*tan(1/2*a)^52 - 101853310806506160*tan(b*x + 4*a)*tan(1/2*a)^53
+ 5911352313614320*tan(1/2*a)^54 - 3952573715797761600*tan(b*x + 4*a)^6*tan
(1/2*a)^46 + 13113192238126665408*tan(b*x + 4*a)^5*tan(1/2*a)^47 - 17080215
895273086480*tan(b*x + 4*a)^4*tan(1/2*a)^48 + 11166956763854634720*tan(b*x
+ 4*a)^3*tan(1/2*a)^49 - 3858112452887270640*tan(b*x + 4*a)^2*tan(1/2*a)^50

+ 666075670217316912*tan(b*x + 4*a)*tan(1/2*a)^51 - 44716273127011152*tan(
1/2*a)^52 + 10845567549201117600*tan(b*x + 4*a)^6*tan(1/2*a)^44 - 404584101
19612759872*tan(b*x + 4*a)^5*tan(1/2*a)^45 + 59335425213582061440*tan(b*x +
 4*a)^4*tan(1/2*a)^46 - 43761763235235147840*tan(b*x + 4*a)^3*tan(1/2*a)^47
 + 17102501830780473660*tan(b*x + 4*a)^2*tan(1/2*a)^48 - 335317836314927016
0*tan(b*x + 4*a)*tan(1/2*a)^49 + 257119896646635120*tan(1/2*a)^50 - 2360665
5837981910848*tan(b*x + 4*a)^6*tan(1/2*a)^42 + 98782934174733211200*tan(b*x
 + 4*a)^5*tan(1/2*a)^43 - 162685440260092255680*tan(b*x + 4*a)^4*tan(1/2*a)
^44 + 134910442121291395520*tan(b*x + 4*a)^3*tan(1/2*a)^45 - 59381984929519
133280*tan(b*x + 4*a)^2*tan(1/2*a)^46 + 13143687759024316704*tan(b*x + 4*a)
*tan(1/2*a)^47 - 1141633895066211960*tan(1/2*a)^48 + 40965300945895635540*t
an(b*x + 4*a)^6*tan(1/2*a)^40 - 191862094942696932288*tan(b*x + 4*a)^5*tan(
1/2*a)^41 + 353993803428323256960*tan(b*x + 4*a)^4*tan(1/2*a)^42 - 32920769
8514963712960*tan(b*x + 4*a)^3*tan(1/2*a)^43 + 162688248623415050640*tan(b*
x + 4*a)^2*tan(1/2*a)^44 - 40487960376937634400*tan(b*x + 4*a)*tan(1/2*a)^4
5 + 3961883425271202144*tan(1/2*a)^46 - 56916645144484965840*tan(b*x + 4*a)
^6*tan(1/2*a)^38 + 297814271756856799920*tan(b*x + 4*a)^5*tan(1/2*a)^39 - 6
14369141282401310520*tan(b*x + 4*a)^4*tan(1/2*a)^40 + 639362819497412266560
*tan(b*x + 4*a)^3*tan(1/2*a)^41 - 353888122537448556000*tan(b*x + 4*a)^2*ta
n(1/2*a)^42 + 98742097283990220000*tan(b*x + 4*a)*tan(1/2*a)^43 - 108461282
70028623840*tan(1/2*a)^44 + 63490416681595775256*tan(b*x + 4*a)^6*tan(1/2*a
)^36 - 370709550220416445008*tan(b*x + 4*a)^5*tan(1/2*a)^37 + 8538128885868
53302080*tan(b*x + 4*a)^4*tan(1/2*a)^38 - 992663809176983006640*tan(b*x + 4
*a)^3*tan(1/2*a)^39 + 614257138968343735410*tan(b*x + 4*a)^2*tan(1/2*a)^40
 - 191755286425161694368*tan(b*x + 4*a)*tan(1/2*a)^41 + 23585525195746656480
*tan(1/2*a)^42 - 56916645144484965840*tan(b*x + 4*a)^6*tan(1/2*a)^34 + 3707
09550220416445008*tan(b*x + 4*a)^5*tan(1/2*a)^35 - 952539854421972061200*ta
n(b*x + 4*a)^4*tan(1/2*a)^36 + 1235898837202727952720*tan(b*x + 4*a)^3*tan(
1/2*a)^37 - 853875925497859030200*tan(b*x + 4*a)^2*tan(1/2*a)^38 + 29778341
9266886959560*tan(b*x + 4*a)*tan(1/2*a)^39 - 40942896085716296868*tan(1/2*a
)^40 + 40965300945895635540*tan(b*x + 4*a)^6*tan(1/2*a)^32 - 29781427175685
6799920*tan(b*x + 4*a)^5*tan(1/2*a)^33 + 853812888586853302080*tan(b*x + 4*
a)^4*tan(1/2*a)^34 - 1235898837202727952720*tan(b*x + 4*a)^3*tan(1/2*a)^35
+ 952725450920718220380*tan(b*x + 4*a)^2*tan(1/2*a)^36 - 370830295637238771
864*tan(b*x + 4*a)*tan(1/2*a)^37 + 56929249131035815800*tan(1/2*a)^38 - 236
06655837981910848*tan(b*x + 4*a)^6*tan(1/2*a)^30 + 191862094942696932288*ta
n(b*x + 4*a)^5*tan(1/2*a)^31 - 614369141282401310520*tan(b*x + 4*a)^4*tan(1
/2*a)^32 + 992663809176983006640*tan(b*x + 4*a)^3*tan(1/2*a)^33 - 853875925
497859030200*tan(b*x + 4*a)^2*tan(1/2*a)^34 + 370830295637238771864*tan(b*x
 + 4*a)*tan(1/2*a)^35 - 63527544212529374408*tan(1/2*a)^36 + 10845567549201
117600*tan(b*x + 4*a)^6*tan(1/2*a)^28 - 98782934174733211200*tan(b*x + 4*a)
^5*tan(1/2*a)^29 + 353993803428323256960*tan(b*x + 4*a)^4*tan(1/2*a)^30 - 6
39362819497412266560*tan(b*x + 4*a)^3*tan(1/2*a)^31 + 614257138968343735410
*tan(b*x + 4*a)^2*tan(1/2*a)^32 - 297783419266886959560*tan(b*x + 4*a)*tan(
1/2*a)^33 + 56929249131035815800*tan(1/2*a)^34 - 3952573715797761600*tan(b*
x + 4*a)^6*tan(1/2*a)^26 + 40458410119612759872*tan(b*x + 4*a)^5*tan(1/2*a)
^27 - 162685440260092255680*tan(b*x + 4*a)^4*tan(1/2*a)^28 + 32920769851496
3712960*tan(b*x + 4*a)^3*tan(1/2*a)^29 - 353888122537448556000*tan(b*x + 4*
a)^2*tan(1/2*a)^30 + 191755286425161694368*tan(b*x + 4*a)*tan(1/2*a)^31 - 4
0942896085716296868*tan(1/2*a)^32 + 1137175039130480280*tan(b*x + 4*a)^6*ta
n(1/2*a)^24 - 13113192238126665408*tan(b*x + 4*a)^5*tan(1/2*a)^25 + 5933542
5213582061440*tan(b*x + 4*a)^4*tan(1/2*a)^26 - 134910442121291395520*tan(b*
x + 4*a)^3*tan(1/2*a)^27 + 162688248623415050640*tan(b*x + 4*a)^2*tan(1/2*a
)^28 - 98742097283990220000*tan(b*x + 4*a)*tan(1/2*a)^29 + 2358552519574665
6480*tan(1/2*a)^30 - 257401775291824800*tan(b*x + 4*a)^6*tan(1/2*a)^22 + 33
46975359162280800*tan(b*x + 4*a)^5*tan(1/2*a)^23 - 17080215895273086480*tan
(b*x + 4*a)^4*tan(1/2*a)^24 + 43761763235235147840*tan(b*x + 4*a)^3*tan(1/2
*a)^25 - 59381984929519133280*tan(b*x + 4*a)^2*tan(1/2*a)^26 + 404879603769
37634400*tan(b*x + 4*a)*tan(1/2*a)^27 - 10846128270028623840*tan(1/2*a)^28

+ 45773543210194800*tan(b*x + 4*a)^6*tan(1/2*a)^20 - 670176117430046880*tan
(b*x + 4*a)^5*tan(1/2*a)^21 + 3859521230041977600*tan(b*x + 4*a)^4*tan(1/2*
a)^22 - 11166956763854634720*tan(b*x + 4*a)^3*tan(1/2*a)^23 + 1710250183078
0473660*tan(b*x + 4*a)^2*tan(1/2*a)^24 - 13143687759024316704*tan(b*x + 4*a
)*tan(1/2*a)^25 + 3961883425271202144*tan(1/2*a)^26 - 6393695038795680*tan(
b*x + 4*a)^6*tan(1/2*a)^18 + 105001341586308000*tan(b*x + 4*a)^5*tan(1/2*a)
^19 - 681255358380496800*tan(b*x + 4*a)^4*tan(1/2*a)^20 + 22270113256060756
80*tan(b*x + 4*a)^3*tan(1/2*a)^21 - 3858112452887270640*tan(b*x + 4*a)^2*ta
n(1/2*a)^22 + 3353178363149270160*tan(b*x + 4*a)*tan(1/2*a)^23 - 1141633895
066211960*tan(1/2*a)^24 + 701137434828600*tan(b*x + 4*a)^6*tan(1/2*a)^16 -
12843489105103968*tan(b*x + 4*a)^5*tan(1/2*a)^17 + 93508763139878400*tan(b*
x + 4*a)^4*tan(1/2*a)^18 - 344752124474645280*tan(b*x + 4*a)^3*tan(1/2*a)^1
9 + 675970919512787160*tan(b*x + 4*a)^2*tan(1/2*a)^20 - 666075670217316912*
tan(b*x + 4*a)*tan(1/2*a)^21 + 257119896646635120*tan(1/2*a)^22 - 602127930
09600*tan(b*x + 4*a)^6*tan(1/2*a)^14 + 1222155524626752*tan(b*x + 4*a)^5*ta
n(1/2*a)^15 - 9928727737010640*tan(b*x + 4*a)^4*tan(1/2*a)^16 + 41119960482
804960*tan(b*x + 4*a)^3*tan(1/2*a)^17 - 91096096904718960*tan(b*x + 4*a)^2*
tan(1/2*a)^18 + 101853310806506160*tan(b*x + 4*a)*tan(1/2*a)^19 - 447162731
27011152*tan(1/2*a)^20 + 4024901399904*tan(b*x + 4*a)^6*tan(1/2*a)^12 - 898
80504373440*tan(b*x + 4*a)^5*tan(1/2*a)^13 + 809309015383680*tan(b*x + 4*a)
^4*tan(1/2*a)^14 - 3743548943593920*tan(b*x + 4*a)^3*tan(1/2*a)^15 + 933282
2656724940*tan(b*x + 4*a)^2*tan(1/2*a)^16 - 11822570313904656*tan(b*x + 4*a
)*tan(1/2*a)^17 + 5911352313614320*tan(1/2*a)^18 - 206987901120*tan(b*x + 4
*a)^6*tan(1/2*a)^10 + 5048602582464*tan(b*x + 4*a)^5*tan(1/2*a)^11 - 500184
73204800*tan(b*x + 4*a)^4*tan(1/2*a)^12 + 256626846135360*tan(b*x + 4*a)^3*
tan(1/2*a)^13 - 715723706535840*tan(b*x + 4*a)^2*tan(1/2*a)^14 + 1023101781
887712*tan(b*x + 4*a)*tan(1/2*a)^15 - 581936260957848*tan(1/2*a)^16 + 80272
89270*tan(b*x + 4*a)^6*tan(1/2*a)^8 - 212284461120*tan(b*x + 4*a)^5*tan(1/2
*a)^9 + 2296406448000*tan(b*x + 4*a)^4*tan(1/2*a)^10 - 12967578538560*tan(b
*x + 4*a)^3*tan(1/2*a)^11 + 40162301142000*tan(b*x + 4*a)^2*tan(1/2*a)^12 -
64379802333600*tan(b*x + 4*a)*tan(1/2*a)^13 + 41487214960800*tan(1/2*a)^14
- 227030328*tan(b*x + 4*a)^6*tan(1/2*a)^6 + 6460519176*tan(b*x + 4*a)^5*ta
n(1/2*a)^7 - 75682808100*tan(b*x + 4*a)^4*tan(1/2*a)^8 + 466347413440*tan(b
*x + 4*a)^3*tan(1/2*a)^9 - 1590087843360*tan(b*x + 4*a)^2*tan(1/2*a)^10 + 2
834816578848*tan(b*x + 4*a)*tan(1/2*a)^11 - 2055305635104*tan(1/2*a)^12 + 4
418820*tan(b*x + 4*a)^6*tan(1/2*a)^4 - 134207352*tan(b*x + 4*a)^5*tan(1/2*a
)^5 + 1686348960*tan(b*x + 4*a)^4*tan(1/2*a)^6 - 11222034120*tan(b*x + 4*a)
^3*tan(1/2*a)^7 + 41675854455*tan(b*x + 4*a)^2*tan(1/2*a)^8 - 81753694560*t
an(b*x + 4*a)*tan(1/2*a)^9 + 66006729504*tan(1/2*a)^10 - 52920*tan(b*x + 4*
a)^6*tan(1/2*a)^2 + 1699320*tan(b*x + 4*a)^5*tan(1/2*a)^3 - 22587480*tan(b*
x + 4*a)^4*tan(1/2*a)^4 + 159789240*tan(b*x + 4*a)^3*tan(1/2*a)^5 - 6357097
80*tan(b*x + 4*a)^2*tan(1/2*a)^6 + 1349403948*tan(b*x + 4*a)*tan(1/2*a)^7 -
1193590350*tan(1/2*a)^8 + 294*tan(b*x + 4*a)^6 - 9864*tan(b*x + 4*a)^5*tan
(1/2*a) + 133920*tan(b*x + 4*a)^4*tan(1/2*a)^2 - 963000*tan(b*x + 4*a)^3*ta
n(1/2*a)^3 + 3912570*tan(b*x + 4*a)^2*tan(1/2*a)^4 - 8559972*tan(b*x + 4*a)
*tan(1/2*a)^5 + 7900020*tan(1/2*a)^6 + 60*tan(b*x + 4*a)^4 - 1080*tan(b*x +
4*a)^3*tan(1/2*a) + 7020*tan(b*x + 4*a)^2*tan(1/2*a)^2 - 20700*tan(b*x + 4
*a)*tan(1/2*a)^3 + 23220*tan(1/2*a)^4 + 15*tan(b*x + 4*a)^2 - 108*tan(b*x +
4*a)*tan(1/2*a) + 180*tan(1/2*a)^2 + 2)/((tan(1/2*a)^36 - 90*tan(1/2*a)^34
+ 3465*tan(1/2*a)^32 - 74256*tan(1/2*a)^30 + 965700*tan(1/2*a)^28 - 781020
0*tan(1/2*a)^26 + 39025140*tan(1/2*a)^24 - 119084400*tan(1/2*a)^22 + 228441
150*tan(1/2*a)^20 - 282933020*tan(1/2*a)^18 + 228441150*tan(1/2*a)^16 - 119
084400*tan(1/2*a)^14 + 39025140*tan(1/2*a)^12 - 7810200*tan(1/2*a)^10 + 965
700*tan(1/2*a)^8 - 74256*tan(1/2*a)^6 + 3465*tan(1/2*a)^4 - 90*tan(1/2*a)^2
+ 1)*(tan(b*x + 4*a)*tan(1/2*a)^6 - 15*tan(b*x + 4*a)*tan(1/2*a)^4 + 6*tan
(1/2*a)^5 + 15*tan(b*x + 4*a)*tan(1/2*a)^2 - 20*tan(1/2*a)^3 - tan(b*x + 4*
a) + 6*tan(1/2*a))^6) + 1920*log(abs(tan(b*x + 4*a)*tan(1/2*a)^6 - 15*tan(b
*x + 4*a)*tan(1/2*a)^4 + 6*tan(1/2*a)^5 + 15*tan(b*x + 4*a)*tan(1/2*a)^2 -
20*tan(1/2*a)^3 - tan(b*x + 4*a) + 6*tan(1/2*a))) - 1920*log(abs(6*tan(b*x

```
+ 4*a)*tan(1/2*a)^5 - tan(1/2*a)^6 - 20*tan(b*x + 4*a)*tan(1/2*a)^3 + 15*ta
n(1/2*a)^4 + 6*tan(b*x + 4*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)))/b
```

3.58 $\int \csc^2(a+bx)\csc^6(2a+2bx)\,dx$

Optimal. Leaf size=102

$$\frac{\tan^5(a+bx)}{320b}+\frac{\tan^3(a+bx)}{32b}+\frac{15\tan(a+bx)}{64b}-\frac{\cot^7(a+bx)}{448b}-\frac{3\cot^5(a+bx)}{160b}-\frac{5\cot^3(a+bx)}{64b}-\frac{5\cot(a+bx)}{16b}$$

```
[Out] (-5*Cot[a + b*x])/(16*b) - (5*Cot[a + b*x]^3)/(64*b) - (3*Cot[a + b*x]^5)/(
160*b) - Cot[a + b*x]^7/(448*b) + (15*Tan[a + b*x])/(64*b) + Tan[a + b*x]^3
/(32*b) + Tan[a + b*x]^5/(320*b)
```

Rubi [A] time = 0.0843532, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4288, 2620, 270}

$$\frac{\tan^5(a+bx)}{320b}+\frac{\tan^3(a+bx)}{32b}+\frac{15\tan(a+bx)}{64b}-\frac{\cot^7(a+bx)}{448b}-\frac{3\cot^5(a+bx)}{160b}-\frac{5\cot^3(a+bx)}{64b}-\frac{5\cot(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^6,x]
```

```
[Out] (-5*Cot[a + b*x])/(16*b) - (5*Cot[a + b*x]^3)/(64*b) - (3*Cot[a + b*x]^5)/(
160*b) - Cot[a + b*x]^7/(448*b) + (15*Tan[a + b*x])/(64*b) + Tan[a + b*x]^3
/(32*b) + Tan[a + b*x]^5/(320*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \csc^2(a+bx)\csc^6(2a+2bx)\,dx = \frac{1}{64}\int \csc^8(a+bx)\sec^6(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x^2)^6}{x^8}\,dx, x, \tan(a+bx)\right)}{64b}$$
$$= \frac{\text{Subst}\left(\int \left(15+\frac{1}{x^8}+\frac{6}{x^6}+\frac{15}{x^4}+\frac{20}{x^2}+6x^2+x^4\right)dx, x, \tan(a+bx)\right)}{64b}$$
$$= -\frac{5\cot(a+bx)}{16b}-\frac{5\cot^3(a+bx)}{64b}-\frac{3\cot^5(a+bx)}{160b}-\frac{\cot^7(a+bx)}{448b}+\frac{15\tan(a+bx)}{64b}$$

Mathematica [A] time = 0.0702229, size = 132, normalized size = 1.29

$$\frac{33\tan(a+bx)}{160b}-\frac{281\cot(a+bx)}{1120b}-\frac{\cot(a+bx)\csc^6(a+bx)}{448b}-\frac{27\cot(a+bx)\csc^4(a+bx)}{2240b}-\frac{53\cot(a+bx)\csc^2(a+bx)}{1120b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^6,x]
```

```
[Out] (-281*Cot[a + b*x])/(1120*b) - (53*Cot[a + b*x]*Csc[a + b*x]^2)/(1120*b) -
(27*Cot[a + b*x]*Csc[a + b*x]^4)/(2240*b) - (Cot[a + b*x]*Csc[a + b*x]^6)/(
448*b) + (33*Tan[a + b*x])/(160*b) + (Sec[a + b*x]^2*Tan[a + b*x])/(40*b) +
 (Sec[a + b*x]^4*Tan[a + b*x])/(320*b)
```

Maple [A] time = 0.041, size = 123, normalized size = 1.2

$$\frac{1}{64\,b}\left(-\frac{1}{7\,(\sin(bx+a))^7(\cos(bx+a))^5}+\frac{12}{35\,(\sin(bx+a))^5(\cos(bx+a))^5}-\frac{24}{35\,(\sin(bx+a))^5(\cos(bx+a))^3}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*csc(2*b*x+2*a)^6,x)
```

```
[Out] 1/64/b*(-1/7/sin(b*x+a)^7/cos(b*x+a)^5+12/35/sin(b*x+a)^5/cos(b*x+a)^5-24/3
5/sin(b*x+a)^5/cos(b*x+a)^3+64/35/sin(b*x+a)^3/cos(b*x+a)^3-128/35/sin(b*x+
a)^3/cos(b*x+a)+512/35/sin(b*x+a)/cos(b*x+a)-1024/35*cot(b*x+a))
```

Maxima [B] time = 1.65418, size = 3659, normalized size = 35.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^6,x, algorithm="maxima")
```

```
[Out] -32/35*((20*sin(10*b*x + 10*a) - 5*sin(8*b*x + 8*a) - 10*sin(6*b*x + 6*a) +
 4*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*cos(24*b*x + 24*a) - 2*(20*sin(10
*b*x + 10*a) - 5*sin(8*b*x + 8*a) - 10*sin(6*b*x + 6*a) + 4*sin(4*b*x + 4*a
) + 2*sin(2*b*x + 2*a))*cos(22*b*x + 22*a) - 4*(20*sin(10*b*x + 10*a) - 5*s
in(8*b*x + 8*a) - 10*sin(6*b*x + 6*a) + 4*sin(4*b*x + 4*a) + 2*sin(2*b*x +
```

```
2*a))*cos(20*b*x + 20*a) + 10*(20*sin(10*b*x + 10*a) - 5*sin(8*b*x + 8*a) -
 10*sin(6*b*x + 6*a) + 4*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*cos(18*b*x
+ 18*a) + 5*(20*sin(10*b*x + 10*a) - 5*sin(8*b*x + 8*a) - 10*sin(6*b*x + 6*
a) + 4*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*cos(16*b*x + 16*a) - 20*(20*s
in(10*b*x + 10*a) - 5*sin(8*b*x + 8*a) - 10*sin(6*b*x + 6*a) + 4*sin(4*b*x
+ 4*a) + 2*sin(2*b*x + 2*a))*cos(14*b*x + 14*a) - (20*cos(10*b*x + 10*a) -
5*cos(8*b*x + 8*a) - 10*cos(6*b*x + 6*a) + 4*cos(4*b*x + 4*a) + 2*cos(2*b*x
 + 2*a) - 1)*sin(24*b*x + 24*a) + 2*(20*cos(10*b*x + 10*a) - 5*cos(8*b*x +
8*a) - 10*cos(6*b*x + 6*a) + 4*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*s
in(22*b*x + 22*a) + 4*(20*cos(10*b*x + 10*a) - 5*cos(8*b*x + 8*a) - 10*cos(
6*b*x + 6*a) + 4*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*sin(20*b*x + 20
*a) - 10*(20*cos(10*b*x + 10*a) - 5*cos(8*b*x + 8*a) - 10*cos(6*b*x + 6*a)
+ 4*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*sin(18*b*x + 18*a) - 5*(20*c
os(10*b*x + 10*a) - 5*cos(8*b*x + 8*a) - 10*cos(6*b*x + 6*a) + 4*cos(4*b*x
+ 4*a) + 2*cos(2*b*x + 2*a) - 1)*sin(16*b*x + 16*a) + 20*(20*cos(10*b*x + 1
0*a) - 5*cos(8*b*x + 8*a) - 10*cos(6*b*x + 6*a) + 4*cos(4*b*x + 4*a) + 2*co
s(2*b*x + 2*a) - 1)*sin(14*b*x + 14*a))/(b*cos(24*b*x + 24*a)^2 + 4*b*cos(2
2*b*x + 22*a)^2 + 16*b*cos(20*b*x + 20*a)^2 + 100*b*cos(18*b*x + 18*a)^2 +
25*b*cos(16*b*x + 16*a)^2 + 400*b*cos(14*b*x + 14*a)^2 + 400*b*cos(10*b*x +
 10*a)^2 + 25*b*cos(8*b*x + 8*a)^2 + 100*b*cos(6*b*x + 6*a)^2 + 16*b*cos(4*
b*x + 4*a)^2 + 4*b*cos(2*b*x + 2*a)^2 + b*sin(24*b*x + 24*a)^2 + 4*b*sin(22
*b*x + 22*a)^2 + 16*b*sin(20*b*x + 20*a)^2 + 100*b*sin(18*b*x + 18*a)^2 + 2
5*b*sin(16*b*x + 16*a)^2 + 400*b*sin(14*b*x + 14*a)^2 + 400*b*sin(10*b*x +
10*a)^2 + 25*b*sin(8*b*x + 8*a)^2 + 100*b*sin(6*b*x + 6*a)^2 + 16*b*sin(4*b
*x + 4*a)^2 + 16*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)
^2 - 2*(2*b*cos(22*b*x + 22*a) + 4*b*cos(20*b*x + 20*a) - 10*b*cos(18*b*x +
 18*a) - 5*b*cos(16*b*x + 16*a) + 20*b*cos(14*b*x + 14*a) - 20*b*cos(10*b*x
 + 10*a) + 5*b*cos(8*b*x + 8*a) + 10*b*cos(6*b*x + 6*a) - 4*b*cos(4*b*x + 4
*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(24*b*x + 24*a) + 4*(4*b*cos(20*b*x + 20
*a) - 10*b*cos(18*b*x + 18*a) - 5*b*cos(16*b*x + 16*a) + 20*b*cos(14*b*x +
14*a) - 20*b*cos(10*b*x + 10*a) + 5*b*cos(8*b*x + 8*a) + 10*b*cos(6*b*x + 6
*a) - 4*b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(22*b*x + 22*a) -
 8*(10*b*cos(18*b*x + 18*a) + 5*b*cos(16*b*x + 16*a) - 20*b*cos(14*b*x + 14
*a) + 20*b*cos(10*b*x + 10*a) - 5*b*cos(8*b*x + 8*a) - 10*b*cos(6*b*x + 6*a
) + 4*b*cos(4*b*x + 4*a) + 2*b*cos(2*b*x + 2*a) - b)*cos(20*b*x + 20*a) + 2
0*(5*b*cos(16*b*x + 16*a) - 20*b*cos(14*b*x + 14*a) + 20*b*cos(10*b*x + 10*
a) - 5*b*cos(8*b*x + 8*a) - 10*b*cos(6*b*x + 6*a) + 4*b*cos(4*b*x + 4*a) +
2*b*cos(2*b*x + 2*a) - b)*cos(18*b*x + 18*a) - 10*(20*b*cos(14*b*x + 14*a)
- 20*b*cos(10*b*x + 10*a) + 5*b*cos(8*b*x + 8*a) + 10*b*cos(6*b*x + 6*a) -
4*b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(16*b*x + 16*a) - 40*(2
0*b*cos(10*b*x + 10*a) - 5*b*cos(8*b*x + 8*a) - 10*b*cos(6*b*x + 6*a) + 4*b
*cos(4*b*x + 4*a) + 2*b*cos(2*b*x + 2*a) - b)*cos(14*b*x + 14*a) - 40*(5*b*
cos(8*b*x + 8*a) + 10*b*cos(6*b*x + 6*a) - 4*b*cos(4*b*x + 4*a) - 2*b*cos(2
*b*x + 2*a) + b)*cos(10*b*x + 10*a) + 10*(10*b*cos(6*b*x + 6*a) - 4*b*cos(4
*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) - 20*(4*b*cos(4*b*
x + 4*a) + 2*b*cos(2*b*x + 2*a) - b)*cos(6*b*x + 6*a) + 8*(2*b*cos(2*b*x +
2*a) - b)*cos(4*b*x + 4*a) - 4*b*cos(2*b*x + 2*a) - 2*(2*b*sin(22*b*x + 22*
a) + 4*b*sin(20*b*x + 20*a) - 10*b*sin(18*b*x + 18*a) - 5*b*sin(16*b*x + 16
*a) + 20*b*sin(14*b*x + 14*a) - 20*b*sin(10*b*x + 10*a) + 5*b*sin(8*b*x + 8
*a) + 10*b*sin(6*b*x + 6*a) - 4*b*sin(4*b*x + 4*a) - 2*b*sin(2*b*x + 2*a))*
sin(24*b*x + 24*a) + 4*(4*b*sin(20*b*x + 20*a) - 10*b*sin(18*b*x + 18*a) -
5*b*sin(16*b*x + 16*a) + 20*b*sin(14*b*x + 14*a) - 20*b*sin(10*b*x + 10*a)
+ 5*b*sin(8*b*x + 8*a) + 10*b*sin(6*b*x + 6*a) - 4*b*sin(4*b*x + 4*a) - 2*b
*sin(2*b*x + 2*a))*sin(22*b*x + 22*a) - 8*(10*b*sin(18*b*x + 18*a) + 5*b*si
n(16*b*x + 16*a) - 20*b*sin(14*b*x + 14*a) + 20*b*sin(10*b*x + 10*a) - 5*b*
sin(8*b*x + 8*a) - 10*b*sin(6*b*x + 6*a) + 4*b*sin(4*b*x + 4*a) + 2*b*sin(2
*b*x + 2*a))*sin(20*b*x + 20*a) + 20*(5*b*sin(16*b*x + 16*a) - 20*b*sin(14*
b*x + 14*a) + 20*b*sin(10*b*x + 10*a) - 5*b*sin(8*b*x + 8*a) - 10*b*sin(6*b
*x + 6*a) + 4*b*sin(4*b*x + 4*a) + 2*b*sin(2*b*x + 2*a))*sin(18*b*x + 18*a)
```

```
 - 10*(20*b*sin(14*b*x + 14*a) - 20*b*sin(10*b*x + 10*a) + 5*b*sin(8*b*x +
8*a) + 10*b*sin(6*b*x + 6*a) - 4*b*sin(4*b*x + 4*a) - 2*b*sin(2*b*x + 2*a))
*sin(16*b*x + 16*a) - 40*(20*b*sin(10*b*x + 10*a) - 5*b*sin(8*b*x + 8*a) -
10*b*sin(6*b*x + 6*a) + 4*b*sin(4*b*x + 4*a) + 2*b*sin(2*b*x + 2*a))*sin(14
*b*x + 14*a) - 40*(5*b*sin(8*b*x + 8*a) + 10*b*sin(6*b*x + 6*a) - 4*b*sin(4
*b*x + 4*a) - 2*b*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + 20*(5*b*sin(6*b*x
+ 6*a) - 2*b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 40*(
2*b*sin(4*b*x + 4*a) + b*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + b)
```

Fricas [A] time = 0.517398, size = 324, normalized size = 3.18

$$-\frac{1024 \cos(bx+a)^{12} - 3584 \cos(bx+a)^{10} + 4480 \cos(bx+a)^{8} - 2240 \cos(bx+a)^{6} + 280 \cos(bx+a)^{4} + 28 \cos(bx}{2240 \left(b\cos(bx+a)^{11} - 3\,b\cos(bx+a)^{9} + 3\,b\cos(bx+a)^{7} - b\cos(bx+a)^{5}\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^6,x, algorithm="fricas")
```

```
[Out] -1/2240*(1024*cos(b*x + a)^12 - 3584*cos(b*x + a)^10 + 4480*cos(b*x + a)^8
- 2240*cos(b*x + a)^6 + 280*cos(b*x + a)^4 + 28*cos(b*x + a)^2 + 7)/((b*cos
(b*x + a)^11 - 3*b*cos(b*x + a)^9 + 3*b*cos(b*x + a)^7 - b*cos(b*x + a)^5)*
sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*csc(2*b*x+2*a)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 13.2479, size = 10094, normalized size = 98.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^6,x, algorithm="giac")
```

```
[Out] -1/71680*(7*(6480*tan(b*x + 4*a)^4*tan(1/2*a)^56 - 2160*tan(b*x + 4*a)^3*ta
n(1/2*a)^57 + 360*tan(b*x + 4*a)^2*tan(1/2*a)^58 - 30*tan(b*x + 4*a)*tan(1/
2*a)^59 + tan(1/2*a)^60 + 963360*tan(b*x + 4*a)^4*tan(1/2*a)^54 - 451440*ta
n(b*x + 4*a)^3*tan(1/2*a)^55 + 84000*tan(b*x + 4*a)^2*tan(1/2*a)^56 - 6650*
tan(b*x + 4*a)*tan(1/2*a)^57 + 210*tan(1/2*a)^58 + 76410000*tan(b*x + 4*a)^
4*tan(1/2*a)^52 - 46604880*tan(b*x + 4*a)^3*tan(1/2*a)^53 + 11606800*tan(b*
x + 4*a)^2*tan(1/2*a)^54 - 1351380*tan(b*x + 4*a)*tan(1/2*a)^55 + 61875*tan
(1/2*a)^56 - 1737755520*tan(b*x + 4*a)^4*tan(1/2*a)^50 + 1779239600*tan(b*x
 + 4*a)^3*tan(1/2*a)^51 - 594557280*tan(b*x + 4*a)^2*tan(1/2*a)^52 + 841797
00*tan(b*x + 4*a)*tan(1/2*a)^53 - 4366140*tan(1/2*a)^54 + 14743616480*tan(b
```

*x + 4*a)^4*tan(1/2*a)^48 - 21984264480*tan(b*x + 4*a)^3*tan(1/2*a)^49 + 10
388004600*tan(b*x + 4*a)^2*tan(1/2*a)^50 - 1902997490*tan(b*x + 4*a)*tan(1/
2*a)^51 + 120469005*tan(1/2*a)^52 - 61028652480*tan(b*x + 4*a)^4*tan(1/2*a)
^46 + 130174560480*tan(b*x + 4*a)^3*tan(1/2*a)^47 - 86273730880*tan(b*x + 4
*a)^2*tan(1/2*a)^48 + 21462983130*tan(b*x + 4*a)*tan(1/2*a)^49 - 1737306714
*tan(1/2*a)^50 + 103180381920*tan(b*x + 4*a)^4*tan(1/2*a)^44 - 371517010400
*tan(b*x + 4*a)^3*tan(1/2*a)^45 + 365221080480*tan(b*x + 4*a)^2*tan(1/2*a)^
46 - 127981989480*tan(b*x + 4*a)*tan(1/2*a)^47 + 14001839215*tan(1/2*a)^48
+ 111160976000*tan(b*x + 4*a)^4*tan(1/2*a)^42 + 211586603040*tan(b*x + 4*a)
^3*tan(1/2*a)^43 - 637111944000*tan(b*x + 4*a)^2*tan(1/2*a)^44 + 3760488381
20*tan(b*x + 4*a)*tan(1/2*a)^45 - 60608749080*tan(1/2*a)^46 - 724773023760*
tan(b*x + 4*a)^4*tan(1/2*a)^40 + 1597018455600*tan(b*x + 4*a)^3*tan(1/2*a)^
41 - 663483964760*tan(b*x + 4*a)^2*tan(1/2*a)^42 - 227449079550*tan(b*x + 4
*a)*tan(1/2*a)^43 + 109533462525*tan(1/2*a)^44 + 663892404960*tan(b*x + 4*a
)^4*tan(1/2*a)^38 - 3633046154960*tan(b*x + 4*a)^3*tan(1/2*a)^39 + 44420531
39040*tan(b*x + 4*a)^2*tan(1/2*a)^40 - 1628950998330*tan(b*x + 4*a)*tan(1/2
*a)^41 + 110076988450*tan(1/2*a)^42 + 1382243479600*tan(b*x + 4*a)^4*tan(1/
2*a)^36 - 662161010160*tan(b*x + 4*a)^3*tan(1/2*a)^37 - 3947253799440*tan(b
*x + 4*a)^2*tan(1/2*a)^38 + 3687432890100*tan(b*x + 4*a)*tan(1/2*a)^39 - 75
7631165865*tan(1/2*a)^40 - 2679939290880*tan(b*x + 4*a)^4*tan(1/2*a)^34 + 9
953265569040*tan(b*x + 4*a)^3*tan(1/2*a)^35 - 8576804580640*tan(b*x + 4*a)^
2*tan(1/2*a)^36 + 813320615340*tan(b*x + 4*a)*tan(1/2*a)^37 + 647844828540*
tan(1/2*a)^38 - 755338211520*tan(b*x + 4*a)^4*tan(1/2*a)^32 - 5830705497280
*tan(b*x + 4*a)^3*tan(1/2*a)^33 + 15767445241080*tan(b*x + 4*a)^2*tan(1/2*a
)^34 - 9992067869250*tan(b*x + 4*a)*tan(1/2*a)^35 + 1474839074345*tan(1/2*a
)^36 + 3975568027520*tan(b*x + 4*a)^4*tan(1/2*a)^30 - 11953155299520*tan(b*
x + 4*a)^3*tan(1/2*a)^31 + 4861415944320*tan(b*x + 4*a)^2*tan(1/2*a)^32 + 5
463157048330*tan(b*x + 4*a)*tan(1/2*a)^33 - 2566729120410*tan(1/2*a)^34 - 7
55338211520*tan(b*x + 4*a)^4*tan(1/2*a)^28 + 11953155299520*tan(b*x + 4*a)^
3*tan(1/2*a)^29 - 23059287629120*tan(b*x + 4*a)^2*tan(1/2*a)^30 + 117070930
56720*tan(b*x + 4*a)*tan(1/2*a)^31 - 840595305645*tan(1/2*a)^32 - 267993929
0880*tan(b*x + 4*a)^4*tan(1/2*a)^26 + 5830705497280*tan(b*x + 4*a)^3*tan(1/
2*a)^27 + 4861415944320*tan(b*x + 4*a)^2*tan(1/2*a)^28 - 11707093056720*tan
(b*x + 4*a)*tan(1/2*a)^29 + 3742852321200*tan(1/2*a)^30 + 1382243479600*tan
(b*x + 4*a)^4*tan(1/2*a)^24 - 9953265569040*tan(b*x + 4*a)^3*tan(1/2*a)^25
+ 15767445241080*tan(b*x + 4*a)^2*tan(1/2*a)^26 - 5463157048330*tan(b*x + 4
*a)*tan(1/2*a)^27 - 840595305645*tan(1/2*a)^28 + 663892404960*tan(b*x + 4*a
)^4*tan(1/2*a)^22 + 662161010160*tan(b*x + 4*a)^3*tan(1/2*a)^23 - 857680458
0640*tan(b*x + 4*a)^2*tan(1/2*a)^24 + 9992067869250*tan(b*x + 4*a)*tan(1/2*
a)^25 - 2566729120410*tan(1/2*a)^26 - 724773023760*tan(b*x + 4*a)^4*tan(1/2
*a)^20 + 3633046154960*tan(b*x + 4*a)^3*tan(1/2*a)^21 - 3947253799440*tan(b
*x + 4*a)^2*tan(1/2*a)^22 - 813320615340*tan(b*x + 4*a)*tan(1/2*a)^23 + 147
4839074345*tan(1/2*a)^24 + 111160976000*tan(b*x + 4*a)^4*tan(1/2*a)^18 - 15
97018455600*tan(b*x + 4*a)^3*tan(1/2*a)^19 + 4442053139040*tan(b*x + 4*a)^2
*tan(1/2*a)^20 - 3687432890100*tan(b*x + 4*a)*tan(1/2*a)^21 + 647844828540*
tan(1/2*a)^22 + 103180381920*tan(b*x + 4*a)^4*tan(1/2*a)^16 - 211586603040*
tan(b*x + 4*a)^3*tan(1/2*a)^17 - 663483964760*tan(b*x + 4*a)^2*tan(1/2*a)^1
8 + 1628950998330*tan(b*x + 4*a)*tan(1/2*a)^19 - 757631165865*tan(1/2*a)^20
 - 61028652480*tan(b*x + 4*a)^4*tan(1/2*a)^14 + 371517010400*tan(b*x + 4*a)
^3*tan(1/2*a)^15 - 637111944000*tan(b*x + 4*a)^2*tan(1/2*a)^16 + 2274490795
50*tan(b*x + 4*a)*tan(1/2*a)^17 + 110076988450*tan(1/2*a)^18 + 14743616480*
tan(b*x + 4*a)^4*tan(1/2*a)^12 - 130174560480*tan(b*x + 4*a)^3*tan(1/2*a)^1
3 + 365221080480*tan(b*x + 4*a)^2*tan(1/2*a)^14 - 376048838120*tan(b*x + 4*
a)*tan(1/2*a)^15 + 109533462525*tan(1/2*a)^16 - 1737755520*tan(b*x + 4*a)^4
*tan(1/2*a)^10 + 21984264480*tan(b*x + 4*a)^3*tan(1/2*a)^11 - 86273730880*t
an(b*x + 4*a)^2*tan(1/2*a)^12 + 127981989480*tan(b*x + 4*a)*tan(1/2*a)^13 -
 60608749080*tan(1/2*a)^14 + 76410000*tan(b*x + 4*a)^4*tan(1/2*a)^8 - 17792
39600*tan(b*x + 4*a)^3*tan(1/2*a)^9 + 10388004600*tan(b*x + 4*a)^2*tan(1/2*
a)^10 - 21462983130*tan(b*x + 4*a)*tan(1/2*a)^11 + 14001839215*tan(1/2*a)^1

2 + 963360*tan(b*x + 4*a)^4*tan(1/2*a)^6 + 46604880*tan(b*x + 4*a)^3*tan(1/
2*a)^7 - 594557280*tan(b*x + 4*a)^2*tan(1/2*a)^8 + 1902997490*tan(b*x + 4*a
)*tan(1/2*a)^9 - 1737306714*tan(1/2*a)^10 + 6480*tan(b*x + 4*a)^4*tan(1/2*a
)^4 + 451440*tan(b*x + 4*a)^3*tan(1/2*a)^5 + 11606800*tan(b*x + 4*a)^2*tan(
1/2*a)^6 - 84179700*tan(b*x + 4*a)*tan(1/2*a)^7 + 120469005*tan(1/2*a)^8 +
2160*tan(b*x + 4*a)^3*tan(1/2*a)^3 + 84000*tan(b*x + 4*a)^2*tan(1/2*a)^4 +
1351380*tan(b*x + 4*a)*tan(1/2*a)^5 - 4366140*tan(1/2*a)^6 + 360*tan(b*x +
4*a)^2*tan(1/2*a)^2 + 6650*tan(b*x + 4*a)*tan(1/2*a)^3 + 61875*tan(1/2*a)^4
 + 30*tan(b*x + 4*a)*tan(1/2*a) + 210*tan(1/2*a)^2 + 1)/((243*tan(1/2*a)^25
 - 4050*tan(1/2*a)^23 + 28215*tan(1/2*a)^21 - 106200*tan(1/2*a)^19 + 233430
*tan(1/2*a)^17 - 304300*tan(1/2*a)^15 + 233430*tan(1/2*a)^13 - 106200*tan(1
/2*a)^11 + 28215*tan(1/2*a)^9 - 4050*tan(1/2*a)^7 + 243*tan(1/2*a)^5)*(6*ta
n(b*x + 4*a)*tan(1/2*a)^5 - tan(1/2*a)^6 - 20*tan(b*x + 4*a)*tan(1/2*a)^3 +
 15*tan(1/2*a)^4 + 6*tan(b*x + 4*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)^5) +
32*(700*tan(b*x + 4*a)^6*tan(1/2*a)^84 - 102900*tan(b*x + 4*a)^6*tan(1/2*a)
^82 + 22050*tan(b*x + 4*a)^5*tan(1/2*a)^83 + 175*tan(b*x + 4*a)^4*tan(1/2*a
)^84 + 7200060*tan(b*x + 4*a)^6*tan(1/2*a)^80 - 3103170*tan(b*x + 4*a)^5*ta
n(1/2*a)^81 + 280770*tan(b*x + 4*a)^4*tan(1/2*a)^82 + 2940*tan(b*x + 4*a)^3
*tan(1/2*a)^83 + 42*tan(b*x + 4*a)^2*tan(1/2*a)^84 - 316457680*tan(b*x + 4*
a)^6*tan(1/2*a)^78 + 206229240*tan(b*x + 4*a)^5*tan(1/2*a)^79 - 38821965*ta
n(b*x + 4*a)^4*tan(1/2*a)^80 + 1904140*tan(b*x + 4*a)^3*tan(1/2*a)^81 + 176
40*tan(b*x + 4*a)^2*tan(1/2*a)^82 + 294*tan(b*x + 4*a)*tan(1/2*a)^83 + 5*ta
n(1/2*a)^84 + 9709765800*tan(b*x + 4*a)^6*tan(1/2*a)^76 - 8530152120*tan(b*
x + 4*a)^5*tan(1/2*a)^77 + 2449598200*tan(b*x + 4*a)^4*tan(1/2*a)^78 - 2592
55080*tan(b*x + 4*a)^3*tan(1/2*a)^79 + 7360290*tan(b*x + 4*a)^2*tan(1/2*a)^
80 + 47530*tan(b*x + 4*a)*tan(1/2*a)^81 + 462*tan(1/2*a)^82 - 218895118680*
tan(b*x + 4*a)^6*tan(1/2*a)^74 + 243907782300*tan(b*x + 4*a)^5*tan(1/2*a)^7
5 - 94820555130*tan(b*x + 4*a)^4*tan(1/2*a)^76 + 15442018200*tan(b*x + 4*a)
^3*tan(1/2*a)^77 - 976159968*tan(b*x + 4*a)^2*tan(1/2*a)^78 + 15463224*tan(
b*x + 4*a)*tan(1/2*a)^79 + 47985*tan(1/2*a)^80 + 3722263347880*tan(b*x + 4*
a)^6*tan(1/2*a)^72 - 5072116579740*tan(b*x + 4*a)^5*tan(1/2*a)^73 + 2508017
550780*tan(b*x + 4*a)^4*tan(1/2*a)^74 - 555145935120*tan(b*x + 4*a)^3*tan(1
/2*a)^75 + 54416410020*tan(b*x + 4*a)^2*tan(1/2*a)^76 - 1964255832*tan(b*x
+ 4*a)*tan(1/2*a)^77 + 13827464*tan(1/2*a)^78 - 48306853319760*tan(b*x + 4*
a)^6*tan(1/2*a)^70 + 78691575627000*tan(b*x + 4*a)^5*tan(1/2*a)^71 - 476992
27086490*tan(b*x + 4*a)^4*tan(1/2*a)^72 + 13463793136080*tan(b*x + 4*a)^3*t
an(1/2*a)^73 - 1800097458768*tan(b*x + 4*a)^2*tan(1/2*a)^74 + 101416919940*
tan(b*x + 4*a)*tan(1/2*a)^75 - 1647303294*tan(1/2*a)^76 + 478852179070860*t
an(b*x + 4*a)^6*tan(1/2*a)^68 - 920259549793080*tan(b*x + 4*a)^5*tan(1/2*a)
^69 + 668627834124600*tan(b*x + 4*a)^4*tan(1/2*a)^70 - 231870649855320*tan(
b*x + 4*a)^3*tan(1/2*a)^71 + 39617163658212*tan(b*x + 4*a)^2*tan(1/2*a)^72
- 3053123538948*tan(b*x + 4*a)*tan(1/2*a)^73 + 77871263652*tan(1/2*a)^74 -
3590491658734820*tan(b*x + 4*a)^6*tan(1/2*a)^66 + 8099233169503770*tan(b*x
+ 4*a)^5*tan(1/2*a)^67 - 6967640085374745*tan(b*x + 4*a)^4*tan(1/2*a)^68 +
2903918016468840*tan(b*x + 4*a)^3*tan(1/2*a)^69 - 610729793019360*tan(b*x +
 4*a)^2*tan(1/2*a)^70 + 60236954245272*tan(b*x + 4*a)*tan(1/2*a)^71 - 21053
89907166*tan(1/2*a)^72 + 19896259338686220*tan(b*x + 4*a)^6*tan(1/2*a)^64 -
 52908136354185210*tan(b*x + 4*a)^5*tan(1/2*a)^65 + 53736728263219610*tan(b
*x + 4*a)^4*tan(1/2*a)^66 - 26624174150101500*tan(b*x + 4*a)^3*tan(1/2*a)^6
7 + 6746600775679002*tan(b*x + 4*a)^2*tan(1/2*a)^68 - 820187477390456*tan(b
*x + 4*a)*tan(1/2*a)^69 + 36705931718472*tan(1/2*a)^70 - 77771578076512320*
tan(b*x + 4*a)^6*tan(1/2*a)^62 + 248694609613265760*tan(b*x + 4*a)^5*tan(1/
2*a)^63 - 300960195956707245*tan(b*x + 4*a)^4*tan(1/2*a)^64 + 1773024391563
31860*tan(b*x + 4*a)^3*tan(1/2*a)^65 - 53651600975241528*tan(b*x + 4*a)^2*t
an(1/2*a)^66 + 7877252164835550*tan(b*x + 4*a)*tan(1/2*a)^67 - 434705306175
315*tan(1/2*a)^68 + 192844994494822880*tan(b*x + 4*a)^6*tan(1/2*a)^60 - 787
509809051626080*tan(b*x + 4*a)^5*tan(1/2*a)^61 + 1173510851306099040*tan(b*
x + 4*a)^4*tan(1/2*a)^62 - 835950959826956960*tan(b*x + 4*a)^3*tan(1/2*a)^6
3 + 303563975744062530*tan(b*x + 4*a)^2*tan(1/2*a)^64 - 53507434964136174*t

an(b*x + 4*a)*tan(1/2*a)^65 + 3573865030470070*tan(1/2*a)^66 - 200273988698
142240*tan(b*x + 4*a)^6*tan(1/2*a)^58 + 1398151947986755920*tan(b*x + 4*a)^
5*tan(1/2*a)^59 - 2880661846092155160*tan(b*x + 4*a)^4*tan(1/2*a)^60 + 2628
180105420371040*tan(b*x + 4*a)^3*tan(1/2*a)^61 - 1179839882273931648*tan(b*
x + 4*a)^2*tan(1/2*a)^62 + 252811149558104928*tan(b*x + 4*a)*tan(1/2*a)^63
- 20413710218743023*tan(1/2*a)^64 - 371753255147504160*tan(b*x + 4*a)^6*tan
(1/2*a)^56 - 1005666833635111120*tan(b*x + 4*a)^5*tan(1/2*a)^57 + 2954417021
031525840*tan(b*x + 4*a)^4*tan(1/2*a)^58 - 4615789529427552000*tan(b*x + 4*
a)^3*tan(1/2*a)^59 + 2869139270141795952*tan(b*x + 4*a)^2*tan(1/2*a)^60 - 7
89182123619651552*tan(b*x + 4*a)*tan(1/2*a)^61 + 79047902485397664*tan(1/2*
a)^62 + 1588175918421104320*tan(b*x + 4*a)^6*tan(1/2*a)^54 - 57052883200744
46880*tan(b*x + 4*a)^5*tan(1/2*a)^55 + 5543116031154667560*tan(b*x + 4*a)^4
*tan(1/2*a)^56 + 291827962982908800*tan(b*x + 4*a)^3*tan(1/2*a)^57 - 290703
5119602048960*tan(b*x + 4*a)^2*tan(1/2*a)^58 + 1372067944595797680*tan(b*x
+ 4*a)*tan(1/2*a)^59 - 190524603741625608*tan(1/2*a)^60 - 17642684176828174
80*tan(b*x + 4*a)^6*tan(1/2*a)^52 + 11851507413382482720*tan(b*x + 4*a)^5*t
an(1/2*a)^53 - 23636877153497649440*tan(b*x + 4*a)^4*tan(1/2*a)^54 + 188525
55003710349600*tan(b*x + 4*a)^3*tan(1/2*a)^55 - 5514689294400576528*tan(b*x
 + 4*a)^2*tan(1/2*a)^56 - 74535612123099568*tan(b*x + 4*a)*tan(1/2*a)^57 +
190797055644582576*tan(1/2*a)^58 - 1248453345588419880*tan(b*x + 4*a)^6*tan
(1/2*a)^50 - 4107878664125019420*tan(b*x + 4*a)^5*tan(1/2*a)^51 + 265217801
08972032990*tan(b*x + 4*a)^4*tan(1/2*a)^52 - 39323126797282860000*tan(b*x +
 4*a)^3*tan(1/2*a)^53 + 23453137263033690240*tan(b*x + 4*a)^2*tan(1/2*a)^54
 - 5608964036362402464*tan(b*x + 4*a)*tan(1/2*a)^55 + 366017355229752504*ta
n(1/2*a)^56 + 5286270479895116600*tan(b*x + 4*a)^6*tan(1/2*a)^48 - 21060700
379637099300*tan(b*x + 4*a)^5*tan(1/2*a)^49 + 18283250157335063460*tan(b*x
+ 4*a)^4*tan(1/2*a)^50 + 14094690574782962200*tan(b*x + 4*a)^3*tan(1/2*a)^5
1 - 26565501133755260460*tan(b*x + 4*a)^2*tan(1/2*a)^52 + 11739546935802688
800*tan(b*x + 4*a)*tan(1/2*a)^53 - 1551520053876868320*tan(1/2*a)^54 - 4019
457841895096160*tan(b*x + 4*a)^6*tan(1/2*a)^46 + 33873365085581604240*tan(b
*x + 4*a)^5*tan(1/2*a)^47 - 79355355678133689210*tan(b*x + 4*a)^4*tan(1/2*a
)^48 + 69758331625489475640*tan(b*x + 4*a)^3*tan(1/2*a)^49 - 17832092647389
129648*tan(b*x + 4*a)^2*tan(1/2*a)^50 - 4344999459760312724*tan(b*x + 4*a)*
tan(1/2*a)^51 + 1772974039101333642*tan(1/2*a)^52 - 3363472645057689360*tan
(b*x + 4*a)^6*tan(1/2*a)^44 - 4196346243738904080*tan(b*x + 4*a)^5*tan(1/2*
a)^45 + 61023004440064909200*tan(b*x + 4*a)^4*tan(1/2*a)^46 - 1135940055490
24014000*tan(b*x + 4*a)^3*tan(1/2*a)^47 + 79397061913802193540*tan(b*x + 4*
a)^2*tan(1/2*a)^48 - 20785250550683362380*tan(b*x + 4*a)*tan(1/2*a)^49 + 11
58344161574237820*tan(1/2*a)^50 + 7922839707121600240*tan(b*x + 4*a)^6*tan(
1/2*a)^42 - 40062672006803795160*tan(b*x + 4*a)^5*tan(1/2*a)^43 + 504790970
14227689220*tan(b*x + 4*a)^4*tan(1/2*a)^44 + 14711499739562948560*tan(b*x +
 4*a)^3*tan(1/2*a)^45 - 61779677247183389760*tan(b*x + 4*a)^2*tan(1/2*a)^46
 + 34283272085462710800*tan(b*x + 4*a)*tan(1/2*a)^47 - 5294375878659948990*
tan(1/2*a)^48 - 3363472645057689360*tan(b*x + 4*a)^6*tan(1/2*a)^40 + 400626
72006803795160*tan(b*x + 4*a)^5*tan(1/2*a)^41 - 119703426365609444120*tan(b
*x + 4*a)^4*tan(1/2*a)^42 + 134372289532573058400*tan(b*x + 4*a)^3*tan(1/2*
a)^43 - 50496361003895322600*tan(b*x + 4*a)^2*tan(1/2*a)^44 - 4643613116469
393360*tan(b*x + 4*a)*tan(1/2*a)^45 + 4170913759258711920*tan(1/2*a)^46 - 4
019457841895096160*tan(b*x + 4*a)^6*tan(1/2*a)^38 + 4196346243738904080*tan
(b*x + 4*a)^5*tan(1/2*a)^39 + 50479097014227689220*tan(b*x + 4*a)^4*tan(1/2
*a)^40 - 134372289532573058400*tan(b*x + 4*a)^3*tan(1/2*a)^41 + 12059954353
0513343520*tan(b*x + 4*a)^2*tan(1/2*a)^42 - 40569490418667454440*tan(b*x +
4*a)*tan(1/2*a)^43 + 3366765108190708140*tan(1/2*a)^44 + 528627047989511660
0*tan(b*x + 4*a)^6*tan(1/2*a)^36 - 33873365085581604240*tan(b*x + 4*a)^5*ta
n(1/2*a)^37 + 61023004440064909200*tan(b*x + 4*a)^4*tan(1/2*a)^38 - 1471149
9739562948560*tan(b*x + 4*a)^3*tan(1/2*a)^39 - 50496361003895322600*tan(b*x
 + 4*a)^2*tan(1/2*a)^40 + 40569490418667454440*tan(b*x + 4*a)*tan(1/2*a)^41
 - 8102375952750405800*tan(1/2*a)^42 - 1248453345588419880*tan(b*x + 4*a)^6
*tan(1/2*a)^34 + 21060700379637099300*tan(b*x + 4*a)^5*tan(1/2*a)^35 - 7935

5355678133689210*tan(b*x + 4*a)^4*tan(1/2*a)^36 + 1135940055490240140000*tan
(b*x + 4*a)^3*tan(1/2*a)^37 - 6177967724718338976O*tan(b*x + 4*a)^2*tan(1/2
*a)^38 + 4643613116469393360*tan(b*x + 4*a)*tan(1/2*a)^39 + 336676510819070
8140*tan(1/2*a)^40 - 1764268417682817480*tan(b*x + 4*a)^6*tan(1/2*a)^32 + 4
107878664125019420*tan(b*x + 4*a)^5*tan(1/2*a)^33 + 1828325015733506346O*ta
n(b*x + 4*a)^4*tan(1/2*a)^34 - 69758331625489475640*tan(b*x + 4*a)^3*tan(1/
2*a)^35 + 79397061913802193540*tan(b*x + 4*a)^2*tan(1/2*a)^36 - 34283272085
462710800*tan(b*x + 4*a)*tan(1/2*a)^37 + 4170913759258711920*tan(1/2*a)^38
+ 1588175918421104320*tan(b*x + 4*a)^6*tan(1/2*a)^30 - 11851507413382482720
*tan(b*x + 4*a)^5*tan(1/2*a)^31 + 26521780108972032990*tan(b*x + 4*a)^4*tan
(1/2*a)^32 - 14094690574782962200*tan(b*x + 4*a)^3*tan(1/2*a)^33 - 17832092
647389129648*tan(b*x + 4*a)^2*tan(1/2*a)^34 + 20785250550683362380*tan(b*x
+ 4*a)*tan(1/2*a)^35 - 5294375878659948990*tan(1/2*a)^36 - 3717532551475041
60*tan(b*x + 4*a)^6*tan(1/2*a)^28 + 5705288320074446880*tan(b*x + 4*a)^5*ta
n(1/2*a)^29 - 23636877153497649440*tan(b*x + 4*a)^4*tan(1/2*a)^30 + 3932312
6797282860000*tan(b*x + 4*a)^3*tan(1/2*a)^31 - 26565501133755260460*tan(b*x
 + 4*a)^2*tan(1/2*a)^32 + 4344999459760312724*tan(b*x + 4*a)*tan(1/2*a)^33
+ 1158344161574237820*tan(1/2*a)^34 - 200273988698142240*tan(b*x + 4*a)^6*t
an(1/2*a)^26 + 100566683363511120*tan(b*x + 4*a)^5*tan(1/2*a)^27 + 55431160
31154667560*tan(b*x + 4*a)^4*tan(1/2*a)^28 - 18852555003710349600*tan(b*x +
 4*a)^3*tan(1/2*a)^29 + 23453137263033690240*tan(b*x + 4*a)^2*tan(1/2*a)^30
 - 11739546935802688800*tan(b*x + 4*a)*tan(1/2*a)^31 + 1772974039101333642*
tan(1/2*a)^32 + 192844994494822880*tan(b*x + 4*a)^6*tan(1/2*a)^24 - 1398151
947986755920*tan(b*x + 4*a)^5*tan(1/2*a)^25 + 2954417021031525840*tan(b*x +
 4*a)^4*tan(1/2*a)^26 - 291827962982908800*tan(b*x + 4*a)^3*tan(1/2*a)^27 -
 5514689294400576528*tan(b*x + 4*a)^2*tan(1/2*a)^28 + 5608964036362402464*t
an(b*x + 4*a)*tan(1/2*a)^29 - 1551520053876868320*tan(1/2*a)^30 - 777715780
76512320*tan(b*x + 4*a)^6*tan(1/2*a)^22 + 787509809051626080*tan(b*x + 4*a)
^5*tan(1/2*a)^23 - 2880661846092155160*tan(b*x + 4*a)^4*tan(1/2*a)^24 + 461
5789529427552000*tan(b*x + 4*a)^3*tan(1/2*a)^25 - 2907035119602048960*tan(b
*x + 4*a)^2*tan(1/2*a)^26 + 74535612123099568*tan(b*x + 4*a)*tan(1/2*a)^27
+ 366017355229752504*tan(1/2*a)^28 + 19896259338686220*tan(b*x + 4*a)^6*tan
(1/2*a)^20 - 248694609613265760*tan(b*x + 4*a)^5*tan(1/2*a)^21 + 1173510851
306099040*tan(b*x + 4*a)^4*tan(1/2*a)^22 - 2628180105420371040*tan(b*x + 4*
a)^3*tan(1/2*a)^23 + 2869139270141795952*tan(b*x + 4*a)^2*tan(1/2*a)^24 - 1
372067944595797680*tan(b*x + 4*a)*tan(1/2*a)^25 + 190797055644582576*tan(1/
2*a)^26 - 3590491658734820*tan(b*x + 4*a)^6*tan(1/2*a)^18 + 529081363541852
10*tan(b*x + 4*a)^5*tan(1/2*a)^19 - 300960195956707245*tan(b*x + 4*a)^4*tan
(1/2*a)^20 + 835950959826956960*tan(b*x + 4*a)^3*tan(1/2*a)^21 - 1179839882
273931648*tan(b*x + 4*a)^2*tan(1/2*a)^22 + 789182123619651552*tan(b*x + 4*a
)*tan(1/2*a)^23 - 190524603741625608*tan(1/2*a)^24 + 478852179070860*tan(b*
x + 4*a)^6*tan(1/2*a)^16 - 8099233169503770*tan(b*x + 4*a)^5*tan(1/2*a)^17
+ 53736728263219610*tan(b*x + 4*a)^4*tan(1/2*a)^18 - 177302439156331860*tan
(b*x + 4*a)^3*tan(1/2*a)^19 + 303563975744062530*tan(b*x + 4*a)^2*tan(1/2*a
)^20 - 252811149558104928*tan(b*x + 4*a)*tan(1/2*a)^21 + 79047902485397664*
tan(1/2*a)^22 - 48306853319760*tan(b*x + 4*a)^6*tan(1/2*a)^14 + 92025954979
3080*tan(b*x + 4*a)^5*tan(1/2*a)^15 - 6967640085374745*tan(b*x + 4*a)^4*tan
(1/2*a)^16 + 26624174150101500*tan(b*x + 4*a)^3*tan(1/2*a)^17 - 53651600975
241528*tan(b*x + 4*a)^2*tan(1/2*a)^18 + 53507434964136174*tan(b*x + 4*a)*ta
n(1/2*a)^19 - 20413710218743023*tan(1/2*a)^20 + 3722263347880*tan(b*x + 4*a
)^6*tan(1/2*a)^12 - 78691575627000*tan(b*x + 4*a)^5*tan(1/2*a)^13 + 6686278
34124600*tan(b*x + 4*a)^4*tan(1/2*a)^14 - 2903918016468840*tan(b*x + 4*a)^3
*tan(1/2*a)^15 + 6746600775679002*tan(b*x + 4*a)^2*tan(1/2*a)^16 - 78772521
64835550*tan(b*x + 4*a)*tan(1/2*a)^17 + 3573865030470070*tan(1/2*a)^18 - 21
8895118680*tan(b*x + 4*a)^6*tan(1/2*a)^10 + 5072116579740*tan(b*x + 4*a)^5*
tan(1/2*a)^11 - 47699227086490*tan(b*x + 4*a)^4*tan(1/2*a)^12 + 23187064985
5320*tan(b*x + 4*a)^3*tan(1/2*a)^13 - 610729793019360*tan(b*x + 4*a)^2*tan(
1/2*a)^14 + 820187477390456*tan(b*x + 4*a)*tan(1/2*a)^15 - 434705306175315*
tan(1/2*a)^16 + 9709765800*tan(b*x + 4*a)^6*tan(1/2*a)^8 - 243907782300*tan

(b*x + 4*a)^5*tan(1/2*a)^9 + 2508017550780*tan(b*x + 4*a)^4*tan(1/2*a)^10 - 13463793136080*tan(b*x + 4*a)^3*tan(1/2*a)^11 + 39617163658212*tan(b*x + 4*a)^2*tan(1/2*a)^12 - 60236954245272*tan(b*x + 4*a)*tan(1/2*a)^13 + 36705931718472*tan(1/2*a)^14 - 316457680*tan(b*x + 4*a)^6*tan(1/2*a)^6 + 8530152120*tan(b*x + 4*a)^5*tan(1/2*a)^7 - 94820555130*tan(b*x + 4*a)^4*tan(1/2*a)^8 + 555145935120*tan(b*x + 4*a)^3*tan(1/2*a)^9 - 1800097458768*tan(b*x + 4*a)^2*tan(1/2*a)^10 + 3053123538948*tan(b*x + 4*a)*tan(1/2*a)^11 - 2105389907166*tan(1/2*a)^12 + 7200060*tan(b*x + 4*a)^6*tan(1/2*a)^4 - 206229240*tan(b*x + 4*a)^5*tan(1/2*a)^5 + 2449598200*tan(b*x + 4*a)^4*tan(1/2*a)^6 - 15442018200*tan(b*x + 4*a)^3*tan(1/2*a)^7 + 54416410020*tan(b*x + 4*a)^2*tan(1/2*a)^8 - 101416919940*tan(b*x + 4*a)*tan(1/2*a)^9 + 77871263652*tan(1/2*a)^10 - 102900*tan(b*x + 4*a)^6*tan(1/2*a)^2 + 3103170*tan(b*x + 4*a)^5*tan(1/2*a)^3 - 38821965*tan(b*x + 4*a)^4*tan(1/2*a)^4 + 259255080*tan(b*x + 4*a)^3*tan(1/2*a)^5 - 976159968*tan(b*x + 4*a)^2*tan(1/2*a)^6 + 1964255832*tan(b*x + 4*a)*tan(1/2*a)^7 - 1647303294*tan(1/2*a)^8 + 700*tan(b*x + 4*a)^6 - 22050*tan(b*x + 4*a)^5*tan(1/2*a) + 280770*tan(b*x + 4*a)^4*tan(1/2*a)^2 - 1904140*tan(b*x + 4*a)^3*tan(1/2*a)^3 + 7360290*tan(b*x + 4*a)^2*tan(1/2*a)^4 - 15463224*tan(b*x + 4*a)*tan(1/2*a)^5 + 13827464*tan(1/2*a)^6 + 175*tan(b*x + 4*a)^4 - 2940*tan(b*x + 4*a)^3*tan(1/2*a) + 17640*tan(b*x + 4*a)^2*tan(1/2*a)^2 - 47530*tan(b*x + 4*a)*tan(1/2*a)^3 + 47985*tan(1/2*a)^4 + 42*tan(b*x + 4*a)^2 - 294*tan(b*x + 4*a)*tan(1/2*a) + 462*tan(1/2*a)^2 + 5)/((tan(1/2*a)^42 - 105*tan(1/2*a)^40 + 4830*tan(1/2*a)^38 - 127582*tan(1/2*a)^36 + 2131605*tan(1/2*a)^34 - 23413005*tan(1/2*a)^32 + 170737896*tan(1/2*a)^30 - 822580200*tan(1/2*a)^28 + 2607894450*tan(1/2*a)^26 - 5534841410*tan(1/2*a)^24 + 8018138100*tan(1/2*a)^22 - 8018138100*tan(1/2*a)^20 + 5534841410*tan(1/2*a)^18 - 2607894450*tan(1/2*a)^16 + 822580200*tan(1/2*a)^14 - 170737896*tan(1/2*a)^12 + 23413005*tan(1/2*a)^10 - 2131605*tan(1/2*a)^8 + 127582*tan(1/2*a)^6 - 4830*tan(1/2*a)^4 + 105*tan(1/2*a)^2 - 1)*(tan(b*x + 4*a)*tan(1/2*a)^6 - 15*tan(b*x + 4*a)*tan(1/2*a)^4 + 6*tan(1/2*a)^5 + 15*tan(b*x + 4*a)*tan(1/2*a)^2 - 20*tan(1/2*a)^3 - tan(b*x + 4*a) + 6*tan(1/2*a))^7))/b

3.59 $\int \csc^3(a+bx)\sin^{10}(2a+2bx)\,dx$

Optimal. Leaf size=61

$$\frac{1024\cos^{17}(a+bx)}{17b}-\frac{1024\cos^{15}(a+bx)}{5b}+\frac{3072\cos^{13}(a+bx)}{13b}-\frac{1024\cos^{11}(a+bx)}{11b}$$

```
[Out] (-1024*Cos[a + b*x]^11)/(11*b) + (3072*Cos[a + b*x]^13)/(13*b) - (1024*Cos[
a + b*x]^15)/(5*b) + (1024*Cos[a + b*x]^17)/(17*b)
```

Rubi [A] time = 0.0722782, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4288, 2565, 270}

$$\frac{1024\cos^{17}(a+bx)}{17b}-\frac{1024\cos^{15}(a+bx)}{5b}+\frac{3072\cos^{13}(a+bx)}{13b}-\frac{1024\cos^{11}(a+bx)}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^10,x]
```

```
[Out] (-1024*Cos[a + b*x]^11)/(11*b) + (3072*Cos[a + b*x]^13)/(13*b) - (1024*Cos[
a + b*x]^15)/(5*b) + (1024*Cos[a + b*x]^17)/(17*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^3(a+bx)\sin^{10}(2a+2bx)\,dx &= 1024\int \cos^{10}(a+bx)\sin^7(a+bx)\,dx\\
&= -\frac{1024\,\text{Subst}\left(\int x^{10}\left(1-x^2\right)^3\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{1024\,\text{Subst}\left(\int \left(x^{10}-3x^{12}+3x^{14}-x^{16}\right)dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{1024\cos^{11}(a+bx)}{11b}+\frac{3072\cos^{13}(a+bx)}{13b}-\frac{1024\cos^{15}(a+bx)}{5b}+\frac{1024\cos^{17}(a+bx)}{17b}\end{aligned}$$

Mathematica [A] time = 0.145892, size = 119, normalized size = 1.95

$$-\frac{35\cos(a+bx)}{32b}-\frac{7\cos(3(a+bx))}{16b}+\frac{7\cos(5(a+bx))}{80b}+\frac{\cos(7(a+bx))}{8b}-\frac{5\cos(11(a+bx))}{176b}-\frac{\cos(13(a+bx))}{208b}+\frac{\cos(15(a+bx))}{320b}+\frac{\cos(17(a+bx))}{1088b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^10,x]
```

```
[Out] (-35*Cos[a + b*x])/(32*b) - (7*Cos[3*(a + b*x)])/(16*b) + (7*Cos[5*(a + b*x
)])/(80*b) + Cos[7*(a + b*x)]/(8*b) - (5*Cos[11*(a + b*x)])/(176*b) - Cos[1
3*(a + b*x)]/(208*b) + Cos[15*(a + b*x)]/(320*b) + Cos[17*(a + b*x)]/(1088*
b)
```

Maple [A] time = 0.04, size = 71, normalized size = 1.2

$$1024\,\frac{1}{b}\left(-1/17\,(\sin(bx+a))^6(\cos(bx+a))^{11}-\frac{2\,(\sin(bx+a))^4(\cos(bx+a))^{11}}{85}-\frac{8\,(\sin(bx+a))^2(\cos(bx+a))^{11}}{1105}-\frac{16\,(\cos(bx+a))^{11}}{12155}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^10,x)
```

```
[Out] 1024/b*(-1/17*sin(b*x+a)^6*cos(b*x+a)^11-2/85*sin(b*x+a)^4*cos(b*x+a)^11-8/
1105*sin(b*x+a)^2*cos(b*x+a)^11-16/12155*cos(b*x+a)^11)
```

Maxima [A] time = 1.06209, size = 123, normalized size = 2.02

$$\frac{715\cos(17\,bx+17\,a)+2431\cos(15\,bx+15\,a)-3740\cos(13\,bx+13\,a)-22100\cos(11\,bx+11\,a)+97240\cos(7\,bx+7\,a)+68068\cos(5\,bx+5\,a)-340340\cos(3\,bx+3\,a)-850850\cos(bx+a)}{777920\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^10,x, algorithm="maxima")
```

```
[Out] 1/777920*(715*cos(17*b*x + 17*a) + 2431*cos(15*b*x + 15*a) - 3740*cos(13*b*
x + 13*a) - 22100*cos(11*b*x + 11*a) + 97240*cos(7*b*x + 7*a) + 68068*cos(5
*b*x + 5*a) - 340340*cos(3*b*x + 3*a) - 850850*cos(b*x + a))/b
```

Fricas [A] time = 0.569647, size = 142, normalized size = 2.33

$$\frac{1024\left(715\,\cos(bx+a)^{17}-2431\,\cos(bx+a)^{15}+2805\,\cos(bx+a)^{13}-1105\,\cos(bx+a)^{11}\right)}{12155\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^10,x, algorithm="fricas")
```

```
[Out] 1024/12155*(715*cos(b*x + a)^17 - 2431*cos(b*x + a)^15 + 2805*cos(b*x + a)^
13 - 1105*cos(b*x + a)^11)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**10,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.30865, size = 424, normalized size = 6.95

$$-\frac{32768\left(\frac{17\,(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{136\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{680\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{9775\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+\frac{71825\,(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5}+\frac{221000\,(\cos(bx+a)-1)^6}{(\cos(bx+a)+1)^6}+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^10,x, algorithm="giac")
```

```
[Out] -32768/12155*(17*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 136*(cos(b*x + a)
- 1)^2/(cos(b*x + a) + 1)^2 + 680*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3
 + 9775*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 71825*(cos(b*x + a) - 1
)^5/(cos(b*x + a) + 1)^5 + 221000*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6
 + 486200*(cos(b*x + a) - 1)^7/(cos(b*x + a) + 1)^7 + 668525*(cos(b*x + a)
- 1)^8/(cos(b*x + a) + 1)^8 + 692835*(cos(b*x + a) - 1)^9/(cos(b*x + a) + 1
)^9 + 466752*(cos(b*x + a) - 1)^10/(cos(b*x + a) + 1)^10 + 233376*(cos(b*x
+ a) - 1)^11/(cos(b*x + a) + 1)^11 + 65637*(cos(b*x + a) - 1)^12/(cos(b*x +
 a) + 1)^12 + 12155*(cos(b*x + a) - 1)^13/(cos(b*x + a) + 1)^13 - 1)/(b*((c
os(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^17)
```

3.60 $\int \csc^3(a+bx)\sin^9(2a+2bx)\,dx$

Optimal. Leaf size=76

$$\frac{512\sin^{15}(a+bx)}{15b}-\frac{2048\sin^{13}(a+bx)}{13b}+\frac{3072\sin^{11}(a+bx)}{11b}-\frac{2048\sin^{9}(a+bx)}{9b}+\frac{512\sin^{7}(a+bx)}{7b}$$

```
[Out] (512*Sin[a + b*x]^7)/(7*b) - (2048*Sin[a + b*x]^9)/(9*b) + (3072*Sin[a + b*
x]^11)/(11*b) - (2048*Sin[a + b*x]^13)/(13*b) + (512*Sin[a + b*x]^15)/(15*b
)
```

Rubi [A] time = 0.0773139, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4288, 2564, 270}

$$\frac{512\sin^{15}(a+bx)}{15b}-\frac{2048\sin^{13}(a+bx)}{13b}+\frac{3072\sin^{11}(a+bx)}{11b}-\frac{2048\sin^{9}(a+bx)}{9b}+\frac{512\sin^{7}(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^9,x]
```

```
[Out] (512*Sin[a + b*x]^7)/(7*b) - (2048*Sin[a + b*x]^9)/(9*b) + (3072*Sin[a + b*
x]^11)/(11*b) - (2048*Sin[a + b*x]^13)/(13*b) + (512*Sin[a + b*x]^15)/(15*b
)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \csc^3(a+bx)\sin^9(2a+2bx)\,dx = 512\int \cos^9(a+bx)\sin^6(a+bx)\,dx$$

$$= \frac{512\,\text{Subst}\left(\int x^6\left(1-x^2\right)^4\,dx, x, \sin(a+bx)\right)}{b}$$

$$= \frac{512\,\text{Subst}\left(\int \left(x^6-4x^8+6x^{10}-4x^{12}+x^{14}\right)dx, x, \sin(a+bx)\right)}{b}$$

$$= \frac{512\sin^7(a+bx)}{7b} - \frac{2048\sin^9(a+bx)}{9b} + \frac{3072\sin^{11}(a+bx)}{11b} - \frac{2048\sin^{13}(a+bx)}{13b} + \ldots$$

Mathematica [A] time = 0.329026, size = 58, normalized size = 0.76

$$\frac{512\left(3003\sin^{15}(a+bx)-13860\sin^{13}(a+bx)+24570\sin^{11}(a+bx)-20020\sin^9(a+bx)+6435\sin^7(a+bx)\right)}{45045b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^9,x]
```

```
[Out] (512*(6435*Sin[a + b*x]^7 - 20020*Sin[a + b*x]^9 + 24570*Sin[a + b*x]^11 -
13860*Sin[a + b*x]^13 + 3003*Sin[a + b*x]^15))/(45045*b)
```

Maple [A] time = 0.058, size = 107, normalized size = 1.4

$$512\,\frac{1}{b}\left(-1/15\,(\sin(bx+a))^5(\cos(bx+a))^{10}-1/39\,(\sin(bx+a))^3(\cos(bx+a))^{10}-\frac{\sin(bx+a)(\cos(bx+a))^{10}}{143}+\frac{\sin(\ldots}{12\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^9,x)
```

```
[Out] 512/b*(-1/15*sin(b*x+a)^5*cos(b*x+a)^10-1/39*sin(b*x+a)^3*cos(b*x+a)^10-1/1
43*sin(b*x+a)*cos(b*x+a)^10+1/1287*(128/35+cos(b*x+a)^8+8/7*cos(b*x+a)^6+48
/35*cos(b*x+a)^4+64/35*cos(b*x+a)^2)*sin(b*x+a))
```

Maxima [A] time = 1.07465, size = 123, normalized size = 1.62

$$-\frac{3003\,\sin(15\,bx+15\,a)+10395\,\sin(13\,bx+13\,a)-12285\,\sin(11\,bx+11\,a)-85085\,\sin(9\,bx+9\,a)-19305\,\sin(7\,bx\ldots}{1441440\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^9,x, algorithm="maxima")
```

```
[Out] -1/1441440*(3003*sin(15*b*x + 15*a) + 10395*sin(13*b*x + 13*a) - 12285*sin(
11*b*x + 11*a) - 85085*sin(9*b*x + 9*a) - 19305*sin(7*b*x + 7*a) + 351351*s
in(5*b*x + 5*a) + 375375*sin(3*b*x + 3*a) - 2027025*sin(b*x + a))/b
```

Fricas [A] time = 0.529794, size = 246, normalized size = 3.24

$$-\frac{512\left(3003\cos\left(bx+a\right)^{14}-7161\cos\left(bx+a\right)^{12}+4473\cos\left(bx+a\right)^{10}-35\cos\left(bx+a\right)^{8}-40\cos\left(bx+a\right)^{6}-48\cos\right.}{45045\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^9,x, algorithm="fricas")
```

```
[Out] -512/45045*(3003*cos(b*x + a)^14 - 7161*cos(b*x + a)^12 + 4473*cos(b*x + a)
^10 - 35*cos(b*x + a)^8 - 40*cos(b*x + a)^6 - 48*cos(b*x + a)^4 - 64*cos(b*
x + a)^2 - 128)*sin(b*x + a)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**9,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.1526, size = 76, normalized size = 1.

$$\frac{512\left(3003\sin\left(bx+a\right)^{15}-13860\sin\left(bx+a\right)^{13}+24570\sin\left(bx+a\right)^{11}-20020\sin\left(bx+a\right)^{9}+6435\sin\left(bx+a\right)^{7}\right)}{45045\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^9,x, algorithm="giac")
```

```
[Out] 512/45045*(3003*sin(b*x + a)^15 - 13860*sin(b*x + a)^13 + 24570*sin(b*x + a
)^11 - 20020*sin(b*x + a)^9 + 6435*sin(b*x + a)^7)/b
```

3.61 $\int \csc^3(a+bx)\sin^8(2a+2bx)\,dx$

Optimal. Leaf size=46

$$-\frac{256\cos^{13}(a+bx)}{13b}+\frac{512\cos^{11}(a+bx)}{11b}-\frac{256\cos^9(a+bx)}{9b}$$

```
[Out] (-256*Cos[a + b*x]^9)/(9*b) + (512*Cos[a + b*x]^11)/(11*b) - (256*Cos[a + b
*x]^13)/(13*b)
```

Rubi [A] time = 0.0672123, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4288, 2565, 270}

$$-\frac{256\cos^{13}(a+bx)}{13b}+\frac{512\cos^{11}(a+bx)}{11b}-\frac{256\cos^9(a+bx)}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^8,x]
```

```
[Out] (-256*Cos[a + b*x]^9)/(9*b) + (512*Cos[a + b*x]^11)/(11*b) - (256*Cos[a + b
*x]^13)/(13*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \csc^3(a+bx)\sin^8(2a+2bx)\,dx &= 256\int \cos^8(a+bx)\sin^5(a+bx)\,dx\\
&= -\frac{256\,\text{Subst}\left(\int x^8\left(1-x^2\right)^2\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{256\,\text{Subst}\left(\int \left(x^8-2x^{10}+x^{12}\right)\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{256\cos^9(a+bx)}{9b}+\frac{512\cos^{11}(a+bx)}{11b}-\frac{256\cos^{13}(a+bx)}{13b}
\end{aligned}$$

Mathematica [B] time = 0.0982023, size = 104, normalized size = 2.26

$$-\frac{5\cos(a+bx)}{4b}-\frac{25\cos(3(a+bx))}{48b}+\frac{\cos(5(a+bx))}{16b}+\frac{\cos(7(a+bx))}{8b}+\frac{\cos(9(a+bx))}{72b}-\frac{3\cos(11(a+bx))}{176b}-\frac{\cos(1}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^8,x]
```

```
[Out] (-5*Cos[a + b*x])/(4*b) - (25*Cos[3*(a + b*x)])/(48*b) + Cos[5*(a + b*x)]/(
16*b) + Cos[7*(a + b*x)]/(8*b) + Cos[9*(a + b*x)]/(72*b) - (3*Cos[11*(a + b
*x)])/(176*b) - Cos[13*(a + b*x)]/(208*b)
```

Maple [A] time = 0.032, size = 53, normalized size = 1.2

$$256\,\frac{1}{b}\left(-1/13\,(\sin(bx+a))^4(\cos(bx+a))^9-\frac{4\,(\sin(bx+a))^2(\cos(bx+a))^9}{143}-\frac{8\,(\cos(bx+a))^9}{1287}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^8,x)
```

```
[Out] 256/b*(-1/13*sin(b*x+a)^4*cos(b*x+a)^9-4/143*sin(b*x+a)^2*cos(b*x+a)^9-8/12
87*cos(b*x+a)^9)
```

Maxima [A] time = 1.05838, size = 108, normalized size = 2.35

$$-\frac{99\cos(13\,bx+13\,a)+351\cos(11\,bx+11\,a)-286\cos(9\,bx+9\,a)-2574\cos(7\,bx+7\,a)-1287\cos(5\,bx+5\,a)}{20592\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^8,x, algorithm="maxima")
```

```
[Out] -1/20592*(99*cos(13*b*x + 13*a) + 351*cos(11*b*x + 11*a) - 286*cos(9*b*x +
9*a) - 2574*cos(7*b*x + 7*a) - 1287*cos(5*b*x + 5*a) + 10725*cos(3*b*x + 3*
a) + 25740*cos(b*x + a))/b
```

Fricas [A] time = 0.523297, size = 104, normalized size = 2.26

$$-\frac{256\left(99\cos(bx+a)^{13}-234\cos(bx+a)^{11}+143\cos(bx+a)^9\right)}{1287\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^8,x, algorithm="fricas")
```

```
[Out] -256/1287*(99*cos(b*x + a)^13 - 234*cos(b*x + a)^11 + 143*cos(b*x + a)^9)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.8694, size = 335, normalized size = 7.28

$$-\frac{4096\left(\frac{13\,(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{78\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-\frac{572\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}-\frac{3718\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}-\frac{7722\,(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5}-\frac{13728\,(\cos(bx+a)-1)^6}{(\cos(bx+a)+1)^6}-1201\ldots}{1287\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^8,x, algorithm="giac")
```

```
[Out] -4096/1287*(13*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 78*(cos(b*x + a) - 1
)^2/(cos(b*x + a) + 1)^2 - 572*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 -
3718*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 7722*(cos(b*x + a) - 1)^5/
(cos(b*x + a) + 1)^5 - 13728*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 - 12
012*(cos(b*x + a) - 1)^7/(cos(b*x + a) + 1)^7 - 9009*(cos(b*x + a) - 1)^8/(
cos(b*x + a) + 1)^8 - 3003*(cos(b*x + a) - 1)^9/(cos(b*x + a) + 1)^9 - 858*
(cos(b*x + a) - 1)^10/(cos(b*x + a) + 1)^10 - 1)/(b*((cos(b*x + a) - 1)/(co
s(b*x + a) + 1) - 1)^13)
```

3.62 $\int \csc^3(a+bx)\sin^7(2a+2bx)\,dx$

Optimal. Leaf size=61

$$-\frac{128\sin^{11}(a+bx)}{11b}+\frac{128\sin^9(a+bx)}{3b}-\frac{384\sin^7(a+bx)}{7b}+\frac{128\sin^5(a+bx)}{5b}$$

```
[Out] (128*Sin[a + b*x]^5)/(5*b) - (384*Sin[a + b*x]^7)/(7*b) + (128*Sin[a + b*x]
^9)/(3*b) - (128*Sin[a + b*x]^11)/(11*b)
```

Rubi [A] time = 0.0712247, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4288, 2564, 270}

$$-\frac{128\sin^{11}(a+bx)}{11b}+\frac{128\sin^9(a+bx)}{3b}-\frac{384\sin^7(a+bx)}{7b}+\frac{128\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (128*Sin[a + b*x]^5)/(5*b) - (384*Sin[a + b*x]^7)/(7*b) + (128*Sin[a + b*x]
^9)/(3*b) - (128*Sin[a + b*x]^11)/(11*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^3(a+bx)\sin^7(2a+2bx)\,dx &= 128\int \cos^7(a+bx)\sin^4(a+bx)\,dx\\ &= \frac{128\,\text{Subst}\left(\int x^4\left(1-x^2\right)^3\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{128\,\text{Subst}\left(\int \left(x^4-3x^6+3x^8-x^{10}\right)dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{128\sin^5(a+bx)}{5b}-\frac{384\sin^7(a+bx)}{7b}+\frac{128\sin^9(a+bx)}{3b}-\frac{128\sin^{11}(a+bx)}{11b}\end{aligned}$$

Mathematica [A] time = 0.166942, size = 48, normalized size = 0.79

$$\frac{128\left(-105\sin^{11}(a+bx)+385\sin^{9}(a+bx)-495\sin^{7}(a+bx)+231\sin^{5}(a+bx)\right)}{1155b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (128*(231*Sin[a + b*x]^5 - 495*Sin[a + b*x]^7 + 385*Sin[a + b*x]^9 - 105*Si
n[a + b*x]^11))/(1155*b)
```

Maple [A] time = 0.029, size = 79, normalized size = 1.3

$$128\,\frac{1}{b}\left(-1/11\,(\sin(bx+a))^{3}(\cos(bx+a))^{8}-1/33\,\sin(bx+a)(\cos(bx+a))^{8}+\frac{\sin(bx+a)}{231}\left(\frac{16}{5}+(\cos(bx+a))^{6}+6/5\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^7,x)
```

```
[Out] 128/b*(-1/11*sin(b*x+a)^3*cos(b*x+a)^8-1/33*sin(b*x+a)*cos(b*x+a)^8+1/231*(
16/5+cos(b*x+a)^6+6/5*cos(b*x+a)^4+8/5*cos(b*x+a)^2)*sin(b*x+a))
```

Maxima [A] time = 1.10357, size = 93, normalized size = 1.52

$$\frac{105\,\sin(11\,bx+11\,a)+385\,\sin(9\,bx+9\,a)-165\,\sin(7\,bx+7\,a)-2541\,\sin(5\,bx+5\,a)-2310\,\sin(3\,bx+3\,a)+16170}{9240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^7,x, algorithm="maxima")
```

```
[Out] 1/9240*(105*sin(11*b*x + 11*a) + 385*sin(9*b*x + 9*a) - 165*sin(7*b*x + 7*a
) - 2541*sin(5*b*x + 5*a) - 2310*sin(3*b*x + 3*a) + 16170*sin(b*x + a))/b
```

Fricas [A] time = 0.510657, size = 176, normalized size = 2.89

$$\frac{128\left(105\,\cos(bx+a)^{10}-140\,\cos(bx+a)^{8}+5\,\cos(bx+a)^{6}+6\,\cos(bx+a)^{4}+8\,\cos(bx+a)^{2}+16\right)\sin(bx+a)}{1155\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^7,x, algorithm="fricas")
```

```
[Out] 128/1155*(105*cos(b*x + a)^10 - 140*cos(b*x + a)^8 + 5*cos(b*x + a)^6 + 6*c
os(b*x + a)^4 + 8*cos(b*x + a)^2 + 16)*sin(b*x + a)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**7,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.73332, size = 62, normalized size = 1.02

$$-\frac{128\left(105\,\sin\left(bx+a\right)^{11}-385\,\sin\left(bx+a\right)^{9}+495\,\sin\left(bx+a\right)^{7}-231\,\sin\left(bx+a\right)^{5}\right)}{1155\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^7,x, algorithm="giac")
```

```
[Out] -128/1155*(105*sin(b*x + a)^11 - 385*sin(b*x + a)^9 + 495*sin(b*x + a)^7 -
231*sin(b*x + a)^5)/b
```

3.63 $\int \csc^3(a+bx)\sin^6(2a+2bx)\,dx$

Optimal. Leaf size=31

$$\frac{64\cos^9(a+bx)}{9b}-\frac{64\cos^7(a+bx)}{7b}$$

```
[Out] (-64*Cos[a + b*x]^7)/(7*b) + (64*Cos[a + b*x]^9)/(9*b)
```

Rubi [A] time = 0.0607875, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4288, 2565, 14}

$$\frac{64\cos^9(a+bx)}{9b}-\frac{64\cos^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^6,x]
```

```
[Out] (-64*Cos[a + b*x]^7)/(7*b) + (64*Cos[a + b*x]^9)/(9*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \csc^3(a+bx)\sin^6(2a+2bx)\,dx &= 64\int \cos^6(a+bx)\sin^3(a+bx)\,dx\\ &= -\frac{64\,\text{Subst}\left(\int x^6\left(1-x^2\right)\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{64\,\text{Subst}\left(\int \left(x^6-x^8\right)\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{64\cos^7(a+bx)}{7b}+\frac{64\cos^9(a+bx)}{9b}\end{aligned}$$

Mathematica [A] time = 0.145513, size = 27, normalized size = 0.87

$$\frac{32\cos^7(a+bx)(7\cos(2(a+bx))-11)}{63b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^6,x]
```

```
[Out] (32*Cos[a + b*x]^7*(-11 + 7*Cos[2*(a + b*x)]))/(63*b)
```

Maple [A] time = 0.029, size = 35, normalized size = 1.1

$$64\,\frac{1}{b}\left(-1/9\,(\sin(bx+a))^2(\cos(bx+a))^7-\frac{2\,(\cos(bx+a))^7}{63}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^6,x)
```

```
[Out] 64/b*(-1/9*sin(b*x+a)^2*cos(b*x+a)^7-2/63*cos(b*x+a)^7)
```

Maxima [A] time = 1.04111, size = 63, normalized size = 2.03

$$\frac{7\,\cos(9\,bx+9\,a)+27\,\cos(7\,bx+7\,a)-168\,\cos(3\,bx+3\,a)-378\,\cos(bx+a)}{252\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^6,x, algorithm="maxima")
```

```
[Out] 1/252*(7*cos(9*b*x + 9*a) + 27*cos(7*b*x + 7*a) - 168*cos(3*b*x + 3*a) - 37
8*cos(b*x + a))/b
```

Fricas [A] time = 0.492255, size = 63, normalized size = 2.03

$$\frac{64\left(7\,\cos(bx+a)^9-9\,\cos(bx+a)^7\right)}{63\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^6,x, algorithm="fricas")
```

```
[Out] 64/63*(7*cos(b*x + a)^9 - 9*cos(b*x + a)^7)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.65083, size = 246, normalized size = 7.94

$$-\frac{256\left(\frac{9(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{27(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{189(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{189(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+\frac{315(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5}+\frac{105(\cos(bx+a)-1)^6}{(\cos(bx+a)+1)^6}+\cdots\right.}{63\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^6,x, algorithm="giac")
```

```
[Out] -256/63*(9*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 27*(cos(b*x + a) - 1)^2/
(cos(b*x + a) + 1)^2 + 189*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 189*
(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 315*(cos(b*x + a) - 1)^5/(cos(b
*x + a) + 1)^5 + 105*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 + 63*(cos(b*
x + a) - 1)^7/(cos(b*x + a) + 1)^7 - 1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a
) + 1) - 1)^9)
```

3.64 $\int \csc^3(a+bx)\sin^5(2a+2bx)\,dx$

Optimal. Leaf size=46

$$\frac{32\sin^7(a+bx)}{7b}-\frac{64\sin^5(a+bx)}{5b}+\frac{32\sin^3(a+bx)}{3b}$$

```
[Out] (32*Sin[a + b*x]^3)/(3*b) - (64*Sin[a + b*x]^5)/(5*b) + (32*Sin[a + b*x]^7)
/(7*b)
```

Rubi [A] time = 0.0656032, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4288, 2564, 270}

$$\frac{32\sin^7(a+bx)}{7b}-\frac{64\sin^5(a+bx)}{5b}+\frac{32\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (32*Sin[a + b*x]^3)/(3*b) - (64*Sin[a + b*x]^5)/(5*b) + (32*Sin[a + b*x]^7)
/(7*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^3(a+bx)\sin^5(2a+2bx)\,dx &= 32\int \cos^5(a+bx)\sin^2(a+bx)\,dx\\ &= \frac{32\,\text{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{32\,\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{32\sin^3(a+bx)}{3b}-\frac{64\sin^5(a+bx)}{5b}+\frac{32\sin^7(a+bx)}{7b}\end{aligned}$$

Mathematica [A] time = 0.108067, size = 37, normalized size = 0.8

$$\frac{4\sin^3(a+bx)(108\cos(2(a+bx))+15\cos(4(a+bx))+157)}{105b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (4*(157 + 108*Cos[2*(a + b*x)] + 15*Cos[4*(a + b*x)])*Sin[a + b*x]^3)/(105*
b)
```

Maple [A] time = 0.029, size = 51, normalized size = 1.1

$$32\,\frac{-1/7\,\sin(bx+a)(\cos(bx+a))^6+1/35\,\left(8/3+(\cos(bx+a))^4+4/3\,(\cos(bx+a))^2\right)\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^5,x)
```

```
[Out] 32/b*(-1/7*sin(b*x+a)*cos(b*x+a)^6+1/35*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)
*sin(b*x+a))
```

Maxima [A] time = 1.03676, size = 63, normalized size = 1.37

$$-\frac{15\,\sin(7\,bx+7\,a)+63\,\sin(5\,bx+5\,a)+35\,\sin(3\,bx+3\,a)-525\,\sin(bx+a)}{210\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] -1/210*(15*sin(7*b*x + 7*a) + 63*sin(5*b*x + 5*a) + 35*sin(3*b*x + 3*a) - 5
25*sin(b*x + a))/b
```

Fricas [A] time = 0.484142, size = 116, normalized size = 2.52

$$-\frac{32\left(15\,\cos(bx+a)^6-3\,\cos(bx+a)^4-4\,\cos(bx+a)^2-8\right)\sin(bx+a)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] -32/105*(15*cos(b*x + a)^6 - 3*cos(b*x + a)^4 - 4*cos(b*x + a)^2 - 8)*sin(b
*x + a)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36548, size = 49, normalized size = 1.07

$$\frac{32\left(15 \sin\left(bx+a\right)^{7}-42 \sin\left(bx+a\right)^{5}+35 \sin\left(bx+a\right)^{3}\right)}{105\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] 32/105*(15*sin(b*x + a)^7 - 42*sin(b*x + a)^5 + 35*sin(b*x + a)^3)/b
```

3.65 $\int \csc^3(a+bx)\sin^4(2a+2bx)\,dx$

Optimal. Leaf size=15

$$-\frac{16\cos^5(a+bx)}{5b}$$

```
[Out] (-16*Cos[a + b*x]^5)/(5*b)
```

Rubi [A] time = 0.0444241, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4288, 2565, 30}

$$-\frac{16\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (-16*Cos[a + b*x]^5)/(5*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \csc^3(a+bx)\sin^4(2a+2bx)\,dx &= 16\int \cos^4(a+bx)\sin(a+bx)\,dx\\ &= -\frac{16\,\text{Subst}\left(\int x^4\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{16\cos^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.0096055, size = 15, normalized size = 1.

$$-\frac{16\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (-16*Cos[a + b*x]^5)/(5*b)
```

Maple [A] time = 0.023, size = 14, normalized size = 0.9

$$-\frac{16\left(\cos\left(bx+a\right)\right)^{5}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^4,x)
```

```
[Out] -16/5*cos(b*x+a)^5/b
```

Maxima [B] time = 1.06807, size = 46, normalized size = 3.07

$$-\frac{\cos\left(5\,bx+5\,a\right)+5\,\cos\left(3\,bx+3\,a\right)+10\,\cos\left(bx+a\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] -1/5*(cos(5*b*x + 5*a) + 5*cos(3*b*x + 3*a) + 10*cos(b*x + a))/b
```

Fricas [A] time = 0.479734, size = 32, normalized size = 2.13

$$-\frac{16\,\cos\left(bx+a\right)^{5}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] -16/5*cos(b*x + a)^5/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.42222, size = 100, normalized size = 6.67

$$\frac{32\left(\frac{10\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{5\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+1\right)}{5\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] 32/5*(10*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 + 5*(cos(b*x + a) - 1)^4
/(cos(b*x + a) + 1)^4 + 1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^5
)
```

3.66 $\int \csc^3(a+bx)\sin^3(2a+2bx)\,dx$

Optimal. Leaf size=27

$$\frac{8\sin(a+bx)}{b}-\frac{8\sin^3(a+bx)}{3b}$$

```
[Out] (8*Sin[a + b*x])/b - (8*Sin[a + b*x]^3)/(3*b)
```

Rubi [A] time = 0.0389608, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4288, 2633}

$$\frac{8\sin(a+bx)}{b}-\frac{8\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (8*Sin[a + b*x])/b - (8*Sin[a + b*x]^3)/(3*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^3(a+bx)\sin^3(2a+2bx)\,dx &= 8\int \cos^3(a+bx)\,dx\\ &= -\frac{8\,\text{Subst}\left(\int \left(1-x^2\right)dx, x, -\sin(a+bx)\right)}{b}\\ &= \frac{8\sin(a+bx)}{b}-\frac{8\sin^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0114353, size = 28, normalized size = 1.04

$$8\left(\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] 8*(Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b))
```

Maple [A] time = 0.025, size = 22, normalized size = 0.8

$$\frac{\left(16+8\,\left(\cos\left(bx+a\right)\right)^{2}\right)\sin\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^3,x)
```

```
[Out] 8/3/b*(2+cos(b*x+a)^2)*sin(b*x+a)
```

Maxima [A] time = 1.06617, size = 31, normalized size = 1.15

$$\frac{2\left(\sin\left(3\,bx+3\,a\right)+9\,\sin\left(bx+a\right)\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 2/3*(sin(3*b*x + 3*a) + 9*sin(b*x + a))/b
```

Fricas [A] time = 0.473609, size = 55, normalized size = 2.04

$$\frac{8\left(\cos\left(bx+a\right)^{2}+2\right)\sin\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] 8/3*(cos(b*x + a)^2 + 2)*sin(b*x + a)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35142, size = 30, normalized size = 1.11

$$-\frac{8\left(\sin\left(bx+a\right)^{3}-3\,\sin\left(bx+a\right)\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] -8/3*(sin(b*x + a)^3 - 3*sin(b*x + a))/b
```

3.67 $\int \csc^3(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=24

$$\frac{4\cos(a+bx)}{b}-\frac{4\tanh^{-1}(\cos(a+bx))}{b}$$

```
[Out] (-4*ArcTanh[Cos[a + b*x]])/b + (4*Cos[a + b*x])/b
```

Rubi [A] time = 0.0430228, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4288, 2592, 321, 206}

$$\frac{4\cos(a+bx)}{b}-\frac{4\tanh^{-1}(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (-4*ArcTanh[Cos[a + b*x]])/b + (4*Cos[a + b*x])/b
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \csc^3(a+bx)\sin^2(2a+2bx)\,dx &= 4\int \cos(a+bx)\cot(a+bx)\,dx \\
&= -\frac{4\,\mathrm{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos(a+bx)\right)}{b} \\
&= \frac{4\cos(a+bx)}{b} - \frac{4\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(a+bx)\right)}{b} \\
&= -\frac{4\tanh^{-1}(\cos(a+bx))}{b} + \frac{4\cos(a+bx)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.0256678, size = 44, normalized size = 1.83

$$
4\left(\frac{\cos(a+bx)}{b} + \frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b} - \frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] 4*(Cos[a + b*x]/b - Log[Cos[(a + b*x)/2]]/b + Log[Sin[(a + b*x)/2]]/b)
```

Maple [A] time = 0.029, size = 34, normalized size = 1.4

$$
4\,\frac{\cos(bx+a)}{b} + 4\,\frac{\ln(\csc(bx+a) - \cot(bx+a))}{b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^2,x)
```

```
[Out] 4*cos(b*x+a)/b+4/b*ln(csc(b*x+a)-cot(b*x+a))
```

Maxima [B] time = 1.05241, size = 124, normalized size = 5.17

$$
\frac{2\left(2\cos(bx+a) - \log\left(\cos(bx)^2 + 2\cos(bx)\cos(a) + \cos(a)^2 + \sin(bx)^2 - 2\sin(bx)\sin(a) + \sin(a)^2\right) + \log\left(\cos\right.\right.}{b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] 2*(2*cos(b*x + a) - log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x
)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + log(cos(b*x)^2 - 2*cos(b*x)*cos(a) +
cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b
```

Fricas [A] time = 0.503847, size = 112, normalized size = 4.67

$$
\frac{2\left(2\cos(bx+a) - \log\left(\frac{1}{2}\cos(bx+a) + \frac{1}{2}\right) + \log\left(-\frac{1}{2}\cos(bx+a) + \frac{1}{2}\right)\right)}{b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] 2*(2*cos(b*x + a) - log(1/2*cos(b*x + a) + 1/2) + log(-1/2*cos(b*x + a) + 1
/2))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24985, size = 77, normalized size = 3.21

$$-\frac{2\left(\frac{4}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1}-\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] -2*(4/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1) - log(abs(-cos(b*x + a) +
 1)/abs(cos(b*x + a) + 1)))/b
```

3.68 $\int \csc^3(a+bx)\sin(2a+2bx)\,dx$

Optimal. Leaf size=11

$$-\frac{2\csc(a+bx)}{b}$$

```
[Out] (-2*Csc[a + b*x])/b
```

Rubi [A] time = 0.0255341, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4288, 2606, 8}

$$-\frac{2\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x],x]

[Out] (-2*Csc[a + b*x])/b
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \csc^3(a+bx)\sin(2a+2bx)\,dx &= 2\int \cot(a+bx)\csc(a+bx)\,dx\\ &= -\frac{2\,\text{Subst}\left(\int 1\,dx, x, \csc(a+bx)\right)}{b}\\ &= -\frac{2\csc(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0103081, size = 11, normalized size = 1.

$$-\frac{2\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x],x]
```

```
[Out] (-2*Csc[a + b*x])/b
```

Maple [A] time = 0.019, size = 14, normalized size = 1.3

$$-2\,{\frac {1}{b\sin \left(bx+a \right) }}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a),x)
```

```
[Out] -2/b/sin(b*x+a)
```

Maxima [B] time = 1.03779, size = 113, normalized size = 10.27

$$-\frac{4\,\left(\cos \left(b x + a\right) \sin \left(2\,b x + 2\,a\right) - \cos \left(2\,b x + 2\,a\right) \sin \left(b x + a\right) + \sin \left(b x + a\right)\right)}{b \cos \left(2\,b x + 2\,a\right)^{2} + b \sin \left(2\,b x + 2\,a\right)^{2} - 2\,b \cos \left(2\,b x + 2\,a\right) + b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -4*(cos(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x
 + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 - 2*b*cos(2*b*x + 2*a)
+ b)
```

Fricas [A] time = 0.453849, size = 28, normalized size = 2.55

$$-\frac{2}{b \sin \left(b x + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] -2/(b*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2038, size = 18, normalized size = 1.64

$$-\frac{2}{b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] -2/(b*sin(b*x + a))
```

3.69 $\int \csc^3(a+bx)\csc(2a+2bx)\,dx$

Optimal. Leaf size=43

$$-\frac{\csc^3(a+bx)}{6b}-\frac{\csc(a+bx)}{2b}+\frac{\tanh^{-1}(\sin(a+bx))}{2b}$$

```
[Out] ArcTanh[Sin[a + b*x]]/(2*b) - Csc[a + b*x]/(2*b) - Csc[a + b*x]^3/(6*b)
```

Rubi [A] time = 0.049419, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4288, 2621, 302, 207}

$$-\frac{\csc^3(a+bx)}{6b}-\frac{\csc(a+bx)}{2b}+\frac{\tanh^{-1}(\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Csc[2*a + 2*b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(2*b) - Csc[a + b*x]/(2*b) - Csc[a + b*x]^3/(6*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^3(a+bx)\csc(2a+2bx)\,dx = \frac{1}{2}\int \csc^4(a+bx)\sec(a+bx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{2b}$$
$$= -\frac{\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \csc(a+bx)\right)}{2b}$$
$$= -\frac{\csc(a+bx)}{2b} - \frac{\csc^3(a+bx)}{6b} - \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{2b}$$
$$= \frac{\tanh^{-1}(\sin(a+bx))}{2b} - \frac{\csc(a+bx)}{2b} - \frac{\csc^3(a+bx)}{6b}$$

Mathematica [C] time = 0.0177232, size = 31, normalized size = 0.72

$$-\frac{\csc^3(a+bx)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\sin^2(a+bx)\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Csc[2*a + 2*b*x],x]

[Out] -(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[a + b*x]^2])/(6*b)
```

Maple [A] time = 0.032, size = 47, normalized size = 1.1

$$-\frac{1}{6\,b\,(\sin(bx+a))^3} - \frac{1}{2\,b\sin(bx+a)} + \frac{\ln(\sec(bx+a)+\tan(bx+a))}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*csc(2*b*x+2*a),x)

[Out] -1/6/b/sin(b*x+a)^3-1/2/b/sin(b*x+a)+1/2/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [B] time = 1.78185, size = 1126, normalized size = 26.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a),x, algorithm="maxima")

[Out] 1/12*(4*(3*sin(5*b*x + 5*a) - 10*sin(3*b*x + 3*a) + 3*sin(b*x + a))*cos(6*b
*x + 6*a) + 36*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(5*b*x + 5*a) + 12*
(10*sin(3*b*x + 3*a) - 3*sin(b*x + a))*cos(4*b*x + 4*a) + 3*(2*(3*cos(4*b*x
 + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) - cos(6*b*x + 6*a)^2 + 6
*(3*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 9*cos(4*b*x + 4*a)^2 - 9*cos(2
*b*x + 2*a)^2 + 6*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(6*b*x + 6*a) -
sin(6*b*x + 6*a)^2 - 9*sin(4*b*x + 4*a)^2 + 18*sin(4*b*x + 4*a)*sin(2*b*x +
```

```
 2*a) - 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2*a) - 1)*log((cos(b*x + 2*a)^
2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a
)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a)
 + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(3*cos(5*b*x
 + 5*a) - 10*cos(3*b*x + 3*a) + 3*cos(b*x + a))*sin(6*b*x + 6*a) - 12*(3*co
s(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*sin(5*b*x + 5*a) - 12*(10*cos(3*b*
x + 3*a) - 3*cos(b*x + a))*sin(4*b*x + 4*a) - 40*(3*cos(2*b*x + 2*a) - 1)*s
in(3*b*x + 3*a) + 120*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) - 36*cos(b*x + a)*s
in(2*b*x + 2*a) + 36*cos(2*b*x + 2*a)*sin(b*x + a) - 12*sin(b*x + a))/(b*co
s(6*b*x + 6*a)^2 + 9*b*cos(4*b*x + 4*a)^2 + 9*b*cos(2*b*x + 2*a)^2 + b*sin(
6*b*x + 6*a)^2 + 9*b*sin(4*b*x + 4*a)^2 - 18*b*sin(4*b*x + 4*a)*sin(2*b*x +
 2*a) + 9*b*sin(2*b*x + 2*a)^2 - 2*(3*b*cos(4*b*x + 4*a) - 3*b*cos(2*b*x +
2*a) + b)*cos(6*b*x + 6*a) - 6*(3*b*cos(2*b*x + 2*a) - b)*cos(4*b*x + 4*a)
- 6*b*cos(2*b*x + 2*a) - 6*(b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*sin(6*
b*x + 6*a) + b)
```

Fricas [B] time = 0.502252, size = 254, normalized size = 5.91

$$\frac{3\left(\cos\left(bx+a\right)^{2}-1\right)\log\left(\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-3\left(\cos\left(bx+a\right)^{2}-1\right)\log\left(-\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-6\cos}{12\left(b\cos\left(bx+a\right)^{2}-b\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*(cos(b*x + a)^2 - 1)*log(sin(b*x + a) + 1)*sin(b*x + a) - 3*(cos(b*
x + a)^2 - 1)*log(-sin(b*x + a) + 1)*sin(b*x + a) - 6*cos(b*x + a)^2 + 8)/(
(b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \csc^{3}\left(a+bx\right)\csc\left(2a+2bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*csc(2*b*x+2*a),x)
```

```
[Out] Integral(csc(a + b*x)**3*csc(2*a + 2*b*x), x)
```

Giac [B] time = 2.10202, size = 2830, normalized size = 65.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] -1/48*((27*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^34 - 9*tan(1/2*b*x + 2*a)^4*tan(
1/2*a)^35 + tan(1/2*b*x + 2*a)^3*tan(1/2*a)^36 + 630*tan(1/2*b*x + 2*a)^5*t
an(1/2*a)^32 - 267*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^33 + 18*tan(1/2*b*x + 2*
a)^3*tan(1/2*a)^34 + 9*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^35 - 25458*tan(1/2*b
```

*x + 2*a)^5*tan(1/2*a)^30 + 28818*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^31 - 1005
3*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^32 + 1077*tan(1/2*b*x + 2*a)^2*tan(1/2*a)
^33 + 27*tan(1/2*b*x + 2*a)*tan(1/2*a)^34 + 202086*tan(1/2*b*x + 2*a)^5*tan
(1/2*a)^28 - 396738*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^29 + 244788*tan(1/2*b*x
 + 2*a)^3*tan(1/2*a)^30 - 58788*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^31 + 4518*t
an(1/2*b*x + 2*a)*tan(1/2*a)^32 + 54*tan(1/2*a)^33 - 644166*tan(1/2*b*x + 2
*a)^5*tan(1/2*a)^26 + 2016678*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^27 - 1980792*
tan(1/2*b*x + 2*a)^3*tan(1/2*a)^28 + 780948*tan(1/2*b*x + 2*a)^2*tan(1/2*a)
^29 - 123954*tan(1/2*b*x + 2*a)*tan(1/2*a)^30 + 5778*tan(1/2*a)^31 + 616590
*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^24 - 4024566*tan(1/2*b*x + 2*a)^4*tan(1/2*
a)^25 + 6570108*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^26 - 4036848*tan(1/2*b*x +
2*a)^2*tan(1/2*a)^27 + 993942*tan(1/2*b*x + 2*a)*tan(1/2*a)^28 - 80658*tan(
1/2*a)^29 + 1290870*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^22 - 545670*tan(1/2*b*x
 + 2*a)^4*tan(1/2*a)^23 - 6240000*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^24 + 8134
776*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^25 - 3273030*tan(1/2*b*x + 2*a)*tan(1/2
*a)^26 + 402730*tan(1/2*a)^27 - 3012066*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^20
+ 12765366*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^21 - 13579740*tan(1/2*b*x + 2*a)
^3*tan(1/2*a)^22 + 1241820*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^23 + 3103230*tan
(1/2*b*x + 2*a)*tan(1/2*a)^24 - 796770*tan(1/2*a)^25 - 10040220*tan(1/2*b*x
 + 2*a)^4*tan(1/2*a)^19 + 29284146*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^20 - 255
75756*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^21 + 6696630*tan(1/2*b*x + 2*a)*tan(1
/2*a)^22 - 99990*tan(1/2*a)^23 + 3012066*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^16
 - 10040220*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^17 + 19709370*tan(1/2*b*x + 2*a
)^2*tan(1/2*a)^19 - 14753394*tan(1/2*b*x + 2*a)*tan(1/2*a)^20 + 2525334*tan
(1/2*a)^21 - 1290870*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^14 + 12765366*tan(1/2*
b*x + 2*a)^4*tan(1/2*a)^15 - 29284146*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^16 +
19709370*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^17 - 2087550*tan(1/2*a)^19 - 61659
0*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^12 - 545670*tan(1/2*b*x + 2*a)^4*tan(1/2*
a)^13 + 13579740*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^14 - 25575756*tan(1/2*b*x
+ 2*a)^2*tan(1/2*a)^15 + 14753394*tan(1/2*b*x + 2*a)*tan(1/2*a)^16 - 208755
0*tan(1/2*a)^17 + 644166*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^10 - 4024566*tan(1
/2*b*x + 2*a)^4*tan(1/2*a)^11 + 6240000*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^12
+ 1241820*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^13 - 6696630*tan(1/2*b*x + 2*a)*t
an(1/2*a)^14 + 2525334*tan(1/2*a)^15 - 202086*tan(1/2*b*x + 2*a)^5*tan(1/2*
a)^8 + 2016678*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^9 - 6570108*tan(1/2*b*x + 2*
a)^3*tan(1/2*a)^10 + 8134776*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^11 - 3103230*t
an(1/2*b*x + 2*a)*tan(1/2*a)^12 - 99990*tan(1/2*a)^13 + 25458*tan(1/2*b*x +
 2*a)^5*tan(1/2*a)^6 - 396738*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^7 + 1980792*t
an(1/2*b*x + 2*a)^3*tan(1/2*a)^8 - 4036848*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^
9 + 3273030*tan(1/2*b*x + 2*a)*tan(1/2*a)^10 - 796770*tan(1/2*a)^11 - 630*t
an(1/2*b*x + 2*a)^5*tan(1/2*a)^4 + 28818*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^5
- 244788*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^6 + 780948*tan(1/2*b*x + 2*a)^2*ta
n(1/2*a)^7 - 993942*tan(1/2*b*x + 2*a)*tan(1/2*a)^8 + 402730*tan(1/2*a)^9 -
 27*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^2 - 267*tan(1/2*b*x + 2*a)^4*tan(1/2*a)
^3 + 10053*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^4 - 58788*tan(1/2*b*x + 2*a)^2*t
an(1/2*a)^5 + 123954*tan(1/2*b*x + 2*a)*tan(1/2*a)^6 - 80658*tan(1/2*a)^7 -
 9*tan(1/2*b*x + 2*a)^4*tan(1/2*a) - 18*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^2 +
 1077*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 - 4518*tan(1/2*b*x + 2*a)*tan(1/2*a
)^4 + 5778*tan(1/2*a)^5 - tan(1/2*b*x + 2*a)^3 + 9*tan(1/2*b*x + 2*a)^2*tan
(1/2*a) - 27*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 + 54*tan(1/2*a)^3)/((27*tan(1/
2*a)^15 - 270*tan(1/2*a)^13 + 981*tan(1/2*a)^11 - 1540*tan(1/2*a)^9 + 981*t
an(1/2*a)^7 - 270*tan(1/2*a)^5 + 27*tan(1/2*a)^3)*(3*tan(1/2*b*x + 2*a)^2*t
an(1/2*a)^5 - tan(1/2*b*x + 2*a)*tan(1/2*a)^6 - 10*tan(1/2*b*x + 2*a)^2*tan
(1/2*a)^3 + 15*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 - 3*tan(1/2*a)^5 + 3*tan(1/2
*b*x + 2*a)^2*tan(1/2*a) - 15*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 + 10*tan(1/2*
a)^3 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a))^3) - 24*log(abs(tan(1/2*b*x + 2*a
)*tan(1/2*a)^3 + 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - tan(1/2*a)^3 - 3*tan(1
/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 - tan(1/2*b*x + 2*a) + 3*tan(1/2*
a) - 1)) + 24*log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a

```
)*tan(1/2*a)^2 + tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2
*a)^2 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a) - 1)))/b
```

3.70 $\int \csc^3(a+bx)\csc^2(2a+2bx)\,dx$

Optimal. Leaf size=70

$$\frac{15\sec(a+bx)}{32b}-\frac{15\tanh^{-1}(\cos(a+bx))}{32b}-\frac{\csc^4(a+bx)\sec(a+bx)}{16b}-\frac{5\csc^2(a+bx)\sec(a+bx)}{32b}$$

```
[Out] (-15*ArcTanh[Cos[a + b*x]])/(32*b) + (15*Sec[a + b*x])/(32*b) - (5*Csc[a +
b*x]^2*Sec[a + b*x])/(32*b) - (Csc[a + b*x]^4*Sec[a + b*x])/(16*b)
```

Rubi [A] time = 0.0726865, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4288, 2622, 288, 321, 207}

$$\frac{15\sec(a+bx)}{32b}-\frac{15\tanh^{-1}(\cos(a+bx))}{32b}-\frac{\csc^4(a+bx)\sec(a+bx)}{16b}-\frac{5\csc^2(a+bx)\sec(a+bx)}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] (-15*ArcTanh[Cos[a + b*x]])/(32*b) + (15*Sec[a + b*x])/(32*b) - (5*Csc[a +
b*x]^2*Sec[a + b*x])/(32*b) - (Csc[a + b*x]^4*Sec[a + b*x])/(16*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \csc^3(a+bx)\csc^2(2a+2bx)\,dx &= \frac{1}{4}\int \csc^5(a+bx)\sec^2(a+bx)\,dx \\
&= \frac{\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^3}\,dx, x, \sec(a+bx)\right)}{4b} \\
&= -\frac{\csc^4(a+bx)\sec(a+bx)}{16b} + \frac{5\,\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, \sec(a+bx)\right)}{16b} \\
&= -\frac{5\csc^2(a+bx)\sec(a+bx)}{32b} - \frac{\csc^4(a+bx)\sec(a+bx)}{16b} + \frac{15\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \text{se}\right.}{32b} \\
&= \frac{15\sec(a+bx)}{32b} - \frac{5\csc^2(a+bx)\sec(a+bx)}{32b} - \frac{\csc^4(a+bx)\sec(a+bx)}{16b} + \frac{15\,\text{Subst}\,(}{} \\
&= -\frac{15\tanh^{-1}(\cos(a+bx))}{32b} + \frac{15\sec(a+bx)}{32b} - \frac{5\csc^2(a+bx)\sec(a+bx)}{32b} - \frac{\csc^4(a+}{}
\end{aligned}$$

Mathematica [A] time = 4.63286, size = 129, normalized size = 1.84

$$-\frac{\csc^4\left(\frac{1}{2}(a+bx)\right)+14\csc^2\left(\frac{1}{2}(a+bx)\right)+\frac{\sec^2\left(\frac{1}{2}(a+bx)\right)\left(-14\tan^2\left(\frac{1}{2}(a+bx)\right)+\cos(a+bx)\left(\sec^4\left(\frac{1}{2}(a+bx)\right)-8\left(-15\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)+15\log\left(\cos\right.\right.\right.\right.}{\tan^2\left(\frac{1}{2}(a+bx)\right)-1}}{256b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] -(14*Csc[(a + b*x)/2]^2 + Csc[(a + b*x)/2]^4 + (Sec[(a + b*x)/2]^2*(78 + Co
s[a + b*x]*(-8*(8 + 15*Log[Cos[(a + b*x)/2]] - 15*Log[Sin[(a + b*x)/2]]) +
Sec[(a + b*x)/2]^4) - 14*Tan[(a + b*x)/2]^2))/(-1 + Tan[(a + b*x)/2]^2))/(2
56*b)
```

Maple [A] time = 0.036, size = 78, normalized size = 1.1

$$-\frac{1}{16\,b\,(\sin(bx+a))^4\cos(bx+a)} - \frac{5}{32\,b\,(\sin(bx+a))^2\cos(bx+a)} + \frac{15}{32\,b\cos(bx+a)} + \frac{15\,\ln(\csc(bx+a)-\cot(bx+}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*csc(2*b*x+2*a)^2,x)
```

```
[Out] -1/16/b/sin(b*x+a)^4/cos(b*x+a)-5/32/b/sin(b*x+a)^2/cos(b*x+a)+15/32/b/cos(
b*x+a)+15/32/b*ln(csc(b*x+a)-cot(b*x+a))
```

Maxima [B] time = 1.34277, size = 3020, normalized size = 43.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] 1/64*(4*(15*cos(9*b*x + 9*a) - 40*cos(7*b*x + 7*a) + 18*cos(5*b*x + 5*a) -
40*cos(3*b*x + 3*a) + 15*cos(b*x + a))*cos(10*b*x + 10*a) - 60*(3*cos(8*b*x
 + 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) - 1)
*cos(9*b*x + 9*a) + 12*(40*cos(7*b*x + 7*a) - 18*cos(5*b*x + 5*a) + 40*cos(
3*b*x + 3*a) - 15*cos(b*x + a))*cos(8*b*x + 8*a) - 160*(2*cos(6*b*x + 6*a)
+ 2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(7*b*x + 7*a) + 8*(18*cos
(5*b*x + 5*a) - 40*cos(3*b*x + 3*a) + 15*cos(b*x + a))*cos(6*b*x + 6*a) + 7
2*(2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(5*b*x + 5*a) - 40*(8*co
s(3*b*x + 3*a) - 3*cos(b*x + a))*cos(4*b*x + 4*a) + 160*(3*cos(2*b*x + 2*a)
 - 1)*cos(3*b*x + 3*a) - 180*cos(2*b*x + 2*a)*cos(b*x + a) + 15*(2*(3*cos(8
*b*x + 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a)
- 1)*cos(10*b*x + 10*a) - cos(10*b*x + 10*a)^2 + 6*(2*cos(6*b*x + 6*a) + 2*
cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - 9*cos(8*b*x +
 8*a)^2 - 4*(2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a)
- 4*cos(6*b*x + 6*a)^2 + 4*(3*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 4*co
s(4*b*x + 4*a)^2 - 9*cos(2*b*x + 2*a)^2 + 2*(3*sin(8*b*x + 8*a) - 2*sin(6*b
*x + 6*a) - 2*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - s
in(10*b*x + 10*a)^2 + 6*(2*sin(6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - 3*sin(2*
b*x + 2*a))*sin(8*b*x + 8*a) - 9*sin(8*b*x + 8*a)^2 - 4*(2*sin(4*b*x + 4*a)
 - 3*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 4*sin(6*b*x + 6*a)^2 - 4*sin(4*b*
x + 4*a)^2 + 12*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 9*sin(2*b*x + 2*a)^2 +
6*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin
(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 15*(2*(3*cos(8*b*x + 8*a) - 2*cos
(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) - 1)*cos(10*b*x + 1
0*a) - cos(10*b*x + 10*a)^2 + 6*(2*cos(6*b*x + 6*a) + 2*cos(4*b*x + 4*a) -
3*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - 9*cos(8*b*x + 8*a)^2 - 4*(2*cos(
4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) - 4*cos(6*b*x + 6*a
)^2 + 4*(3*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 4*cos(4*b*x + 4*a)^2 -
9*cos(2*b*x + 2*a)^2 + 2*(3*sin(8*b*x + 8*a) - 2*sin(6*b*x + 6*a) - 2*sin(4
*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - sin(10*b*x + 10*a)^2
 + 6*(2*sin(6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(8*b
*x + 8*a) - 9*sin(8*b*x + 8*a)^2 - 4*(2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*
a))*sin(6*b*x + 6*a) - 4*sin(6*b*x + 6*a)^2 - 4*sin(4*b*x + 4*a)^2 + 12*sin
(4*b*x + 4*a)*sin(2*b*x + 2*a) - 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2*a)
- 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x
)*sin(a) + sin(a)^2) + 4*(15*sin(9*b*x + 9*a) - 40*sin(7*b*x + 7*a) + 18*si
n(5*b*x + 5*a) - 40*sin(3*b*x + 3*a) + 15*sin(b*x + a))*sin(10*b*x + 10*a)
- 60*(3*sin(8*b*x + 8*a) - 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) + 3*sin(
2*b*x + 2*a))*sin(9*b*x + 9*a) + 12*(40*sin(7*b*x + 7*a) - 18*sin(5*b*x + 5
*a) + 40*sin(3*b*x + 3*a) - 15*sin(b*x + a))*sin(8*b*x + 8*a) - 160*(2*sin(
6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(7*b*x + 7*a) +
8*(18*sin(5*b*x + 5*a) - 40*sin(3*b*x + 3*a) + 15*sin(b*x + a))*sin(6*b*x +
 6*a) + 72*(2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(5*b*x + 5*a) - 40*
(8*sin(3*b*x + 3*a) - 3*sin(b*x + a))*sin(4*b*x + 4*a) + 480*sin(3*b*x + 3*
a)*sin(2*b*x + 2*a) - 180*sin(2*b*x + 2*a)*sin(b*x + a) + 60*cos(b*x + a))/
(b*cos(10*b*x + 10*a)^2 + 9*b*cos(8*b*x + 8*a)^2 + 4*b*cos(6*b*x + 6*a)^2 +
 4*b*cos(4*b*x + 4*a)^2 + 9*b*cos(2*b*x + 2*a)^2 + b*sin(10*b*x + 10*a)^2 +
 9*b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 + 4*b*sin(4*b*x + 4*a)^2 -
 12*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 9*b*sin(2*b*x + 2*a)^2 - 2*(3*b*c
os(8*b*x + 8*a) - 2*b*cos(6*b*x + 6*a) - 2*b*cos(4*b*x + 4*a) + 3*b*cos(2*b
*x + 2*a) - b)*cos(10*b*x + 10*a) - 6*(2*b*cos(6*b*x + 6*a) + 2*b*cos(4*b*x
```

```
 + 4*a) - 3*b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) + 4*(2*b*cos(4*b*x + 4
*a) - 3*b*cos(2*b*x + 2*a) + b)*cos(6*b*x + 6*a) - 4*(3*b*cos(2*b*x + 2*a)
- b)*cos(4*b*x + 4*a) - 6*b*cos(2*b*x + 2*a) - 2*(3*b*sin(8*b*x + 8*a) - 2*
b*sin(6*b*x + 6*a) - 2*b*sin(4*b*x + 4*a) + 3*b*sin(2*b*x + 2*a))*sin(10*b*
x + 10*a) - 6*(2*b*sin(6*b*x + 6*a) + 2*b*sin(4*b*x + 4*a) - 3*b*sin(2*b*x
+ 2*a))*sin(8*b*x + 8*a) + 4*(2*b*sin(4*b*x + 4*a) - 3*b*sin(2*b*x + 2*a))*
sin(6*b*x + 6*a) + b)
```

Fricas [B] time = 0.508706, size = 374, normalized size = 5.34

$$\frac{30\,\cos(bx+a)^4-50\,\cos(bx+a)^2-15\left(\cos(bx+a)^5-2\,\cos(bx+a)^3+\cos(bx+a)\right)\log\left(\frac{1}{2}\,\cos(bx+a)+\frac{1}{2}\right)+15\left(\mathrm{c}\right.}{64\left(b\cos(bx+a)^5-2\,b\cos(bx+a)^3+b\cos(bx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] 1/64*(30*cos(b*x + a)^4 - 50*cos(b*x + a)^2 - 15*(cos(b*x + a)^5 - 2*cos(b*
x + a)^3 + cos(b*x + a))*log(1/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^5 -
 2*cos(b*x + a)^3 + cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2) + 16)/(b*cos
(b*x + a)^5 - 2*b*cos(b*x + a)^3 + b*cos(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \csc^3(a+bx)\csc^2(2a+2bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*csc(2*b*x+2*a)**2,x)
```

```
[Out] Integral(csc(a + b*x)**3*csc(2*a + 2*b*x)**2, x)
```

Giac [B] time = 4.4033, size = 5096, normalized size = 72.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] -1/256*(128*(6*tan(1/2*b*x + 2*a)*tan(1/2*a)^11 - tan(1/2*a)^12 - 2*tan(1/2
*b*x + 2*a)*tan(1/2*a)^9 + 12*tan(1/2*a)^10 - 36*tan(1/2*b*x + 2*a)*tan(1/2
*a)^7 + 27*tan(1/2*a)^8 - 36*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - 2*tan(1/2*b*
x + 2*a)*tan(1/2*a)^3 - 27*tan(1/2*a)^4 + 6*tan(1/2*b*x + 2*a)*tan(1/2*a) -
 12*tan(1/2*a)^2 + 1)/((tan(1/2*b*x + 2*a)^2*tan(1/2*a)^6 - 15*tan(1/2*b*x
+ 2*a)^2*tan(1/2*a)^4 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - tan(1/2*a)^6 +
 15*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2 - 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3
+ 15*tan(1/2*a)^4 - tan(1/2*b*x + 2*a)^2 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a)
 - 15*tan(1/2*a)^2 + 1)*(tan(1/2*a)^6 - 15*tan(1/2*a)^4 + 15*tan(1/2*a)^2 -
 1)) + (108*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^45 - 54*tan(1/2*b*x + 2*a)^6*ta
```

n(1/2*a)^46 + 12*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^47 - tan(1/2*b*x + 2*a)^4*
tan(1/2*a)^48 + 4428*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^43 - 2880*tan(1/2*b*x
+ 2*a)^6*tan(1/2*a)^44 + 536*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^45 + 48*tan(1/
2*b*x + 2*a)^4*tan(1/2*a)^46 - 12*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^47 - 1532
16*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^41 + 210486*tan(1/2*b*x + 2*a)^6*tan(1/2
*a)^42 - 95832*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^43 + 16740*tan(1/2*b*x + 2*a
)^4*tan(1/2*a)^44 - 536*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^45 - 54*tan(1/2*b*x
 + 2*a)^2*tan(1/2*a)^46 + 2486912*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^39 - 4777
344*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^40 + 3337020*tan(1/2*b*x + 2*a)^5*tan(1
/2*a)^41 - 1046736*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^42 + 142488*tan(1/2*b*x
+ 2*a)^3*tan(1/2*a)^43 - 5472*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^44 - 108*tan(
1/2*b*x + 2*a)*tan(1/2*a)^45 - 20891364*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^37
+ 54465762*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^38 - 52518388*tan(1/2*b*x + 2*a)
^5*tan(1/2*a)^39 + 23995332*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^40 - 5409324*ta
n(1/2*b*x + 2*a)^3*tan(1/2*a)^41 + 546366*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^4
2 - 16092*tan(1/2*b*x + 2*a)*tan(1/2*a)^43 - 162*tan(1/2*a)^44 + 95841468*t
an(1/2*b*x + 2*a)^7*tan(1/2*a)^35 - 336486592*tan(1/2*b*x + 2*a)^6*tan(1/2*
a)^36 + 436639944*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^37 - 272279280*tan(1/2*b*
x + 2*a)^4*tan(1/2*a)^38 + 87480580*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^39 - 14
094720*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^40 + 977472*tan(1/2*b*x + 2*a)*tan(1
/2*a)^41 - 17280*tan(1/2*a)^42 - 234315648*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^
33 + 1144864350*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^34 - 2010477960*tan(1/2*b*x
 + 2*a)^5*tan(1/2*a)^35 + 1680996460*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^36 - 7
29548040*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^37 + 164195226*tan(1/2*b*x + 2*a)^
2*tan(1/2*a)^38 - 17453120*tan(1/2*b*x + 2*a)*tan(1/2*a)^39 + 642384*tan(1/
2*a)^40 + 211641984*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^31 - 1888388352*tan(1/2
*b*x + 2*a)^6*tan(1/2*a)^32 + 4935828252*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^33
 - 5726288880*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^34 + 3352125960*tan(1/2*b*x +
 2*a)^3*tan(1/2*a)^35 - 1012326432*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^36 + 147
189588*tan(1/2*b*x + 2*a)*tan(1/2*a)^37 - 7902336*tan(1/2*a)^38 + 314808792
*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^29 + 96589076*tan(1/2*b*x + 2*a)^6*tan(1/2
*a)^30 - 4440981672*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^31 + 9447759099*tan(1/2
*b*x + 2*a)^4*tan(1/2*a)^32 - 8215340892*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^33
 + 3430559358*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^34 - 671389308*tan(1/2*b*x +
2*a)*tan(1/2*a)^35 + 48570730*tan(1/2*a)^36 - 939066728*tan(1/2*b*x + 2*a)^
7*tan(1/2*a)^27 + 4808309376*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^28 - 665521204
8*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^29 - 466539808*tan(1/2*b*x + 2*a)^4*tan(1
/2*a)^30 + 7397865576*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^31 - 5649253632*tan(1
/2*b*x + 2*a)^2*tan(1/2*a)^32 + 1635149568*tan(1/2*b*x + 2*a)*tan(1/2*a)^33
 - 162842112*tan(1/2*a)^34 + 564924672*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^25 -
 6391344852*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^26 + 19663795984*tan(1/2*b*x +
2*a)^5*tan(1/2*a)^27 - 24029264184*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^28 + 110
50166160*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^29 + 310355316*tan(1/2*b*x + 2*a)^
2*tan(1/2*a)^30 - 1477416960*tan(1/2*b*x + 2*a)*tan(1/2*a)^31 + 267141600*t
an(1/2*a)^32 + 564924672*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^23 - 11879791560*t
an(1/2*b*x + 2*a)^5*tan(1/2*a)^25 + 31957055712*tan(1/2*b*x + 2*a)^4*tan(1/
2*a)^26 - 32793412368*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^27 + 14381865792*tan(
1/2*b*x + 2*a)^2*tan(1/2*a)^28 - 2181545880*tan(1/2*b*x + 2*a)*tan(1/2*a)^2
9 - 17553920*tan(1/2*a)^30 - 939066728*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^21 +
 6391344852*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^22 - 11879791560*tan(1/2*b*x +
2*a)^5*tan(1/2*a)^23 + 19856454696*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^25 - 192
58200036*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^26 + 6571946504*tan(1/2*b*x + 2*a)
*tan(1/2*a)^27 - 680794644*tan(1/2*a)^28 + 314808792*tan(1/2*b*x + 2*a)^7*t
an(1/2*a)^19 - 4808309376*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^20 + 19663795984*
tan(1/2*b*x + 2*a)^5*tan(1/2*a)^21 - 31957055712*tan(1/2*b*x + 2*a)^4*tan(1
/2*a)^22 + 19856454696*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^23 - 4012160256*tan(
1/2*b*x + 2*a)*tan(1/2*a)^25 + 925883136*tan(1/2*a)^26 + 211641984*tan(1/2*
b*x + 2*a)^7*tan(1/2*a)^17 - 96589076*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^18 -
6655212048*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^19 + 24029264184*tan(1/2*b*x + 2

```
*a)^4*tan(1/2*a)^20 - 32793412368*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^21 + 1925
8200036*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^22 - 4012160256*tan(1/2*b*x + 2*a)*
tan(1/2*a)^23 - 234315648*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^15 + 1888388352*t
an(1/2*b*x + 2*a)^6*tan(1/2*a)^16 - 4440981672*tan(1/2*b*x + 2*a)^5*tan(1/2
*a)^17 + 466539808*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^18 + 11050166160*tan(1/2
*b*x + 2*a)^3*tan(1/2*a)^19 - 14381865792*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2
0 + 6571946504*tan(1/2*b*x + 2*a)*tan(1/2*a)^21 - 925883136*tan(1/2*a)^22 +
 95841468*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^13 - 1144864350*tan(1/2*b*x + 2*a
)^6*tan(1/2*a)^14 + 4935828252*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^15 - 9447759
099*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^16 + 7397865576*tan(1/2*b*x + 2*a)^3*ta
n(1/2*a)^17 - 310355316*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^18 - 2181545880*tan
(1/2*b*x + 2*a)*tan(1/2*a)^19 + 680794644*tan(1/2*a)^20 - 20891364*tan(1/2*
b*x + 2*a)^7*tan(1/2*a)^11 + 336486592*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^12 -
 2010477960*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^13 + 5726288880*tan(1/2*b*x + 2
*a)^4*tan(1/2*a)^14 - 8215340892*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^15 + 56492
53632*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^16 - 1477416960*tan(1/2*b*x + 2*a)*ta
n(1/2*a)^17 + 17553920*tan(1/2*a)^18 + 2486912*tan(1/2*b*x + 2*a)^7*tan(1/2
*a)^9 - 54465762*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^10 + 436639944*tan(1/2*b*x
 + 2*a)^5*tan(1/2*a)^11 - 1680996460*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^12 + 3
352125960*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^13 - 3430559358*tan(1/2*b*x + 2*a
)^2*tan(1/2*a)^14 + 1635149568*tan(1/2*b*x + 2*a)*tan(1/2*a)^15 - 267141600
*tan(1/2*a)^16 - 153216*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^7 + 4777344*tan(1/2
*b*x + 2*a)^6*tan(1/2*a)^8 - 52518388*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^9 + 2
72279280*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^10 - 729548040*tan(1/2*b*x + 2*a)^
3*tan(1/2*a)^11 + 1012326432*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^12 - 671389308
*tan(1/2*b*x + 2*a)*tan(1/2*a)^13 + 162842112*tan(1/2*a)^14 + 4428*tan(1/2*
b*x + 2*a)^7*tan(1/2*a)^5 - 210486*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^6 + 3337
020*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^7 - 23995332*tan(1/2*b*x + 2*a)^4*tan(1
/2*a)^8 + 87480580*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^9 - 164195226*tan(1/2*b*
x + 2*a)^2*tan(1/2*a)^10 + 147189588*tan(1/2*b*x + 2*a)*tan(1/2*a)^11 - 485
70730*tan(1/2*a)^12 + 108*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^3 + 2880*tan(1/2*
b*x + 2*a)^6*tan(1/2*a)^4 - 95832*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^5 + 10467
36*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^6 - 5409324*tan(1/2*b*x + 2*a)^3*tan(1/2
*a)^7 + 14094720*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^8 - 17453120*tan(1/2*b*x +
 2*a)*tan(1/2*a)^9 + 7902336*tan(1/2*a)^10 + 54*tan(1/2*b*x + 2*a)^6*tan(1/
2*a)^2 + 536*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^3 - 16740*tan(1/2*b*x + 2*a)^4
*tan(1/2*a)^4 + 142488*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^5 - 546366*tan(1/2*b
*x + 2*a)^2*tan(1/2*a)^6 + 977472*tan(1/2*b*x + 2*a)*tan(1/2*a)^7 - 642384*
tan(1/2*a)^8 + 12*tan(1/2*b*x + 2*a)^5*tan(1/2*a) - 48*tan(1/2*b*x + 2*a)^4
*tan(1/2*a)^2 - 536*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^3 + 5472*tan(1/2*b*x +
2*a)^2*tan(1/2*a)^4 - 16092*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 + 17280*tan(1/2
*a)^6 + tan(1/2*b*x + 2*a)^4 - 12*tan(1/2*b*x + 2*a)^3*tan(1/2*a) + 54*tan(
1/2*b*x + 2*a)^2*tan(1/2*a)^2 - 108*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 162*t
an(1/2*a)^4)/((81*tan(1/2*a)^20 - 1080*tan(1/2*a)^18 + 5724*tan(1/2*a)^16 -
 15240*tan(1/2*a)^14 + 21286*tan(1/2*a)^12 - 15240*tan(1/2*a)^10 + 5724*tan
(1/2*a)^8 - 1080*tan(1/2*a)^6 + 81*tan(1/2*a)^4)*(3*tan(1/2*b*x + 2*a)^2*ta
n(1/2*a)^5 - tan(1/2*b*x + 2*a)*tan(1/2*a)^6 - 10*tan(1/2*b*x + 2*a)^2*tan(
1/2*a)^3 + 15*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 - 3*tan(1/2*a)^5 + 3*tan(1/2*
b*x + 2*a)^2*tan(1/2*a) - 15*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 + 10*tan(1/2*a
)^3 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a))^4) + 120*log(abs(tan(1/2*b*x + 2*a
)*tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 - 1)) - 1
20*log(abs(3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - tan(1/2*a)^3 - tan(1/2*b*x +
 2*a) + 3*tan(1/2*a))))/b
```

3.71 $\int \csc^3(a+bx)\csc^3(2a+2bx)\,dx$

Optimal. Leaf size=81

$$-\frac{7\csc^5(a+bx)}{80b}-\frac{7\csc^3(a+bx)}{48b}-\frac{7\csc(a+bx)}{16b}+\frac{7\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\csc^5(a+bx)\sec^2(a+bx)}{16b}$$

```
[Out] (7*ArcTanh[Sin[a + b*x]])/(16*b) - (7*Csc[a + b*x])/(16*b) - (7*Csc[a + b*x
]^3)/(48*b) - (7*Csc[a + b*x]^5)/(80*b) + (Csc[a + b*x]^5*Sec[a + b*x]^2)/(
16*b)
```

Rubi [A] time = 0.0731432, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4288, 2621, 288, 302, 207}

$$-\frac{7\csc^5(a+bx)}{80b}-\frac{7\csc^3(a+bx)}{48b}-\frac{7\csc(a+bx)}{16b}+\frac{7\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\csc^5(a+bx)\sec^2(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] (7*ArcTanh[Sin[a + b*x]])/(16*b) - (7*Csc[a + b*x])/(16*b) - (7*Csc[a + b*x
]^3)/(48*b) - (7*Csc[a + b*x]^5)/(80*b) + (Csc[a + b*x]^5*Sec[a + b*x]^2)/(
16*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^3(a+bx)\csc^3(2a+2bx)\,dx = \frac{1}{8}\int \csc^6(a+bx)\sec^3(a+bx)\,dx$$

$$= -\frac{\text{Subst}\left(\int \frac{x^8}{(-1+x^2)^2}\,dx, x, \csc(a+bx)\right)}{8b}$$

$$= \frac{\csc^5(a+bx)\sec^2(a+bx)}{16b} - \frac{7\,\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{16b}$$

$$= \frac{\csc^5(a+bx)\sec^2(a+bx)}{16b} - \frac{7\,\text{Subst}\left(\int \left(1+x^2+x^4+\frac{1}{-1+x^2}\right)dx, x, \csc(a+bx)\right)}{16b}$$

$$= -\frac{7\csc(a+bx)}{16b} - \frac{7\csc^3(a+bx)}{48b} - \frac{7\csc^5(a+bx)}{80b} + \frac{\csc^5(a+bx)\sec^2(a+bx)}{16b} - \frac{7\,\text{S}}{}$$

$$= \frac{7\tanh^{-1}(\sin(a+bx))}{16b} - \frac{7\csc(a+bx)}{16b} - \frac{7\csc^3(a+bx)}{48b} - \frac{7\csc^5(a+bx)}{80b} + \frac{\csc^5(a+}{}$$

Mathematica [C] time = 0.0347099, size = 31, normalized size = 0.38

$$-\frac{\csc^5(a+bx)\text{Hypergeometric2F1}\left(-\frac{5}{2},2,-\frac{3}{2},\sin^2(a+bx)\right)}{40b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] -(Csc[a + b*x]^5*Hypergeometric2F1[-5/2, 2, -3/2, Sin[a + b*x]^2])/(40*b)
```

Maple [A] time = 0.039, size = 97, normalized size = 1.2

$$-\frac{1}{40\,b\,(\sin(bx+a))^5(\cos(bx+a))^2} - \frac{7}{120\,b\,(\sin(bx+a))^3(\cos(bx+a))^2} + \frac{7}{48\,b\sin(bx+a)(\cos(bx+a))^2} - \frac{7}{16\,b\sin(b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*csc(2*b*x+2*a)^3,x)
```

```
[Out] -1/40/b/sin(b*x+a)^5/cos(b*x+a)^2-7/120/b/sin(b*x+a)^3/cos(b*x+a)^2+7/48/b/
sin(b*x+a)/cos(b*x+a)^2-7/16/b/sin(b*x+a)+7/16/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [B] time = 2.32027, size = 4178, normalized size = 51.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 1/480*(4*(105*sin(13*b*x + 13*a) - 350*sin(11*b*x + 11*a) + 231*sin(9*b*x +
 9*a) + 412*sin(7*b*x + 7*a) + 231*sin(5*b*x + 5*a) - 350*sin(3*b*x + 3*a)
+ 105*sin(b*x + a))*cos(14*b*x + 14*a) + 420*(3*sin(12*b*x + 12*a) - sin(10
*b*x + 10*a) - 5*sin(8*b*x + 8*a) + 5*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) -
 3*sin(2*b*x + 2*a))*cos(13*b*x + 13*a) + 12*(350*sin(11*b*x + 11*a) - 231*
sin(9*b*x + 9*a) - 412*sin(7*b*x + 7*a) - 231*sin(5*b*x + 5*a) + 350*sin(3*
b*x + 3*a) - 105*sin(b*x + a))*cos(12*b*x + 12*a) + 1400*(sin(10*b*x + 10*a
) + 5*sin(8*b*x + 8*a) - 5*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) + 3*sin(2*b*
x + 2*a))*cos(11*b*x + 11*a) + 4*(231*sin(9*b*x + 9*a) + 412*sin(7*b*x + 7*
a) + 231*sin(5*b*x + 5*a) - 350*sin(3*b*x + 3*a) + 105*sin(b*x + a))*cos(10
*b*x + 10*a) - 924*(5*sin(8*b*x + 8*a) - 5*sin(6*b*x + 6*a) - sin(4*b*x + 4
*a) + 3*sin(2*b*x + 2*a))*cos(9*b*x + 9*a) + 20*(412*sin(7*b*x + 7*a) + 231
*sin(5*b*x + 5*a) - 350*sin(3*b*x + 3*a) + 105*sin(b*x + a))*cos(8*b*x + 8*
a) + 1648*(5*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*cos(
7*b*x + 7*a) - 140*(33*sin(5*b*x + 5*a) - 50*sin(3*b*x + 3*a) + 15*sin(b*x
+ a))*cos(6*b*x + 6*a) + 924*(sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*cos(5*
b*x + 5*a) + 140*(10*sin(3*b*x + 3*a) - 3*sin(b*x + a))*cos(4*b*x + 4*a) +
105*(2*(3*cos(12*b*x + 12*a) - cos(10*b*x + 10*a) - 5*cos(8*b*x + 8*a) + 5*
cos(6*b*x + 6*a) + cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(14*b*x +
14*a) - cos(14*b*x + 14*a)^2 + 6*(cos(10*b*x + 10*a) + 5*cos(8*b*x + 8*a) -
 5*cos(6*b*x + 6*a) - cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) - 1)*cos(12*b*x
 + 12*a) - 9*cos(12*b*x + 12*a)^2 - 2*(5*cos(8*b*x + 8*a) - 5*cos(6*b*x + 6
*a) - cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) - 1)*cos(10*b*x + 10*a) - cos(1
0*b*x + 10*a)^2 + 10*(5*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) - 3*cos(2*b*x +
 2*a) + 1)*cos(8*b*x + 8*a) - 25*cos(8*b*x + 8*a)^2 - 10*(cos(4*b*x + 4*a)
- 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) - 25*cos(6*b*x + 6*a)^2 + 2*(3*c
os(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 9*cos(2*b*x +
2*a)^2 + 2*(3*sin(12*b*x + 12*a) - sin(10*b*x + 10*a) - 5*sin(8*b*x + 8*a)
+ 5*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(14*b*x +
14*a) - sin(14*b*x + 14*a)^2 + 6*(sin(10*b*x + 10*a) + 5*sin(8*b*x + 8*a) -
 5*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(12*b*x + 1
2*a) - 9*sin(12*b*x + 12*a)^2 - 2*(5*sin(8*b*x + 8*a) - 5*sin(6*b*x + 6*a)
- sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - sin(10*b*x +
10*a)^2 + 10*(5*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*s
in(8*b*x + 8*a) - 25*sin(8*b*x + 8*a)^2 - 10*(sin(4*b*x + 4*a) - 3*sin(2*b*
x + 2*a))*sin(6*b*x + 6*a) - 25*sin(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 + 6
*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2
*a) - 1)*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b
*x + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a
)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a)
+ sin(a)^2)) - 4*(105*cos(13*b*x + 13*a) - 350*cos(11*b*x + 11*a) + 231*cos
(9*b*x + 9*a) + 412*cos(7*b*x + 7*a) + 231*cos(5*b*x + 5*a) - 350*cos(3*b*x
 + 3*a) + 105*cos(b*x + a))*sin(14*b*x + 14*a) - 420*(3*cos(12*b*x + 12*a)
- cos(10*b*x + 10*a) - 5*cos(8*b*x + 8*a) + 5*cos(6*b*x + 6*a) + cos(4*b*x
+ 4*a) - 3*cos(2*b*x + 2*a) + 1)*sin(13*b*x + 13*a) - 12*(350*cos(11*b*x +
11*a) - 231*cos(9*b*x + 9*a) - 412*cos(7*b*x + 7*a) - 231*cos(5*b*x + 5*a)
+ 350*cos(3*b*x + 3*a) - 105*cos(b*x + a))*sin(12*b*x + 12*a) - 1400*(cos(1
0*b*x + 10*a) + 5*cos(8*b*x + 8*a) - 5*cos(6*b*x + 6*a) - cos(4*b*x + 4*a)
+ 3*cos(2*b*x + 2*a) - 1)*sin(11*b*x + 11*a) - 4*(231*cos(9*b*x + 9*a) + 41
2*cos(7*b*x + 7*a) + 231*cos(5*b*x + 5*a) - 350*cos(3*b*x + 3*a) + 105*cos(
b*x + a))*sin(10*b*x + 10*a) + 924*(5*cos(8*b*x + 8*a) - 5*cos(6*b*x + 6*a)
 - cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) - 1)*sin(9*b*x + 9*a) - 20*(412*co
s(7*b*x + 7*a) + 231*cos(5*b*x + 5*a) - 350*cos(3*b*x + 3*a) + 105*cos(b*x
+ a))*sin(8*b*x + 8*a) - 1648*(5*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) - 3*co
s(2*b*x + 2*a) + 1)*sin(7*b*x + 7*a) + 140*(33*cos(5*b*x + 5*a) - 50*cos(3*
b*x + 3*a) + 15*cos(b*x + a))*sin(6*b*x + 6*a) - 924*(cos(4*b*x + 4*a) - 3*
cos(2*b*x + 2*a) + 1)*sin(5*b*x + 5*a) - 140*(10*cos(3*b*x + 3*a) - 3*cos(b
```

```
*x + a))*sin(4*b*x + 4*a) - 1400*(3*cos(2*b*x + 2*a) - 1)*sin(3*b*x + 3*a)
+ 4200*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) - 1260*cos(b*x + a)*sin(2*b*x + 2*
a) + 1260*cos(2*b*x + 2*a)*sin(b*x + a) - 420*sin(b*x + a))/(b*cos(14*b*x +
 14*a)^2 + 9*b*cos(12*b*x + 12*a)^2 + b*cos(10*b*x + 10*a)^2 + 25*b*cos(8*b
*x + 8*a)^2 + 25*b*cos(6*b*x + 6*a)^2 + b*cos(4*b*x + 4*a)^2 + 9*b*cos(2*b*
x + 2*a)^2 + b*sin(14*b*x + 14*a)^2 + 9*b*sin(12*b*x + 12*a)^2 + b*sin(10*b
*x + 10*a)^2 + 25*b*sin(8*b*x + 8*a)^2 + 25*b*sin(6*b*x + 6*a)^2 + b*sin(4*
b*x + 4*a)^2 - 6*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 9*b*sin(2*b*x + 2*a)
^2 - 2*(3*b*cos(12*b*x + 12*a) - b*cos(10*b*x + 10*a) - 5*b*cos(8*b*x + 8*a
) + 5*b*cos(6*b*x + 6*a) + b*cos(4*b*x + 4*a) - 3*b*cos(2*b*x + 2*a) + b)*c
os(14*b*x + 14*a) - 6*(b*cos(10*b*x + 10*a) + 5*b*cos(8*b*x + 8*a) - 5*b*co
s(6*b*x + 6*a) - b*cos(4*b*x + 4*a) + 3*b*cos(2*b*x + 2*a) - b)*cos(12*b*x
+ 12*a) + 2*(5*b*cos(8*b*x + 8*a) - 5*b*cos(6*b*x + 6*a) - b*cos(4*b*x + 4*
a) + 3*b*cos(2*b*x + 2*a) - b)*cos(10*b*x + 10*a) - 10*(5*b*cos(6*b*x + 6*a
) + b*cos(4*b*x + 4*a) - 3*b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) + 10*(b
*cos(4*b*x + 4*a) - 3*b*cos(2*b*x + 2*a) + b)*cos(6*b*x + 6*a) - 2*(3*b*cos
(2*b*x + 2*a) - b)*cos(4*b*x + 4*a) - 6*b*cos(2*b*x + 2*a) - 2*(3*b*sin(12*
b*x + 12*a) - b*sin(10*b*x + 10*a) - 5*b*sin(8*b*x + 8*a) + 5*b*sin(6*b*x +
 6*a) + b*sin(4*b*x + 4*a) - 3*b*sin(2*b*x + 2*a))*sin(14*b*x + 14*a) - 6*(
b*sin(10*b*x + 10*a) + 5*b*sin(8*b*x + 8*a) - 5*b*sin(6*b*x + 6*a) - b*sin(
4*b*x + 4*a) + 3*b*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) + 2*(5*b*sin(8*b*x
+ 8*a) - 5*b*sin(6*b*x + 6*a) - b*sin(4*b*x + 4*a) + 3*b*sin(2*b*x + 2*a))*
sin(10*b*x + 10*a) - 10*(5*b*sin(6*b*x + 6*a) + b*sin(4*b*x + 4*a) - 3*b*si
n(2*b*x + 2*a))*sin(8*b*x + 8*a) + 10*(b*sin(4*b*x + 4*a) - 3*b*sin(2*b*x +
 2*a))*sin(6*b*x + 6*a) + b)
```

Fricas [B] time = 0.516404, size = 458, normalized size = 5.65

$$-\frac{210\cos(bx+a)^6-490\cos(bx+a)^4-105\left(\cos(bx+a)^6-2\cos(bx+a)^4+\cos(bx+a)^2\right)\log(\sin(bx+a)+1)\sin(}{480\left(b\cos(bx+a)^6-2b\cos(bx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] -1/480*(210*cos(b*x + a)^6 - 490*cos(b*x + a)^4 - 105*(cos(b*x + a)^6 - 2*c
os(b*x + a)^4 + cos(b*x + a)^2)*log(sin(b*x + a) + 1)*sin(b*x + a) + 105*(c
os(b*x + a)^6 - 2*cos(b*x + a)^4 + cos(b*x + a)^2)*log(-sin(b*x + a) + 1)*s
in(b*x + a) + 322*cos(b*x + a)^2 - 30)/((b*cos(b*x + a)^6 - 2*b*cos(b*x + a
)^4 + b*cos(b*x + a)^2)*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*csc(2*b*x+2*a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 11.8856, size = 8529, normalized size = 105.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] -1/3840*(480*(tan(1/2*b*x + 2*a)^3*tan(1/2*a)^24 + 30*tan(1/2*b*x + 2*a)^3*
tan(1/2*a)^22 - 6*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^23 + tan(1/2*b*x + 2*a)*t
an(1/2*a)^24 - 756*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^20 + 614*tan(1/2*b*x + 2
*a)^2*tan(1/2*a)^21 - 114*tan(1/2*b*x + 2*a)*tan(1/2*a)^22 + 6*tan(1/2*a)^2
3 + 2058*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^18 - 4578*tan(1/2*b*x + 2*a)^2*tan
(1/2*a)^19 + 1932*tan(1/2*b*x + 2*a)*tan(1/2*a)^20 - 182*tan(1/2*a)^21 - 27
*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^16 + 6210*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^
17 - 7462*tan(1/2*b*x + 2*a)*tan(1/2*a)^18 + 1554*tan(1/2*a)^19 - 9396*tan(
1/2*b*x + 2*a)^3*tan(1/2*a)^14 + 15588*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^15 -
 2331*tan(1/2*b*x + 2*a)*tan(1/2*a)^16 - 2178*tan(1/2*a)^17 - 21924*tan(1/2
*b*x + 2*a)^2*tan(1/2*a)^13 + 26028*tan(1/2*b*x + 2*a)*tan(1/2*a)^14 - 5668
*tan(1/2*a)^15 + 9396*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^10 - 21924*tan(1/2*b*
x + 2*a)^2*tan(1/2*a)^11 + 6468*tan(1/2*a)^13 + 27*tan(1/2*b*x + 2*a)^3*tan
(1/2*a)^8 + 15588*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^9 - 26028*tan(1/2*b*x + 2
*a)*tan(1/2*a)^10 + 6468*tan(1/2*a)^11 - 2058*tan(1/2*b*x + 2*a)^3*tan(1/2*
a)^6 + 6210*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^7 + 2331*tan(1/2*b*x + 2*a)*tan
(1/2*a)^8 - 5668*tan(1/2*a)^9 + 756*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^4 - 457
8*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 + 7462*tan(1/2*b*x + 2*a)*tan(1/2*a)^6
 - 2178*tan(1/2*a)^7 - 30*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^2 + 614*tan(1/2*b*
x + 2*a)^2*tan(1/2*a)^3 - 1932*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 1554*tan(1
/2*a)^5 - tan(1/2*b*x + 2*a)^3 - 6*tan(1/2*b*x + 2*a)^2*tan(1/2*a) + 114*ta
n(1/2*b*x + 2*a)*tan(1/2*a)^2 - 182*tan(1/2*a)^3 - tan(1/2*b*x + 2*a) + 6*t
an(1/2*a))/((tan(1/2*a)^12 - 30*tan(1/2*a)^10 + 255*tan(1/2*a)^8 - 452*tan(
1/2*a)^6 + 255*tan(1/2*a)^4 - 30*tan(1/2*a)^2 + 1)*(tan(1/2*b*x + 2*a)^2*ta
n(1/2*a)^6 - 15*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^4 + 12*tan(1/2*b*x + 2*a)*t
an(1/2*a)^5 - tan(1/2*a)^6 + 15*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^2 - 40*tan(
1/2*b*x + 2*a)*tan(1/2*a)^3 + 15*tan(1/2*a)^4 - tan(1/2*b*x + 2*a)^2 + 12*t
an(1/2*b*x + 2*a)*tan(1/2*a) - 15*tan(1/2*a)^2 + 1)^2) + (1215*tan(1/2*b*x
+ 2*a)^9*tan(1/2*a)^56 - 810*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^57 + 270*tan(1
/2*b*x + 2*a)^7*tan(1/2*a)^58 - 45*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^59 + 3*t
an(1/2*b*x + 2*a)^5*tan(1/2*a)^60 + 71280*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^5
4 - 59940*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^55 + 17640*tan(1/2*b*x + 2*a)^7*t
an(1/2*a)^56 - 795*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^57 - 360*tan(1/2*b*x + 2
*a)^5*tan(1/2*a)^58 + 45*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^59 + 2407725*tan(1
/2*b*x + 2*a)^9*tan(1/2*a)^52 - 2568780*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^53
+ 1013880*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^54 - 144900*tan(1/2*b*x + 2*a)^6*
tan(1/2*a)^55 - 1635*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^56 + 795*tan(1/2*b*x +
 2*a)^4*tan(1/2*a)^57 + 270*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^58 - 164020320*
tan(1/2*b*x + 2*a)^9*tan(1/2*a)^50 + 294234150*tan(1/2*b*x + 2*a)^8*tan(1/2
*a)^51 - 204257160*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^52 + 68740740*tan(1/2*b*
x + 2*a)^6*tan(1/2*a)^53 - 10958240*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^54 + 56
7720*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^55 + 17640*tan(1/2*b*x + 2*a)^3*tan(1/
2*a)^56 + 810*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^57 + 2847265170*tan(1/2*b*x +
 2*a)^9*tan(1/2*a)^48 - 7061366340*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^49 + 684
8508750*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^50 - 3360350485*tan(1/2*b*x + 2*a)^
6*tan(1/2*a)^51 + 882126585*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^52 - 115984800*
tan(1/2*b*x + 2*a)^4*tan(1/2*a)^53 + 5446200*tan(1/2*b*x + 2*a)^3*tan(1/2*a
)^54 + 86670*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^55 + 1215*tan(1/2*b*x + 2*a)*t
an(1/2*a)^56 - 25466109120*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^46 + 82199042640
*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^47 - 104745094160*tan(1/2*b*x + 2*a)^7*tan
(1/2*a)^48 + 68829859005*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^49 - 25315144296*t
```

an(1/2*b*x + 2*a)^5*tan(1/2*a)^50 + 5205627685*tan(1/2*b*x + 2*a)^4*tan(1/2
*a)^51 - 546245640*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^52 + 20895030*tan(1/2*b*
x + 2*a)^2*tan(1/2*a)^53 + 187920*tan(1/2*b*x + 2*a)*tan(1/2*a)^54 + 1458*t
an(1/2*a)^55 + 137905936830*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^44 - 5629893348
00*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^45 + 911808828960*tan(1/2*b*x + 2*a)^7*t
an(1/2*a)^46 - 768698936880*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^47 + 3700499622
95*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^48 - 104054449965*tan(1/2*b*x + 2*a)^4*t
an(1/2*a)^49 + 16540981710*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^50 - 1323596190*
tan(1/2*b*x + 2*a)^2*tan(1/2*a)^51 + 37166445*tan(1/2*b*x + 2*a)*tan(1/2*a)
^52 + 172530*tan(1/2*a)^53 - 472782926880*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^4
2 + 2429658984060*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^43 - 4936681833840*tan(1/
2*b*x + 2*a)^7*tan(1/2*a)^44 + 5227899617200*tan(1/2*b*x + 2*a)^6*tan(1/2*a
)^45 - 3184356059520*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^46 + 1153540125240*tan
(1/2*b*x + 2*a)^4*tan(1/2*a)^47 - 245839387280*tan(1/2*b*x + 2*a)^3*tan(1/2
*a)^48 + 28942646220*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^49 - 1600947360*tan(1/
2*b*x + 2*a)*tan(1/2*a)^50 + 25793640*tan(1/2*a)^51 + 985566407445*tan(1/2*
b*x + 2*a)^9*tan(1/2*a)^40 - 6594701846310*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^
41 + 17020352439990*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^42 - 22628389243545*tan
(1/2*b*x + 2*a)^6*tan(1/2*a)^43 + 17235607351635*tan(1/2*b*x + 2*a)^5*tan(1
/2*a)^44 - 7834375816360*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^45 + 2124636283680
*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^46 - 329145687420*tan(1/2*b*x + 2*a)^2*tan
(1/2*a)^47 + 25919270610*tan(1/2*b*x + 2*a)*tan(1/2*a)^48 - 753655320*tan(1
/2*a)^49 - 941937351120*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^38 + 10024213003740
*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^39 - 35619165658440*tan(1/2*b*x + 2*a)^7*t
an(1/2*a)^40 + 61660501611345*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^41 - 59571399
047240*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^42 + 33928676906985*tan(1/2*b*x + 2*
a)^4*tan(1/2*a)^43 - 11501138427120*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^44 + 22
45614722020*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^45 - 228535536960*tan(1/2*b*x +
 2*a)*tan(1/2*a)^46 + 9122039460*tan(1/2*a)^47 - 756859069305*tan(1/2*b*x +
 2*a)^9*tan(1/2*a)^36 - 2938166946540*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^37 +
33963536732760*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^38 - 93824947269780*tan(1/2*
b*x + 2*a)^6*tan(1/2*a)^39 + 124880209066749*tan(1/2*b*x + 2*a)^5*tan(1/2*a
)^40 - 92522313305985*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^41 + 39714192559670*t
an(1/2*b*x + 2*a)^3*tan(1/2*a)^42 - 9707168584980*tan(1/2*b*x + 2*a)^2*tan(
1/2*a)^43 + 1237500036990*tan(1/2*b*x + 2*a)*tan(1/2*a)^44 - 62831824476*ta
n(1/2*a)^45 + 3403775131200*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^34 - 1929084665
1030*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^35 + 27663473408040*tan(1/2*b*x + 2*a)
^7*tan(1/2*a)^36 + 27109136872980*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^37 - 1189
39479927840*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^38 + 140811947469040*tan(1/2*b*
x + 2*a)^4*tan(1/2*a)^39 - 83200102704840*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^4
0 + 26404824927630*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^41 - 4255065187040*tan(1
/2*b*x + 2*a)*tan(1/2*a)^42 + 270445183560*tan(1/2*a)^43 - 3650513198580*ta
n(1/2*b*x + 2*a)^9*tan(1/2*a)^32 + 36323787336840*tan(1/2*b*x + 2*a)^8*tan(
1/2*a)^33 - 122659045084170*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^34 + 1807661256
59295*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^35 - 97447951446495*tan(1/2*b*x + 2*a
)^5*tan(1/2*a)^36 - 40571312613240*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^37 + 792
76457792280*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^38 - 40160124218470*tan(1/2*b*x
 + 2*a)^2*tan(1/2*a)^39 + 8888251621845*tan(1/2*b*x + 2*a)*tan(1/2*a)^40 -
731644643640*tan(1/2*a)^41 - 19469403725760*tan(1/2*b*x + 2*a)^8*tan(1/2*a)
^31 + 130628272644000*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^32 - 338314460497335*
tan(1/2*b*x + 2*a)^6*tan(1/2*a)^33 + 429435328442040*tan(1/2*b*x + 2*a)^5*t
an(1/2*a)^34 - 271342502839935*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^35 + 6480904
4724520*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^36 + 11678391455970*tan(1/2*b*x + 2
*a)^2*tan(1/2*a)^37 - 8484705136560*tan(1/2*b*x + 2*a)*tan(1/2*a)^38 + 1111
051397230*tan(1/2*a)^39 + 3650513198580*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^28
- 19469403725760*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^29 + 180715347146880*tan(1
/2*b*x + 2*a)^6*tan(1/2*a)^31 - 456090384021345*tan(1/2*b*x + 2*a)^5*tan(1/
2*a)^32 + 507272799301335*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^33 - 286258153276
170*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^34 + 77334987640230*tan(1/2*b*x + 2*a)^

```
2*tan(1/2*a)^35 - 6866537353465*tan(1/2*b*x + 2*a)*tan(1/2*a)^36 - 32944462
2930*tan(1/2*a)^37 - 3403775131200*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^26 + 363
23787336840*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^27 - 130628272644000*tan(1/2*b*
x + 2*a)^7*tan(1/2*a)^28 + 180715347146880*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^
29 - 270808903495440*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^31 + 304337156718240*t
an(1/2*b*x + 2*a)^3*tan(1/2*a)^32 - 145126844230520*tan(1/2*b*x + 2*a)^2*ta
n(1/2*a)^33 + 30650585964480*tan(1/2*b*x + 2*a)*tan(1/2*a)^34 - 21355470067
68*tan(1/2*a)^35 + 756859069305*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^24 - 192908
46651030*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^25 + 122659045084170*tan(1/2*b*x +
 2*a)^7*tan(1/2*a)^26 - 338314460497335*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^27
+ 456090384021345*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^28 - 270808903495440*tan(
1/2*b*x + 2*a)^4*tan(1/2*a)^29 + 77638534829400*tan(1/2*b*x + 2*a)^2*tan(1/
2*a)^31 - 32750159843700*tan(1/2*b*x + 2*a)*tan(1/2*a)^32 + 4042939650000*t
an(1/2*a)^33 + 941937351120*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^22 - 2938166946
540*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^23 - 27663473408040*tan(1/2*b*x + 2*a)^
7*tan(1/2*a)^24 + 180766125659295*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^25 - 4294
35328442040*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^26 + 507272799301335*tan(1/2*b*
x + 2*a)^4*tan(1/2*a)^27 - 304337156718240*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^
28 + 77638534829400*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^29 - 2175308641800*tan(
1/2*a)^31 - 985566407445*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^20 + 1002421300374
0*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^21 - 33963536732760*tan(1/2*b*x + 2*a)^7*
tan(1/2*a)^22 + 27109136872980*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^23 + 9744795
1446495*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^24 - 271342502839935*tan(1/2*b*x +
2*a)^4*tan(1/2*a)^25 + 286258153276170*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^26 -
 145126844230520*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^27 + 32750159843700*tan(1/
2*b*x + 2*a)*tan(1/2*a)^28 - 2175308641800*tan(1/2*a)^29 + 472782926880*tan
(1/2*b*x + 2*a)^9*tan(1/2*a)^18 - 6594701846310*tan(1/2*b*x + 2*a)^8*tan(1/
2*a)^19 + 35619165658440*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^20 - 9382494726978
0*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^21 + 118939479927840*tan(1/2*b*x + 2*a)^5
*tan(1/2*a)^22 - 40571312613240*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^23 - 648090
44724520*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^24 + 77334987640230*tan(1/2*b*x +
2*a)^2*tan(1/2*a)^25 - 30650585964480*tan(1/2*b*x + 2*a)*tan(1/2*a)^26 + 40
42939650000*tan(1/2*a)^27 - 137905936830*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^16
 + 2429658984060*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^17 - 17020352439990*tan(1/
2*b*x + 2*a)^7*tan(1/2*a)^18 + 61660501611345*tan(1/2*b*x + 2*a)^6*tan(1/2*
a)^19 - 124880209066749*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^20 + 14081194746904
0*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^21 - 79276457792280*tan(1/2*b*x + 2*a)^3*
tan(1/2*a)^22 + 11678391455970*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^23 + 6866537
353465*tan(1/2*b*x + 2*a)*tan(1/2*a)^24 - 2135547006768*tan(1/2*a)^25 + 254
66109120*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^14 - 562989334800*tan(1/2*b*x + 2*
a)^8*tan(1/2*a)^15 + 4936681833840*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^16 - 226
28389243545*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^17 + 59571399047240*tan(1/2*b*x
 + 2*a)^5*tan(1/2*a)^18 - 92522313305985*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^19
 + 83200102704840*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^20 - 40160124218470*tan(1
/2*b*x + 2*a)^2*tan(1/2*a)^21 + 8484705136560*tan(1/2*b*x + 2*a)*tan(1/2*a)
^22 - 329444622930*tan(1/2*a)^23 - 2847265170*tan(1/2*b*x + 2*a)^9*tan(1/2*
a)^12 + 82199042640*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^13 - 911808828960*tan(1
/2*b*x + 2*a)^7*tan(1/2*a)^14 + 5227899617200*tan(1/2*b*x + 2*a)^6*tan(1/2*
a)^15 - 17235607351635*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^16 + 33928676906985*
tan(1/2*b*x + 2*a)^4*tan(1/2*a)^17 - 39714192559670*tan(1/2*b*x + 2*a)^3*ta
n(1/2*a)^18 + 26404824927630*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^19 - 888825162
1845*tan(1/2*b*x + 2*a)*tan(1/2*a)^20 + 1111051397230*tan(1/2*a)^21 + 16402
0320*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^10 - 7061366340*tan(1/2*b*x + 2*a)^8*t
an(1/2*a)^11 + 104745094160*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^12 - 7686989368
80*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^13 + 3184356059520*tan(1/2*b*x + 2*a)^5*
tan(1/2*a)^14 - 7834375816360*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^15 + 11501138
427120*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^16 - 9707168584980*tan(1/2*b*x + 2*a
)^2*tan(1/2*a)^17 + 4255065187040*tan(1/2*b*x + 2*a)*tan(1/2*a)^18 - 731644
643640*tan(1/2*a)^19 - 2407725*tan(1/2*b*x + 2*a)^9*tan(1/2*a)^8 + 29423415
```

0*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^9 - 6848508750*tan(1/2*b*x + 2*a)^7*tan(1
/2*a)^10 + 68829859005*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^11 - 370049962295*ta
n(1/2*b*x + 2*a)^5*tan(1/2*a)^12 + 1153540125240*tan(1/2*b*x + 2*a)^4*tan(1
/2*a)^13 - 2124636283680*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^14 + 2245614722020
*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^15 - 1237500036990*tan(1/2*b*x + 2*a)*tan(
1/2*a)^16 + 270445183560*tan(1/2*a)^17 - 71280*tan(1/2*b*x + 2*a)^9*tan(1/2
*a)^6 - 2568780*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^7 + 204257160*tan(1/2*b*x +
 2*a)^7*tan(1/2*a)^8 - 3360350485*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^9 + 25315
144296*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^10 - 104054449965*tan(1/2*b*x + 2*a)
^4*tan(1/2*a)^11 + 245839387280*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^12 - 329145
687420*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^13 + 228535536960*tan(1/2*b*x + 2*a)
*tan(1/2*a)^14 - 62831824476*tan(1/2*a)^15 - 1215*tan(1/2*b*x + 2*a)^9*tan(
1/2*a)^4 - 59940*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^5 - 1013880*tan(1/2*b*x +
2*a)^7*tan(1/2*a)^6 + 68740740*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^7 - 88212658
5*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^8 + 5205627685*tan(1/2*b*x + 2*a)^4*tan(1
/2*a)^9 - 16540981710*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^10 + 28942646220*tan(
1/2*b*x + 2*a)^2*tan(1/2*a)^11 - 25919270610*tan(1/2*b*x + 2*a)*tan(1/2*a)^
12 + 9122039460*tan(1/2*a)^13 - 810*tan(1/2*b*x + 2*a)^8*tan(1/2*a)^3 - 176
40*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^4 - 144900*tan(1/2*b*x + 2*a)^6*tan(1/2*
a)^5 + 10958240*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^6 - 115984800*tan(1/2*b*x +
 2*a)^4*tan(1/2*a)^7 + 546245640*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^8 - 132359
6190*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^9 + 1600947360*tan(1/2*b*x + 2*a)*tan(
1/2*a)^10 - 753655320*tan(1/2*a)^11 - 270*tan(1/2*b*x + 2*a)^7*tan(1/2*a)^2
 - 795*tan(1/2*b*x + 2*a)^6*tan(1/2*a)^3 + 1635*tan(1/2*b*x + 2*a)^5*tan(1/
2*a)^4 + 567720*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^5 - 5446200*tan(1/2*b*x + 2
*a)^3*tan(1/2*a)^6 + 20895030*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^7 - 37166445*
tan(1/2*b*x + 2*a)*tan(1/2*a)^8 + 25793640*tan(1/2*a)^9 - 45*tan(1/2*b*x +
2*a)^6*tan(1/2*a) + 360*tan(1/2*b*x + 2*a)^5*tan(1/2*a)^2 + 795*tan(1/2*b*x
 + 2*a)^4*tan(1/2*a)^3 - 17640*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^4 + 86670*ta
n(1/2*b*x + 2*a)^2*tan(1/2*a)^5 - 187920*tan(1/2*b*x + 2*a)*tan(1/2*a)^6 +
172530*tan(1/2*a)^7 - 3*tan(1/2*b*x + 2*a)^5 + 45*tan(1/2*b*x + 2*a)^4*tan(
1/2*a) - 270*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^2 + 810*tan(1/2*b*x + 2*a)^2*t
an(1/2*a)^3 - 1215*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 1458*tan(1/2*a)^5)/((2
43*tan(1/2*a)^25 - 4050*tan(1/2*a)^23 + 28215*tan(1/2*a)^21 - 106200*tan(1/
2*a)^19 + 233430*tan(1/2*a)^17 - 304300*tan(1/2*a)^15 + 233430*tan(1/2*a)^1
3 - 106200*tan(1/2*a)^11 + 28215*tan(1/2*a)^9 - 4050*tan(1/2*a)^7 + 243*tan
(1/2*a)^5)*(3*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 - tan(1/2*b*x + 2*a)*tan(1/
2*a)^6 - 10*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 + 15*tan(1/2*b*x + 2*a)*tan(1
/2*a)^4 - 3*tan(1/2*a)^5 + 3*tan(1/2*b*x + 2*a)^2*tan(1/2*a) - 15*tan(1/2*b
*x + 2*a)*tan(1/2*a)^2 + 10*tan(1/2*a)^3 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a
))^5) - 1680*log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 3*tan(1/2*b*x + 2*a)
*tan(1/2*a)^2 - tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*
a)^2 - tan(1/2*b*x + 2*a) + 3*tan(1/2*a) - 1)) + 1680*log(abs(tan(1/2*b*x +
 2*a)*tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 + tan(1/2*a)^3 - 3*t
an(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 + tan(1/2*b*x + 2*a) - 3*tan(
1/2*a) - 1)))/b

3.72 $\int \csc^3(a+bx)\csc^4(2a+2bx)\,dx$

Optimal. Leaf size=112

$$\frac{35\sec^3(a+bx)}{256b}+\frac{105\sec(a+bx)}{256b}-\frac{105\tanh^{-1}(\cos(a+bx))}{256b}-\frac{\csc^6(a+bx)\sec^3(a+bx)}{96b}-\frac{3\csc^4(a+bx)\sec^3(a+b}{128b}$$

```
[Out] (-105*ArcTanh[Cos[a + b*x]])/(256*b) + (105*Sec[a + b*x])/(256*b) + (35*Sec
[a + b*x]^3)/(256*b) - (21*Csc[a + b*x]^2*Sec[a + b*x]^3)/(256*b) - (3*Csc[
a + b*x]^4*Sec[a + b*x]^3)/(128*b) - (Csc[a + b*x]^6*Sec[a + b*x]^3)/(96*b)
```

Rubi [A] time = 0.0870985, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4288, 2622, 288, 302, 207}

$$\frac{35\sec^3(a+bx)}{256b}+\frac{105\sec(a+bx)}{256b}-\frac{105\tanh^{-1}(\cos(a+bx))}{256b}-\frac{\csc^6(a+bx)\sec^3(a+bx)}{96b}-\frac{3\csc^4(a+bx)\sec^3(a+b}{128b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] (-105*ArcTanh[Cos[a + b*x]])/(256*b) + (105*Sec[a + b*x])/(256*b) + (35*Sec
[a + b*x]^3)/(256*b) - (21*Csc[a + b*x]^2*Sec[a + b*x]^3)/(256*b) - (3*Csc[
a + b*x]^4*Sec[a + b*x]^3)/(128*b) - (Csc[a + b*x]^6*Sec[a + b*x]^3)/(96*b)
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^3(a+bx)\csc^4(2a+2bx)\,dx = \frac{1}{16}\int \csc^7(a+bx)\sec^4(a+bx)\,dx$$

$$= \frac{\text{Subst}\left(\int \frac{x^{10}}{\left(-1+x^2\right)^4}\,dx, x, \sec(a+bx)\right)}{16b}$$

$$= -\frac{\csc^6(a+bx)\sec^3(a+bx)}{96b} + \frac{3\,\text{Subst}\left(\int \frac{x^8}{\left(-1+x^2\right)^3}\,dx, x, \sec(a+bx)\right)}{32b}$$

$$= -\frac{3\csc^4(a+bx)\sec^3(a+bx)}{128b} - \frac{\csc^6(a+bx)\sec^3(a+bx)}{96b} + \frac{21\,\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^2}\,dx,\right.}{128b}$$

$$= -\frac{21\csc^2(a+bx)\sec^3(a+bx)}{256b} - \frac{3\csc^4(a+bx)\sec^3(a+bx)}{128b} - \frac{\csc^6(a+bx)\sec^3(a+}{96b}$$

$$= -\frac{21\csc^2(a+bx)\sec^3(a+bx)}{256b} - \frac{3\csc^4(a+bx)\sec^3(a+bx)}{128b} - \frac{\csc^6(a+bx)\sec^3(a+}{96b}$$

$$= \frac{105\sec(a+bx)}{256b} + \frac{35\sec^3(a+bx)}{256b} - \frac{21\csc^2(a+bx)\sec^3(a+bx)}{256b} - \frac{3\csc^4(a+bx)\,\mathrm{s}}{128b}$$

$$= -\frac{105\tanh^{-1}(\cos(a+bx))}{256b} + \frac{105\sec(a+bx)}{256b} + \frac{35\sec^3(a+bx)}{256b} - \frac{21\csc^2(a+bx)\,\mathrm{se}}{256b}$$

Mathematica [B] time = 0.833866, size = 278, normalized size = 2.48

$$\frac{\csc^{12}(a+bx)\left(-4752\cos(2(a+bx))+1600\cos(3(a+bx))+504\cos(4(a+bx))+1680\cos(6(a+bx))-600\cos(7(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] (Csc[a + b*x]^12*(1150 - 4752*Cos[2*(a + b*x)] + 1600*Cos[3*(a + b*x)] + 50
4*Cos[4*(a + b*x)] + 1680*Cos[6*(a + b*x)] - 600*Cos[7*(a + b*x)] - 630*Cos
[8*(a + b*x)] + 200*Cos[9*(a + b*x)] + 2520*Cos[3*(a + b*x)]*Log[Cos[(a + b
*x)/2]] - 945*Cos[7*(a + b*x)]*Log[Cos[(a + b*x)/2]] + 315*Cos[9*(a + b*x)]
*Log[Cos[(a + b*x)/2]] - 30*Cos[a + b*x]*(40 + 63*Log[Cos[(a + b*x)/2]] - 6
3*Log[Sin[(a + b*x)/2]]) - 2520*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] + 94
5*Cos[7*(a + b*x)]*Log[Sin[(a + b*x)/2]] - 315*Cos[9*(a + b*x)]*Log[Sin[(a
+ b*x)/2]]))/(3072*b*(Csc[(a + b*x)/2]^2 - Sec[(a + b*x)/2]^2)^3)
```

Maple [A] time = 0.04, size = 120, normalized size = 1.1

$$-\frac{1}{96\,b\,(\sin(bx+a))^6\,(\cos(bx+a))^3} - \frac{3}{128\,b\,(\sin(bx+a))^4\,(\cos(bx+a))^3} + \frac{7}{128\,b\,(\sin(bx+a))^2\,(\cos(bx+a))^3} - \frac{}{256\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*csc(2*b*x+2*a)^4,x)
```

```
[Out] -1/96/b/sin(b*x+a)^6/cos(b*x+a)^3-3/128/b/sin(b*x+a)^4/cos(b*x+a)^3+7/128/b
/sin(b*x+a)^2/cos(b*x+a)^3-35/256/b/sin(b*x+a)^2/cos(b*x+a)+105/256/b/cos(b
*x+a)+105/256/b*ln(csc(b*x+a)-cot(b*x+a))
```

<u>Maxima</u> [B] time = 1.93999, size = 5762, normalized size = 51.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/1536*(4*(315*cos(17*b*x + 17*a) - 840*cos(15*b*x + 15*a) - 252*cos(13*b*x
 + 13*a) + 2376*cos(11*b*x + 11*a) - 1150*cos(9*b*x + 9*a) + 2376*cos(7*b*x
 + 7*a) - 252*cos(5*b*x + 5*a) - 840*cos(3*b*x + 3*a) + 315*cos(b*x + a))*c
os(18*b*x + 18*a) - 1260*(3*cos(16*b*x + 16*a) - 8*cos(12*b*x + 12*a) + 6*c
os(10*b*x + 10*a) + 6*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(2*b*x +
 2*a) - 1)*cos(17*b*x + 17*a) + 12*(840*cos(15*b*x + 15*a) + 252*cos(13*b*x
 + 13*a) - 2376*cos(11*b*x + 11*a) + 1150*cos(9*b*x + 9*a) - 2376*cos(7*b*x
 + 7*a) + 252*cos(5*b*x + 5*a) + 840*cos(3*b*x + 3*a) - 315*cos(b*x + a))*c
os(16*b*x + 16*a) - 3360*(8*cos(12*b*x + 12*a) - 6*cos(10*b*x + 10*a) - 6*c
os(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 3*cos(2*b*x + 2*a) + 1)*cos(15*b*x +
 15*a) - 1008*(8*cos(12*b*x + 12*a) - 6*cos(10*b*x + 10*a) - 6*cos(8*b*x +
8*a) + 8*cos(6*b*x + 6*a) - 3*cos(2*b*x + 2*a) + 1)*cos(13*b*x + 13*a) + 32
*(2376*cos(11*b*x + 11*a) - 1150*cos(9*b*x + 9*a) + 2376*cos(7*b*x + 7*a) -
 252*cos(5*b*x + 5*a) - 840*cos(3*b*x + 3*a) + 315*cos(b*x + a))*cos(12*b*x
 + 12*a) - 9504*(6*cos(10*b*x + 10*a) + 6*cos(8*b*x + 8*a) - 8*cos(6*b*x +
6*a) + 3*cos(2*b*x + 2*a) - 1)*cos(11*b*x + 11*a) + 24*(1150*cos(9*b*x + 9*
a) - 2376*cos(7*b*x + 7*a) + 252*cos(5*b*x + 5*a) + 840*cos(3*b*x + 3*a) -
315*cos(b*x + a))*cos(10*b*x + 10*a) + 4600*(6*cos(8*b*x + 8*a) - 8*cos(6*b
*x + 6*a) + 3*cos(2*b*x + 2*a) - 1)*cos(9*b*x + 9*a) - 72*(792*cos(7*b*x +
7*a) - 84*cos(5*b*x + 5*a) - 280*cos(3*b*x + 3*a) + 105*cos(b*x + a))*cos(8
*b*x + 8*a) + 9504*(8*cos(6*b*x + 6*a) - 3*cos(2*b*x + 2*a) + 1)*cos(7*b*x
+ 7*a) - 672*(12*cos(5*b*x + 5*a) + 40*cos(3*b*x + 3*a) - 15*cos(b*x + a))*
cos(6*b*x + 6*a) + 1008*(3*cos(2*b*x + 2*a) - 1)*cos(5*b*x + 5*a) + 3360*(3
*cos(2*b*x + 2*a) - 1)*cos(3*b*x + 3*a) - 3780*cos(2*b*x + 2*a)*cos(b*x + a
) + 315*(2*(3*cos(16*b*x + 16*a) - 8*cos(12*b*x + 12*a) + 6*cos(10*b*x + 10
*a) + 6*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(2*b*x + 2*a) - 1)*cos
(18*b*x + 18*a) - cos(18*b*x + 18*a)^2 + 6*(8*cos(12*b*x + 12*a) - 6*cos(10
*b*x + 10*a) - 6*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 3*cos(2*b*x + 2*a)
 + 1)*cos(16*b*x + 16*a) - 9*cos(16*b*x + 16*a)^2 + 16*(6*cos(10*b*x + 10*a
) + 6*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(2*b*x + 2*a) - 1)*cos(1
2*b*x + 12*a) - 64*cos(12*b*x + 12*a)^2 - 12*(6*cos(8*b*x + 8*a) - 8*cos(6*
b*x + 6*a) + 3*cos(2*b*x + 2*a) - 1)*cos(10*b*x + 10*a) - 36*cos(10*b*x + 1
0*a)^2 + 12*(8*cos(6*b*x + 6*a) - 3*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a)
- 36*cos(8*b*x + 8*a)^2 + 16*(3*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 64
*cos(6*b*x + 6*a)^2 - 9*cos(2*b*x + 2*a)^2 + 2*(3*sin(16*b*x + 16*a) - 8*si
n(12*b*x + 12*a) + 6*sin(10*b*x + 10*a) + 6*sin(8*b*x + 8*a) - 8*sin(6*b*x
+ 6*a) + 3*sin(2*b*x + 2*a))*sin(18*b*x + 18*a) - sin(18*b*x + 18*a)^2 + 6*
(8*sin(12*b*x + 12*a) - 6*sin(10*b*x + 10*a) - 6*sin(8*b*x + 8*a) + 8*sin(6
*b*x + 6*a) - 3*sin(2*b*x + 2*a))*sin(16*b*x + 16*a) - 9*sin(16*b*x + 16*a)
^2 + 16*(6*sin(10*b*x + 10*a) + 6*sin(8*b*x + 8*a) - 8*sin(6*b*x + 6*a) + 3
*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - 64*sin(12*b*x + 12*a)^2 - 12*(6*sin
(8*b*x + 8*a) - 8*sin(6*b*x + 6*a) + 3*sin(2*b*x + 2*a))*sin(10*b*x + 10*a)
 - 36*sin(10*b*x + 10*a)^2 + 12*(8*sin(6*b*x + 6*a) - 3*sin(2*b*x + 2*a))*s
```

```
in(8*b*x + 8*a) - 36*sin(8*b*x + 8*a)^2 - 64*sin(6*b*x + 6*a)^2 + 48*sin(6*
b*x + 6*a)*sin(2*b*x + 2*a) - 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2*a) - 1
)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*s
in(a) + sin(a)^2) - 315*(2*(3*cos(16*b*x + 16*a) - 8*cos(12*b*x + 12*a) + 6
*cos(10*b*x + 10*a) + 6*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(2*b*x
 + 2*a) - 1)*cos(18*b*x + 18*a) - cos(18*b*x + 18*a)^2 + 6*(8*cos(12*b*x +
12*a) - 6*cos(10*b*x + 10*a) - 6*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 3*
cos(2*b*x + 2*a) + 1)*cos(16*b*x + 16*a) - 9*cos(16*b*x + 16*a)^2 + 16*(6*c
os(10*b*x + 10*a) + 6*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(2*b*x +
 2*a) - 1)*cos(12*b*x + 12*a) - 64*cos(12*b*x + 12*a)^2 - 12*(6*cos(8*b*x +
 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(2*b*x + 2*a) - 1)*cos(10*b*x + 10*a) - 3
6*cos(10*b*x + 10*a)^2 + 12*(8*cos(6*b*x + 6*a) - 3*cos(2*b*x + 2*a) + 1)*c
os(8*b*x + 8*a) - 36*cos(8*b*x + 8*a)^2 + 16*(3*cos(2*b*x + 2*a) - 1)*cos(6
*b*x + 6*a) - 64*cos(6*b*x + 6*a)^2 - 9*cos(2*b*x + 2*a)^2 + 2*(3*sin(16*b*
x + 16*a) - 8*sin(12*b*x + 12*a) + 6*sin(10*b*x + 10*a) + 6*sin(8*b*x + 8*a
) - 8*sin(6*b*x + 6*a) + 3*sin(2*b*x + 2*a))*sin(18*b*x + 18*a) - sin(18*b*
x + 18*a)^2 + 6*(8*sin(12*b*x + 12*a) - 6*sin(10*b*x + 10*a) - 6*sin(8*b*x
+ 8*a) + 8*sin(6*b*x + 6*a) - 3*sin(2*b*x + 2*a))*sin(16*b*x + 16*a) - 9*si
n(16*b*x + 16*a)^2 + 16*(6*sin(10*b*x + 10*a) + 6*sin(8*b*x + 8*a) - 8*sin(
6*b*x + 6*a) + 3*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - 64*sin(12*b*x + 12*
a)^2 - 12*(6*sin(8*b*x + 8*a) - 8*sin(6*b*x + 6*a) + 3*sin(2*b*x + 2*a))*si
n(10*b*x + 10*a) - 36*sin(10*b*x + 10*a)^2 + 12*(8*sin(6*b*x + 6*a) - 3*sin
(2*b*x + 2*a))*sin(8*b*x + 8*a) - 36*sin(8*b*x + 8*a)^2 - 64*sin(6*b*x + 6*
a)^2 + 48*sin(6*b*x + 6*a)*sin(2*b*x + 2*a) - 9*sin(2*b*x + 2*a)^2 + 6*cos(
2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^
2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*(315*sin(17*b*x + 17*a) - 840*sin(15*
b*x + 15*a) - 252*sin(13*b*x + 13*a) + 2376*sin(11*b*x + 11*a) - 1150*sin(9
*b*x + 9*a) + 2376*sin(7*b*x + 7*a) - 252*sin(5*b*x + 5*a) - 840*sin(3*b*x
+ 3*a) + 315*sin(b*x + a))*sin(18*b*x + 18*a) - 1260*(3*sin(16*b*x + 16*a)
- 8*sin(12*b*x + 12*a) + 6*sin(10*b*x + 10*a) + 6*sin(8*b*x + 8*a) - 8*sin(
6*b*x + 6*a) + 3*sin(2*b*x + 2*a))*sin(17*b*x + 17*a) + 12*(840*sin(15*b*x
+ 15*a) + 252*sin(13*b*x + 13*a) - 2376*sin(11*b*x + 11*a) + 1150*sin(9*b*x
 + 9*a) - 2376*sin(7*b*x + 7*a) + 252*sin(5*b*x + 5*a) + 840*sin(3*b*x + 3*
a) - 315*sin(b*x + a))*sin(16*b*x + 16*a) - 3360*(8*sin(12*b*x + 12*a) - 6*
sin(10*b*x + 10*a) - 6*sin(8*b*x + 8*a) + 8*sin(6*b*x + 6*a) - 3*sin(2*b*x
+ 2*a))*sin(15*b*x + 15*a) - 1008*(8*sin(12*b*x + 12*a) - 6*sin(10*b*x + 10
*a) - 6*sin(8*b*x + 8*a) + 8*sin(6*b*x + 6*a) - 3*sin(2*b*x + 2*a))*sin(13*
b*x + 13*a) + 32*(2376*sin(11*b*x + 11*a) - 1150*sin(9*b*x + 9*a) + 2376*si
n(7*b*x + 7*a) - 252*sin(5*b*x + 5*a) - 840*sin(3*b*x + 3*a) + 315*sin(b*x
+ a))*sin(12*b*x + 12*a) - 9504*(6*sin(10*b*x + 10*a) + 6*sin(8*b*x + 8*a)
- 8*sin(6*b*x + 6*a) + 3*sin(2*b*x + 2*a))*sin(11*b*x + 11*a) + 24*(1150*si
n(9*b*x + 9*a) - 2376*sin(7*b*x + 7*a) + 252*sin(5*b*x + 5*a) + 840*sin(3*b
*x + 3*a) - 315*sin(b*x + a))*sin(10*b*x + 10*a) + 4600*(6*sin(8*b*x + 8*a)
 - 8*sin(6*b*x + 6*a) + 3*sin(2*b*x + 2*a))*sin(9*b*x + 9*a) - 72*(792*sin(
7*b*x + 7*a) - 84*sin(5*b*x + 5*a) - 280*sin(3*b*x + 3*a) + 105*sin(b*x + a
))*sin(8*b*x + 8*a) + 9504*(8*sin(6*b*x + 6*a) - 3*sin(2*b*x + 2*a))*sin(7*
b*x + 7*a) - 672*(12*sin(5*b*x + 5*a) + 40*sin(3*b*x + 3*a) - 15*sin(b*x +
a))*sin(6*b*x + 6*a) + 3024*sin(5*b*x + 5*a)*sin(2*b*x + 2*a) + 10080*sin(3
*b*x + 3*a)*sin(2*b*x + 2*a) - 3780*sin(2*b*x + 2*a)*sin(b*x + a) + 1260*co
s(b*x + a))/(b*cos(18*b*x + 18*a)^2 + 9*b*cos(16*b*x + 16*a)^2 + 64*b*cos(1
2*b*x + 12*a)^2 + 36*b*cos(10*b*x + 10*a)^2 + 36*b*cos(8*b*x + 8*a)^2 + 64*
b*cos(6*b*x + 6*a)^2 + 9*b*cos(2*b*x + 2*a)^2 + b*sin(18*b*x + 18*a)^2 + 9*
b*sin(16*b*x + 16*a)^2 + 64*b*sin(12*b*x + 12*a)^2 + 36*b*sin(10*b*x + 10*a
)^2 + 36*b*sin(8*b*x + 8*a)^2 + 64*b*sin(6*b*x + 6*a)^2 - 48*b*sin(6*b*x +
6*a)*sin(2*b*x + 2*a) + 9*b*sin(2*b*x + 2*a)^2 - 2*(3*b*cos(16*b*x + 16*a)
- 8*b*cos(12*b*x + 12*a) + 6*b*cos(10*b*x + 10*a) + 6*b*cos(8*b*x + 8*a) -
8*b*cos(6*b*x + 6*a) + 3*b*cos(2*b*x + 2*a) - b)*cos(18*b*x + 18*a) - 6*(8*
b*cos(12*b*x + 12*a) - 6*b*cos(10*b*x + 10*a) - 6*b*cos(8*b*x + 8*a) + 8*b*
cos(6*b*x + 6*a) - 3*b*cos(2*b*x + 2*a) + b)*cos(16*b*x + 16*a) - 16*(6*b*c
```

```
os(10*b*x + 10*a) + 6*b*cos(8*b*x + 8*a) - 8*b*cos(6*b*x + 6*a) + 3*b*cos(2
*b*x + 2*a) - b)*cos(12*b*x + 12*a) + 12*(6*b*cos(8*b*x + 8*a) - 8*b*cos(6*
b*x + 6*a) + 3*b*cos(2*b*x + 2*a) - b)*cos(10*b*x + 10*a) - 12*(8*b*cos(6*b
*x + 6*a) - 3*b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) - 16*(3*b*cos(2*b*x
+ 2*a) - b)*cos(6*b*x + 6*a) - 6*b*cos(2*b*x + 2*a) - 2*(3*b*sin(16*b*x + 1
6*a) - 8*b*sin(12*b*x + 12*a) + 6*b*sin(10*b*x + 10*a) + 6*b*sin(8*b*x + 8*
a) - 8*b*sin(6*b*x + 6*a) + 3*b*sin(2*b*x + 2*a))*sin(18*b*x + 18*a) - 6*(8
*b*sin(12*b*x + 12*a) - 6*b*sin(10*b*x + 10*a) - 6*b*sin(8*b*x + 8*a) + 8*b
*sin(6*b*x + 6*a) - 3*b*sin(2*b*x + 2*a))*sin(16*b*x + 16*a) - 16*(6*b*sin(
10*b*x + 10*a) + 6*b*sin(8*b*x + 8*a) - 8*b*sin(6*b*x + 6*a) + 3*b*sin(2*b*
x + 2*a))*sin(12*b*x + 12*a) + 12*(6*b*sin(8*b*x + 8*a) - 8*b*sin(6*b*x + 6
*a) + 3*b*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 12*(8*b*sin(6*b*x + 6*a) -
 3*b*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + b)
```

Fricas [A] time = 0.532339, size = 529, normalized size = 4.72

$$\frac{630 \cos(bx+a)^8 - 1680 \cos(bx+a)^6 + 1386 \cos(bx+a)^4 - 288 \cos(bx+a)^2 - 315\left(\cos(bx+a)^9 - 3 \cos(bx+a)\right.}{1536\left(b\cos(bx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] 1/1536*(630*cos(b*x + a)^8 - 1680*cos(b*x + a)^6 + 1386*cos(b*x + a)^4 - 28
8*cos(b*x + a)^2 - 315*(cos(b*x + a)^9 - 3*cos(b*x + a)^7 + 3*cos(b*x + a)^
5 - cos(b*x + a)^3)*log(1/2*cos(b*x + a) + 1/2) + 315*(cos(b*x + a)^9 - 3*c
os(b*x + a)^7 + 3*cos(b*x + a)^5 - cos(b*x + a)^3)*log(-1/2*cos(b*x + a) +
1/2) - 32)/(b*cos(b*x + a)^9 - 3*b*cos(b*x + a)^7 + 3*b*cos(b*x + a)^5 - b*
cos(b*x + a)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*csc(2*b*x+2*a)**4,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.73 $\int \sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=136

$$\frac{5\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{24b}-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{6b}-\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{32b}-\frac{5\sqrt{\sin(2a+2bx)}\cos(}{16b}$$

```
[Out] (-5*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(32*b) + (5*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(32*b) - (5*Cos[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(16*b) + (5*Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(24*b) - (Cos[a
 + b*x]*Sin[2*a + 2*b*x]^(5/2))/(6*b)
```

Rubi [A] time = 0.0943819, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4302, 4301, 4305}

$$\frac{5\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{24b}-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{6b}-\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{32b}-\frac{5\sqrt{\sin(2a+2bx)}\cos(}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```


```
[Out] (-5*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(32*b) + (5*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(32*b) - (5*Cos[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(16*b) + (5*Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(24*b) - (Cos[a
 + b*x]*Sin[2*a + 2*b*x]^(5/2))/(6*b)
```


Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = -\frac{\cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b} + \frac{5}{6}\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$= \frac{5\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{24b} - \frac{\cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b} + \frac{5}{8}\int \sin(a+bx)\sqrt{\mathrm{si}$$
$$= -\frac{5\cos(a+bx)\sqrt{\sin(2a+2bx)}}{16b} + \frac{5\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{24b} - \frac{\cos(a+bx)\sin^{\frac{5}{2}}}{6b}$$
$$= -\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{32b} + \frac{5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a}\right.}{32b}$$

Mathematica [A] time = 0.287945, size = 98, normalized size = 0.72

$$\frac{15\left(\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(a+bx)\right)-\sin^{-1}(\cos(a+bx)-\sin(a+bx))\right)-2\sqrt{\sin(2(a+bx))}(14\cos(}{96b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (15*(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]]) - 2*(14*Cos[a + b*x] + 3*Cos[3*(a + b*x)] - 2*C
os[5*(a + b*x)])*Sqrt[Sin[2*(a + b*x)]])/(96*b)
```

Maple [B] time = 59.26, size = 183661406, normalized size = 1350451.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^(5/2)*sin(b*x + a), x)
```

Fricas [B] time = 0.563799, size = 803, normalized size = 5.9

$$8\sqrt{2}\left(32\cos\left(bx+a\right)^{5}-52\cos\left(bx+a\right)^{3}+5\cos\left(bx+a\right)\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+30\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(b}}{\cos(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/384*(8*sqrt(2)*(32*cos(b*x + a)^5 - 52*cos(b*x + a)^3 + 5*cos(b*x + a))*s
qrt(cos(b*x + a)*sin(b*x + a)) + 30*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(
b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b
*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) - 30*arctan(-(2*sqrt(2)*sqrt(
cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - s
in(b*x + a))) - 15*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (
4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(
b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(2bx + 2a)^{\frac{5}{2}} \sin(bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^(5/2)*sin(b*x + a), x)
```

3.74 $\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=110

$$\frac{3\sin(a+bx)\sqrt{\sin(2a+2bx)}}{8b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}-\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{16b}-\frac{3\log\left(\sin(a+bx)\right.}{}$$

```
[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(16*b) - (3*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(16*b) + (3*Sin[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(8*b) - (Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(4*b)
```

Rubi [A] time = 0.0675042, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4302, 4301, 4306}

$$\frac{3\sin(a+bx)\sqrt{\sin(2a+2bx)}}{8b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}-\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{16b}-\frac{3\log\left(\sin(a+bx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]
```


```
[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(16*b) - (3*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(16*b) + (3*Sin[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(8*b) - (Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(4*b)
```


Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = -\frac{\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b} + \frac{3}{4}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$
$$= \frac{3\sin(a+bx)\sqrt{\sin(2a+2bx)}}{8b} - \frac{\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b} + \frac{3}{8}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,d$$
$$= -\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{16b} - \frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2\right.}}{16b}$$

Mathematica [A] time = 0.194416, size = 86, normalized size = 0.78

$$\frac{2\sqrt{\sin(2(a+bx))}(2\sin(a+bx)-\sin(3(a+bx)))-3\left(\sin^{-1}(\cos(a+bx)-\sin(a+bx))+\log\left(\sin(a+bx)+\sqrt{\sin(2(a+b\right.}\right.}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-3*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]) + 2*Sqrt[Sin[2*(a + b*x)]]*(2*Sin[a + b*x] - Sin
[3*(a + b*x)]))/(16*b)
```

Maple [B] time = 15.386, size = 65166864, normalized size = 592426.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(2\,bx+2\,a)^{\frac{3}{2}}\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^(3/2)*sin(b*x + a), x)
```

Fricas [B] time = 0.555751, size = 771, normalized size = 7.01

$$-\frac{8\sqrt{2}\left(4\cos(bx+a)^2-3\right)\sqrt{\cos(bx+a)\sin(bx+a)}\sin(bx+a)-6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(8*sqrt(2)*(4*cos(b*x + a)^2 - 3)*sqrt(cos(b*x + a)*sin(b*x + a))*sin
(b*x + a) - 6*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a
) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x
+ a)*sin(b*x + a) - 1)) + 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x +
a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - 3*log(-
32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*si
n(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x +
 a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(2\,bx + 2\,a)^{\frac{3}{2}} \sin(bx + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^(3/2)*sin(b*x + a), x)
```

3.75 $\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal. Leaf size=84

$$-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b}-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}+\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{4b}$$

```
[Out] -ArcSin[Cos[a + b*x] - Sin[a + b*x]]/(4*b) + Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*a + 2*b*x]]]/(4*b) - (Cos[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(
2*b)
```

Rubi [A] time = 0.0431275, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4302, 4305}

$$-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b}-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}+\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -ArcSin[Cos[a + b*x] - Sin[a + b*x]]/(4*b) + Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*a + 2*b*x]]]/(4*b) - (Cos[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(
2*b)
```

Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx &= -\frac{\cos(a+bx)\sqrt{\sin(2a+2bx)}}{2b}+\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= -\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b}+\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx}\right.}{4b}\end{aligned}$$

Mathematica [A] time = 0.0721811, size = 72, normalized size = 0.86

$$\frac{-\sin^{-1}(\cos(a+bx)-\sin(a+bx))-2\sqrt{\sin(2(a+bx))}\cos(a+bx)+\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(a+bx)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x] + S
qrt[Sin[2*(a + b*x)]]] - 2*Cos[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/(4*b)
```

Maple [B] time = 1.171, size = 6219390, normalized size = 74040.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sin\left(2\,bx + 2\,a\right)}\sin\left(bx + a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sin(2*b*x + 2*a))*sin(b*x + a), x)
```

Fricas [B] time = 0.538193, size = 737, normalized size = 8.77

$$-\frac{8\,\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}\cos\left(bx+a\right) - 2\,\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\,\cos(bx+a)\sin(bx+a)-1}\right) + \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/16*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*cos(b*x + a) - 2*arctan(-(
sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos
(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1))
 + 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - si
n(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + log(-32*cos(b*x + a)^4 + 4*sqr
t(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x +
a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*s
in(b*x + a) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sin(2bx+2a)}\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(2*b*x + 2*a))*sin(b*x + a), x)
```

3.76 $\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal. Leaf size=58

$$-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}$$

```
[Out] -ArcSin[Cos[a + b*x] - Sin[a + b*x]]/(2*b) - Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b)
```

Rubi [A] time = 0.021149, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {4306}

$$-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -ArcSin[Cos[a + b*x] - Sin[a + b*x]]/(2*b) - Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b)
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx=-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{2b}$$

Mathematica [A] time = 0.0488717, size = 50, normalized size = 0.86

$$-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))+\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(a+bx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x] + S
qrt[Sin[2*(a + b*x)]]])/(2*b)
```

Maple [B] time = 1.424, size = 15979424, normalized size = 275507.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)}{\sqrt{\sin(2\,bx+2\,a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)/sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [B] time = 0.526008, size = 657, normalized size = 11.33

$$\frac{2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)-2\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}-\cos(bx+a)-\sin(bx+a)}{\cos(bx+a)-\sin(bx+a)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin
(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*si
n(b*x + a) - 1)) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - c
os(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + log(-32*cos(b*
x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a
) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 1
6*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.77 $\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=23

$$\frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

```
[Out] Sin[a + b*x]/(b*Sqrt[Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.0176177, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {4292}

$$\frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] Sin[a + b*x]/(b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A] time = 0.0184345, size = 22, normalized size = 0.96

$$\frac{\sin(a+bx)}{b\sqrt{\sin(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] Sin[a + b*x]/(b*Sqrt[Sin[2*(a + b*x)]])
```

Maple [B] time = 8.56, size = 59131270, normalized size = 2570924.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin (bx+a)}{\sin (2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)/sin(2*b*x + 2*a)^(3/2), x)
```

Fricas [A] time = 0.486828, size = 107, normalized size = 4.65

$$\frac{\sqrt{2}\sqrt{\cos (bx+a)\sin (bx+a)}+\cos (bx+a)}{2\,b\cos (bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) + cos(b*x + a))/(b*cos(b*x + a
))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.78 $\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)} dx$

Optimal. Leaf size=53

$$\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)} - \frac{2\cos(a+bx)}{3b\sqrt{\sin(2a+2bx)}}$$

```
[Out] Sin[a + b*x]/(3*b*Sin[2*a + 2*b*x]^(3/2)) - (2*Cos[a + b*x])/(3*b*Sqrt[Sin[
2*a + 2*b*x]])
```

Rubi [A] time = 0.0395403, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4304, 4291}

$$\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)} - \frac{2\cos(a+bx)}{3b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] Sin[a + b*x]/(3*b*Sin[2*a + 2*b*x]^(3/2)) - (2*Cos[a + b*x])/(3*b*Sqrt[Sin[
2*a + 2*b*x]])
```

Rule 4304

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> -Simp[(Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dis
t[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4291

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> -Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*
m), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/
b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)} dx &= \frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{2}{3}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)} dx \\ &= \frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)} - \frac{2\cos(a+bx)}{3b\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [A] time = 0.102788, size = 43, normalized size = 0.81

$$\frac{\sqrt{\sin(2(a+bx))}\left(\frac{1}{12}\tan(a+bx)\sec(a+bx) - \frac{1}{4}\csc(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (Sqrt[Sin[2*(a + b*x)]]*(-Csc[a + b*x]/4 + (Sec[a + b*x]*Tan[a + b*x])/12))
/b
```

Maple [C] time = 18.474, size = 597, normalized size = 11.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] 1/8/b*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(6*(tan(1/2*b*x+
1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan
(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticE((tan(1/2*b*x
+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^2-3*(tan(1/2*b*x+1/2*a)*(t
an(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+
1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1
)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^2+6*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*
x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)
^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticE((tan(1/2*b*x+1/2*a)+1)^(1/2),1
/2*2^(1/2))-3*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*
b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(
1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))+2*(tan(1/2*b*x+1/2
*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*tan(1/2*b*x+1/2*a)^4+2*(tan(1/2*b*x+1/2
*a)^3-tan(1/2*b*x+1/2*a))^(1/2)*tan(1/2*b*x+1/2*a)^4-2*(tan(1/2*b*x+1/2*a)*
(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*tan(1/2*b*x+1/2*a)^2-2*(tan(1/2*b*x+1/2*a)^
3-tan(1/2*b*x+1/2*a))^(1/2))/tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^3-tan(1
/2*b*x+1/2*a))^(1/2)/(tan(1/2*b*x+1/2*a)^2+1)
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)}{\sin(2bx+2a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)/sin(2*b*x + 2*a)^(5/2), x)
```

Fricas [A] time = 0.509805, size = 184, normalized size = 3.47

$$-\frac{4\cos(bx+a)^2\sin(bx+a)+\sqrt{2}\left(4\cos(bx+a)^2-1\right)\sqrt{\cos(bx+a)\sin(bx+a)}}{12\,b\cos(bx+a)^2\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(4*cos(b*x + a)^2*sin(b*x + a) + sqrt(2)*(4*cos(b*x + a)^2 - 1)*sqrt(
cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^2*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.79 $\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)} dx$

Optimal. Leaf size=79

$$\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{8\sin(a+bx)}{15b\sqrt{\sin(2a+2bx)}} - \frac{4\cos(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}$$

```
[Out] Sin[a + b*x]/(5*b*Sin[2*a + 2*b*x]^(5/2)) - (4*Cos[a + b*x])/(15*b*Sin[2*a
+ 2*b*x]^(3/2)) + (8*Sin[a + b*x])/(15*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.0586601, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4304, 4303, 4292}

$$\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{8\sin(a+bx)}{15b\sqrt{\sin(2a+2bx)}} - \frac{4\cos(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] Sin[a + b*x]/(5*b*Sin[2*a + 2*b*x]^(5/2)) - (4*Cos[a + b*x])/(15*b*Sin[2*a
+ 2*b*x]^(3/2)) + (8*Sin[a + b*x])/(15*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4304

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> -Simp[(Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dis
t[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4303

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{4}{5}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$$
$$= \frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{4\cos(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{8}{15}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$$
$$= \frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{4\cos(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{8\sin(a+bx)}{15b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A] time = 0.189159, size = 52, normalized size = 0.66

$$\frac{\sqrt{\sin(2(a+bx))}\left(3\sec(a+bx)\left(\sec^2(a+bx)+9\right)-5\cot(a+bx)\csc(a+bx)\right)}{120b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] ((-5*Cot[a + b*x]*Csc[a + b*x] + 3*Sec[a + b*x]*(9 + Sec[a + b*x]^2))*Sqrt[
Sin[2*(a + b*x)]])/(120*b)
```

Maple [C] time = 257.353, size = 308, normalized size = 3.9

$$-\frac{1}{48\,b}\sqrt{-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2-1\right)^{-1}}\left(\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2-1\right)\left(5\sqrt{\tan(1/2\,bx+a/2)+1}\sqrt{-2\tan(1/2\,bx+a/2)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/sin(2*b*x+2*a)^(7/2),x)
```

```
[Out] -1/48/b*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1
/2*a)^2-1)/tan(1/2*b*x+1/2*a)*(5*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b
*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a
)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a)^6+5*(tan(1/
2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))
^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*
a)-7*tan(1/2*b*x+1/2*a)^4+7*tan(1/2*b*x+1/2*a)^2+1)/(tan(1/2*b*x+1/2*a)*(ta
n(1/2*b*x+1/2*a)^2-1))^(1/2)/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2
)/(tan(1/2*b*x+1/2*a)^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)}{\sin(2\,bx+2\,a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)/sin(2*b*x + 2*a)^(7/2), x)
```

Fricas [A] time = 0.514501, size = 231, normalized size = 2.92

$$\frac{32\cos\left(bx+a\right)^{5}-32\cos\left(bx+a\right)^{3}+\sqrt{2}\left(32\cos\left(bx+a\right)^{4}-24\cos\left(bx+a\right)^{2}-3\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}}{120\left(b\cos\left(bx+a\right)^{5}-b\cos\left(bx+a\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/120*(32*cos(b*x + a)^5 - 32*cos(b*x + a)^3 + sqrt(2)*(32*cos(b*x + a)^4 -
 24*cos(b*x + a)^2 - 3)*sqrt(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^5
- b*cos(b*x + a)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.80 $\int \frac{\sin(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)} dx$

Optimal. Leaf size=105

$$\frac{8\sin(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{6\cos(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{16\cos(a+bx)}{35b\sqrt{\sin(2a+2bx)}}$$

```
[Out] Sin[a + b*x]/(7*b*Sin[2*a + 2*b*x]^(7/2)) - (6*Cos[a + b*x])/(35*b*Sin[2*a
+ 2*b*x]^(5/2)) + (8*Sin[a + b*x])/(35*b*Sin[2*a + 2*b*x]^(3/2)) - (16*Cos[
a + b*x])/(35*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.0833013, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4304, 4303, 4291}

$$\frac{8\sin(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{6\cos(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{16\cos(a+bx)}{35b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] Sin[a + b*x]/(7*b*Sin[2*a + 2*b*x]^(7/2)) - (6*Cos[a + b*x])/(35*b*Sin[2*a
+ 2*b*x]^(5/2)) + (8*Sin[a + b*x])/(35*b*Sin[2*a + 2*b*x]^(3/2)) - (16*Cos[
a + b*x])/(35*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4304

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> -Simp[(Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dis
t[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4303

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4291

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> -Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*
m), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/
b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\sin(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{6}{7}\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$$
$$= \frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} - \frac{6\cos(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{24}{35}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$$
$$= \frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} - \frac{6\cos(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{8\sin(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{16}{35}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$$
$$= \frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} - \frac{6\cos(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{8\sin(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)} - \frac{16\cos(a+bx)}{35b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A] time = 0.146025, size = 67, normalized size = 0.64

$$\frac{\sqrt{\sin(2(a+bx))}(-10\cos(2(a+bx))+4\cos(4(a+bx))+4\cos(6(a+bx))-5)\csc^3(a+bx)\sec^4(a+bx)}{560b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] ((-5 - 10*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)] + 4*Cos[6*(a + b*x)])*Csc[a
 + b*x]^3*Sec[a + b*x]^4*Sqrt[Sin[2*(a + b*x)]])/(560*b)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \sin(bx+a)(\sin(2bx+2a))^{-\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/sin(2*b*x+2*a)^(9/2),x)
```

```
[Out] int(sin(b*x+a)/sin(2*b*x+2*a)^(9/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)}{\sin(2bx+2a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)/sin(2*b*x + 2*a)^(9/2), x)
```

Fricas [A] time = 0.525527, size = 300, normalized size = 2.86

$$-\frac{\sqrt{2}\left(128\cos(bx+a)^6-160\cos(bx+a)^4+20\cos(bx+a)^2+5\right)\sqrt{\cos(bx+a)\sin(bx+a)}+128\left(\cos(bx+a)^6-\mathrm{c}\right.}{560\left(b\cos(bx+a)^6-b\cos(bx+a)^4\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/560*(sqrt(2)*(128*cos(b*x + a)^6 - 160*cos(b*x + a)^4 + 20*cos(b*x + a)^
2 + 5)*sqrt(cos(b*x + a)*sin(b*x + a)) + 128*(cos(b*x + a)^6 - cos(b*x + a)
^4)*sin(b*x + a))/((b*cos(b*x + a)^6 - b*cos(b*x + a)^4)*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.81 $\int \sin^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=98

$$\frac{5\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{42b}-\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{14b}-\frac{5\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{42b}$$

```
[Out] (5*EllipticF[a - Pi/4 + b*x, 2])/(42*b) - (5*Cos[2*a + 2*b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(42*b) - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(5/2))/(14*b) - Sin[
2*a + 2*b*x]^(9/2)/(18*b)
```

Rubi [A] time = 0.0595323, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4298, 2635, 2641}

$$-\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}+\frac{5F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{42b}-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{14b}-\frac{5\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{42b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (5*EllipticF[a - Pi/4 + b*x, 2])/(42*b) - (5*Cos[2*a + 2*b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(42*b) - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(5/2))/(14*b) - Sin[
2*a + 2*b*x]^(9/2)/(18*b)
```

Rule 4298

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[(e^2*(e*Sin[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p +
1))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Sin[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sin^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = -\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b} + \frac{1}{2}\int \sin^{\frac{7}{2}}(2a+2bx)\,dx$$
$$= -\frac{\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2bx)}{14b} - \frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b} + \frac{5}{14}\int \sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$= -\frac{5\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{42b} - \frac{\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2bx)}{14b} - \frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$
$$= \frac{5F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{42b} - \frac{5\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{42b} - \frac{\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2bx)}{14b}$$

Mathematica [A] time = 0.394278, size = 96, normalized size = 0.98

$$\frac{240\sqrt{\sin(2(a+bx))}\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right) - 70\sin(2(a+bx)) - 156\sin(4(a+bx)) + 35\sin(6(a+bx)) + 18\sin(8(a+}{2016b\sqrt{\sin(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (240*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*(a + b*x)]] - 70*Sin[2*(a + b*
x)] - 156*Sin[4*(a + b*x)] + 35*Sin[6*(a + b*x)] + 18*Sin[8*(a + b*x)] - 7*
Sin[10*(a + b*x)])/(2016*b*Sqrt[Sin[2*(a + b*x)]])
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (\sin(bx+a))^2 (\sin(2bx+2a))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x)
```

```
[Out] int(sin(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(2bx+2a)^{\frac{7}{2}}\sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^(7/2)*sin(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(\cos(bx+a)^2-1\right)\cos(2bx+2a)^2-\cos(bx+a)^2+1\right)\sin(2bx+2a)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(((cos(b*x + a)^2 - 1)*cos(2*b*x + 2*a)^2 - cos(b*x + a)^2 + 1)*sin
(2*b*x + 2*a)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.82 $\int \sin^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=69

$$-\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}+\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{10b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{10b}$$

```
[Out] (3*EllipticE[a - Pi/4 + b*x, 2])/(10*b) - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x
]^(3/2))/(10*b) - Sin[2*a + 2*b*x]^(7/2)/(14*b)
```

Rubi [A] time = 0.0474207, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4298, 2635, 2639}

$$-\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}+\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{10b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (3*EllipticE[a - Pi/4 + b*x, 2])/(10*b) - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x
]^(3/2))/(10*b) - Sin[2*a + 2*b*x]^(7/2)/(14*b)
```

Rule 4298

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[(e^2*(e*Sin[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p +
1))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Sin[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \sin^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx &= -\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}+\frac{1}{2}\int \sin^{\frac{5}{2}}(2a+2bx)\,dx\\
&= -\frac{\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{10b}-\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}+\frac{3}{10}\int \sqrt{\sin(2a+2bx)}\,dx\\
&= \frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{10b}-\frac{\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{10b}-\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}
\end{aligned}$$

Mathematica [A] time = 0.216787, size = 66, normalized size = 0.96

$$\frac{\sqrt{\sin(2(a+bx))}(-15\sin(2(a+bx))-14\sin(4(a+bx))+5\sin(6(a+bx)))+84E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{280b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (84*EllipticE[a - Pi/4 + b*x, 2] + Sqrt[Sin[2*(a + b*x)]]*(-15*Sin[2*(a + b
*x)] - 14*Sin[4*(a + b*x)] + 5*Sin[6*(a + b*x)]))/(280*b)
```

Maple [B] time = 103.249, size = 278672995, normalized size = 4038739.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}\sin\left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^(5/2)*sin(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(\cos\left(bx+a\right)^{2}-1\right)\cos\left(2\,bx+2\,a\right)^{2}-\cos\left(bx+a\right)^{2}+1\right)\sqrt{\sin\left(2\,bx+2\,a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(((cos(b*x + a)^2 - 1)*cos(2*b*x + 2*a)^2 - cos(b*x + a)^2 + 1)*sqr
t(sin(2*b*x + 2*a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(2bx+2a)^{\frac{5}{2}} \sin(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^(5/2)*sin(b*x + a)^2, x)
```

3.83 $\int \sin^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=69

$$\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{6b}-\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{6b}$$

```
[Out] EllipticF[a - Pi/4 + b*x, 2]/(6*b) - (Cos[2*a + 2*b*x]*Sqrt[Sin[2*a + 2*b*x
]])/(6*b) - Sin[2*a + 2*b*x]^(5/2)/(10*b)
```

Rubi [A] time = 0.0474437, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4298, 2635, 2641}

$$-\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}+\frac{F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{6b}-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] EllipticF[a - Pi/4 + b*x, 2]/(6*b) - (Cos[2*a + 2*b*x]*Sqrt[Sin[2*a + 2*b*x
]])/(6*b) - Sin[2*a + 2*b*x]^(5/2)/(10*b)
```

Rule 4298

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[(e^2*(e*Sin[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p +
1))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Sin[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx &= -\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}+\frac{1}{2}\int \sin^{\frac{3}{2}}(2a+2bx)\,dx\\ &= -\frac{\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{6b}-\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}+\frac{1}{6}\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= \frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{6b}-\frac{\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{6b}-\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}\end{aligned}$$

Mathematica [A] time = 0.34521, size = 76, normalized size = 1.1

$$\frac{20\sqrt{\sin(2(a+bx))}\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)-9\sin(2(a+bx))-10\sin(4(a+bx))+3\sin(6(a+bx))}{120b\sqrt{\sin(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (20*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*(a + b*x)]] - 9*Sin[2*(a + b*x)
] - 10*Sin[4*(a + b*x)] + 3*Sin[6*(a + b*x)])/(120*b*Sqrt[Sin[2*(a + b*x)]]
)
```

Maple [B] time = 48.767, size = 172329442, normalized size = 2497528.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}\sin\left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^(3/2)*sin(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos\left(bx+a\right)^{2}-1\right)\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sin(2*b*x + 2*a)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(2\,bx + 2\,a)^{\frac{3}{2}} \sin(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^(3/2)*sin(b*x + a)^2, x)
```

3.84 $\int \sin^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal. Leaf size=40

$$\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}$$

```
[Out] EllipticE[a - Pi/4 + b*x, 2]/(2*b) - Sin[2*a + 2*b*x]^(3/2)/(6*b)
```

Rubi [A] time = 0.0357468, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4298, 2639}

$$\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] EllipticE[a - Pi/4 + b*x, 2]/(2*b) - Sin[2*a + 2*b*x]^(3/2)/(6*b)
```

Rule 4298

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[(e^2*(e*Sin[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p +
1))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Sin[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx &= -\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}+\frac{1}{2}\int \sqrt{\sin(2a+2bx)}\,dx\\ &= \frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}\end{aligned}$$

Mathematica [A] time = 0.0827766, size = 34, normalized size = 0.85

$$-\frac{\sin^{\frac{3}{2}}(2(a+bx))-3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -(-3*EllipticE[a - Pi/4 + b*x, 2] + Sin[2*(a + b*x)]^(3/2))/(6*b)
```

Maple [B] time = 3.85, size = 16542587, normalized size = 413564.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sin\left(2\,bx + 2\,a\right)}\sin\left(bx + a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sin(2*b*x + 2*a))*sin(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(\cos\left(bx + a\right)^{2} - 1\right)\sqrt{\sin\left(2\,bx + 2\,a\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sqrt(sin(2*b*x + 2*a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sin\left(2\,bx + 2\,a\right)}\sin\left(bx + a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(2*b*x + 2*a))*sin(b*x + a)^2, x)
```

3.85 $\int \frac{\sin^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal. Leaf size=40

$$\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{2b}-\frac{\sqrt{\sin(2a+2bx)}}{2b}$$

```
[Out] EllipticF[a - Pi/4 + b*x, 2]/(2*b) - Sqrt[Sin[2*a + 2*b*x]]/(2*b)
```

Rubi [A] time = 0.0369828, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4298, 2641}

$$\frac{F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}-\frac{\sqrt{\sin(2a+2bx)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] EllipticF[a - Pi/4 + b*x, 2]/(2*b) - Sqrt[Sin[2*a + 2*b*x]]/(2*b)
```

Rule 4298

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[(e^2*(e*Sin[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p +
1))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Sin[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx &= -\frac{\sqrt{\sin(2a+2bx)}}{2b}+\frac{1}{2}\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= \frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b}-\frac{\sqrt{\sin(2a+2bx)}}{2b}\end{aligned}$$

Mathematica [A] time = 0.244552, size = 75, normalized size = 1.88

$$-\frac{\frac{\sqrt{2}(\sin(a+bx)+\cos(a+bx))\text{EllipticF}\left(\sin^{-1}(\cos(a+bx)-\sin(a+bx)),\frac{1}{2}\right)}{\sqrt{\sin(2(a+bx))+1}}+2\sqrt{\sin(2(a+bx))}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -(2*Sqrt[Sin[2*(a + b*x)]] + (Sqrt[2]*EllipticF[ArcSin[Cos[a + b*x] - Sin[a
 + b*x]], 1/2]*(Cos[a + b*x] + Sin[a + b*x]))/Sqrt[1 + Sin[2*(a + b*x)]])/(
4*b)
```

Maple [B] time = 8.632, size = 53350427, normalized size = 1333760.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^2}{\sqrt{\sin(2\,bx+2\,a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\cos(bx+a)^2-1}{\sqrt{\sin(2\,bx+2\,a)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)/sqrt(sin(2*b*x + 2*a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.86 $\int \frac{\sin^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)} dx$

Optimal. Leaf size=45

$$\frac{\sin^2(a+bx)}{b\sqrt{\sin(2a+2bx)}} - \frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}$$

```
[Out] -EllipticE[a - Pi/4 + b*x, 2]/(2*b) + Sin[a + b*x]^2/(b*Sqrt[Sin[2*a + 2*b*
x]])
```

Rubi [A] time = 0.0373112, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4296, 2639}

$$\frac{\sin^2(a+bx)}{b\sqrt{\sin(2a+2bx)}} - \frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] -EllipticE[a - Pi/4 + b*x, 2]/(2*b) + Sin[a + b*x]^2/(b*Sqrt[Sin[2*a + 2*b*
x]])
```

Rule 4296

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g
*(p + 1)), x] + Dist[(e^2*(m + 2*p + 2))/(4*g^2*(p + 1)), Int[(e*Sin[a + b*
x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[
p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m
, 2*p]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)} dx &= \frac{\sin^2(a+bx)}{b\sqrt{\sin(2a+2bx)}} - \frac{1}{2}\int \sqrt{\sin(2a+2bx)}\, dx \\
&= -\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b} + \frac{\sin^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}
\end{aligned}$$

Mathematica [A] time = 0.0955152, size = 41, normalized size = 0.91

$$\frac{\sqrt{\sin(2(a+bx))}\tan(a+bx) - E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-EllipticE[a - Pi/4 + b*x, 2] + Sqrt[Sin[2*(a + b*x)]]*Tan[a + b*x])/(2*b)
```

Maple [B] time = 9.329, size = 83117872, normalized size = 1847063.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^2/sin(2*b*x + 2*a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(\cos(bx+a)^2-1\right)\sqrt{\sin(2\,bx+2\,a)}}{\cos(2\,bx+2\,a)^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((cos(b*x + a)^2 - 1)*sqrt(sin(2*b*x + 2*a))/(cos(2*b*x + 2*a)^2 -
1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.87 $\int \frac{\sin^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=48

$$\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{6b}+\frac{\sin^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

```
[Out] EllipticF[a - Pi/4 + b*x, 2]/(6*b) + Sin[a + b*x]^2/(3*b*Sin[2*a + 2*b*x]^(
3/2))
```

Rubi [A] time = 0.0376517, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4296, 2641}

$$\frac{\sin^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] EllipticF[a - Pi/4 + b*x, 2]/(6*b) + Sin[a + b*x]^2/(3*b*Sin[2*a + 2*b*x]^(
3/2))
```

Rule 4296

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g
*(p + 1)), x] + Dist[(e^2*(m + 2*p + 2))/(4*g^2*(p + 1)), Int[(e*Sin[a + b*
x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[
p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m
, 2*p]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx &= \frac{\sin^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{1}{6}\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= \frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{6b}+\frac{\sin^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\end{aligned}$$

Mathematica [A] time = 0.190226, size = 83, normalized size = 1.73

$$\frac{\sqrt{\sin(2(a+bx))}\sec^2(a+bx)-\frac{\sqrt{2}(\sin(a+bx)+\cos(a+bx))\text{EllipticF}\left(\sin^{-1}(\cos(a+bx)-\sin(a+bx)),\frac{1}{2}\right)}{\sqrt{\sin(2(a+bx))+1}}}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (Sec[a + b*x]^2*Sqrt[Sin[2*(a + b*x)]] - (Sqrt[2]*EllipticF[ArcSin[Cos[a +
b*x] - Sin[a + b*x]], 1/2]*(Cos[a + b*x] + Sin[a + b*x]))/Sqrt[1 + Sin[2*(a
 + b*x)]])/(12*b)
```

Maple [A] time = 11.461, size = 123, normalized size = 2.6

$$\frac{1}{12\cos(2bx+2a)b}\left(\sqrt{\sin(2bx+2a)+1}\sqrt{-2\sin(2bx+2a)+2}\sqrt{-\sin(2bx+2a)}\mathrm{EllipticF}\left(\sqrt{\sin(2bx+2a)+1},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] 1/12/sin(2*b*x+2*a)^(3/2)/cos(2*b*x+2*a)*((sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(
2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticF((sin(2*b*x+2*a)+1)^(1
/2),1/2*2^(1/2))*sin(2*b*x+2*a)-2*cos(2*b*x+2*a)^2+2*cos(2*b*x+2*a))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^2}{\sin(2bx+2a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^2/sin(2*b*x + 2*a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cos(bx+a)^2-1}{\left(\cos(2bx+2a)^2-1\right)\sqrt{\sin(2bx+2a)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((cos(b*x + a)^2 - 1)/((cos(2*b*x + 2*a)^2 - 1)*sqrt(sin(2*b*x + 2*
a))), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.88 $\int \frac{\sin^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)} dx$

Optimal. Leaf size=77

$$\frac{\sin^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{10b} - \frac{3\cos(2a+2bx)}{10b\sqrt{\sin(2a+2bx)}}$$

```
[Out] (-3*EllipticE[a - Pi/4 + b*x, 2])/(10*b) + Sin[a + b*x]^2/(5*b*Sin[2*a + 2*
b*x]^(5/2)) - (3*Cos[2*a + 2*b*x])/(10*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.0464883, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4296, 2636, 2639}

$$\frac{\sin^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{10b} - \frac{3\cos(2a+2bx)}{10b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (-3*EllipticE[a - Pi/4 + b*x, 2])/(10*b) + Sin[a + b*x]^2/(5*b*Sin[2*a + 2*
b*x]^(5/2)) - (3*Cos[2*a + 2*b*x])/(10*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4296

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g
*(p + 1)), x] + Dist[(e^2*(m + 2*p + 2))/(4*g^2*(p + 1)), Int[(e*Sin[a + b*
x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[
p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m
, 2*p]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \frac{\sin^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{3}{10}\int \frac{1}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$$
$$= \frac{\sin^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{3\cos(2a+2bx)}{10b\sqrt{\sin(2a+2bx)}} - \frac{3}{10}\int \sqrt{\sin(2a+2bx)}\,dx$$
$$= -\frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{10b} + \frac{\sin^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{3\cos(2a+2bx)}{10b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A] time = 0.815191, size = 66, normalized size = 0.86

$$-\frac{12E\left(a+bx-\frac{\pi}{4}\middle|2\right)+\frac{4\sin^2(a+bx)(6\cos(2(a+bx))+3\cos(4(a+bx))+1)}{\sin^{\frac{5}{2}}(2(a+bx))}}{40b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] -(12*EllipticE[a - Pi/4 + b*x, 2] + (4*(1 + 6*Cos[2*(a + b*x)] + 3*Cos[4*(a
 + b*x)])*Sin[a + b*x]^2)/Sin[2*(a + b*x)]^(5/2))/(40*b)
```

Maple [B] time = 58.739, size = 227, normalized size = 3.

$$\frac{\sqrt{2}}{32\,b}\left(\frac{8\,\sqrt{2}}{5}(\sin(2\,bx+2\,a))^{-\frac{5}{2}}+\frac{4\,\sqrt{2}}{5\,\cos(2\,bx+2\,a)}\left(6\,\sqrt{\sin(2\,bx+2\,a)+1}\sqrt{-2\,\sin(2\,bx+2\,a)+2}\sqrt{-\sin(2\,bx+2\,a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x)
```

```
[Out] 1/32*2^(1/2)*(8/5*2^(1/2)/sin(2*b*x+2*a)^(5/2)+4/5*2^(1/2)/sin(2*b*x+2*a)^(
5/2)*(6*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*
a))^(1/2)*sin(2*b*x+2*a)^2*EllipticE((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))-
3*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1
/2)*sin(2*b*x+2*a)^2*EllipticF((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))+6*sin(
2*b*x+2*a)^4-4*sin(2*b*x+2*a)^2-2)/cos(2*b*x+2*a))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^2/sin(2*b*x + 2*a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(\cos\left(bx+a\right)^{2}-1\right)\sqrt{\sin\left(2\,bx+2\,a\right)}}{\cos\left(2\,bx+2\,a\right)^{4}-2\,\cos\left(2\,bx+2\,a\right)^{2}+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sqrt(sin(2*b*x + 2*a))/(cos(2*b*x + 2*a)^4 -
 2*cos(2*b*x + 2*a)^2 + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.89 $\int \sin^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=136

$$-\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b}+\frac{7\sin(a+bx)\sqrt{\sin(2a+2bx)}}{32b}-\frac{7\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{48b}-\frac{7\sin^{-1}(\cos(a+bx)-\text{:}}{64b}$$

```
[Out] (-7*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(64*b) - (7*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(64*b) + (7*Sin[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(32*b) - (7*Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(48*b) - (Sin[a
 + b*x]*Sin[2*a + 2*b*x]^(5/2))/(12*b)
```

Rubi [A] time = 0.101429, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4298, 4302, 4301, 4306}

$$-\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b}+\frac{7\sin(a+bx)\sqrt{\sin(2a+2bx)}}{32b}-\frac{7\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{48b}-\frac{7\sin^{-1}(\cos(a+bx)-\text{:}}{64b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-7*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(64*b) - (7*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(64*b) + (7*Sin[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(32*b) - (7*Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(48*b) - (Sin[a
 + b*x]*Sin[2*a + 2*b*x]^(5/2))/(12*b)
```

Rule 4298

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[(e^2*(e*Sin[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p +
1))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Sin[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\begin{aligned}\int \sin^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx &= -\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b}+\frac{7}{12}\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx\\ &= -\frac{7\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{48b}-\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b}+\frac{7}{16}\int \cos(a+bx)\sqrt{s\\ &= \frac{7\sin(a+bx)\sqrt{\sin(2a+2bx)}}{32b}-\frac{7\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{48b}-\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a\\ &= -\frac{7\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{64b}-\frac{7\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+}\right.}{64b}\end{aligned}$$

Mathematica [A] time = 0.340175, size = 98, normalized size = 0.72

$$\frac{\frac{2}{3}\sqrt{\sin(2(a+bx))}(10\sin(a+bx)-9\sin(3(a+bx))+2\sin(5(a+bx)))-7\left(\sin^{-1}(\cos(a+bx)-\sin(a+bx))+\log\left(\sin(a\right.\right.}{64b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-7*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]) + (2*Sqrt[Sin[2*(a + b*x)]]*(10*Sin[a + b*x] - 9
*Sin[3*(a + b*x)] + 2*Sin[5*(a + b*x)]))/3)/(64*b)
```

Maple [B] time = 127.454, size = 322248039, normalized size = 2369470.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(2\,bx+2\,a)^{\frac{3}{2}}\sin(bx+a)^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^(3/2)*sin(b*x + a)^3, x)
```

Fricas [B] time = 0.569239, size = 805, normalized size = 5.92

$$\frac{8\sqrt{2}\left(32\cos\left(bx+a\right)^{4}-60\cos\left(bx+a\right)^{2}+21\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}\sin\left(bx+a\right)+42\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(b}}{\cos(}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/768*(8*sqrt(2)*(32*cos(b*x + a)^4 - 60*cos(b*x + a)^2 + 21)*sqrt(cos(b*x
+ a)*sin(b*x + a))*sin(b*x + a) + 42*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin
(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(
b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) - 42*arctan(-(2*sqrt(2)*sqrt
(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) -
sin(b*x + a))) + 21*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 -
(4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin
(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}\sin\left(bx+a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^(3/2)*sin(b*x + a)^3, x)
```

3.90 $\int \sin^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal. Leaf size=110

$$-\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b}-\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{32b}-\frac{5\sqrt{\sin(2a+2bx)}\cos(a+bx)}{16b}+\frac{5\log\left(\sin(a+bx)+\right.}{}$$

```
[Out] (-5*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(32*b) + (5*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(32*b) - (5*Cos[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(16*b) - (Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(8*b)
```

Rubi [A] time = 0.0782088, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4298, 4302, 4305}

$$-\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b}-\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{32b}-\frac{5\sqrt{\sin(2a+2bx)}\cos(a+bx)}{16b}+\frac{5\log\left(\sin(a+bx)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```


```
[Out] (-5*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(32*b) + (5*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(32*b) - (5*Cos[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(16*b) - (Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(8*b)
```


Rule 4298

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[(e^2*(e*Sin[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p +
1))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Sin[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \sin^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = -\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b} + \frac{5}{8}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$
$$= -\frac{5\cos(a+bx)\sqrt{\sin(2a+2bx)}}{16b} - \frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b} + \frac{5}{16}\int \frac{\cos(a+b}{\sqrt{\sin(2a+}}$$
$$= -\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{32b} + \frac{5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2}\right.}{32b}$$

Mathematica [A] time = 0.219184, size = 86, normalized size = 0.78

$$\frac{2\sqrt{\sin(2(a+bx))}(\cos(3(a+bx))-6\cos(a+bx))+5\left(\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(a+bx)\right)-\sin^{-1}(\cos\right.}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (5*(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]) + 2*(-6*Cos[a + b*x] + Cos[3*(a + b*x)])*Sqrt[Si
n[2*(a + b*x)]])/(32*b)
```

Maple [B] time = 15.251, size = 47430975, normalized size = 431190.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sin(2\,bx+2\,a)}\sin(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sin(2*b*x + 2*a))*sin(b*x + a)^3, x)
```

Fricas [B] time = 0.550574, size = 774, normalized size = 7.04

$$\frac{8\sqrt{2}\left(4\cos(bx+a)^3-9\cos(bx+a)\right)\sqrt{\cos(bx+a)\sin(bx+a)}+10\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/128*(8*sqrt(2)*(4*cos(b*x + a)^3 - 9*cos(b*x + a))*sqrt(cos(b*x + a)*sin(
b*x + a)) + 10*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x +
a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x
 + a)*sin(b*x + a) - 1)) - 10*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x
+ a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - 5*log
(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*
sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x
 + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.91 $\int \frac{\sin^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal. Leaf size=84

$$-\frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{4b}-\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{8b}-\frac{3\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+b\right.}{8b}$$

```
[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(8*b) - (3*Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(8*b) - (Sin[a + b*x]*Sqrt[Sin[2*a + 2*
b*x]])/(4*b)
```

Rubi [A] time = 0.0541423, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4298, 4306}

$$-\frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{4b}-\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{8b}-\frac{3\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+b\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(8*b) - (3*Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(8*b) - (Sin[a + b*x]*Sqrt[Sin[2*a + 2*
b*x]])/(4*b)
```

Rule 4298

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[(e^2*(e*Sin[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p +
1))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Sin[a
+ b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p},
x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] && GtQ[m, 1] && NeQ
[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx &= -\frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{4b}+\frac{3}{4}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= -\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{8b}-\frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{8b}-\end{aligned}$$

Mathematica [A] time = 0.140747, size = 74, normalized size = 0.88

$$-\frac{2\sin(a+bx)\sqrt{\sin(2(a+bx))}+3\sin^{-1}(\cos(a+bx)-\sin(a+bx))+3\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(a+\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -(3*ArcSin[Cos[a + b*x] - Sin[a + b*x]] + 3*Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]] + 2*Sin[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/(8*b)
```

Maple [B] time = 33.727, size = 155734626, normalized size = 1853983.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^3}{\sqrt{\sin(2\,bx+2\,a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^3/sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [B] time = 0.547127, size = 740, normalized size = 8.81

$$-\frac{8\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\sin(bx+a) - 6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right) + 6\,a\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/32*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*sin(b*x + a) - 6*arctan(-(
sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos
(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1))
 + 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - si
n(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - 3*log(-32*cos(b*x + a)^4 + 4*s
qrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x
+ a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)
*sin(b*x + a) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.92 $\int \frac{\sin^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=81

$$\frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}+\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{4b}$$

```
[Out] ArcSin[Cos[a + b*x] - Sin[a + b*x]]/(4*b) - Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*a + 2*b*x]]]/(4*b) + Sin[a + b*x]/(b*Sqrt[Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.0778138, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4294, 4308, 4305}

$$\frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}+\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] ArcSin[Cos[a + b*x] - Sin[a + b*x]]/(4*b) - Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*a + 2*b*x]]]/(4*b) + Sin[a + b*x]/(b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4294

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[(e^2*(e*Sin[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p +
1))/(2*b*g*(p + 1)), x] + Dist[(e^4*(m + p - 1))/(4*g^2*(p + 1)), Int[(e*Si
n[a + b*x])^(m - 4)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d,
e, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 2]
 && LtQ[p, -1] && (GtQ[m, 3] || EqQ[p, -3/2]) && IntegersQ[2*m, 2*p]
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}} - \frac{1}{4}\int \csc(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$
$$= \frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}} - \frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$$
$$= \frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b} - \frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{4b} + \frac{\mathrm{si}}{b\sqrt{\mathrm{s}}}$$

Mathematica [A] time = 0.0905944, size = 72, normalized size = 0.89

$$\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx)) + 2\sqrt{\sin(2(a+bx))}\sec(a+bx) - \log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(a+bx)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (ArcSin[Cos[a + b*x] - Sin[a + b*x]] - Log[Cos[a + b*x] + Sin[a + b*x] + Sq
rt[Sin[2*(a + b*x)]]] + 2*Sec[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/(4*b)
```

Maple [B] time = 30.918, size = 149404972, normalized size = 1844505.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^3/sin(2*b*x + 2*a)^(3/2), x)
```

Fricas [B] time = 0.535688, size = 815, normalized size = 10.06

$$-\frac{2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)\cos(bx+a) - 2\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}}{\cos(bx+a)-\mathrm{si}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/16*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - s
in(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*
sin(b*x + a) - 1))*cos(b*x + a) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*si
n(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a)))*c
os(b*x + a) - cos(b*x + a)*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x +
a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x +
 a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1) -
 8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - 8*cos(b*x + a))/(b*cos(b*x + a
))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.93 $\int \frac{\sin^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=28

$$\frac{\sin^3(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

```
[Out] Sin[a + b*x]^3/(3*b*Sin[2*a + 2*b*x]^(3/2))
```

Rubi [A] time = 0.0280523, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {4292}

$$\frac{\sin^3(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]

[Out] Sin[a + b*x]^3/(3*b*Sin[2*a + 2*b*x]^(3/2))
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \frac{\sin^3(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

Mathematica [A] time = 0.0542326, size = 27, normalized size = 0.96

$$\frac{\sin^3(a+bx)}{3b\sin^{\frac{3}{2}}(2(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]

[Out] Sin[a + b*x]^3/(3*b*Sin[2*(a + b*x)]^(3/2))
```

Maple [C] time = 44.19, size = 727, normalized size = 26.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] -1/48*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1/2
*a)^2-1)*(6*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-
tan(1/2*b*x+1/2*a))^(1/2)*EllipticE((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2
))*tan(1/2*b*x+1/2*a)^6-3*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*
a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1
/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^6+18*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*t
an(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticE((tan(1/2*b
*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^4-9*(tan(1/2*b*x+1/2*a)+
1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*Ellipt
icF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^4+6*tan(1/
2*b*x+1/2*a)^8+18*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1
/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticE((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*
2^(1/2))*tan(1/2*b*x+1/2*a)^2-9*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*
x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)
+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^2-2*tan(1/2*b*x+1/2*a)^6+6*(tan(1
/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a)
)^(1/2)*EllipticE((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))-3*(tan(1/2*b*x+
1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)
*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))+10*tan(1/2*b*x+1/2*a)^
4-14*tan(1/2*b*x+1/2*a)^2)/(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1
/2)/(tan(1/2*b*x+1/2*a)^2+1)^3/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1
/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(bx + a\right)^{3}}{\sin\left(2\,bx + 2\,a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^3/sin(2*b*x + 2*a)^(5/2), x)
```

Fricas [B] time = 0.493171, size = 132, normalized size = 4.71

$$-\frac{\cos\left(bx + a\right)^{2} - \sqrt{2}\sqrt{\cos\left(bx + a\right)\sin\left(bx + a\right)}\sin\left(bx + a\right)}{12\,b\cos\left(bx + a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(cos(b*x + a)^2 - sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*sin(b*x + a
))/(b*cos(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.94 $\int \frac{\sin^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=55

$$\frac{\sin^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{\sin(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

```
[Out] Sin[a + b*x]^3/(5*b*Sin[2*a + 2*b*x]^(5/2)) + Sin[a + b*x]/(5*b*Sqrt[Sin[2*
a + 2*b*x]])
```

Rubi [A] time = 0.0486463, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4296, 4292}

$$\frac{\sin^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{\sin(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] Sin[a + b*x]^3/(5*b*Sin[2*a + 2*b*x]^(5/2)) + Sin[a + b*x]/(5*b*Sqrt[Sin[2*
a + 2*b*x]])
```

Rule 4296

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g
*(p + 1)), x] + Dist[(e^2*(m + 2*p + 2))/(4*g^2*(p + 1)), Int[(e*Sin[a + b*
x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[
p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m
, 2*p]
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx&=\frac{\sin^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{1}{5}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx\\&=\frac{\sin^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{\sin(a+bx)}{5b\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [A] time = 0.0961792, size = 35, normalized size = 0.64

$$\frac{\sqrt{\sin(2(a+bx))}\sec(a+bx)\left(\sec^2(a+bx)+4\right)}{40b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (Sec[a + b*x]*(4 + Sec[a + b*x]^2)*Sqrt[Sin[2*(a + b*x)]])/(40*b)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (\sin (bx+a))^3 (\sin (2bx+2a))^{-\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x)
```

```
[Out] int(sin(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin (bx+a)^3}{\sin (2bx+2a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^3/sin(2*b*x + 2*a)^(7/2), x)
```

Fricas [A] time = 0.503384, size = 147, normalized size = 2.67

$$\frac{4 \cos (bx+a)^3 + \sqrt{2}\left(4 \cos (bx+a)^2 + 1\right)\sqrt{\cos (bx+a)\sin (bx+a)}}{40\, b \cos (bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/40*(4*cos(b*x + a)^3 + sqrt(2)*(4*cos(b*x + a)^2 + 1)*sqrt(cos(b*x + a)*s
in(b*x + a)))/(b*cos(b*x + a)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.95 $\int \frac{\sin^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=81

$$\frac{\sin^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}+\frac{2\sin(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{4\cos(a+bx)}{21b\sqrt{\sin(2a+2bx)}}$$

```
[Out] Sin[a + b*x]^3/(7*b*Sin[2*a + 2*b*x]^(7/2)) + (2*Sin[a + b*x])/(21*b*Sin[2*
a + 2*b*x]^(3/2)) - (4*Cos[a + b*x])/(21*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.0691508, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4296, 4304, 4291}

$$\frac{\sin^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}+\frac{2\sin(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{4\cos(a+bx)}{21b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] Sin[a + b*x]^3/(7*b*Sin[2*a + 2*b*x]^(7/2)) + (2*Sin[a + b*x])/(21*b*Sin[2*
a + 2*b*x]^(3/2)) - (4*Cos[a + b*x])/(21*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4296

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g
*(p + 1)), x] + Dist[(e^2*(m + 2*p + 2))/(4*g^2*(p + 1)), Int[(e*Sin[a + b*
x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[
p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m
, 2*p]
```

Rule 4304

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> -Simp[(Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dis
t[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4291

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> -Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*
m), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/
b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = \frac{\sin^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{2}{7}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$$
$$= \frac{\sin^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{2\sin(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{4}{21}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$$
$$= \frac{\sin^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{2\sin(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)} - \frac{4\cos(a+bx)}{21b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A] time = 0.106691, size = 55, normalized size = 0.68

$$-\frac{\sqrt{\sin(2(a+bx))}(12\cos(2(a+bx))+4\cos(4(a+bx))+5)\csc(a+bx)\sec^4(a+bx)}{336b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] -((5 + 12*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Csc[a + b*x]*Sec[a + b*x]^
4*Sqrt[Sin[2*(a + b*x)]])/(336*b)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (\sin(bx+a))^3 (\sin(2bx+2a))^{-\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x)
```

```
[Out] int(sin(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^3}{\sin(2bx+2a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^3/sin(2*b*x + 2*a)^(9/2), x)
```

Fricas [A] time = 0.517084, size = 213, normalized size = 2.63

$$-\frac{32\cos(bx+a)^4\sin(bx+a) + \sqrt{2}\left(32\cos(bx+a)^4 - 8\cos(bx+a)^2 - 3\right)\sqrt{\cos(bx+a)\sin(bx+a)}}{336\,b\cos(bx+a)^4\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/336*(32*cos(b*x + a)^4*sin(b*x + a) + sqrt(2)*(32*cos(b*x + a)^4 - 8*cos
(b*x + a)^2 - 3)*sqrt(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^4*sin(b*x
 + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.96 $\int \frac{\sin^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=107

$$\frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}+\frac{\sin(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{8\sin(a+bx)}{45b\sqrt{\sin(2a+2bx)}}-\frac{4\cos(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}$$

```
[Out] Sin[a + b*x]^3/(9*b*Sin[2*a + 2*b*x]^(9/2)) + Sin[a + b*x]/(15*b*Sin[2*a +
2*b*x]^(5/2)) - (4*Cos[a + b*x])/(45*b*Sin[2*a + 2*b*x]^(3/2)) + (8*Sin[a +
 b*x])/(45*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.0924541, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4296, 4304, 4303, 4292}

$$\frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}+\frac{\sin(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{8\sin(a+bx)}{45b\sqrt{\sin(2a+2bx)}}-\frac{4\cos(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(11/2),x]
```

```
[Out] Sin[a + b*x]^3/(9*b*Sin[2*a + 2*b*x]^(9/2)) + Sin[a + b*x]/(15*b*Sin[2*a +
2*b*x]^(5/2)) - (4*Cos[a + b*x])/(45*b*Sin[2*a + 2*b*x]^(3/2)) + (8*Sin[a +
 b*x])/(45*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4296

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g
*(p + 1)), x] + Dist[(e^2*(m + 2*p + 2))/(4*g^2*(p + 1)), Int[(e*Sin[a + b*
x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[
p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m
, 2*p]
```

Rule 4304

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> -Simp[(Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dis
t[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4303

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx &= \frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}+\frac{1}{3}\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx\\
&= \frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}+\frac{\sin(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{4}{15}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx\\
&= \frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}+\frac{\sin(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{4\cos(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{8}{45}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx\\
&= \frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}+\frac{\sin(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{4\cos(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{8\sin(a+bx)}{45b\sqrt{\sin(2a+2bx)}}
\end{aligned}
$$

Mathematica [A] time = 0.163097, size = 62, normalized size = 0.58

$$
\frac{\sqrt{\sin(2(a+bx))}\left(5\sec^5(a+bx)+17\sec^3(a+bx)+113\sec(a+bx)-15\cot(a+bx)\csc(a+bx)\right)}{1440b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(11/2),x]
```

```
[Out] ((-15*Cot[a + b*x]*Csc[a + b*x] + 113*Sec[a + b*x] + 17*Sec[a + b*x]^3 + 5*
Sec[a + b*x]^5)*Sqrt[Sin[2*(a + b*x)]])/(1440*b)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$
\int (\sin(bx+a))^3(\sin(2bx+2a))^{-\frac{11}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x)
```

```
[Out] int(sin(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\sin(bx+a)^3}{\sin(2bx+2a)^{\frac{11}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^3/sin(2*b*x + 2*a)^(11/2), x)
```

Fricas [A] time = 0.535726, size = 263, normalized size = 2.46

$$\frac{128 \cos(bx+a)^7 - 128 \cos(bx+a)^5 + \sqrt{2}\left(128 \cos(bx+a)^6 - 96 \cos(bx+a)^4 - 12 \cos(bx+a)^2 - 5\right)\sqrt{\cos(bx+a)\sin(bx+a)}}{1440\left(b\cos(bx+a)^7 - b\cos(bx+a)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/1440*(128*cos(b*x + a)^7 - 128*cos(b*x + a)^5 + sqrt(2)*(128*cos(b*x + a)
^6 - 96*cos(b*x + a)^4 - 12*cos(b*x + a)^2 - 5)*sqrt(cos(b*x + a)*sin(b*x +
 a)))/(b*cos(b*x + a)^7 - b*cos(b*x + a)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.97 $\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=136

$$\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{3b}+\frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{8b}-\frac{5\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{12b}-\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{16b}$$

```
[Out] (-5*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(16*b) - (5*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(16*b) + (5*Sin[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(8*b) - (5*Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(12*b) + (Sin[a
+ b*x]*Sin[2*a + 2*b*x]^(5/2))/(3*b)
```

Rubi [A] time = 0.122951, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4308, 4301, 4302, 4306}

$$\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{3b}+\frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{8b}-\frac{5\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{12b}-\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (-5*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(16*b) - (5*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(16*b) + (5*Sin[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(8*b) - (5*Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(12*b) + (Sin[a
+ b*x]*Sin[2*a + 2*b*x]^(5/2))/(3*b)
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] &
& IntegerQ[2*p]
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
```

```
a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = 2\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$$
$$= \frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{3b} + \frac{5}{3}\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$= -\frac{5\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{12b} + \frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{3b} + \frac{5}{4}\int \cos(a+bx)\sqrt{\sin$$
$$= \frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{8b} - \frac{5\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{12b} + \frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+$$
$$= -\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{16b} - \frac{5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+}\right.}{16b}$$

Mathematica [A] time = 0.321705, size = 98, normalized size = 0.72

$$\frac{\frac{2}{3}\sqrt{\sin(2(a+bx))}(14\sin(a+bx)-3\sin(3(a+bx))-2\sin(5(a+bx)))-5\left(\sin^{-1}(\cos(a+bx)-\sin(a+bx))+\log\left(\sin(a\right.\right.}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (-5*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]) + (2*Sqrt[Sin[2*(a + b*x)]]*(14*Sin[a + b*x] - 3
*Sin[3*(a + b*x)] - 2*Sin[5*(a + b*x)]))/3)/(16*b)
```

Maple [C] time = 4.362, size = 973, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^(7/2),x)
```

```
[Out] -16/5/b*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(6*(tan(1/2*b*
x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/
2)*EllipticE((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*b*x+1/2*a)*
(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2*a)+1))^(1/2)*tan(1/2*b*x+1/2*a)^4-3
*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x
+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*(tan(1/2
*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2*a)+1))^(1/2)*tan(1/2*b*
x+1/2*a)^4-12*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*
(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticE((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1
/2))*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2*a)+1))^(1/
2)*tan(1/2*b*x+1/2*a)^2+6*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*
a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1
/2),1/2*2^(1/2))*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/
2*a)+1))^(1/2)*tan(1/2*b*x+1/2*a)^2+6*(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2
*a))^(1/2)*tan(1/2*b*x+1/2*a)^6+6*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*
```

```
b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticE((tan(1/2*b*x+1/2*
a)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/
2*b*x+1/2*a)+1))^(1/2)-3*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a
)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/
2),1/2*2^(1/2))*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2
*a)+1))^(1/2)-12*(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)*tan(1/2*b*
x+1/2*a)^4-4*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2*a)
+1))^(1/2)*tan(1/2*b*x+1/2*a)^4+6*(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))
^(1/2)*tan(1/2*b*x+1/2*a)^2-4*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(t
an(1/2*b*x+1/2*a)+1))^(1/2)*tan(1/2*b*x+1/2*a)^2)/(tan(1/2*b*x+1/2*a)*(tan(
1/2*b*x+1/2*a)^2-1))^(1/2)/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)/
(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2*a)+1))^(1/2)/(t
an(1/2*b*x+1/2*a)-1)/(tan(1/2*b*x+1/2*a)+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(bx+a)\sin(2\,bx+2\,a)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)*sin(2*b*x + 2*a)^(7/2), x)
```

Fricas [B] time = 0.579119, size = 806, normalized size = 5.93

$$-\frac{8\,\sqrt{2}\left(32\,\cos(bx+a)^4-12\,\cos(bx+a)^2-15\right)\sqrt{\cos(bx+a)\sin(bx+a)}\,\sin(bx+a)-30\,\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(}}{\cos}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/192*(8*sqrt(2)*(32*cos(b*x + a)^4 - 12*cos(b*x + a)^2 - 15)*sqrt(cos(b*x
 + a)*sin(b*x + a))*sin(b*x + a) - 30*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*si
n(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos
(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) + 30*arctan(-(2*sqrt(2)*sqr
t(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) -
 sin(b*x + a))) - 15*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 -
 (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*si
n(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.98 $\int \csc(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=110

$$\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{2b}-\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{8b}-\frac{3\sqrt{\sin(2a+2bx)}\cos(a+bx)}{4b}+\frac{3\log\left(\sin(a+bx)\right.}{}$$

```
[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(8*b) + (3*Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(8*b) - (3*Cos[a + b*x]*Sqrt[Sin[2*a +
2*b*x]])/(4*b) + (Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(2*b)
```

Rubi [A] time = 0.095278, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4308, 4301, 4302, 4305}

$$\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{2b}-\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{8b}-\frac{3\sqrt{\sin(2a+2bx)}\cos(a+bx)}{4b}+\frac{3\log\left(\sin(a+bx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```


```
[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(8*b) + (3*Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(8*b) - (3*Cos[a + b*x]*Sqrt[Sin[2*a +
2*b*x]])/(4*b) + (Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(2*b)
```


Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \csc(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = 2\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$= \frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{2b} + \frac{3}{2}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$
$$= -\frac{3\cos(a+bx)\sqrt{\sin(2a+2bx)}}{4b} + \frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{2b} + \frac{3}{4}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}$$
$$= -\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{8b} + \frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+\right.}}{8b}$$

Mathematica [A] time = 0.190355, size = 86, normalized size = 0.78

$$\frac{3\left(\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(a+bx)\right)-\sin^{-1}(\cos(a+bx)-\sin(a+bx))\right)-2\sqrt{\sin(2(a+bx))}(2\cos(a+b}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (3*(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]) - 2*(2*Cos[a + b*x] + Cos[3*(a + b*x)])*Sqrt[Sin
[2*(a + b*x)]])/(8*b)
```

Maple [C] time = 2.911, size = 243, normalized size = 2.2

$$-\frac{8}{3b}\sqrt{-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2-1\right)^{-1}}\left(\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\sqrt{-2\tan(1/2\,bx+a/2)+2}\sqrt{-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}\text{Ellip}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] -8/3/b*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*((tan(1/2*b*x+1
/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*
EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^2-(t
an(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/
2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))+2*tan(1/2*b
*x+1/2*a)^3+2*tan(1/2*b*x+1/2*a))/(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2
-1))^(1/2)/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(bx+a)\sin(2\,bx+2\,a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)*sin(2*b*x + 2*a)^(5/2), x)
```

Fricas [B] time = 0.55652, size = 768, normalized size = 6.98

$$-\frac{8\sqrt{2}\left(4\cos\left(bx+a\right)^{3}-\cos\left(bx+a\right)\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}-6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cdots}{\cos(bx+a)^{2}+2\cos(bx+a)\sin(bx+a\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/32*(8*sqrt(2)*(4*cos(b*x + a)^3 - cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*
x + a)) - 6*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a)
- sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x +
a)*sin(b*x + a) - 1)) + 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)
) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + 3*log(-32
*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(
b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a
)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \csc\left(bx+a\right)\sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)*sin(2*b*x + 2*a)^(5/2), x)
```

3.99 $\int \csc(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=81

$$\frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{b}-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}$$

```
[Out] -ArcSin[Cos[a + b*x] - Sin[a + b*x]]/(2*b) - Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b) + (Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/b
```

Rubi [A] time = 0.0722475, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4308, 4301, 4306}

$$\frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{b}-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] -ArcSin[Cos[a + b*x] - Sin[a + b*x]]/(2*b) - Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b) + (Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/b
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\begin{aligned}\int \csc(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx &= 2\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx\\ &= \frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{b}+\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= -\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right.}{2b}\end{aligned}$$

Mathematica [A] time = 0.0863831, size = 70, normalized size = 0.86

$$-\frac{-2\sin(a+bx)\sqrt{\sin(2(a+bx))}+\sin^{-1}(\cos(a+bx)-\sin(a+bx))+\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(a+bx\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] -(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x] + S
qrt[Sin[2*(a + b*x)]]] - 2*Sin[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/(2*b)
```

Maple [C] time = 1.805, size = 362, normalized size = 4.5

$$4\frac{\left((\tan(1/2\,bx+a/2))^2-1\right)\left(2\sqrt{\tan(1/2\,bx+a/2)+1}\sqrt{-2\,\tan(1/2\,bx+a/2)+2}\sqrt{-\tan(1/2\,bx+a/2)}\text{EllipticE}\left(\sqrt{t}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] 4/b*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1/2*a
)^2-1)*(2*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-ta
n(1/2*b*x+1/2*a))^(1/2)*EllipticE((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))
*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2*a)+1))^(1/2)-(
tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1
/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*b
*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2*a)+1))^(1/2)+2*(tan(1/2*b
*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)*tan(1/2*b*x+1/2*a)^2)/(tan(1/2*b*x+1/
2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*
a))^(1/2)/(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2*a)+1)
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(bx+a)\sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)*sin(2*b*x + 2*a)^(3/2), x)
```

Fricas [B] time = 0.541782, size = 734, normalized size = 9.06

$$\frac{8\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\sin(bx+a)+2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)-2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*sin(b*x + a) + 2*arctan(-(sq
rt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b
*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) -
 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(
b*x + a))/(cos(b*x + a) - sin(b*x + a))) + log(-32*cos(b*x + a)^4 + 4*sqrt(
2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a)
)*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin
(b*x + a) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(bx+a)\sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)*sin(2*b*x + 2*a)^(3/2), x)
```

3.100 $\int \csc(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal. Leaf size=53

$$\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{b}-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{b}$$

```
[Out] -(ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b) + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*a + 2*b*x]]]/b
```

Rubi [A] time = 0.0472142, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4308, 4305}

$$\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{b}-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -(ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b) + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*a + 2*b*x]]]/b
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\begin{aligned}\int \csc(a+bx)\sqrt{\sin(2a+2bx)}\,dx &= 2\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= -\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{b}+\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+\right.}{b}\end{aligned}$$

Mathematica [A] time = 0.0406555, size = 52, normalized size = 0.98

$$\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(a+bx)\right)}{b}-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -(ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b) + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]/b
```

Maple [C] time = 1.042, size = 157, normalized size = 3.

$$2\frac{\left((\tan(1/2\,bx+a/2))^2-1\right)\sqrt{\tan(1/2\,bx+a/2)+1}\sqrt{-2\,\tan(1/2\,bx+a/2)+2}\sqrt{-\tan(1/2\,bx+a/2)}\mathrm{EllipticF}\left(\sqrt{\tan(1/2\,bx+a/2)+1},1/2\sqrt{2}\right)}{b\sqrt{\tan(1/2\,bx+a/2)\left((\tan(1/2\,bx+a/2))^2-1\right)}\sqrt{(\tan(1/2\,bx+a/2))^3-\tan(1/2\,bx+a/2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] 2/b*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1/2*a
)^2-1)/(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1/2
*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)/(t
an(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)
+1)^(1/2),1/2*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(bx+a)\sqrt{\sin(2\,bx+2\,a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)*sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [B] time = 0.520019, size = 657, normalized size = 12.4

$$\frac{2\,\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\,\cos(bx+a)\sin(bx+a)-1}\right)-2\,\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}-\cos(bx+a)-\sin(bx+a)}{\cos(bx+a)-\sin(bx+a)}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin
(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*si
n(b*x + a) - 1)) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - c
os(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - log(-32*cos(b*
x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a
) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 1
6*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(bx+a)\sqrt{\sin(2bx+2a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)*sqrt(sin(2*b*x + 2*a)), x)
```

3.101 $\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal. Leaf size=24

$$-\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)}{b}$$

```
[Out] -((Csc[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/b)
```

Rubi [A] time = 0.0238121, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {4292}

$$-\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -((Csc[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/b)
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{\csc(a+bx)\sqrt{\sin(2a+2bx)}}{b}$$

Mathematica [A] time = 0.0467407, size = 23, normalized size = 0.96

$$-\frac{\sqrt{\sin(2(a+bx))}\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -((Csc[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/b)
```

Maple [C] time = 1.546, size = 308, normalized size = 12.8

$$\frac{1}{b}\sqrt{-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2-1\right)^{-1}}\left(2\sqrt{\tan(1/2\,bx+a/2)\left((\tan(1/2\,bx+a/2))^2-1\right)}\sqrt{\tan(1/2\,bx+a/2)+1}\sqrt{-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] 1/b*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(2*(tan(1/2*b*x+1/
2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1
/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticE((tan(1/2*b*x+1
/2*a)+1)^(1/2),1/2*2^(1/2))-(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(
1/2)*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2
*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))+(tan
(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)*tan(1/2*b*x+1/2*a)^2-(tan(1/2*b
*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2))/tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*
a)^3-tan(1/2*b*x+1/2*a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)}{\sqrt{\sin(2bx+2a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)/sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [A] time = 0.491203, size = 103, normalized size = 4.29

$$-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}+\sin(bx+a)}{b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) + sin(b*x + a))/(b*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)}{\sqrt{\sin(2bx+2a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)/sqrt(sin(2*b*x + 2*a)), x)
```

3.102 $\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=53

$$\frac{4\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

```
[Out] (-2*Cos[a + b*x])/(3*b*Sin[2*a + 2*b*x]^(3/2)) + (4*Sin[a + b*x])/(3*b*Sqrt
[Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.0642377, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4308, 4303, 4292}

$$\frac{4\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(3*b*Sin[2*a + 2*b*x]^(3/2)) + (4*Sin[a + b*x])/(3*b*Sqrt
[Sin[2*a + 2*b*x]])
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rule 4303

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = 2\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$$
$$= -\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{4}{3}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$$
$$= -\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{4\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A] time = 0.100315, size = 43, normalized size = 0.81

$$\frac{\sqrt{\sin(2(a+bx))}\left(\frac{1}{2}\sec(a+bx)-\frac{1}{6}\cot(a+bx)\csc(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] ((-(Cot[a + b*x]*Csc[a + b*x])/6 + Sec[a + b*x]/2)*Sqrt[Sin[2*(a + b*x)]])/
b
```

Maple [C] time = 2.469, size = 194, normalized size = 3.7

$$-\frac{1}{12\,b}\sqrt{-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2-1\right)^{-1}}\left(\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2-1\right)\left(2\sqrt{\tan(1/2\,bx+a/2)+1}\sqrt{-2\tan(1/2\,bx+a/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] -1/12/b*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1
/2*a)^2-1)/tan(1/2*b*x+1/2*a)*(2*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b
*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a
)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)-tan(1/2*b*x+1/2*a)^4+1)/(tan(1/2
*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b
*x+1/2*a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(3/2), x)
```

Fricas [A] time = 0.498337, size = 192, normalized size = 3.62

$$\frac{4\cos(bx+a)^3+\sqrt{2}\left(4\cos(bx+a)^2-3\right)\sqrt{\cos(bx+a)\sin(bx+a)}-4\cos(bx+a)}{6\left(b\cos(bx+a)^3-b\cos(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*cos(b*x + a)^3 + sqrt(2)*(4*cos(b*x + a)^2 - 3)*sqrt(cos(b*x + a)*si
n(b*x + a)) - 4*cos(b*x + a))/(b*cos(b*x + a)^3 - b*cos(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(3/2), x)
```

3.103 $\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)} dx$

Optimal. Leaf size=79

$$\frac{8\sin(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)} - \frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{16\cos(a+bx)}{15b\sqrt{\sin(2a+2bx)}}$$

```
[Out] (-2*Cos[a + b*x])/(5*b*Sin[2*a + 2*b*x]^(5/2)) + (8*Sin[a + b*x])/(15*b*Sin
[2*a + 2*b*x]^(3/2)) - (16*Cos[a + b*x])/(15*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.0860161, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4308, 4303, 4304, 4291}

$$\frac{8\sin(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)} - \frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{16\cos(a+bx)}{15b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(5*b*Sin[2*a + 2*b*x]^(5/2)) + (8*Sin[a + b*x])/(15*b*Sin
[2*a + 2*b*x]^(3/2)) - (16*Cos[a + b*x])/(15*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rule 4303

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4304

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> -Simp[(Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dis
t[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4291

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> -Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*
m), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/
b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx &= 2\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx\\ &= -\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{8}{5}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx\\ &= -\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{8\sin(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{16}{15}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx\\ &= -\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{8\sin(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{16\cos(a+bx)}{15b\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [A] time = 0.124077, size = 52, normalized size = 0.66

$$-\frac{\sqrt{\sin(2(a+bx))}\left(3\csc^3(a+bx)+27\csc(a+bx)-5\tan(a+bx)\sec(a+bx)\right)}{60b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] -(Sqrt[Sin[2*(a + b*x)]]*(27*Csc[a + b*x] + 3*Csc[a + b*x]^3 - 5*Sec[a + b*
x]*Tan[a + b*x]))/(60*b)
```

Maple [C] time = 5.486, size = 481, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] -1/80/b*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)/tan(1/2*b*x+1/
2*a)^3*(24*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x
+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2
)*EllipticE((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^2-
12*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1/2*a)+
1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*Ellipt
icF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^2+(tan(1/2
*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*tan(1/2*b*x+1/2*a)^6-(tan(1/2*b
*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*tan(1/2*b*x+1/2*a)^4+12*(tan(1/2*
b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)*tan(1/2*b*x+1/2*a)^4-(tan(1/2*b*x+1/
2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*tan(1/2*b*x+1/2*a)^2-12*(tan(1/2*b*x+1
/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)*tan(1/2*b*x+1/2*a)^2+(tan(1/2*b*x+1/2*a)*
(tan(1/2*b*x+1/2*a)^2-1))^(1/2))/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(5/2), x)
```

Fricas [A] time = 0.514205, size = 267, normalized size = 3.38

$$-\frac{\sqrt{2}\left(32\cos\left(bx+a\right)^{4}-40\cos\left(bx+a\right)^{2}+5\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+32\left(\cos\left(bx+a\right)^{4}-\cos\left(bx+a\right)^{2}\right)\sin\left(bx+a\right)}{60\left(b\cos\left(bx+a\right)^{4}-b\cos\left(bx+a\right)^{2}\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/60*(sqrt(2)*(32*cos(b*x + a)^4 - 40*cos(b*x + a)^2 + 5)*sqrt(cos(b*x + a
)*sin(b*x + a)) + 32*(cos(b*x + a)^4 - cos(b*x + a)^2)*sin(b*x + a))/((b*co
s(b*x + a)^4 - b*cos(b*x + a)^2)*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(bx+a\right)}{\sin\left(2\,bx+2\,a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(5/2), x)
```

3.104 $\int \frac{\csc(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=105

$$\frac{12\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{32\sin(a+bx)}{35b\sqrt{\sin(2a+2bx)}}-\frac{16\cos(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

```
[Out] (-2*Cos[a + b*x])/(7*b*Sin[2*a + 2*b*x]^(7/2)) + (12*Sin[a + b*x])/(35*b*Si
n[2*a + 2*b*x]^(5/2)) - (16*Cos[a + b*x])/(35*b*Sin[2*a + 2*b*x]^(3/2)) + (
32*Sin[a + b*x])/(35*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.107893, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4308, 4303, 4304, 4292}

$$\frac{12\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{32\sin(a+bx)}{35b\sqrt{\sin(2a+2bx)}}-\frac{16\cos(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(7*b*Sin[2*a + 2*b*x]^(7/2)) + (12*Sin[a + b*x])/(35*b*Si
n[2*a + 2*b*x]^(5/2)) - (16*Cos[a + b*x])/(35*b*Sin[2*a + 2*b*x]^(3/2)) + (
32*Sin[a + b*x])/(35*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rule 4303

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4304

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> -Simp[(Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dis
t[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
```

```
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\csc(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = 2\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$$
$$= -\frac{2\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{12}{7}\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$$
$$= -\frac{2\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{12\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{48}{35}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$$
$$= -\frac{2\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{12\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{16\cos(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{32}{35}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$$
$$= -\frac{2\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{12\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{16\cos(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{32\sin(a+bx)}{35b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A] time = 0.13797, size = 67, normalized size = 0.64

$$\frac{\sqrt{\sin(2(a+bx))}(-10\cos(2(a+bx)) - 4\cos(4(a+bx)) + 4\cos(6(a+bx)) + 5)\csc^4(a+bx)\sec^3(a+bx)}{280b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] ((5 - 10*Cos[2*(a + b*x)] - 4*Cos[4*(a + b*x)] + 4*Cos[6*(a + b*x)])*Csc[a
+ b*x]^4*Sec[a + b*x]^3*Sqrt[Sin[2*(a + b*x)]])/(280*b)
```

Maple [C] time = 30.944, size = 222, normalized size = 2.1

$$\frac{1}{1344\,b}\sqrt{-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2-1\right)^{-1}}\left(\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2-1\right)\left(3\,(\tan(1/2\,bx+a/2))^8+40\sqrt{\tan(1/2\,bx+a/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/sin(2*b*x+2*a)^(7/2),x)
```

```
[Out] 1/1344/b*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+
1/2*a)^2-1)/tan(1/2*b*x+1/2*a)^3*(3*tan(1/2*b*x+1/2*a)^8+40*(tan(1/2*b*x+1/
2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*E
llipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^3-26*
tan(1/2*b*x+1/2*a)^6+26*tan(1/2*b*x+1/2*a)^2-3)/(tan(1/2*b*x+1/2*a)*(tan(1/
2*b*x+1/2*a)^2-1))^(1/2)/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)}{\sin(2\,bx+2\,a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(7/2), x)
```

Fricas [A] time = 0.530232, size = 320, normalized size = 3.05

$$\frac{128 \cos\left(bx+a\right)^{7} - 256 \cos\left(bx+a\right)^{5} + 128 \cos\left(bx+a\right)^{3} + \sqrt{2}\left(128 \cos\left(bx+a\right)^{6} - 224 \cos\left(bx+a\right)^{4} + 84 \cos\left(bx+\right.\right.}{280\left(b\cos\left(bx+a\right)^{7} - 2\,b\cos\left(bx+a\right)^{5} + b\cos\left(bx+a\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/280*(128*cos(b*x + a)^7 - 256*cos(b*x + a)^5 + 128*cos(b*x + a)^3 + sqrt(
2)*(128*cos(b*x + a)^6 - 224*cos(b*x + a)^4 + 84*cos(b*x + a)^2 + 7)*sqrt(c
os(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*cos(b
*x + a)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(bx+a\right)}{\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(7/2), x)
```

3.105 $\int \csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=106

$$\frac{6E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b}-\frac{2\sin^{\frac{7}{2}}(2a+2bx)\cos(2a+2bx)}{7b}-\frac{2\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^2(a+bx)}{7b}$$

```
[Out] (6*EllipticE[a - Pi/4 + b*x, 2])/(5*b) - (2*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*
x]^(3/2))/(5*b) - (2*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(7/2))/(7*b) + (Csc[
a + b*x]^2*Sin[2*a + 2*b*x]^(11/2))/(7*b)
```

Rubi [A] time = 0.0584837, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4300, 2635, 2639}

$$\frac{6E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b}-\frac{2\sin^{\frac{7}{2}}(2a+2bx)\cos(2a+2bx)}{7b}-\frac{2\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^2(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] (6*EllipticE[a - Pi/4 + b*x, 2])/(5*b) - (2*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*
x]^(3/2))/(5*b) - (2*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(7/2))/(7*b) + (Csc[
a + b*x]^2*Sin[2*a + 2*b*x]^(11/2))/(7*b)
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx = \frac{\csc^2(a+bx)\sin^{\frac{11}{2}}(2a+2bx)}{7b} + \frac{18}{7}\int \sin^{\frac{9}{2}}(2a+2bx)\,dx$$

$$= -\frac{2\cos(2a+2bx)\sin^{\frac{7}{2}}(2a+2bx)}{7b} + \frac{\csc^2(a+bx)\sin^{\frac{11}{2}}(2a+2bx)}{7b} + 2\int \sin^{\frac{5}{2}}(2a$$

$$= -\frac{2\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{5b} - \frac{2\cos(2a+2bx)\sin^{\frac{7}{2}}(2a+2bx)}{7b} + \frac{\csc^2(a+b}{}$$

$$= \frac{6E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{5b} - \frac{2\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{5b} - \frac{2\cos(2a+2bx)\sin^{\frac{7}{2}}(2a}{7b}$$

Mathematica [A] time = 0.2952, size = 66, normalized size = 0.62

$$\frac{\sqrt{\sin(2(a+bx))}(15\sin(2(a+bx)) - 14\sin(4(a+bx)) - 5\sin(6(a+bx))) + 84E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{70b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] (84*EllipticE[a - Pi/4 + b*x, 2] + Sqrt[Sin[2*(a + b*x)]]*(15*Sin[2*(a + b*
x)] - 14*Sin[4*(a + b*x)] - 5*Sin[6*(a + b*x)]))/(70*b)
```

Maple [A] time = 7.493, size = 204, normalized size = 1.9

$$8\frac{\sqrt{2}}{b}\left(\frac{\sqrt{2}(\sin(2bx+2a))^{7/2}}{56} - \frac{\sqrt{2}\left(6\sqrt{\sin(2bx+2a)+1}\sqrt{-2\sin(2bx+2a)+2}\sqrt{-\sin(2bx+2a)}\text{EllipticE}\left(\sqrt{s}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^(9/2),x)
```

```
[Out] 8*2^(1/2)*(1/56*2^(1/2)*sin(2*b*x+2*a)^(7/2)-1/80*2^(1/2)*(6*(sin(2*b*x+2*a
)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticE((s
in(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))-3*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*
x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticF((sin(2*b*x+2*a)+1)^(1/2),
1/2*2^(1/2))-2*sin(2*b*x+2*a)^4+2*sin(2*b*x+2*a)^2)/cos(2*b*x+2*a)/sin(2*b*
x+2*a)^(1/2))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(bx+a)^2 \sin(2bx+2a)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\cos\left(2\,bx+2\,a\right)^{4}-2\,\cos\left(2\,bx+2\,a\right)^{2}+1\right)\csc\left(bx+a\right)^{2}\sqrt{\sin\left(2\,bx+2\,a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((cos(2*b*x + 2*a)^4 - 2*cos(2*b*x + 2*a)^2 + 1)*csc(b*x + a)^2*sqr
t(sin(2*b*x + 2*a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.106 $\int \csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=106

$$\frac{2\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3b}-\frac{2\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{5b}-\frac{2\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc}{5b}$$

```
[Out] (2*EllipticF[a - Pi/4 + b*x, 2])/(3*b) - (2*Cos[2*a + 2*b*x]*Sqrt[Sin[2*a +
 2*b*x]])/(3*b) - (2*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(5/2))/(5*b) + (Csc[
a + b*x]^2*Sin[2*a + 2*b*x]^(9/2))/(5*b)
```

Rubi [A] time = 0.059291, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4300, 2635, 2641}

$$\frac{2F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{3b}-\frac{2\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{5b}-\frac{2\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^2(a+bx}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (2*EllipticF[a - Pi/4 + b*x, 2])/(3*b) - (2*Cos[2*a + 2*b*x]*Sqrt[Sin[2*a +
 2*b*x]])/(3*b) - (2*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(5/2))/(5*b) + (Csc[
a + b*x]^2*Sin[2*a + 2*b*x]^(9/2))/(5*b)
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \frac{\csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)}{5b} + \frac{14}{5}\int \sin^{\frac{7}{2}}(2a+2bx)\,dx$$
$$= -\frac{2\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2bx)}{5b} + \frac{\csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)}{5b} + 2\int \sin^{\frac{3}{2}}(2a+2\ldots$$
$$= -\frac{2\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{3b} - \frac{2\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2bx)}{5b} + \frac{\csc^2(a+bx)\ldots}{5\ldots}$$
$$= \frac{2F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{3b} - \frac{2\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{3b} - \frac{2\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2\ldots}{5b}$$

Mathematica [A] time = 0.257858, size = 76, normalized size = 0.72

$$\frac{20\sqrt{\sin(2(a+bx))}\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)+9\sin(2(a+bx))-10\sin(4(a+bx))-3\sin(6(a+bx))}{30b\sqrt{\sin(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (20*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*(a + b*x)]] + 9*Sin[2*(a + b*x)
] - 10*Sin[4*(a + b*x)] - 3*Sin[6*(a + b*x)])/(30*b*Sqrt[Sin[2*(a + b*x)]])
```

Maple [A] time = 5.161, size = 139, normalized size = 1.3

$$4\frac{\sqrt{2}}{b}\left(1/20\sqrt{2}(\sin(2bx+2a))^{5/2}+1/24\frac{\sqrt{2}\left(\sqrt{\sin(2bx+2a)+1}\sqrt{-2\sin(2bx+2a)+2}\sqrt{-\sin(2bx+2a)}\text{EllipticF}\ldots\right.}{\cos(2bx+2a)\sqrt{\text{s}\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x)
```

```
[Out] 4*2^(1/2)*(1/20*2^(1/2)*sin(2*b*x+2*a)^(5/2)+1/24*2^(1/2)*((sin(2*b*x+2*a)+
1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticF((sin
(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))+2*sin(2*b*x+2*a)^3-2*sin(2*b*x+2*a))/cos(
2*b*x+2*a)/sin(2*b*x+2*a)^(1/2))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(bx+a)^2 \sin(2bx+2a)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos\left(2\,bx+2\,a\right)^{2}-1\right)\csc\left(bx+a\right)^{2}\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(2*b*x + 2*a)^2 - 1)*csc(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.107 $\int \csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=75

$$\frac{2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b}-\frac{2\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{3b}+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^2(a+bx)}{3b}$$

```
[Out] (2*EllipticE[a - Pi/4 + b*x, 2])/b - (2*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(
3/2))/(3*b) + (Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2))/(3*b)
```

Rubi [A] time = 0.0469874, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4300, 2635, 2639}

$$\frac{2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b}-\frac{2\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{3b}+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (2*EllipticE[a - Pi/4 + b*x, 2])/b - (2*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(
3/2))/(3*b) + (Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2))/(3*b)
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx=\frac{\csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{3b}+\frac{10}{3}\int \sin^{\frac{5}{2}}(2a+2bx)\,dx$$

$$=-\frac{2\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{3b}+\frac{\csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{3b}+2\int \sqrt{\sin(2a+2$$

$$=\frac{2E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b}-\frac{2\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{3b}+\frac{\csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{3b}$$

Mathematica [A] time = 0.0784348, size = 34, normalized size = 0.45

$$\frac{2\left(\sin^{\frac{3}{2}}(2(a+bx))+3E\left(a+bx-\frac{\pi}{4}\middle|2\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (2*(3*EllipticE[a - Pi/4 + b*x, 2] + Sin[2*(a + b*x)]^(3/2)))/(3*b)
```

Maple [A] time = 3.971, size = 137, normalized size = 1.8

$$2\,\frac{\sqrt{2}}{b}\left(1/6\,\sqrt{2}\,(\sin(2\,bx+2\,a))^{3/2}-1/4\,\frac{\sqrt{2}\sqrt{\sin(2\,bx+2\,a)+1}\sqrt{-2\,\sin(2\,bx+2\,a)+2}\sqrt{-\sin(2\,bx+2\,a)}\,(2\,\mathrm{Elliptic}}{\cos(2\,bx+2\,a)\,\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] 2*2^(1/2)*(1/6*2^(1/2)*sin(2*b*x+2*a)^(3/2)-1/4*2^(1/2)*(sin(2*b*x+2*a)+1)^
(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*(2*EllipticE((sin
(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))-EllipticF((sin(2*b*x+2*a)+1)^(1/2),1/2*2^
(1/2)))/cos(2*b*x+2*a)/sin(2*b*x+2*a)^(1/2))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(bx+a)^2 \sin(2\,bx+2\,a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(\cos(2\,bx+2\,a)^2-1\right)\csc(bx+a)^2\sqrt{\sin(2\,bx+2\,a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(2*b*x + 2*a)^2 - 1)*csc(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(bx+a)^2 \sin(2bx+2a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(5/2), x)
```

3.108 $\int \csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=70

$$\frac{2\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{b}-\frac{2\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{b}+\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^2(a+bx)}{b}$$

```
[Out] (2*EllipticF[a - Pi/4 + b*x, 2])/b - (2*Cos[2*a + 2*b*x]*Sqrt[Sin[2*a + 2*b
*x]])/b + (Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2))/b
```

Rubi [A] time = 0.0482702, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4300, 2635, 2641}

$$\frac{2F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b}-\frac{2\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{b}+\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^2(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (2*EllipticF[a - Pi/4 + b*x, 2])/b - (2*Cos[2*a + 2*b*x]*Sqrt[Sin[2*a + 2*b
*x]])/b + (Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2))/b
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx &= \frac{\csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{b}+6\int \sin^{\frac{3}{2}}(2a+2bx)\,dx\\ &= -\frac{2\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{b}+\frac{\csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{b}+2\int \frac{1}{\sqrt{\sin(2a}}\\ &= \frac{2F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b}-\frac{2\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{b}+\frac{\csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2}{b}\end{aligned}$$

Mathematica [A] time = 0.859969, size = 73, normalized size = 1.04

$$\frac{2\sqrt{\sin(2(a+bx))}-\frac{\sqrt{2}(\sin(a+bx)+\cos(a+bx))\text{EllipticF}\left(\sin^{-1}(\cos(a+bx)-\sin(a+bx)),\frac{1}{2}\right)}{\sqrt{\sin(2(a+bx))+1}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (2*Sqrt[Sin[2*(a + b*x)]] - (Sqrt[2]*EllipticF[ArcSin[Cos[a + b*x] - Sin[a
+ b*x]], 1/2]*(Cos[a + b*x] + Sin[a + b*x]))/Sqrt[1 + Sin[2*(a + b*x)]])/b
```

Maple [A] time = 2.622, size = 111, normalized size = 1.6

$$\frac{\sqrt{2}}{b}\left(\sqrt{2}\sqrt{\sin(2bx+2a)}+\frac{\sqrt{2}}{2\cos(2bx+2a)}\sqrt{\sin(2bx+2a)+1}\sqrt{-2\sin(2bx+2a)+2}\sqrt{-\sin(2bx+2a)}\text{EllipticF}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] 2^(1/2)*(2^(1/2)*sin(2*b*x+2*a)^(1/2)+1/2*2^(1/2)*(sin(2*b*x+2*a)+1)^(1/2)*
(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticF((sin(2*b*x+2*
a)+1)^(1/2),1/2*2^(1/2))/cos(2*b*x+2*a)/sin(2*b*x+2*a)^(1/2))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(bx+a)^2 \sin(2bx+2a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\csc(bx+a)^2 \sin(2bx+2a)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(bx+a)^2 \sin(2bx+2a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

3.109 $\int \csc^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal. Leaf size=44

$$-\frac{2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\csc^2(a+bx)}{b}$$

```
[Out] (-2*EllipticE[a - Pi/4 + b*x, 2])/b - (Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2
))/b
```

Rubi [A] time = 0.0361731, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4300, 2639}

$$-\frac{2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\csc^2(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-2*EllipticE[a - Pi/4 + b*x, 2])/b - (Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2
))/b
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx &= -\frac{\csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{b}-2\int \sqrt{\sin(2a+2bx)}\,dx\\ &= -\frac{2E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b}-\frac{\csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.12709, size = 37, normalized size = 0.84

$$-\frac{2\left(E\left(a+bx-\frac{\pi}{4}\middle|2\right)+\sqrt{\sin(2(a+bx))}\cot(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-2*(EllipticE[a - Pi/4 + b*x, 2] + Cot[a + b*x]*Sqrt[Sin[2*(a + b*x)]]))/b
```

Maple [B] time = 2.096, size = 176, normalized size = 4.

$$\frac{1}{b\cos(2bx+2a)}\left(2\sqrt{\sin(2bx+2a)+1}\sqrt{-2\sin(2bx+2a)+2}\sqrt{-\sin(2bx+2a)}\mathrm{EllipticE}\left(\sqrt{\sin(2bx+2a)+1},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] 1/cos(2*b*x+2*a)/sin(2*b*x+2*a)^(1/2)*(2*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2
*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticE((sin(2*b*x+2*a)+1)^(1/
2),1/2*2^(1/2))-(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(
2*b*x+2*a))^(1/2)*EllipticF((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))-2*cos(2*b
*x+2*a)^2-2*cos(2*b*x+2*a))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(bx+a)^2\sqrt{\sin(2bx+2a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\csc(bx+a)^2\sqrt{\sin(2bx+2a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(bx+a)^2 \sqrt{\sin(2bx+2a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x)
```

3.110 $\int \frac{\csc^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal. Leaf size=48

$$\frac{2\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3b}-\frac{\sqrt{\sin(2a+2bx)}\csc^2(a+bx)}{3b}$$

```
[Out] (2*EllipticF[a - Pi/4 + b*x, 2])/(3*b) - (Csc[a + b*x]^2*Sqrt[Sin[2*a + 2*b
*x]])/(3*b)
```

Rubi [A] time = 0.0367026, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4300, 2641}

$$\frac{2F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{3b}-\frac{\sqrt{\sin(2a+2bx)}\csc^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (2*EllipticF[a - Pi/4 + b*x, 2])/(3*b) - (Csc[a + b*x]^2*Sqrt[Sin[2*a + 2*b
*x]])/(3*b)
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx &= -\frac{\csc^2(a+bx)\sqrt{\sin(2a+2bx)}}{3b}+\frac{2}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= \frac{2F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{3b}-\frac{\csc^2(a+bx)\sqrt{\sin(2a+2bx)}}{3b}\end{aligned}$$

Mathematica [A] time = 0.951406, size = 82, normalized size = 1.71

$$-\frac{\frac{\sqrt{2}(\sin(a+bx)+\cos(a+bx))\text{EllipticF}\left(\sin^{-1}(\cos(a+bx)-\sin(a+bx)),\frac{1}{2}\right)}{\sqrt{\sin(2(a+bx))+1}}+\sqrt{\sin(2(a+bx))}\csc^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -(Csc[a + b*x]^2*Sqrt[Sin[2*(a + b*x)]] + (Sqrt[2]*EllipticF[ArcSin[Cos[a +
 b*x] - Sin[a + b*x]], 1/2]*(Cos[a + b*x] + Sin[a + b*x]))/Sqrt[1 + Sin[2*(
a + b*x)]])/(3*b)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (\csc (bx + a))^2 \frac{1}{\sqrt{\sin (2\,bx + 2\,a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] int(csc(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc (bx + a)^2}{\sqrt{\sin (2\,bx + 2\,a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc (bx + a)^2}{\sqrt{\sin (2\,bx + 2\,a)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^2}{\sqrt{\sin(2\,bx+2\,a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

3.111 $\int \frac{\csc^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=77

$$-\frac{6E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b}-\frac{6\cos(2a+2bx)}{5b\sqrt{\sin(2a+2bx)}}-\frac{\csc^2(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

```
[Out] (-6*EllipticE[a - Pi/4 + b*x, 2])/(5*b) - (6*Cos[2*a + 2*b*x])/(5*b*Sqrt[Si
n[2*a + 2*b*x]]) - Csc[a + b*x]^2/(5*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.0466341, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4300, 2636, 2639}

$$-\frac{6E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b}-\frac{6\cos(2a+2bx)}{5b\sqrt{\sin(2a+2bx)}}-\frac{\csc^2(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-6*EllipticE[a - Pi/4 + b*x, 2])/(5*b) - (6*Cos[2*a + 2*b*x])/(5*b*Sqrt[Si
n[2*a + 2*b*x]]) - Csc[a + b*x]^2/(5*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{\csc^2(a+bx)}{5b\sqrt{\sin(2a+2bx)}} + \frac{6}{5}\int \frac{1}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$$
$$= -\frac{6\cos(2a+2bx)}{5b\sqrt{\sin(2a+2bx)}} - \frac{\csc^2(a+bx)}{5b\sqrt{\sin(2a+2bx)}} - \frac{6}{5}\int \sqrt{\sin(2a+2bx)}\,dx$$
$$= -\frac{6E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{5b} - \frac{6\cos(2a+2bx)}{5b\sqrt{\sin(2a+2bx)}} - \frac{\csc^2(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A] time = 0.580404, size = 64, normalized size = 0.83

$$\frac{\frac{2(-6\cos(2(a+bx))+3\cos(4(a+bx))+1)\cot(a+bx)}{\sin^{\frac{3}{2}}(2(a+bx))} - 12E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-12*EllipticE[a - Pi/4 + b*x, 2] + (2*(1 - 6*Cos[2*(a + b*x)] + 3*Cos[4*(a
 + b*x)])*Cot[a + b*x])/Sin[2*(a + b*x)]^(3/2))/(10*b)
```

Maple [B] time = 6.056, size = 227, normalized size = 3.

$$\frac{\sqrt{2}}{8b}\left(-\frac{8\sqrt{2}}{5}(\sin(2bx+2a))^{-\frac{5}{2}} + \frac{4\sqrt{2}}{5\cos(2bx+2a)}\left(6\sqrt{\sin(2bx+2a)+1}\sqrt{-2\sin(2bx+2a)+2}\sqrt{-\sin(2bx+2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] 1/8*2^(1/2)*(-8/5*2^(1/2)/sin(2*b*x+2*a)^(5/2)+4/5*2^(1/2)/sin(2*b*x+2*a)^(
5/2)*(6*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*
a))^(1/2)*sin(2*b*x+2*a)^2*EllipticE((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))-
3*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1
/2)*sin(2*b*x+2*a)^2*EllipticF((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))+6*sin(
2*b*x+2*a)^4-4*sin(2*b*x+2*a)^2-2)/cos(2*b*x+2*a))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^2}{\sin(2bx+2a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\csc (bx+a)^2\sqrt{\sin (2\,bx+2\,a)}}{\cos (2\,bx+2\,a)^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-csc(b*x + a)^2*sqrt(sin(2*b*x + 2*a))/(cos(2*b*x + 2*a)^2 - 1), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc (bx+a)^2}{\sin (2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(3/2), x)
```

3.112 $\int \frac{\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=77

$$\frac{10\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{21b}-\frac{10\cos(2a+2bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{7b\sin^{\frac{3}{2}}(2a+2bx)}$$

```
[Out] (10*EllipticF[a - Pi/4 + b*x, 2])/(21*b) - (10*Cos[2*a + 2*b*x])/(21*b*Sin[
2*a + 2*b*x]^(3/2)) - Csc[a + b*x]^2/(7*b*Sin[2*a + 2*b*x]^(3/2))
```

Rubi [A] time = 0.0478643, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4300, 2636, 2641}

$$\frac{10F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{21b}-\frac{10\cos(2a+2bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{7b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (10*EllipticF[a - Pi/4 + b*x, 2])/(21*b) - (10*Cos[2*a + 2*b*x])/(21*b*Sin[
2*a + 2*b*x]^(3/2)) - Csc[a + b*x]^2/(7*b*Sin[2*a + 2*b*x]^(3/2))
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{\csc^2(a+bx)}{7b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{10}{7}\int \frac{1}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$$
$$= -\frac{10\cos(2a+2bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)} - \frac{\csc^2(a+bx)}{7b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{10}{21}\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx$$
$$= \frac{10F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{21b} - \frac{10\cos(2a+2bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)} - \frac{\csc^2(a+bx)}{7b\sin^{\frac{3}{2}}(2a+2bx)}$$

Mathematica [A] time = 0.46149, size = 66, normalized size = 0.86

$$\frac{40\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)+\sqrt{\sin(2(a+bx))}\left(-3\csc^4(a+bx)-13\csc^2(a+bx)+7\sec^2(a+bx)\right)}{84b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (40*EllipticF[a - Pi/4 + b*x, 2] + (-13*Csc[a + b*x]^2 - 3*Csc[a + b*x]^4 +
 7*Sec[a + b*x]^2)*Sqrt[Sin[2*(a + b*x)]])/(84*b)
```

Maple [A] time = 9.525, size = 154, normalized size = 2.

$$\frac{\sqrt{2}}{16b}\left(-\frac{16\sqrt{2}}{7}(\sin(2bx+2a))^{-\frac{7}{2}}+\frac{8\sqrt{2}}{21\cos(2bx+2a)}\left(5\sqrt{\sin(2bx+2a)+1}\sqrt{-2\sin(2bx+2a)+2}\sqrt{-\sin(2bx+2a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] 1/16*2^(1/2)*(-16/7*2^(1/2)/sin(2*b*x+2*a)^(7/2)+8/21*2^(1/2)/sin(2*b*x+2*a
)^(7/2)*(5*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x
+2*a))^(1/2)*EllipticF((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))*sin(2*b*x+2*a)
^3+10*sin(2*b*x+2*a)^4-4*sin(2*b*x+2*a)^2-6)/cos(2*b*x+2*a))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^2}{\sin(2bx+2a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\csc(bx+a)^2}{\left(\cos(2bx+2a)^2-1\right)\sqrt{\sin(2bx+2a)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-csc(b*x + a)^2/((cos(2*b*x + 2*a)^2 - 1)*sqrt(sin(2*b*x + 2*a))),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(5/2), x)
```

3.113 $\int \frac{\csc^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=106

$$-\frac{14E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{15b}-\frac{14\cos(2a+2bx)}{45b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{14\cos(2a+2bx)}{15b\sqrt{\sin(2a+2bx)}}-\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}$$

```
[Out] (-14*EllipticE[a - Pi/4 + b*x, 2])/(15*b) - (14*Cos[2*a + 2*b*x])/(45*b*Sin
[2*a + 2*b*x]^(5/2)) - Csc[a + b*x]^2/(9*b*Sin[2*a + 2*b*x]^(5/2)) - (14*Co
s[2*a + 2*b*x])/(15*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.0580496, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4300, 2636, 2639}

$$-\frac{14E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{15b}-\frac{14\cos(2a+2bx)}{45b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{14\cos(2a+2bx)}{15b\sqrt{\sin(2a+2bx)}}-\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (-14*EllipticE[a - Pi/4 + b*x, 2])/(15*b) - (14*Cos[2*a + 2*b*x])/(45*b*Sin
[2*a + 2*b*x]^(5/2)) - Csc[a + b*x]^2/(9*b*Sin[2*a + 2*b*x]^(5/2)) - (14*Co
s[2*a + 2*b*x])/(15*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx &= -\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{14}{9}\int \frac{1}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx \\
&= -\frac{14\cos(2a+2bx)}{45b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{14}{15}\int \frac{1}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx \\
&= -\frac{14\cos(2a+2bx)}{45b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{14\cos(2a+2bx)}{15b\sqrt{\sin(2a+2bx)}}-\frac{14}{15}\int \sqrt{\sin(2a+2bx)}\,dx \\
&= -\frac{14E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{15b}-\frac{14\cos(2a+2bx)}{45b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{14\cos(2a+2bx)}{15b\sqrt{\sin(2a+2bx)}}
\end{aligned}
$$

Mathematica [A] time = 0.814174, size = 85, normalized size = 0.8

$$
-\frac{336E\left(a+bx-\frac{\pi}{4}\middle|2\right)+\frac{(98\cos(2(a+bx))-28\cos(4(a+bx))-42\cos(6(a+bx))+21\cos(8(a+bx))-9)\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2(a+bx))}}{360b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] -(336*EllipticE[a - Pi/4 + b*x, 2] + ((-9 + 98*Cos[2*(a + b*x)] - 28*Cos[4*
(a + b*x)] - 42*Cos[6*(a + b*x)] + 21*Cos[8*(a + b*x)])*Csc[a + b*x]^2)/Sin
[2*(a + b*x)]^(5/2))/(360*b)
```

Maple [B] time = 30.493, size = 240, normalized size = 2.3

$$
\frac{\sqrt{2}}{32\,b}\left(-\frac{32\,\sqrt{2}}{9}\left(\sin\left(2\,bx+2\,a\right)\right)^{-\frac{9}{2}}+\frac{16\,\sqrt{2}}{45\,\cos\left(2\,bx+2\,a\right)}\left(42\,\sqrt{\sin\left(2\,bx+2\,a\right)+1}\sqrt{-2\,\sin\left(2\,bx+2\,a\right)+2}\sqrt{-\sin\left(2\,bx\right.}\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x)
```

```
[Out] 1/32*2^(1/2)*(-32/9*2^(1/2)/sin(2*b*x+2*a)^(9/2)+16/45*2^(1/2)/sin(2*b*x+2*
a)^(9/2)*(42*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b
*x+2*a))^(1/2)*sin(2*b*x+2*a)^4*EllipticE((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1
/2))-21*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*
a))^(1/2)*sin(2*b*x+2*a)^4*EllipticF((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))+
42*sin(2*b*x+2*a)^6-28*sin(2*b*x+2*a)^4-4*sin(2*b*x+2*a)^2-10)/cos(2*b*x+2*
a))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\csc\left(bx+a\right)^2}{\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx+a)^2\sqrt{\sin(2\,bx+2\,a)}}{\cos(2\,bx+2\,a)^4-2\,\cos(2\,bx+2\,a)^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sqrt(sin(2*b*x + 2*a))/(cos(2*b*x + 2*a)^4 - 2*cos(
2*b*x + 2*a)^2 + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(7/2), x)
```

3.114 $\int \frac{\csc^2(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)} \, dx$

Optimal. Leaf size=106

$$\frac{30\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{77b}-\frac{30\cos(2a+2bx)}{77b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{18\cos(2a+2bx)}{77b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)}$$

```
[Out] (30*EllipticF[a - Pi/4 + b*x, 2])/(77*b) - (18*Cos[2*a + 2*b*x])/(77*b*Sin[
2*a + 2*b*x]^(7/2)) - Csc[a + b*x]^2/(11*b*Sin[2*a + 2*b*x]^(7/2)) - (30*Co
s[2*a + 2*b*x])/(77*b*Sin[2*a + 2*b*x]^(3/2))
```

Rubi [A] time = 0.059475, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4300, 2636, 2641}

$$\frac{30F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{77b}-\frac{30\cos(2a+2bx)}{77b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{18\cos(2a+2bx)}{77b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] (30*EllipticF[a - Pi/4 + b*x, 2])/(77*b) - (18*Cos[2*a + 2*b*x])/(77*b*Sin[
2*a + 2*b*x]^(7/2)) - Csc[a + b*x]^2/(11*b*Sin[2*a + 2*b*x]^(7/2)) - (30*Co
s[2*a + 2*b*x])/(77*b*Sin[2*a + 2*b*x]^(3/2))
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}dx = -\frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{18}{11}\int \frac{1}{\sin^{\frac{9}{2}}(2a+2bx)}dx$$
$$= -\frac{18\cos(2a+2bx)}{77b\sin^{\frac{7}{2}}(2a+2bx)} - \frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{90}{77}\int \frac{1}{\sin^{\frac{5}{2}}(2a+2bx)}dx$$
$$= -\frac{18\cos(2a+2bx)}{77b\sin^{\frac{7}{2}}(2a+2bx)} - \frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)} - \frac{30\cos(2a+2bx)}{77b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{30}{77}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx$$
$$= \frac{30F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{77b} - \frac{18\cos(2a+2bx)}{77b\sin^{\frac{7}{2}}(2a+2bx)} - \frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)} - \frac{30\cos(2a+2bx)}{77b\sin^{\frac{3}{2}}(2a+2bx)}$$

Mathematica [A] time = 0.343713, size = 86, normalized size = 0.81

$$\frac{480\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)+\sqrt{\sin(2(a+bx))}\left(-7\csc^6(a+bx)-32\csc^4(a+bx)-141\csc^2(a+bx)+11\sec^2(a+bx)\right(}{1232b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] (480*EllipticF[a - Pi/4 + b*x, 2] + (-141*Csc[a + b*x]^2 - 32*Csc[a + b*x]^
4 - 7*Csc[a + b*x]^6 + 11*Sec[a + b*x]^2*(9 + Sec[a + b*x]^2))*Sqrt[Sin[2*(
a + b*x)]])/(1232*b)
```

Maple [A] time = 47.27, size = 167, normalized size = 1.6

$$\frac{\sqrt{2}}{64b}\left(-\frac{64\sqrt{2}}{11}(\sin(2bx+2a))^{-\frac{11}{2}}+\frac{32\sqrt{2}}{77\cos(2bx+2a)}\left(15\sqrt{\sin(2bx+2a)+1}\sqrt{-2\sin(2bx+2a)+2}\sqrt{-\sin(2bx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/sin(2*b*x+2*a)^(9/2),x)
```

```
[Out] 1/64*2^(1/2)*(-64/11*2^(1/2)/sin(2*b*x+2*a)^(11/2)+32/77*2^(1/2)/sin(2*b*x+
2*a)^(11/2)*(15*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(
2*b*x+2*a))^(1/2)*EllipticF((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))*sin(2*b*x
+2*a)^5+30*sin(2*b*x+2*a)^6-12*sin(2*b*x+2*a)^4-4*sin(2*b*x+2*a)^2-14)/cos(
2*b*x+2*a))/b
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx+a)^2}{\left(\cos(2\,bx+2\,a)^4-2\,\cos(2\,bx+2\,a)^2+1\right)\sqrt{\sin(2\,bx+2\,a)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2/((cos(2*b*x + 2*a)^4 - 2*cos(2*b*x + 2*a)^2 + 1)*sq
rt(sin(2*b*x + 2*a))), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/sin(2*b*x+2*a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(9/2), x)
```

3.115 $\int \csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=190

$$\frac{4\sin(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{5b}+\frac{7\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{6b}-\frac{14\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{15b}-\frac{7\sin^{-1}(\cos(a+bx)-\sin$$

```
[Out] (-7*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(8*b) + (7*Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(8*b) - (7*Cos[a + b*x]*Sqrt[Sin[2*a +
2*b*x]])/(4*b) + (7*Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(6*b) - (14*Cos[a
+ b*x]*Sin[2*a + 2*b*x]^(5/2))/(15*b) + (4*Sin[a + b*x]*Sin[2*a + 2*b*x]^(7
/2))/(5*b) + (Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(11/2))/(5*b)
```

Rubi [A] time = 0.186188, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4300, 4308, 4301, 4302, 4305}

$$\frac{4\sin(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{5b}+\frac{7\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{6b}-\frac{14\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{15b}-\frac{7\sin^{-1}(\cos(a+bx)-\sin$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(9/2),x]
```


```
[Out] (-7*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(8*b) + (7*Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(8*b) - (7*Cos[a + b*x]*Sqrt[Sin[2*a +
2*b*x]])/(4*b) + (7*Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(6*b) - (14*Cos[a
+ b*x]*Sin[2*a + 2*b*x]^(5/2))/(15*b) + (4*Sin[a + b*x]*Sin[2*a + 2*b*x]^(7
/2))/(5*b) + (Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(11/2))/(5*b)
```


Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$
\begin{aligned}
\int \csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx &= \frac{\csc^3(a+bx)\sin^{\frac{11}{2}}(2a+2bx)}{5b}+\frac{16}{5}\int \csc(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx\\
&= \frac{\csc^3(a+bx)\sin^{\frac{11}{2}}(2a+2bx)}{5b}+\frac{32}{5}\int \cos(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx\\
&= \frac{4\sin(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{5b}+\frac{\csc^3(a+bx)\sin^{\frac{11}{2}}(2a+2bx)}{5b}+\frac{28}{5}\int \sin(a+bx)\\
&= -\frac{14\cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{15b}+\frac{4\sin(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{5b}+\frac{\csc^3(a+bx)\sin}{5}\\
&= \frac{7\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{6b}-\frac{14\cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{15b}+\frac{4\sin(a+bx)\sin}{5b}\\
&= -\frac{7\cos(a+bx)\sqrt{\sin(2a+2bx)}}{4b}+\frac{7\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{6b}-\frac{14\cos(a+bx)}{15}\\
&= -\frac{7\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{8b}+\frac{7\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2}\right.}{8b}
\end{aligned}
$$

Mathematica [A] time = 0.434573, size = 100, normalized size = 0.53

$$
\frac{7\left(\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(a+bx)\right)-\sin^{-1}(\cos(a+bx)-\sin(a+bx))\right)-\frac{2}{3}\sqrt{\sin(2(a+bx))}(10\cos(}{8b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] (7*(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]) - (2*(10*Cos[a + b*x] + 9*Cos[3*(a + b*x)] + 2*C
os[5*(a + b*x)])*Sqrt[Sin[2*(a + b*x)]])/3)/(8*b)
```

Maple [C] time = 26.75, size = 441, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^(9/2),x)
```

```
[Out] -64/21*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*((tan(1/2*b*x+1
/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*
EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^6-3*
(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+
1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b
*x+1/2*a)^4+2*tan(1/2*b*x+1/2*a)^7+3*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1
/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1
/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^2+10*tan(1/2*b*x+1/2*a)^5-(t
an(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/
2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))+10*tan(1/2*
b*x+1/2*a)^3+2*tan(1/2*b*x+1/2*a))/(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^
2-1))^(1/2)/(tan(1/2*b*x+1/2*a)+1)^2/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*
a))^(1/2)/(tan(1/2*b*x+1/2*a)-1)^2/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(bx+a)^3 \sin(2\,bx+2\,a)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(9/2), x)
```

Fricas [A] time = 0.573756, size = 802, normalized size = 4.22

$$-\frac{8\sqrt{2}\left(32\cos(bx+a)^5-4\cos(bx+a)^3-7\cos(bx+a)\right)\sqrt{\cos(bx+a)\sin(bx+a)}-42\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}}{\cos(bx+a}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/96*(8*sqrt(2)*(32*cos(b*x + a)^5 - 4*cos(b*x + a)^3 - 7*cos(b*x + a))*sq
rt(cos(b*x + a)*sin(b*x + a)) - 42*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b
*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*
x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) + 42*arctan(-(2*sqrt(2)*sqrt(c
os(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - si
n(b*x + a))) + 21*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4
*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b
*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.116 $\int \csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=164

$$\frac{4\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{3b}+\frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{2b}-\frac{5\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{3b}-\frac{5\sin^{-1}(\cos(a+bx)-\sin$$

```
[Out] (-5*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(4*b) - (5*Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(4*b) + (5*Sin[a + b*x]*Sqrt[Sin[2*a +
2*b*x]])/(2*b) - (5*Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(3*b) + (4*Sin[a +
 b*x]*Sin[2*a + 2*b*x]^(5/2))/(3*b) + (Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(9/2
))/(3*b)
```

Rubi [A] time = 0.156032, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4300, 4308, 4301, 4302, 4306}

$$\frac{4\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{3b}+\frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{2b}-\frac{5\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{3b}-\frac{5\sin^{-1}(\cos(a+bx)-\sin$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(7/2),x]
```


```
[Out] (-5*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(4*b) - (5*Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(4*b) + (5*Sin[a + b*x]*Sqrt[Sin[2*a +
2*b*x]])/(2*b) - (5*Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(3*b) + (4*Sin[a +
 b*x]*Sin[2*a + 2*b*x]^(5/2))/(3*b) + (Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(9/2
))/(3*b)
```


Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] &
& IntegerQ[2*p]
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\begin{aligned}
\int \csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx &= \frac{\csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)}{3b}+4\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx \\
&= \frac{\csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)}{3b}+8\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx \\
&= \frac{4\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{3b}+\frac{\csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)}{3b}+\frac{20}{3}\int \sin(a+bx) \\
&= -\frac{5\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{3b}+\frac{4\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{3b}+\frac{\csc^3(a+bx)\sin}{3b} \\
&= \frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{2b}-\frac{5\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{3b}+\frac{4\sin(a+bx)\sin}{3b} \\
&= -\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b}-\frac{5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2}\right.}{4b}
\end{aligned}$$

Mathematica [A] time = 0.219607, size = 84, normalized size = 0.51

$$\frac{2\sqrt{\sin(2(a+bx))}(6\sin(a+bx)+\sin(3(a+bx)))-5\left(\sin^{-1}(\cos(a+bx)-\sin(a+bx))+\log\left(\sin(a+bx)+\sqrt{\sin(2(a\right.\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (-5*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]) + 2*Sqrt[Sin[2*(a + b*x)]]*(6*Sin[a + b*x] + Sin
[3*(a + b*x)]))/(4*b)
```

Maple [C] time = 19.125, size = 973, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^(7/2),x)
```

```
[Out] 32/5*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(2*(tan(1/2*b*x+1
/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*
```

```
EllipticE((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*b*x+1/2*a)*(ta
n(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2*a)+1))^(1/2)*tan(1/2*b*x+1/2*a)^4-(tan
(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*
a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*b*x+
1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2*a)+1))^(1/2)*tan(1/2*b*x+1/2
*a)^4-4*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(
1/2*b*x+1/2*a))^(1/2)*EllipticE((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*(
tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2*a)+1))^(1/2)*tan
(1/2*b*x+1/2*a)^2+2*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^
(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/
2*2^(1/2))*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2*a)+1
))^(1/2)*tan(1/2*b*x+1/2*a)^2+2*(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(
1/2)*tan(1/2*b*x+1/2*a)^6+2*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/
2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticE((tan(1/2*b*x+1/2*a)+1)^
(1/2),1/2*2^(1/2))*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+
1/2*a)+1))^(1/2)-(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/
2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2
^(1/2))*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2*a)+1))^
(1/2)-4*(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)*tan(1/2*b*x+1/2*a)^
4+2*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2*a)+1))^(1/2
)*tan(1/2*b*x+1/2*a)^4+2*(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)*ta
n(1/2*b*x+1/2*a)^2+2*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*
x+1/2*a)+1))^(1/2)*tan(1/2*b*x+1/2*a)^2)/(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1
/2*a)^2-1))^(1/2)/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)/(tan(1/2*
b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2*a)+1))^(1/2)/(tan(1/2*b*
x+1/2*a)-1)/(tan(1/2*b*x+1/2*a)+1)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(bx+a)^3 \sin(2\,bx+2\,a)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(7/2), x)
```

Fricas [A] time = 0.560269, size = 772, normalized size = 4.71

$$\frac{8\sqrt{2}\left(4\cos(bx+a)^2+5\right)\sqrt{\cos(bx+a)\sin(bx+a)}\sin(bx+a)+10\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/16*(8*sqrt(2)*(4*cos(b*x + a)^2 + 5)*sqrt(cos(b*x + a)*sin(b*x + a))*sin(
b*x + a) + 10*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a
) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x
+ a)*sin(b*x + a) - 1)) - 10*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x +
 a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + 5*log(
-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*s
in(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x
```

```
+ a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.117 $\int \csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=127

$$\frac{4\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{b}-\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{b}-\frac{6\sqrt{\sin(2a+2bx)}\cos(a+bx)}{b}+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^3}{b}$$

```
[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/b + (3*Log[Cos[a + b*x] + Sin[a +
b*x] + Sqrt[Sin[2*a + 2*b*x]]])/b - (6*Cos[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])
/b + (4*Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/b + (Csc[a + b*x]^3*Sin[2*a +
2*b*x]^(7/2))/b
```

Rubi [A] time = 0.130911, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4300, 4308, 4301, 4302, 4305}

$$\frac{4\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{b}-\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{b}-\frac{6\sqrt{\sin(2a+2bx)}\cos(a+bx)}{b}+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^3}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/b + (3*Log[Cos[a + b*x] + Sin[a +
b*x] + Sqrt[Sin[2*a + 2*b*x]]])/b - (6*Cos[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])
/b + (4*Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/b + (Csc[a + b*x]^3*Sin[2*a +
2*b*x]^(7/2))/b
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
```

```
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\begin{aligned}\int \csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx &= \frac{\csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{b}+8\int \csc(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx\\ &= \frac{\csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{b}+16\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx\\ &= \frac{4\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{b}+\frac{\csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{b}+12\int \sin(a+bx)\\ &= -\frac{6\cos(a+bx)\sqrt{\sin(2a+2bx)}}{b}+\frac{4\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{b}+\frac{\csc^3(a+bx)\sin}{b}\\ &= -\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{b}+\frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2}\right.}{b}\end{aligned}$$

Mathematica [A] time = 0.129792, size = 70, normalized size = 0.55

$$\frac{-3\sin^{-1}(\cos(a+bx)-\sin(a+bx))+\sin^{\frac{3}{2}}(2(a+bx))\csc(a+bx)+3\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(a+b\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]] + 3*Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]] + Csc[a + b*x]*Sin[2*(a + b*x)]^(3/2))/b
```

Maple [C] time = 7.616, size = 243, normalized size = 1.9

$$\frac{16}{3b}\sqrt{-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2-1\right)^{-1}}\left(\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\sqrt{-2\,\tan\left(1/2\,bx+a/2\right)+2}\sqrt{-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}\mathrm{Elli}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] 16/3*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*((tan(1/2*b*x+1/2
*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*El
lipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^2-(tan
(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*
```

```
a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))-tan(1/2*b*x+1
/2*a)^3-tan(1/2*b*x+1/2*a))/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)
/(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(bx+a)^3 \sin(2bx+2a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(5/2), x)
```

Fricas [B] time = 0.539785, size = 737, normalized size = 5.8

$$\frac{8\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\cos(bx+a) + 6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right) - 6\,\text{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*cos(b*x + a) + 6*arctan(-(sq
rt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b
*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) -
 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(
b*x + a))/(cos(b*x + a) - sin(b*x + a))) - 3*log(-32*cos(b*x + a)^4 + 4*sqr
t(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x +
a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*s
in(b*x + a) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(bx+a)^3 \sin(2bx+2a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(5/2), x)
```

3.118 $\int \csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=104

$$-\frac{4\sin(a+bx)\sqrt{\sin(2a+2bx)}}{b}+\frac{2\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{b}-\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^3(a+bx)}{b}+\frac{2\log\left(\sin(a+bx)\right.}{}$$

```
[Out] (2*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/b + (2*Log[Cos[a + b*x] + Sin[a + b
*x] + Sqrt[Sin[2*a + 2*b*x]]])/b - (4*Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/
b - (Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(5/2))/b
```

Rubi [A] time = 0.102062, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4300, 4308, 4301, 4306}

$$-\frac{4\sin(a+bx)\sqrt{\sin(2a+2bx)}}{b}+\frac{2\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{b}-\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^3(a+bx)}{b}+\frac{2\log\left(\sin(a+bx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (2*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/b + (2*Log[Cos[a + b*x] + Sin[a + b
*x] + Sqrt[Sin[2*a + 2*b*x]]])/b - (4*Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/
b - (Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(5/2))/b
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
```

```
a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = -\frac{\csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{b} - 4\int \csc(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$= -\frac{\csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{b} - 8\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$
$$= -\frac{4\sin(a+bx)\sqrt{\sin(2a+2bx)}}{b} - \frac{\csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{b} - 4\int \frac{\sin(a+b}{\sqrt{\sin(2a+2}}$$
$$= \frac{2\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{b} + \frac{2\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a}\right.}{b}$$

Mathematica [A] time = 0.0983066, size = 68, normalized size = 0.65

$$\frac{2\left(\sin^{-1}(\cos(a+bx)-\sin(a+bx)) - 2\sqrt{\sin(2(a+bx))}\csc(a+bx) + \log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(a+bx\right.\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (2*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x] +
 Sqrt[Sin[2*(a + b*x)]]] - 2*Csc[a + b*x]*Sqrt[Sin[2*(a + b*x)]]))/b
```

Maple [C] time = 6.01, size = 542, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] 4*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(4*(tan(1/2*b*x+1/2*
a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*(ta
n(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2*a)+1))^(1/2)*(tan(
1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*EllipticE((tan(1/2*b*x+1/2*a
)+1)^(1/2),1/2*2^(1/2))-2*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*
a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/
2*a)-1)*(tan(1/2*b*x+1/2*a)+1))^(1/2)*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*
a)^2-1))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))+(tan(1/2
*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/
2*a)-1)*(tan(1/2*b*x+1/2*a)+1))^(1/2)*tan(1/2*b*x+1/2*a)^2+2*tan(1/2*b*x+1/
2*a)^2*(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)*(tan(1/2*b*x+1/2*a)*
(tan(1/2*b*x+1/2*a)^2-1))^(1/2)-(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(
1/2)*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2*a)+1))^(1/
2))/tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)/(tan
(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)-1)*(tan(1/2*b*x+1/2*a)+1))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(bx+a)^3 \sin(2bx+2a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(3/2), x)
```

Fricas [B] time = 0.530154, size = 814, normalized size = 7.83

$$-\frac{2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)\sin(bx+a)-2\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}-\cos}{\cos(bx+a)-\sin(bx}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - si
n(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*s
in(b*x + a) - 1))*sin(b*x + a) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin
(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a)))*si
n(b*x + a) + log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(
b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x +
a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1)*sin(b*x + a) +
8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) + 8*sin(b*x + a))/(b*sin(b*x + a)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(bx+a)^3 \sin(2bx+2a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(3/2), x)
```

3.119 $\int \csc^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal. Leaf size=28

$$-\frac{\sin^{\frac{3}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$

```
[Out] -(Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2))/(3*b)
```

Rubi [A] time = 0.0281631, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {4292}

$$-\frac{\sin^{\frac{3}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -(Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2))/(3*b)
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \csc^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = -\frac{\csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{3b}$$

Mathematica [A] time = 0.0493045, size = 27, normalized size = 0.96

$$-\frac{\sin^{\frac{3}{2}}(2(a+bx))\csc^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -(Csc[a + b*x]^3*Sin[2*(a + b*x)]^(3/2))/(3*b)
```

Maple [C] time = 3.405, size = 192, normalized size = 6.9

$$\frac{1}{3b}\sqrt{-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2-1\right)^{-1}}\left(\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2-1\right)\left(4\sqrt{\tan(1/2\,bx+a/2)+1}\sqrt{-2\,\tan(1/2\,bx+a/\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] 1/3*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1/2*a
)^2-1)/tan(1/2*b*x+1/2*a)*(4*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1
/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)
^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)+tan(1/2*b*x+1/2*a)^4-1)/(tan(1/2*b*x
+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1
/2*a))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(bx+a)^3 \sqrt{\sin(2bx+2a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^3*sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [B] time = 0.487701, size = 140, normalized size = 5.

$$\frac{2\left(\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\cos(bx+a)+\cos(bx+a)^2-1\right)}{3\left(b\cos(bx+a)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*cos(b*x + a) + cos(b*x + a)^2
- 1)/(b*cos(b*x + a)^2 - b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(bx+a)^3 \sqrt{\sin(2bx+2a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3*sqrt(sin(2*b*x + 2*a)), x)
```

3.120 $\int \frac{\csc^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal. Leaf size=55

$$-\frac{\sqrt{\sin(2a+2bx)}\csc^3(a+bx)}{5b}-\frac{4\sqrt{\sin(2a+2bx)}\csc(a+bx)}{5b}$$

```
[Out] (-4*Csc[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(5*b) - (Csc[a + b*x]^3*Sqrt[Sin[2
*a + 2*b*x]])/(5*b)
```

Rubi [A] time = 0.0533406, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4300, 4292}

$$-\frac{\sqrt{\sin(2a+2bx)}\csc^3(a+bx)}{5b}-\frac{4\sqrt{\sin(2a+2bx)}\csc(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-4*Csc[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(5*b) - (Csc[a + b*x]^3*Sqrt[Sin[2
*a + 2*b*x]])/(5*b)
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx &= -\frac{\csc^3(a+bx)\sqrt{\sin(2a+2bx)}}{5b}+\frac{4}{5}\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= -\frac{4\csc(a+bx)\sqrt{\sin(2a+2bx)}}{5b}-\frac{\csc^3(a+bx)\sqrt{\sin(2a+2bx)}}{5b}\end{aligned}$$

Mathematica [A] time = 0.0919217, size = 35, normalized size = 0.64

$$-\frac{\sqrt{\sin(2(a+bx))}\csc(a+bx)\left(\csc^2(a+bx)+4\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -(Csc[a + b*x]*(4 + Csc[a + b*x]^2)*Sqrt[Sin[2*(a + b*x)]])/(5*b)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (\csc(bx+a))^3 \frac{1}{\sqrt{\sin(2bx+2a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] int(csc(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^3}{\sqrt{\sin(2bx+2a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^3/sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [A] time = 0.498063, size = 198, normalized size = 3.6

$$-\frac{\sqrt{2}\left(4\cos(bx+a)^2-5\right)\sqrt{\cos(bx+a)\sin(bx+a)}+4\left(\cos(bx+a)^2-1\right)\sin(bx+a)}{5\left(b\cos(bx+a)^2-b\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/5*(sqrt(2)*(4*cos(b*x + a)^2 - 5)*sqrt(cos(b*x + a)*sin(b*x + a)) + 4*(c
os(b*x + a)^2 - 1)*sin(b*x + a))/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3/sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^3}{\sqrt{\sin(2\,bx+2\,a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3/sqrt(sin(2*b*x + 2*a)), x)
```

3.121 $\int \frac{\csc^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=81

$$\frac{32\sin(a+bx)}{21b\sqrt{\sin(2a+2bx)}}-\frac{16\cos(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}}$$

```
[Out] (-16*Cos[a + b*x])/(21*b*Sin[2*a + 2*b*x]^(3/2)) - Csc[a + b*x]^3/(7*b*Sqrt
[Sin[2*a + 2*b*x]]) + (32*Sin[a + b*x])/(21*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.0918782, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4300, 4308, 4303, 4292}

$$\frac{32\sin(a+bx)}{21b\sqrt{\sin(2a+2bx)}}-\frac{16\cos(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-16*Cos[a + b*x])/(21*b*Sin[2*a + 2*b*x]^(3/2)) - Csc[a + b*x]^3/(7*b*Sqrt
[Sin[2*a + 2*b*x]]) + (32*Sin[a + b*x])/(21*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rule 4303

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx &= -\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}} + \frac{8}{7}\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx \\
&= -\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}} + \frac{16}{7}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx \\
&= -\frac{16\cos(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)} - \frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}} + \frac{32}{21}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx \\
&= -\frac{16\cos(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)} - \frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}} + \frac{32\sin(a+bx)}{21b\sqrt{\sin(2a+2bx)}}
\end{aligned}
$$

Mathematica [A] time = 0.116, size = 55, normalized size = 0.68

$$\frac{\sqrt{\sin(2(a+bx))}(-12\cos(2(a+bx))+4\cos(4(a+bx))+5)\csc^4(a+bx)\sec(a+bx)}{42b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] ((5 - 12*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Csc[a + b*x]^4*Sec[a + b*x]
*Sqrt[Sin[2*(a + b*x)]])/(42*b)
```

Maple [C] time = 9.663, size = 222, normalized size = 2.7

$$-\frac{1}{336\,b}\sqrt{-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2-1\right)^{-1}}\left(\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2-1\right)\left(-3\,\left(\tan\left(1/2\,bx+a/2\right)\right)^8+16\,\sqrt{\tan\left(1/2\,bx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] -1/336*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1/
2*a)^2-1)/tan(1/2*b*x+1/2*a)^3*(-3*tan(1/2*b*x+1/2*a)^8+16*(tan(1/2*b*x+1/2
*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*El
lipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^3-2*ta
n(1/2*b*x+1/2*a)^6+2*tan(1/2*b*x+1/2*a)^2+3)/(tan(1/2*b*x+1/2*a)*(tan(1/2*b
*x+1/2*a)^2-1))^(1/2)/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(bx+a\right)^3}{\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^3/sin(2*b*x + 2*a)^(3/2), x)
```

Fricas [A] time = 0.506828, size = 281, normalized size = 3.47

$$\frac{32\cos(bx+a)^5 - 64\cos(bx+a)^3 + \sqrt{2}\left(32\cos(bx+a)^4 - 56\cos(bx+a)^2 + 21\right)\sqrt{\cos(bx+a)\sin(bx+a)} + 32\cos(bx+a)}{42\left(b\cos(bx+a)^5 - 2b\cos(bx+a)^3 + b\cos(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/42*(32*cos(b*x + a)^5 - 64*cos(b*x + a)^3 + sqrt(2)*(32*cos(b*x + a)^4 -
56*cos(b*x + a)^2 + 21)*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a))/
(b*cos(b*x + a)^5 - 2*b*cos(b*x + a)^3 + b*cos(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3/sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^3}{\sin(2bx+2a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3/sin(2*b*x + 2*a)^(3/2), x)
```

3.122 $\int \frac{\csc^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)} dx$

Optimal. Leaf size=107

$$\frac{32\sin(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{8\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{64\cos(a+bx)}{45b\sqrt{\sin(2a+2bx)}}-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}$$

```
[Out] (-8*Cos[a + b*x])/(15*b*Sin[2*a + 2*b*x]^(5/2)) - Csc[a + b*x]^3/(9*b*Sin[2
*a + 2*b*x]^(3/2)) + (32*Sin[a + b*x])/(45*b*Sin[2*a + 2*b*x]^(3/2)) - (64*
Cos[a + b*x])/(45*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.115507, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4300, 4308, 4303, 4304, 4291}

$$\frac{32\sin(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{8\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{64\cos(a+bx)}{45b\sqrt{\sin(2a+2bx)}}-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (-8*Cos[a + b*x])/(15*b*Sin[2*a + 2*b*x]^(5/2)) - Csc[a + b*x]^3/(9*b*Sin[2
*a + 2*b*x]^(3/2)) + (32*Sin[a + b*x])/(45*b*Sin[2*a + 2*b*x]^(3/2)) - (64*
Cos[a + b*x])/(45*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4300

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rule 4303

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4304

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> -Simp[(Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dis
t[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
```

```
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4291

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> -Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*
m), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/
b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx &= -\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{4}{3}\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx\\
&= -\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{8}{3}\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx\\
&= -\frac{8\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{32}{15}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx\\
&= -\frac{8\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{32\sin(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{64}{45}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx\\
&= -\frac{8\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{32\sin(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{64\cos(a+bx)}{45b\sqrt{\sin(2a+2bx)}}
\end{aligned}$$

Mathematica [A] time = 0.091329, size = 62, normalized size = 0.58

$$-\frac{\sqrt{\sin(2(a+bx))}\left(5\csc^5(a+bx)+17\csc^3(a+bx)+113\csc(a+bx)-15\tan(a+bx)\sec(a+bx)\right)}{180b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] -(Sqrt[Sin[2*(a + b*x)]]*(113*Csc[a + b*x] + 17*Csc[a + b*x]^3 + 5*Csc[a +
b*x]^5 - 15*Sec[a + b*x]*Tan[a + b*x]))/(180*b)
```

Maple [C] time = 18.636, size = 560, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] -1/2880*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)/tan(1/2*b*x+1/
2*a)^5*(5*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*tan(1/2*b*x+1
/2*a)^10+192*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(
-tan(1/2*b*x+1/2*a))^(1/2)*EllipticE((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/
2))*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*tan(1/2*b*x+1/2*a)^
4-96*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2
*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*(tan
```

```
(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*tan(1/2*b*x+1/2*a)^4-7*tan(
1/2*b*x+1/2*a)^8*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)+96*(ta
n(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)*tan(1/2*b*x+1/2*a)^6+2*(tan(1/
2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*tan(1/2*b*x+1/2*a)^6-96*(tan(1
/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)*tan(1/2*b*x+1/2*a)^4+2*(tan(1/2*b
*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*tan(1/2*b*x+1/2*a)^4-7*(tan(1/2*b
*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*tan(1/2*b*x+1/2*a)^2+5*(tan(1/2*b
*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2))/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*
x+1/2*a))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc (bx+a)^3}{\sin (2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^3/sin(2*b*x + 2*a)^(5/2), x)
```

Fricas [A] time = 0.527352, size = 356, normalized size = 3.33

$$-\frac{\sqrt{2}\left(128\,\cos (bx+a)^6-288\,\cos (bx+a)^4+180\,\cos (bx+a)^2-15\right)\sqrt{\cos (bx+a)\sin (bx+a)}+128\left(\cos (bx+a)^6-2\right.}{180\left(b\cos (bx+a)^6-2\,b\cos (bx+a)^4+b\cos (bx+a)^2\right)\sin (bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/180*(sqrt(2)*(128*cos(b*x + a)^6 - 288*cos(b*x + a)^4 + 180*cos(b*x + a)
^2 - 15)*sqrt(cos(b*x + a)*sin(b*x + a)) + 128*(cos(b*x + a)^6 - 2*cos(b*x
+ a)^4 + cos(b*x + a)^2)*sin(b*x + a))/((b*cos(b*x + a)^6 - 2*b*cos(b*x + a
)^4 + b*cos(b*x + a)^2)*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3/sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc (bx+a)^3}{\sin (2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3/sin(2*b*x + 2*a)^(5/2), x)
```

3.123 $\int \sin^3(a+bx)\sin^m(2a+2bx)\,dx$

Optimal. Leaf size=84

$$\frac{\sin^3(a+bx)\tan(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{m+4}{2},\frac{m+6}{2},\sin^2(a+bx)\right)}{b(m+4)}$$

```
[Out] ((Cos[a + b*x]^2)^((1 - m)/2)*Hypergeometric2F1[(1 - m)/2, (4 + m)/2, (6 +
m)/2, Sin[a + b*x]^2]*Sin[a + b*x]^3*Sin[2*a + 2*b*x]^m*Tan[a + b*x])/(b*(4
 + m))
```

Rubi [A] time = 0.0744873, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4310, 2577}

$$\frac{\sin^3(a+bx)\tan(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{m+4}{2};\frac{m+6}{2};\sin^2(a+bx)\right)}{b(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] ((Cos[a + b*x]^2)^((1 - m)/2)*Hypergeometric2F1[(1 - m)/2, (4 + m)/2, (6 +
m)/2, Sin[a + b*x]^2]*Sin[a + b*x]^3*Sin[2*a + 2*b*x]^m*Tan[a + b*x])/(b*(4
 + m))
```

Rule 4310

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p), Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sin^3(a+bx)\sin^m(2a+2bx)\,dx = \left(\cos^{-m}(a+bx)\sin^{-m}(a+bx)\sin^m(2a+2bx)\right)\int \cos^m(a+bx)\sin^{3+m}(a+bx)\,dx$$
$$= \frac{\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{4+m}{2};\frac{6+m}{2};\sin^2(a+bx)\right)\sin^3(a+bx)\sin^m(2a+2bx)\tan(a}{b(4+m)}$$

Mathematica [C] time = 5.62474, size = 602, normalized size = 7.17

$$\frac{b(m+2)\left(-2(m+4)\cos^2\left(\frac{1}{2}(a+bx)\right)F_1\left(\frac{m}{2}+1;-m,2m+4;\frac{m}{2}+2;\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)+2(\cos(a+b}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] (32*(4 + m)*(AppellF1[1 + m/2, -m, 3 + 2*m, 2 + m/2, Tan[(a + b*x)/2]^2, -T
an[(a + b*x)/2]^2] - AppellF1[1 + m/2, -m, 4 + 2*m, 2 + m/2, Tan[(a + b*x)/
2]^2, -Tan[(a + b*x)/2]^2])*Cos[(a + b*x)/2]^6*Sin[(a + b*x)/2]^4*Sin[2*(a
+ b*x)]^m)/(b*(2 + m)*(-2*(4 + m)*AppellF1[1 + m/2, -m, 4 + 2*m, 2 + m/2, T
an[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^2 + 2*(m*AppellF1[
2 + m/2, 1 - m, 3 + 2*m, 3 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]
- m*AppellF1[2 + m/2, 1 - m, 4 + 2*m, 3 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a
+ b*x)/2]^2] + 3*AppellF1[2 + m/2, -m, 4 + 2*m, 3 + m/2, Tan[(a + b*x)/2]^2
, -Tan[(a + b*x)/2]^2] + 2*m*AppellF1[2 + m/2, -m, 4 + 2*m, 3 + m/2, Tan[(a
 + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 4*AppellF1[2 + m/2, -m, 5 + 2*m, 3 + m
/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 2*m*AppellF1[2 + m/2, -m, 5
+ 2*m, 3 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*(-1 + Cos[a + b*x
]) + (4 + m)*AppellF1[1 + m/2, -m, 3 + 2*m, 2 + m/2, Tan[(a + b*x)/2]^2, -T
an[(a + b*x)/2]^2]*(1 + Cos[a + b*x])))
```

Maple [F] time = 1.214, size = 0, normalized size = 0.

$$\int (\sin (bx+a))^3 (\sin (2bx+2a))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(2*b*x+2*a)^m,x)
```

```
[Out] int(sin(b*x+a)^3*sin(2*b*x+2*a)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin (2bx+2a)^m \sin (bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*sin(b*x + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos (bx+a)^2-1\right) \sin (2bx+2a)^m \sin (bx+a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sin(2*b*x + 2*a)^m*sin(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(2bx + 2a)^m \sin(bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*sin(b*x + a)^3, x)
```

3.124 $\int \sin^2(a+bx)\sin^m(2a+2bx)\,dx$

Optimal. Leaf size=84

$$\frac{\sin^2(a+bx)\tan(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{m+3}{2},\frac{m+5}{2},\sin^2(a+bx)\right)}{b(m+3)}$$

```
[Out] ((Cos[a + b*x]^2)^((1 - m)/2)*Hypergeometric2F1[(1 - m)/2, (3 + m)/2, (5 +
m)/2, Sin[a + b*x]^2]*Sin[a + b*x]^2*Sin[2*a + 2*b*x]^m*Tan[a + b*x])/(b*(3
 + m))
```

Rubi [A] time = 0.0765246, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4310, 2577}

$$\frac{\sin^2(a+bx)\tan(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{m+3}{2};\frac{m+5}{2};\sin^2(a+bx)\right)}{b(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] ((Cos[a + b*x]^2)^((1 - m)/2)*Hypergeometric2F1[(1 - m)/2, (3 + m)/2, (5 +
m)/2, Sin[a + b*x]^2]*Sin[a + b*x]^2*Sin[2*a + 2*b*x]^m*Tan[a + b*x])/(b*(3
 + m))
```

Rule 4310

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p), Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sin^2(a+bx)\sin^m(2a+2bx)\,dx = \left(\cos^{-m}(a+bx)\sin^{-m}(a+bx)\sin^m(2a+2bx)\right)\int \cos^m(a+bx)\sin^{2+m}(a+bx)\,dx$$

$$= \frac{\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{3+m}{2};\frac{5+m}{2};\sin^2(a+bx)\right)\sin^2(a+bx)\sin^m(2a+2bx)\,\text{ta}}{b(3+m)}$$

Mathematica [C] time = 3.59562, size = 602, normalized size = 7.17

$$\overline{b(m+1)\left(-2(m+3)\cos^2\left(\frac{1}{2}(a+bx)\right)F_1\left(\frac{m+1}{2};-m,2m+3;\frac{m+3}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)+2(\cos(a+b\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] (16*(3 + m)*(AppellF1[(1 + m)/2, -m, 2*(1 + m), (3 + m)/2, Tan[(a + b*x)/2]
^2, -Tan[(a + b*x)/2]^2] - AppellF1[(1 + m)/2, -m, 3 + 2*m, (3 + m)/2, Tan[
(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*Cos[(a + b*x)/2]^5*Sin[(a + b*x)/2]^3
*Sin[2*(a + b*x)]^m)/(b*(1 + m)*(-2*(3 + m)*AppellF1[(1 + m)/2, -m, 3 + 2*m
, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^2 +
2*(m*AppellF1[(3 + m)/2, 1 - m, 2*(1 + m), (5 + m)/2, Tan[(a + b*x)/2]^2, -
Tan[(a + b*x)/2]^2] - m*AppellF1[(3 + m)/2, 1 - m, 3 + 2*m, (5 + m)/2, Tan[
(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 3*AppellF1[(3 + m)/2, -m, 2*(2 + m),
 (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 2*m*AppellF1[(3 + m)
/2, -m, 2*(2 + m), (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 2*
AppellF1[(3 + m)/2, -m, 3 + 2*m, (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b
*x)/2]^2] + 2*m*AppellF1[(3 + m)/2, -m, 3 + 2*m, (5 + m)/2, Tan[(a + b*x)/2
]^2, -Tan[(a + b*x)/2]^2])*(-1 + Cos[a + b*x]) + (3 + m)*AppellF1[(1 + m)/2
, -m, 2*(1 + m), (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + C
os[a + b*x])))
```

Maple [F] time = 0.848, size = 0, normalized size = 0.

$$\int \left(\sin \left(bx+a\right)\right)^{2} \left(\sin \left(2\,bx+2\,a\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a)^m,x)
```

```
[Out] int(sin(b*x+a)^2*sin(2*b*x+2*a)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin \left(2\,bx+2\,a\right)^{m} \sin \left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*sin(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(\cos \left(bx+a\right)^{2}-1\right) \sin \left(2\,bx+2\,a\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sin(2*b*x + 2*a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin (2\,bx + 2\,a)^m \sin (bx + a)^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*sin(b*x + a)^2, x)
```

3.125 $\int \sin(a+bx)\sin^m(2a+2bx)\,dx$

Optimal. Leaf size=82

$$\frac{\sin(a+bx)\tan(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{m+2}{2},\frac{m+4}{2},\sin^2(a+bx)\right)}{b(m+2)}$$

```
[Out] ((Cos[a + b*x]^2)^((1 - m)/2)*Hypergeometric2F1[(1 - m)/2, (2 + m)/2, (4 +
m)/2, Sin[a + b*x]^2]*Sin[a + b*x]*Sin[2*a + 2*b*x]^m*Tan[a + b*x])/(b*(2 +
 m))
```

Rubi [A] time = 0.0639865, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4310, 2577}

$$\frac{\sin(a+bx)\tan(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{m+2}{2};\frac{m+4}{2};\sin^2(a+bx)\right)}{b(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] ((Cos[a + b*x]^2)^((1 - m)/2)*Hypergeometric2F1[(1 - m)/2, (2 + m)/2, (4 +
m)/2, Sin[a + b*x]^2]*Sin[a + b*x]*Sin[2*a + 2*b*x]^m*Tan[a + b*x])/(b*(2 +
 m))
```

Rule 4310

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p), Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sin(a+bx)\sin^m(2a+2bx)\,dx = \left(\cos^{-m}(a+bx)\sin^{-m}(a+bx)\sin^m(2a+2bx)\right)\int \cos^m(a+bx)\sin^{1+m}(a+bx)\,dx$$
$$= \frac{\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{2+m}{2};\frac{4+m}{2};\sin^2(a+bx)\right)\sin(a+bx)\sin^m(2a+2bx)\tan(a+}{b(2+m)}$$

Mathematica [C] time = 0.267681, size = 152, normalized size = 1.85

$$-\frac{i2^{-m-1}e^{i(a+bx)}\left(-ie^{-2i(a+bx)}\left(-1+e^{4i(a+bx)}\right)\right)^{m+1}\left((1-2m)\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2m+3),\frac{1}{4}(3-2m),e^{4i(a+bx)}\right)+(2m\right.}{b\left(4m^2-1\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] ((-I)*2^(-1 - m)*E^(I*(a + b*x))*(((-I)*(-1 + E^((4*I)*(a + b*x))))/E^((2*I
)*(a + b*x)))^(1 + m)*((1 - 2*m)*Hypergeometric2F1[1, (3 + 2*m)/4, (3 - 2*m
)/4, E^((4*I)*(a + b*x))] + E^((2*I)*(a + b*x))*(1 + 2*m)*Hypergeometric2F1
[1, (5 + 2*m)/4, (5 - 2*m)/4, E^((4*I)*(a + b*x))]))/(b*(-1 + 4*m^2))
```

Maple [F] time = 0.957, size = 0, normalized size = 0.

$$\int \sin\left(bx+a\right)\left(\sin\left(2\,bx+2\,a\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^m,x)
```

```
[Out] int(sin(b*x+a)*sin(2*b*x+2*a)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(2\,bx+2\,a\right)^{m}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*sin(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sin\left(2\,bx+2\,a\right)^{m}\sin\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

```
[Out] integral(sin(2*b*x + 2*a)^m*sin(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(2bx + 2a)^m \sin(bx + a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*sin(b*x + a), x)
```

3.126 $\int \csc(a+bx)\sin^m(2a+2bx)\,dx$

Optimal. Leaf size=72

$$\frac{\sec(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{m}{2},\frac{m+2}{2},\sin^2(a+bx)\right)}{bm}$$

```
[Out] ((Cos[a + b*x]^2)^((1 - m)/2)*Hypergeometric2F1[(1 - m)/2, m/2, (2 + m)/2,
Sin[a + b*x]^2]*Sec[a + b*x]*Sin[2*a + 2*b*x]^m)/(b*m)
```

Rubi [A] time = 0.0702846, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4310, 2577}

$$\frac{\sec(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{m}{2};\frac{m+2}{2};\sin^2(a+bx)\right)}{bm}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] ((Cos[a + b*x]^2)^((1 - m)/2)*Hypergeometric2F1[(1 - m)/2, m/2, (2 + m)/2,
Sin[a + b*x]^2]*Sec[a + b*x]*Sin[2*a + 2*b*x]^m)/(b*m)
```

Rule 4310

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p), Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \csc(a+bx)\sin^m(2a+2bx)\,dx = \left(\cos^{-m}(a+bx)\sin^{-m}(a+bx)\sin^m(2a+2bx)\right)\int \cos^m(a+bx)\sin^{-1+m}(a+bx)\,dx$$
$$= \frac{\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{m}{2};\frac{2+m}{2};\sin^2(a+bx)\right)\sec(a+bx)\sin^m(2a+2bx)}{bm}$$

Mathematica [C] time = 0.889277, size = 254, normalized size = 3.53

$$\frac{2(m+2)\cos^2\left(\frac{1}{2}(a+bx)\right)\sin^m(2(a+bx))F_1}{bm\left((m+2)(\cos(a+bx)+1)F_1\left(\frac{m}{2};-m,2m;\frac{m+2}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)-4m\sin^2\left(\frac{1}{2}(a+bx)\right)\left(F_1\left(\frac{m}{}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] (2*(2 + m)*AppellF1[m/2, -m, 2*m, (2 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a +
b*x)/2]^2]*Cos[(a + b*x)/2]^2*Sin[2*(a + b*x)]^m)/(b*m*((2 + m)*AppellF1[m/
2, -m, 2*m, (2 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + Cos[a
+ b*x]) - 4*m*(AppellF1[(2 + m)/2, 1 - m, 2*m, (4 + m)/2, Tan[(a + b*x)/2]^
2, -Tan[(a + b*x)/2]^2] + 2*AppellF1[(2 + m)/2, -m, 1 + 2*m, (4 + m)/2, Tan
[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*Sin[(a + b*x)/2]^2))
```

Maple [F] time = 0.517, size = 0, normalized size = 0.

$$\int \csc\left(bx+a\right)\left(\sin\left(2bx+2a\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^m,x)
```

```
[Out] int(csc(b*x+a)*sin(2*b*x+2*a)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(2bx+2a\right)^{m}\csc\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*csc(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sin\left(2bx+2a\right)^{m}\csc\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

```
[Out] integral(sin(2*b*x + 2*a)^m*csc(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin^{m}\left(2a+2bx\right)\csc\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**m,x)
```

```
[Out] Integral(sin(2*a + 2*b*x)**m*csc(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(2\,bx+2\,a)^m \csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*csc(b*x + a), x)
```

3.127 $\int \csc^2(a+bx)\sin^m(2a+2bx)\,dx$

Optimal. Leaf size=85

$$-\frac{\csc(a+bx)\sec(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{m-1}{2},\frac{m+1}{2},\sin^2(a+bx)\right)}{b(1-m)}$$

```
[Out] -(((Cos[a + b*x]^2)^((1 - m)/2)*Csc[a + b*x]*Hypergeometric2F1[(1 - m)/2, (
-1 + m)/2, (1 + m)/2, Sin[a + b*x]^2]*Sec[a + b*x]*Sin[2*a + 2*b*x]^m)/(b*(
1 - m)))
```

Rubi [A] time = 0.0819772, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4310, 2577}

$$-\frac{\csc(a+bx)\sec(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{m-1}{2};\frac{m+1}{2};\sin^2(a+bx)\right)}{b(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] -(((Cos[a + b*x]^2)^((1 - m)/2)*Csc[a + b*x]*Hypergeometric2F1[(1 - m)/2, (
-1 + m)/2, (1 + m)/2, Sin[a + b*x]^2]*Sec[a + b*x]*Sin[2*a + 2*b*x]^m)/(b*(
1 - m)))
```

Rule 4310

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p), Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \csc^2(a+bx)\sin^m(2a+2bx)\,dx = \left(\cos^{-m}(a+bx)\sin^{-m}(a+bx)\sin^m(2a+2bx)\right)\int \cos^m(a+bx)\sin^{-2+m}(a+bx)\,dx$$

$$= -\frac{\cos^2(a+bx)^{\frac{1-m}{2}}\csc(a+bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{1}{2}(-1+m);\frac{1+m}{2};\sin^2(a+bx)\right)\sec(a+bx)\,\text{si}}{b(1-m)}$$

Mathematica [C] time = 5.51468, size = 938, normalized size = 11.04

$$\frac{}{b\left(m(m+1)F_1\left(\frac{m-1}{2};-m,2m;\frac{m+1}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)(3\cos(a+bx)-2)\sec(a+bx)\cot^2\left(\frac{1}{2}(a+bx)\right)\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] (2*((-1 + m)*AppellF1[(1 + m)/2, -m, 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -T
an[(a + b*x)/2]^2] + (1 + m)*AppellF1[(-1 + m)/2, -m, 2*m, (1 + m)/2, Tan[(
a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cot[(a + b*x)/2]^2)*Csc[a + b*x]^2*Sin[
2*(a + b*x)]^m*Tan[(a + b*x)/2])/(b*(m*(1 + m)*AppellF1[(-1 + m)/2, -m, 2*m
, (1 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - (1 + m)*AppellF1[(-
1 + m)/2, -m, 2*m, (1 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Csc[
(a + b*x)/2]^2 + (-1 + m)*AppellF1[(1 + m)/2, -m, 2*m, (3 + m)/2, Tan[(a +
b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2 - 2*(-1 + m)*m*(AppellF1
[(1 + m)/2, 1 - m, 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]
 + 2*AppellF1[(1 + m)/2, -m, 1 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(
a + b*x)/2]^2])*Sec[(a + b*x)/2]^2 + (-1 + m)*m*AppellF1[(1 + m)/2, -m, 2*m
, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(-2 + 3*Cos[a + b*x])
*Sec[a + b*x] + m*(1 + m)*AppellF1[(-1 + m)/2, -m, 2*m, (1 + m)/2, Tan[(a +
 b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(-2 + 3*Cos[a + b*x])*Cot[(a + b*x)/2]^2*S
ec[a + b*x] + (-1 + m)*m*AppellF1[(1 + m)/2, -m, 2*m, (3 + m)/2, Tan[(a + b
*x)/2]^2, -Tan[(a + b*x)/2]^2]*Tan[(a + b*x)/2]^2 - (2*(-1 + m)*m*(1 + m)*(
AppellF1[(3 + m)/2, 1 - m, 2*m, (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*
x)/2]^2] + 2*AppellF1[(3 + m)/2, -m, 1 + 2*m, (5 + m)/2, Tan[(a + b*x)/2]^2
, -Tan[(a + b*x)/2]^2])*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2]^2)/(3 + m) + 2*
m*(1 + m)*AppellF1[(-1 + m)/2, -m, 2*m, (1 + m)/2, Tan[(a + b*x)/2]^2, -Tan
[(a + b*x)/2]^2]*Cot[(a + b*x)/2]*Tan[a + b*x] + 2*(-1 + m)*m*AppellF1[(1 +
 m)/2, -m, 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Tan[(a
+ b*x)/2]*Tan[a + b*x]))
```

Maple [F] time = 0.326, size = 0, normalized size = 0.

$$\int (\csc (bx+a))^2 (\sin (2bx+2a))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^m,x)
```

```
[Out] int(csc(b*x+a)^2*sin(2*b*x+2*a)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin (2bx+2a)^m \csc (bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*csc(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sin (2bx+2a)^m \csc (bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

```
[Out] integral(sin(2*b*x + 2*a)^m*csc(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(2bx + 2a)^m \csc(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*csc(b*x + a)^2, x)
```

3.128 $\int \csc^3(a+bx)\sin^m(2a+2bx)\,dx$

Optimal. Leaf size=85

$$-\frac{\csc^2(a+bx)\sec(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{m-2}{2},\frac{m}{2},\sin^2(a+bx)\right)}{b(2-m)}$$

```
[Out] -(((Cos[a + b*x]^2)^((1 - m)/2)*Csc[a + b*x]^2*Hypergeometric2F1[(1 - m)/2,
 (-2 + m)/2, m/2, Sin[a + b*x]^2]*Sec[a + b*x]*Sin[2*a + 2*b*x]^m)/(b*(2 -
m)))
```

Rubi [A] time = 0.0807039, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4310, 2577}

$$-\frac{\csc^2(a+bx)\sec(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{m-2}{2};\frac{m}{2};\sin^2(a+bx)\right)}{b(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^m,x]
```


```
[Out] -(((Cos[a + b*x]^2)^((1 - m)/2)*Csc[a + b*x]^2*Hypergeometric2F1[(1 - m)/2,
 (-2 + m)/2, m/2, Sin[a + b*x]^2]*Sec[a + b*x]*Sin[2*a + 2*b*x]^m)/(b*(2 -
m)))
```


Rule 4310

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p), Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \csc^3(a+bx)\sin^m(2a+2bx)\,dx = \left(\cos^{-m}(a+bx)\sin^{-m}(a+bx)\sin^m(2a+2bx)\right)\int \cos^m(a+bx)\sin^{-3+m}(a+bx)\,d$$

$$= -\frac{\cos^2(a+bx)^{\frac{1-m}{2}}\csc^2(a+bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{1}{2}(-2+m);\frac{m}{2};\sin^2(a+bx)\right)\sec(a+bx)}{b(2-m)}$$

Mathematica [C] time = 14.4233, size = 2421, normalized size = 28.48

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] (2^(-2 + m)*((-2 + m)*m*AppellF1[1 + m/2, -m, 2*m, 2 + m/2, Tan[(a + b*x)/2
]^2, -Tan[(a + b*x)/2]^2] + (2 + m)*Cot[(a + b*x)/2]^2*(2*(-2 + m)*AppellF1
[m/2, -m, 2*m, 1 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + m*Appell
F1[-1 + m/2, -m, 2*m, m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cot[(a
+ b*x)/2]^2))*Csc[a + b*x]^3*(Sec[(a + b*x)/2]^2)^(2*m)*(Cos[(a + b*x)/2]*(
-Sin[(a + b*x)/2] + Sin[(3*(a + b*x))/2]))^m*Sin[2*(a + b*x)]^m*Tan[(a + b*
x)/2]^2)/(b*m*(-4 + m^2)*(Cos[a + b*x]*Sec[(a + b*x)/2]^2)^m*((2^(-2 + m)*(
(-2 + m)*m*AppellF1[1 + m/2, -m, 2*m, 2 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a
+ b*x)/2]^2] + (2 + m)*Cot[(a + b*x)/2]^2*(2*(-2 + m)*AppellF1[m/2, -m, 2*m
, 1 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + m*AppellF1[-1 + m/2,
-m, 2*m, m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cot[(a + b*x)/2]^2))
*(Sec[(a + b*x)/2]^2)^(1 + 2*m)*(Cos[(a + b*x)/2]*(-Sin[(a + b*x)/2] + Sin[
(3*(a + b*x))/2]))^m*Tan[(a + b*x)/2])/(m*(-4 + m^2)*(Cos[a + b*x]*Sec[(a +
 b*x)/2]^2)^m) + (2^(-2 + m)*((-2 + m)*m*AppellF1[1 + m/2, -m, 2*m, 2 + m/2
, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + (2 + m)*Cot[(a + b*x)/2]^2*(2*
(-2 + m)*AppellF1[m/2, -m, 2*m, 1 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)
/2]^2] + m*AppellF1[-1 + m/2, -m, 2*m, m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b
*x)/2]^2]*Cot[(a + b*x)/2]^2))*(Sec[(a + b*x)/2]^2)^(2*m)*(Cos[(a + b*x)/2]
*(-Sin[(a + b*x)/2] + Sin[(3*(a + b*x))/2]))^(-1 + m)*(Cos[(a + b*x)/2]*(-C
os[(a + b*x)/2]/2 + (3*Cos[(3*(a + b*x))/2])/2) - (Sin[(a + b*x)/2]*(-Sin[(
a + b*x)/2] + Sin[(3*(a + b*x))/2]))/2)*Tan[(a + b*x)/2]^2)/((-4 + m^2)*(Co
s[a + b*x]*Sec[(a + b*x)/2]^2)^m) + (2^(-1 + m)*((-2 + m)*m*AppellF1[1 + m/
2, -m, 2*m, 2 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + (2 + m)*Cot
[(a + b*x)/2]^2*(2*(-2 + m)*AppellF1[m/2, -m, 2*m, 1 + m/2, Tan[(a + b*x)/2
]^2, -Tan[(a + b*x)/2]^2] + m*AppellF1[-1 + m/2, -m, 2*m, m/2, Tan[(a + b*x
)/2]^2, -Tan[(a + b*x)/2]^2]*Cot[(a + b*x)/2]^2))*(Sec[(a + b*x)/2]^2)^(2*m
)*(Cos[(a + b*x)/2]*(-Sin[(a + b*x)/2] + Sin[(3*(a + b*x))/2]))^m*Tan[(a +
b*x)/2]^3)/((-4 + m^2)*(Cos[a + b*x]*Sec[(a + b*x)/2]^2)^m) - (2^(-2 + m)*(
(-2 + m)*m*AppellF1[1 + m/2, -m, 2*m, 2 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a
+ b*x)/2]^2] + (2 + m)*Cot[(a + b*x)/2]^2*(2*(-2 + m)*AppellF1[m/2, -m, 2*m
, 1 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + m*AppellF1[-1 + m/2,
-m, 2*m, m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cot[(a + b*x)/2]^2))
*(Sec[(a + b*x)/2]^2)^(2*m)*(Cos[a + b*x]*Sec[(a + b*x)/2]^2)^(-1 - m)*(Cos
[(a + b*x)/2]*(-Sin[(a + b*x)/2] + Sin[(3*(a + b*x))/2]))^m*Tan[(a + b*x)/2
]^2*(-(Sec[(a + b*x)/2]^2*Sin[a + b*x]) + Cos[a + b*x]*Sec[(a + b*x)/2]^2*T
an[(a + b*x)/2]))/(-4 + m^2) + (2^(-2 + m)*(Sec[(a + b*x)/2]^2)^(2*m)*(Cos[
(a + b*x)/2]*(-Sin[(a + b*x)/2] + Sin[(3*(a + b*x))/2]))^m*Tan[(a + b*x)/2]
^2*(-((2 + m)*Cot[(a + b*x)/2]*(2*(-2 + m)*AppellF1[m/2, -m, 2*m, 1 + m/2,
Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + m*AppellF1[-1 + m/2, -m, 2*m, m/
2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cot[(a + b*x)/2]^2)*Csc[(a + b*
x)/2]^2) + (-2 + m)*m*(-(((1 + m/2)*m*AppellF1[2 + m/2, 1 - m, 2*m, 3 + m/2
, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[(a + b*x)
/2])/(2 + m/2)) - (2*(1 + m/2)*m*AppellF1[2 + m/2, -m, 1 + 2*m, 3 + m/2, Ta
n[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2])
/(2 + m/2)) + (2 + m)*Cot[(a + b*x)/2]^2*(-(m*AppellF1[-1 + m/2, -m, 2*m, m
/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cot[(a + b*x)/2]*Csc[(a + b*x)
/2]^2) + 2*(-2 + m)*(-(m^2*AppellF1[1 + m/2, 1 - m, 2*m, 2 + m/2, Tan[(a +
b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2])/(2*(1
+ m/2)) - (m^2*AppellF1[1 + m/2, -m, 1 + 2*m, 2 + m/2, Tan[(a + b*x)/2]^2,
-Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2])/(1 + m/2)) + m*Co
t[(a + b*x)/2]^2*(-2*(-1 + m/2)*AppellF1[m/2, 1 - m, 2*m, 1 + m/2, Tan[(a +
 b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2] - 4*(-
1 + m/2)*AppellF1[m/2, -m, 1 + 2*m, 1 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a +
b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2]))))/(m*(-4 + m^2)*(Cos[a + b
*x]*Sec[(a + b*x)/2]^2)^m)))
```

Maple [F] time = 0.345, size = 0, normalized size = 0.

$$\int (\csc (bx + a))^3 (\sin (2\,bx + 2\,a))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^m,x)
```

```
[Out] int(csc(b*x+a)^3*sin(2*b*x+2*a)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin (2\,bx + 2\,a)^m \csc (bx + a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*csc(b*x + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sin (2\,bx + 2\,a)^m \csc (bx + a)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

```
[Out] integral(sin(2*b*x + 2*a)^m*csc(b*x + a)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin (2\,bx + 2\,a)^m \csc (bx + a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*csc(b*x + a)^3, x)
```

3.129 $\int \cos(a+bx)\sin^7(2a+2bx)\,dx$

Optimal. Leaf size=61

$$\frac{128\cos^{15}(a+bx)}{15b}-\frac{384\cos^{13}(a+bx)}{13b}+\frac{384\cos^{11}(a+bx)}{11b}-\frac{128\cos^{9}(a+bx)}{9b}$$

```
[Out] (-128*Cos[a + b*x]^9)/(9*b) + (384*Cos[a + b*x]^11)/(11*b) - (384*Cos[a + b
*x]^13)/(13*b) + (128*Cos[a + b*x]^15)/(15*b)
```

Rubi [A] time = 0.0598614, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4287, 2565, 270}

$$\frac{128\cos^{15}(a+bx)}{15b}-\frac{384\cos^{13}(a+bx)}{13b}+\frac{384\cos^{11}(a+bx)}{11b}-\frac{128\cos^{9}(a+bx)}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (-128*Cos[a + b*x]^9)/(9*b) + (384*Cos[a + b*x]^11)/(11*b) - (384*Cos[a + b
*x]^13)/(13*b) + (128*Cos[a + b*x]^15)/(15*b)
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos(a+bx)\sin^7(2a+2bx)\,dx &= 128\int \cos^8(a+bx)\sin^7(a+bx)\,dx\\
&= -\frac{128\,\text{Subst}\left(\int x^8\left(1-x^2\right)^3\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{128\,\text{Subst}\left(\int\left(x^8-3x^{10}+3x^{12}-x^{14}\right)dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{128\cos^9(a+bx)}{9b}+\frac{384\cos^{11}(a+bx)}{11b}-\frac{384\cos^{13}(a+bx)}{13b}+\frac{128\cos^{15}(a+bx)}{15b}
\end{aligned}$$

Mathematica [A] time = 0.454349, size = 47, normalized size = 0.77

$$\frac{4\cos^9(a+bx)(10755\cos(2(a+bx))-3366\cos(4(a+bx))+429\cos(6(a+bx))-8330)}{6435b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (4*Cos[a + b*x]^9*(-8330 + 10755*Cos[2*(a + b*x)] - 3366*Cos[4*(a + b*x)] +
 429*Cos[6*(a + b*x)]))/(6435*b)
```

Maple [B] time = 0.024, size = 111, normalized size = 1.8

$$-\frac{35\cos(bx+a)}{128b}-\frac{35\cos(3bx+3a)}{384b}+\frac{21\cos(5bx+5a)}{640b}+\frac{3\cos(7bx+7a)}{128b}-\frac{7\cos(9bx+9a)}{1152b}-\frac{7\cos(11bx+1\cdots}{1408b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^7,x)
```

```
[Out] -35/128*cos(b*x+a)/b-35/384*cos(3*b*x+3*a)/b+21/640*cos(5*b*x+5*a)/b+3/128*
cos(7*b*x+7*a)/b-7/1152*cos(9*b*x+9*a)/b-7/1408*cos(11*b*x+11*a)/b+1/1664*c
os(13*b*x+13*a)/b+1/1920*cos(15*b*x+15*a)/b
```

Maxima [A] time = 1.08165, size = 123, normalized size = 2.02

$$\frac{429\cos(15bx+15a)+495\cos(13bx+13a)-4095\cos(11bx+11a)-5005\cos(9bx+9a)+19305\cos(7bx+7a\cdots}{823680b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="maxima")
```

```
[Out] 1/823680*(429*cos(15*b*x + 15*a) + 495*cos(13*b*x + 13*a) - 4095*cos(11*b*x
 + 11*a) - 5005*cos(9*b*x + 9*a) + 19305*cos(7*b*x + 7*a) + 27027*cos(5*b*x
 + 5*a) - 75075*cos(3*b*x + 3*a) - 225225*cos(b*x + a))/b
```

Fricas [A] time = 0.538273, size = 136, normalized size = 2.23

$$\frac{128\left(429\cos(bx+a)^{15}-1485\cos(bx+a)^{13}+1755\cos(bx+a)^{11}-715\cos(bx+a)^{9}\right)}{6435b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="fricas")
```

```
[Out] 128/6435*(429*cos(b*x + a)^15 - 1485*cos(b*x + a)^13 + 1755*cos(b*x + a)^11
 - 715*cos(b*x + a)^9)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**7,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.66449, size = 149, normalized size = 2.44

$$\frac{\cos(15\,bx+15\,a)}{1920\,b}+\frac{\cos(13\,bx+13\,a)}{1664\,b}-\frac{7\,\cos(11\,bx+11\,a)}{1408\,b}-\frac{7\,\cos(9\,bx+9\,a)}{1152\,b}+\frac{3\,\cos(7\,bx+7\,a)}{128\,b}+\frac{21\,\cos(5\,bx}{640\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="giac")
```

```
[Out] 1/1920*cos(15*b*x + 15*a)/b + 1/1664*cos(13*b*x + 13*a)/b - 7/1408*cos(11*b
*x + 11*a)/b - 7/1152*cos(9*b*x + 9*a)/b + 3/128*cos(7*b*x + 7*a)/b + 21/64
0*cos(5*b*x + 5*a)/b - 35/384*cos(3*b*x + 3*a)/b - 35/128*cos(b*x + a)/b
```

3.130 $\int \cos(a+bx)\sin^6(2a+2bx)\,dx$

Optimal. Leaf size=61

$$-\frac{64\sin^{13}(a+bx)}{13b}+\frac{192\sin^{11}(a+bx)}{11b}-\frac{64\sin^{9}(a+bx)}{3b}+\frac{64\sin^{7}(a+bx)}{7b}$$

```
[Out] (64*Sin[a + b*x]^7)/(7*b) - (64*Sin[a + b*x]^9)/(3*b) + (192*Sin[a + b*x]^1
1)/(11*b) - (64*Sin[a + b*x]^13)/(13*b)
```

Rubi [A] time = 0.0608379, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4287, 2564, 270}

$$-\frac{64\sin^{13}(a+bx)}{13b}+\frac{192\sin^{11}(a+bx)}{11b}-\frac{64\sin^{9}(a+bx)}{3b}+\frac{64\sin^{7}(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

```
[Out] (64*Sin[a + b*x]^7)/(7*b) - (64*Sin[a + b*x]^9)/(3*b) + (192*Sin[a + b*x]^1
1)/(11*b) - (64*Sin[a + b*x]^13)/(13*b)
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sin^6(2a+2bx)\,dx &= 64\int \cos^7(a+bx)\sin^6(a+bx)\,dx\\ &= \frac{64\,\mathrm{Subst}\left(\int x^6\left(1-x^2\right)^3\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{64\,\mathrm{Subst}\left(\int \left(x^6-3x^8+3x^{10}-x^{12}\right)dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{64\sin^7(a+bx)}{7b}-\frac{64\sin^9(a+bx)}{3b}+\frac{192\sin^{11}(a+bx)}{11b}-\frac{64\sin^{13}(a+bx)}{13b}\end{aligned}$$

Mathematica [A] time = 0.315134, size = 47, normalized size = 0.77

$$\frac{2\sin^7(a+bx)(6377\cos(2(a+bx))+1890\cos(4(a+bx))+231\cos(6(a+bx))+5230)}{3003b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

```
[Out] (2*(5230 + 6377*Cos[2*(a + b*x)] + 1890*Cos[4*(a + b*x)] + 231*Cos[6*(a + b
*x)])*Sin[a + b*x]^7)/(3003*b)
```

Maple [A] time = 0.036, size = 97, normalized size = 1.6

$$\frac{5\sin(bx+a)}{16b}-\frac{5\sin(3bx+3a)}{64b}-\frac{3\sin(5bx+5a)}{64b}+\frac{3\sin(7bx+7a)}{224b}+\frac{\sin(9bx+9a)}{96b}-\frac{\sin(11bx+11a)}{704b}-\mathrm{si}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^6,x)
```

```
[Out] 5/16*sin(b*x+a)/b-5/64*sin(3*b*x+3*a)/b-3/64/b*sin(5*b*x+5*a)+3/224/b*sin(7
*b*x+7*a)+1/96/b*sin(9*b*x+9*a)-1/704/b*sin(11*b*x+11*a)-1/832/b*sin(13*b*x
+13*a)
```

Maxima [A] time = 1.1206, size = 108, normalized size = 1.77

$$-\frac{231\sin(13bx+13a)+273\sin(11bx+11a)-2002\sin(9bx+9a)-2574\sin(7bx+7a)+9009\sin(5bx+5a)}{192192b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="maxima")
```

```
[Out] -1/192192*(231*sin(13*b*x + 13*a) + 273*sin(11*b*x + 11*a) - 2002*sin(9*b*x
 + 9*a) - 2574*sin(7*b*x + 7*a) + 9009*sin(5*b*x + 5*a) + 15015*sin(3*b*x +
 3*a) - 60060*sin(b*x + a))/b
```

Fricas [A] time = 0.519273, size = 205, normalized size = 3.36

$$-\frac{64\left(231\cos(bx+a)^{12}-567\cos(bx+a)^{10}+371\cos(bx+a)^8-5\cos(bx+a)^6-6\cos(bx+a)^4-8\cos(bx+a)^2\right.}{3003b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="fricas")
```

```
[Out] -64/3003*(231*cos(b*x + a)^12 - 567*cos(b*x + a)^10 + 371*cos(b*x + a)^8 -
5*cos(b*x + a)^6 - 6*cos(b*x + a)^4 - 8*cos(b*x + a)^2 - 16)*sin(b*x + a)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.52838, size = 130, normalized size = 2.13

$$-\frac{\sin(13\,bx+13\,a)}{832\,b}-\frac{\sin(11\,bx+11\,a)}{704\,b}+\frac{\sin(9\,bx+9\,a)}{96\,b}+\frac{3\,\sin(7\,bx+7\,a)}{224\,b}-\frac{3\,\sin(5\,bx+5\,a)}{64\,b}-\frac{5\,\sin(3\,bx+3\,a)}{64\,b}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="giac")
```

```
[Out] -1/832*sin(13*b*x + 13*a)/b - 1/704*sin(11*b*x + 11*a)/b + 1/96*sin(9*b*x +
 9*a)/b + 3/224*sin(7*b*x + 7*a)/b - 3/64*sin(5*b*x + 5*a)/b - 5/64*sin(3*b
*x + 3*a)/b + 5/16*sin(b*x + a)/b
```

3.131 $\int \cos(a+bx)\sin^5(2a+2bx)\,dx$

Optimal. Leaf size=46

$$-\frac{32\cos^{11}(a+bx)}{11b}+\frac{64\cos^9(a+bx)}{9b}-\frac{32\cos^7(a+bx)}{7b}$$

```
[Out] (-32*Cos[a + b*x]^7)/(7*b) + (64*Cos[a + b*x]^9)/(9*b) - (32*Cos[a + b*x]^1
1)/(11*b)
```

Rubi [A] time = 0.0554593, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4287, 2565, 270}

$$-\frac{32\cos^{11}(a+bx)}{11b}+\frac{64\cos^9(a+bx)}{9b}-\frac{32\cos^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (-32*Cos[a + b*x]^7)/(7*b) + (64*Cos[a + b*x]^9)/(9*b) - (32*Cos[a + b*x]^1
1)/(11*b)
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos(a+bx)\sin^5(2a+2bx)\,dx &= 32\int \cos^6(a+bx)\sin^5(a+bx)\,dx\\
&= -\frac{32\,\text{Subst}\left(\int x^6\left(1-x^2\right)^2\,dx, x, \cos(a+bx)\right)}{b}\\
&= -\frac{32\,\text{Subst}\left(\int \left(x^6-2x^8+x^{10}\right)dx, x, \cos(a+bx)\right)}{b}\\
&= -\frac{32\cos^7(a+bx)}{7b}+\frac{64\cos^9(a+bx)}{9b}-\frac{32\cos^{11}(a+bx)}{11b}
\end{aligned}$$

Mathematica [A] time = 0.264445, size = 37, normalized size = 0.8

$$\frac{4\cos^7(a+bx)(364\cos(2(a+bx))-63\cos(4(a+bx))-365)}{693b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (4*Cos[a + b*x]^7*(-365 + 364*Cos[2*(a + b*x)] - 63*Cos[4*(a + b*x)]))/(693
*b)
```

Maple [B] time = 0.023, size = 83, normalized size = 1.8

$$-\frac{5\cos(bx+a)}{16b}-\frac{5\cos(3bx+3a)}{48b}+\frac{\cos(5bx+5a)}{32b}+\frac{5\cos(7bx+7a)}{224b}-\frac{\cos(9bx+9a)}{288b}-\frac{\cos(11bx+11a)}{352b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^5,x)
```

```
[Out] -5/16*cos(b*x+a)/b-5/48*cos(3*b*x+3*a)/b+1/32*cos(5*b*x+5*a)/b+5/224*cos(7*
b*x+7*a)/b-1/288*cos(9*b*x+9*a)/b-1/352*cos(11*b*x+11*a)/b
```

Maxima [A] time = 1.02742, size = 93, normalized size = 2.02

$$-\frac{63\cos(11bx+11a)+77\cos(9bx+9a)-495\cos(7bx+7a)-693\cos(5bx+5a)+2310\cos(3bx+3a)+6930}{22176b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] -1/22176*(63*cos(11*b*x + 11*a) + 77*cos(9*b*x + 9*a) - 495*cos(7*b*x + 7*a
) - 693*cos(5*b*x + 5*a) + 2310*cos(3*b*x + 3*a) + 6930*cos(b*x + a))/b
```

Fricas [A] time = 0.50379, size = 99, normalized size = 2.15

$$-\frac{32\left(63\cos(bx+a)^{11}-154\cos(bx+a)^{9}+99\cos(bx+a)^{7}\right)}{693b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] -32/693*(63*cos(b*x + a)^11 - 154*cos(b*x + a)^9 + 99*cos(b*x + a)^7)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.38627, size = 111, normalized size = 2.41

$$-\frac{\cos(11\,bx+11\,a)}{352\,b}-\frac{\cos(9\,bx+9\,a)}{288\,b}+\frac{5\,\cos(7\,bx+7\,a)}{224\,b}+\frac{\cos(5\,bx+5\,a)}{32\,b}-\frac{5\,\cos(3\,bx+3\,a)}{48\,b}-\frac{5\,\cos(bx+a)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] -1/352*cos(11*b*x + 11*a)/b - 1/288*cos(9*b*x + 9*a)/b + 5/224*cos(7*b*x +
7*a)/b + 1/32*cos(5*b*x + 5*a)/b - 5/48*cos(3*b*x + 3*a)/b - 5/16*cos(b*x +
 a)/b
```

3.132 $\int \cos(a+bx)\sin^4(2a+2bx)\,dx$

Optimal. Leaf size=46

$$\frac{16\sin^9(a+bx)}{9b}-\frac{32\sin^7(a+bx)}{7b}+\frac{16\sin^5(a+bx)}{5b}$$

```
[Out] (16*Sin[a + b*x]^5)/(5*b) - (32*Sin[a + b*x]^7)/(7*b) + (16*Sin[a + b*x]^9)
/(9*b)
```

Rubi [A] time = 0.0537951, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4287, 2564, 270}

$$\frac{16\sin^9(a+bx)}{9b}-\frac{32\sin^7(a+bx)}{7b}+\frac{16\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (16*Sin[a + b*x]^5)/(5*b) - (32*Sin[a + b*x]^7)/(7*b) + (16*Sin[a + b*x]^9)
/(9*b)
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sin^4(2a+2bx)\,dx &= 16\int \cos^5(a+bx)\sin^4(a+bx)\,dx\\ &= \frac{16\,\text{Subst}\left(\int x^4\left(1-x^2\right)^2\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{16\,\text{Subst}\left(\int \left(x^4-2x^6+x^8\right)\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{16\sin^5(a+bx)}{5b}-\frac{32\sin^7(a+bx)}{7b}+\frac{16\sin^9(a+bx)}{9b}\end{aligned}$$

Mathematica [A] time = 0.131488, size = 37, normalized size = 0.8

$$\frac{2\sin^5(a+bx)(220\cos(2(a+bx))+35\cos(4(a+bx))+249)}{315b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (2*(249 + 220*Cos[2*(a + b*x)] + 35*Cos[4*(a + b*x)])*Sin[a + b*x]^5)/(315*
b)
```

Maple [A] time = 0.027, size = 69, normalized size = 1.5

$$\frac{3\sin(bx+a)}{8b}-\frac{\sin(3bx+3a)}{12b}-\frac{\sin(5bx+5a)}{20b}+\frac{\sin(7bx+7a)}{112b}+\frac{\sin(9bx+9a)}{144b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^4,x)
```

```
[Out] 3/8*sin(b*x+a)/b-1/12*sin(3*b*x+3*a)/b-1/20/b*sin(5*b*x+5*a)+1/112/b*sin(7*
b*x+7*a)+1/144/b*sin(9*b*x+9*a)
```

Maxima [A] time = 1.05242, size = 78, normalized size = 1.7

$$\frac{35\sin(9bx+9a)+45\sin(7bx+7a)-252\sin(5bx+5a)-420\sin(3bx+3a)+1890\sin(bx+a)}{5040b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/5040*(35*sin(9*b*x + 9*a) + 45*sin(7*b*x + 7*a) - 252*sin(5*b*x + 5*a) -
420*sin(3*b*x + 3*a) + 1890*sin(b*x + a))/b
```

Fricas [A] time = 0.504224, size = 142, normalized size = 3.09

$$\frac{16\left(35\cos(bx+a)^8-50\cos(bx+a)^6+3\cos(bx+a)^4+4\cos(bx+a)^2+8\right)\sin(bx+a)}{315b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] 16/315*(35*cos(b*x + a)^8 - 50*cos(b*x + a)^6 + 3*cos(b*x + a)^4 + 4*cos(b*
x + a)^2 + 8)*sin(b*x + a)/b
```

Sympy [A] time = 76.0474, size = 162, normalized size = 3.52

$$\begin{cases} \frac{107 \sin(a+bx)\sin^4(2a+2bx)}{315b} + \frac{16 \sin(a+bx)\sin^2(2a+2bx)\cos^2(2a+2bx)}{21b} + \frac{128 \sin(a+bx)\cos^4(2a+2bx)}{315b} - \frac{104 \sin^3(2a+2bx)\cos(a+bx)\cos(2a+2bx)}{315b} - \\ x \sin^4(2a)\cos(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**4,x)
```

```
[Out] Piecewise((107*sin(a + b*x)*sin(2*a + 2*b*x)**4/(315*b) + 16*sin(a + b*x)*s
in(2*a + 2*b*x)**2*cos(2*a + 2*b*x)**2/(21*b) + 128*sin(a + b*x)*cos(2*a +
2*b*x)**4/(315*b) - 104*sin(2*a + 2*b*x)**3*cos(a + b*x)*cos(2*a + 2*b*x)/(
315*b) - 64*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**3/(315*b), Ne(b
, 0)), (x*sin(2*a)**4*cos(a), True))
```

Giac [A] time = 1.2651, size = 92, normalized size = 2.

$$\frac{\sin(9\,bx + 9\,a)}{144\,b} + \frac{\sin(7\,bx + 7\,a)}{112\,b} - \frac{\sin(5\,bx + 5\,a)}{20\,b} - \frac{\sin(3\,bx + 3\,a)}{12\,b} + \frac{3\,\sin(bx + a)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] 1/144*sin(9*b*x + 9*a)/b + 1/112*sin(7*b*x + 7*a)/b - 1/20*sin(5*b*x + 5*a)
/b - 1/12*sin(3*b*x + 3*a)/b + 3/8*sin(b*x + a)/b
```

3.133 $\int \cos(a+bx)\sin^3(2a+2bx)\,dx$

Optimal. Leaf size=31

$$\frac{8\cos^7(a+bx)}{7b}-\frac{8\cos^5(a+bx)}{5b}$$

```
[Out] (-8*Cos[a + b*x]^5)/(5*b) + (8*Cos[a + b*x]^7)/(7*b)
```

Rubi [A] time = 0.0500029, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4287, 2565, 14}

$$\frac{8\cos^7(a+bx)}{7b}-\frac{8\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (-8*Cos[a + b*x]^5)/(5*b) + (8*Cos[a + b*x]^7)/(7*b)
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \cos(a+bx)\sin^3(2a+2bx)\,dx &= 8\int \cos^4(a+bx)\sin^3(a+bx)\,dx\\
&= -\frac{8\,\text{Subst}\left(\int x^4\left(1-x^2\right)\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{8\,\text{Subst}\left(\int \left(x^4-x^6\right)\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{8\cos^5(a+bx)}{5b}+\frac{8\cos^7(a+bx)}{7b}
\end{aligned}$$

Mathematica [A] time = 0.0899755, size = 27, normalized size = 0.87

$$\frac{4\cos^5(a+bx)(5\cos(2(a+bx))-9)}{35b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (4*Cos[a + b*x]^5*(-9 + 5*Cos[2*(a + b*x)]))/(35*b)
```

Maple [A] time = 0.019, size = 55, normalized size = 1.8

$$-\frac{3\cos(bx+a)}{8b}-\frac{\cos(3bx+3a)}{8b}+\frac{\cos(5bx+5a)}{40b}+\frac{\cos(7bx+7a)}{56b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^3,x)
```

```
[Out] -3/8*cos(b*x+a)/b-1/8*cos(3*b*x+3*a)/b+1/40*cos(5*b*x+5*a)/b+1/56*cos(7*b*x
+7*a)/b
```

Maxima [A] time = 1.01116, size = 63, normalized size = 2.03

$$\frac{5\cos(7bx+7a)+7\cos(5bx+5a)-35\cos(3bx+3a)-105\cos(bx+a)}{280b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 1/280*(5*cos(7*b*x + 7*a) + 7*cos(5*b*x + 5*a) - 35*cos(3*b*x + 3*a) - 105*
cos(b*x + a))/b
```

Fricas [A] time = 0.478136, size = 62, normalized size = 2.

$$\frac{8\left(5\cos(bx+a)^7-7\cos(bx+a)^5\right)}{35b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] 8/35*(5*cos(b*x + a)^7 - 7*cos(b*x + a)^5)/b
```

Sympy [A] time = 22.7035, size = 128, normalized size = 4.13

$$\begin{cases} -\frac{9\sin(a+bx)\sin^3(2a+2bx)}{35b}-\frac{8\sin(a+bx)\sin(2a+2bx)\cos^2(2a+2bx)}{35b}-\frac{22\sin^2(2a+2bx)\cos(a+bx)\cos(2a+2bx)}{35b}-\frac{16\cos(a+bx)\cos^3(2a+2bx)}{35b} & \text{for} \\ x\sin^3(2a)\cos(a) & \text{oth} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**3,x)
```

```
[Out] Piecewise((-9*sin(a + b*x)*sin(2*a + 2*b*x)**3/(35*b) - 8*sin(a + b*x)*sin(
2*a + 2*b*x)*cos(2*a + 2*b*x)**2/(35*b) - 22*sin(2*a + 2*b*x)**2*cos(a + b*
x)*cos(2*a + 2*b*x)/(35*b) - 16*cos(a + b*x)*cos(2*a + 2*b*x)**3/(35*b), Ne
(b, 0)), (x*sin(2*a)**3*cos(a), True))
```

Giac [B] time = 1.2452, size = 73, normalized size = 2.35

$$\frac{\cos(7\,bx + 7\,a)}{56\,b} + \frac{\cos(5\,bx + 5\,a)}{40\,b} - \frac{\cos(3\,bx + 3\,a)}{8\,b} - \frac{3\,\cos(bx + a)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] 1/56*cos(7*b*x + 7*a)/b + 1/40*cos(5*b*x + 5*a)/b - 1/8*cos(3*b*x + 3*a)/b
- 3/8*cos(b*x + a)/b
```

3.134 $\int \cos(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=31

$$\frac{4\sin^3(a+bx)}{3b}-\frac{4\sin^5(a+bx)}{5b}$$

```
[Out] (4*Sin[a + b*x]^3)/(3*b) - (4*Sin[a + b*x]^5)/(5*b)
```

Rubi [A] time = 0.0483909, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4287, 2564, 14}

$$\frac{4\sin^3(a+bx)}{3b}-\frac{4\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (4*Sin[a + b*x]^3)/(3*b) - (4*Sin[a + b*x]^5)/(5*b)
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \cos(a+bx)\sin^2(2a+2bx)\,dx &= 4\int \cos^3(a+bx)\sin^2(a+bx)\,dx\\
&= \frac{4\,\text{Subst}\left(\int x^2\left(1-x^2\right)dx, x, \sin(a+bx)\right)}{b}\\
&= \frac{4\,\text{Subst}\left(\int \left(x^2-x^4\right)dx, x, \sin(a+bx)\right)}{b}\\
&= \frac{4\sin^3(a+bx)}{3b}-\frac{4\sin^5(a+bx)}{5b}
\end{aligned}$$

Mathematica [A] time = 0.0630134, size = 27, normalized size = 0.87

$$\frac{2\sin^3(a+bx)(3\cos(2(a+bx))+7)}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (2*(7 + 3*Cos[2*(a + b*x)])*Sin[a + b*x]^3)/(15*b)
```

Maple [A] time = 0.022, size = 41, normalized size = 1.3

$$\frac{\sin(bx+a)}{2b}-\frac{\sin(3bx+3a)}{12b}-\frac{\sin(5bx+5a)}{20b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^2,x)
```

```
[Out] 1/2*sin(b*x+a)/b-1/12*sin(3*b*x+3*a)/b-1/20/b*sin(5*b*x+5*a)
```

Maxima [A] time = 1.03342, size = 49, normalized size = 1.58

$$-\frac{3\sin(5bx+5a)+5\sin(3bx+3a)-30\sin(bx+a)}{60b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] -1/60*(3*sin(5*b*x + 5*a) + 5*sin(3*b*x + 3*a) - 30*sin(b*x + a))/b
```

Fricas [A] time = 0.478375, size = 84, normalized size = 2.71

$$-\frac{4\left(3\cos(bx+a)^4-\cos(bx+a)^2-2\right)\sin(bx+a)}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] -4/15*(3*cos(b*x + a)^4 - cos(b*x + a)^2 - 2)*sin(b*x + a)/b
```

Sympy [A] time = 6.3123, size = 90, normalized size = 2.9

$$\begin{cases} \frac{7\sin(a+bx)\sin^2(2a+2bx)}{15b}+\frac{8\sin(a+bx)\cos^2(2a+2bx)}{15b}-\frac{4\sin(2a+2bx)\cos(a+bx)\cos(2a+2bx)}{15b} & \text{for } b\neq 0\\ x\sin^2(2a)\cos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**2,x)
```

```
[Out] Piecewise((7*sin(a + b*x)*sin(2*a + 2*b*x)**2/(15*b) + 8*sin(a + b*x)*cos(2
*a + 2*b*x)**2/(15*b) - 4*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)/(1
5*b), Ne(b, 0)), (x*sin(2*a)**2*cos(a), True))
```

Giac [A] time = 1.25192, size = 54, normalized size = 1.74

$$-\frac{\sin(5\,bx+5\,a)}{20\,b}-\frac{\sin(3\,bx+3\,a)}{12\,b}+\frac{\sin(bx+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] -1/20*sin(5*b*x + 5*a)/b - 1/12*sin(3*b*x + 3*a)/b + 1/2*sin(b*x + a)/b
```

3.135 $\int \cos(a+bx)\sin(2a+2bx)\,dx$

Optimal. Leaf size=30

$$-\frac{\cos(a+bx)}{2b}-\frac{\cos(3a+3bx)}{6b}$$

```
[Out] -Cos[a + b*x]/(2*b) - Cos[3*a + 3*b*x]/(6*b)
```

Rubi [A] time = 0.01103, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4284}

$$-\frac{\cos(a+bx)}{2b}-\frac{\cos(3a+3bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x],x]

[Out] -Cos[a + b*x]/(2*b) - Cos[3*a + 3*b*x]/(6*b)
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(a+bx)\sin(2a+2bx)\,dx=-\frac{\cos(a+bx)}{2b}-\frac{\cos(3a+3bx)}{6b}$$

Mathematica [A] time = 0.0056336, size = 15, normalized size = 0.5

$$-\frac{2\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x],x]

[Out] (-2*Cos[a + b*x]^3)/(3*b)
```

Maple [A] time = 0.014, size = 27, normalized size = 0.9

$$-\frac{\cos(bx+a)}{2b}-\frac{\cos(3bx+3a)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a),x)
```

```
[Out] -1/2*cos(b*x+a)/b-1/6*cos(3*b*x+3*a)/b
```

Maxima [A] time = 1.06651, size = 35, normalized size = 1.17

$$-\frac{\cos(3\,bx+3\,a)}{6\,b}-\frac{\cos(bx+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -1/6*cos(3*b*x + 3*a)/b - 1/2*cos(b*x + a)/b
```

Fricas [A] time = 0.483418, size = 31, normalized size = 1.03

$$-\frac{2\cos(bx+a)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] -2/3*cos(b*x + a)^3/b
```

Sympy [A] time = 1.3295, size = 53, normalized size = 1.77

$$\begin{cases} -\frac{\sin(a+bx)\sin(2a+2bx)}{3b}-\frac{2\cos(a+bx)\cos(2a+2bx)}{3b} & \text{for } b\neq 0\\ x\sin(2a)\cos(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a),x)
```

```
[Out] Piecewise((-sin(a + b*x)*sin(2*a + 2*b*x)/(3*b) - 2*cos(a + b*x)*cos(2*a +
2*b*x)/(3*b), Ne(b, 0)), (x*sin(2*a)*cos(a), True))
```

Giac [A] time = 1.25087, size = 35, normalized size = 1.17

$$-\frac{\cos(3\,bx+3\,a)}{6\,b}-\frac{\cos(bx+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] -1/6*cos(3*b*x + 3*a)/b - 1/2*cos(b*x + a)/b
```

3.136 $\int \cos(a+bx)\csc(2a+2bx)\,dx$

Optimal. Leaf size=14

$$-\frac{\tanh^{-1}(\cos(a+bx))}{2b}$$

```
[Out] -ArcTanh[Cos[a + b*x]]/(2*b)
```

Rubi [A] time = 0.0163863, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4287, 3770}

$$-\frac{\tanh^{-1}(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Csc[2*a + 2*b*x],x]
```

```
[Out] -ArcTanh[Cos[a + b*x]]/(2*b)
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\csc(2a+2bx)\,dx &= \frac{1}{2}\int \csc(a+bx)\,dx\\ &= -\frac{\tanh^{-1}(\cos(a+bx))}{2b}\end{aligned}$$

Mathematica [B] time = 0.0124396, size = 42, normalized size = 3.

$$\frac{1}{2}\left(\frac{\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}-\frac{\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Csc[2*a + 2*b*x],x]
```

```
[Out] (-(Log[Cos[a/2 + (b*x)/2]]/b) + Log[Sin[a/2 + (b*x)/2]]/b)/2
```

Maple [A] time = 0.019, size = 22, normalized size = 1.6

$$\frac{\ln(\csc(bx+a)-\cot(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a),x)
```

```
[Out] 1/2/b*ln(csc(b*x+a)-cot(b*x+a))
```

Maxima [B] time = 1.15764, size = 113, normalized size = 8.07

$$-\frac{\log\left(\cos(bx)^2+2\cos(bx)\cos(a)+\cos(a)^2+\sin(bx)^2-2\sin(bx)\sin(a)+\sin(a)^2\right)-\log\left(\cos(bx)^2-2\cos(bx)\cos\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -1/4*(log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*
x)*sin(a) + sin(a)^2) - log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin
(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b
```

Fricas [B] time = 0.491409, size = 93, normalized size = 6.64

$$-\frac{\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] -1/4*(log(1/2*cos(b*x + a) + 1/2) - log(-1/2*cos(b*x + a) + 1/2))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19284, size = 22, normalized size = 1.57

$$\frac{\log\left(\left|\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)\right|\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(tan(1/2*b*x + 1/2*a)))/b
```

3.137 $\int \cos(a+bx)\csc^2(2a+2bx)\,dx$

Optimal. Leaf size=28

$$\frac{\tanh^{-1}(\sin(a+bx))}{4b}-\frac{\csc(a+bx)}{4b}$$

```
[Out] ArcTanh[Sin[a + b*x]]/(4*b) - Csc[a + b*x]/(4*b)
```

Rubi [A] time = 0.0377685, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4287, 2621, 321, 207}

$$\frac{\tanh^{-1}(\sin(a+bx))}{4b}-\frac{\csc(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(4*b) - Csc[a + b*x]/(4*b)
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos(a+bx)\csc^2(2a+2bx)\,dx = \frac{1}{4}\int \csc^2(a+bx)\sec(a+bx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{4b}$$
$$= -\frac{\csc(a+bx)}{4b} - \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{4b}$$
$$= \frac{\tanh^{-1}(\sin(a+bx))}{4b} - \frac{\csc(a+bx)}{4b}$$

Mathematica [C] time = 0.0177062, size = 29, normalized size = 1.04

$$-\frac{\csc(a+bx)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\sin^2(a+bx)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] -(Csc[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[a + b*x]^2])/(4*b)
```

Maple [A] time = 0.023, size = 34, normalized size = 1.2

$$-\frac{1}{4\,b\sin(bx+a)} + \frac{\ln(\sec(bx+a)+\tan(bx+a))}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a)^2,x)
```

```
[Out] -1/4/b/sin(b*x+a)+1/4/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [B] time = 1.81805, size = 315, normalized size = 11.25

$$-\frac{\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2-2\,\cos(2\,bx+2\,a)+1\right)\log\left(\frac{\cos(bx+2\,a)^2+\cos(a)^2-2\,\cos(a)\sin(bx+2\,a)+\sin(bx+2\,a)^2+2\,\cos}{\cos(bx+2\,a)^2+\cos(a)^2+2\,\cos(a)\sin(bx+2\,a)+\sin(bx+2\,a)^2-2\,\cos}\right.}{8\left(b\cos(2\,bx+2\,a)^2+b\sin(2\,bx+2\,a)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*lo
g((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2
 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos
(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)
) + 4*cos(b*x + a)*sin(2*b*x + 2*a) - 4*cos(2*b*x + 2*a)*sin(b*x + a) + 4*s
in(b*x + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 - 2*b*cos(2*b*x +
 2*a) + b)
```

Fricas [B] time = 0.498404, size = 136, normalized size = 4.86

$$\frac{\log\left(\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-\log\left(-\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-2}{8\,b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(log(sin(b*x + a) + 1)*sin(b*x + a) - log(-sin(b*x + a) + 1)*sin(b*x +
a) - 2)/(b*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23463, size = 51, normalized size = 1.82

$$-\frac{\frac{2}{\sin(bx+a)}-\log\left(\left|\sin\left(bx+a\right)+1\right|\right)+\log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] -1/8*(2/sin(b*x + a) - log(abs(sin(b*x + a) + 1)) + log(abs(sin(b*x + a) -
1)))/b
```

3.138 $\int \cos(a+bx)\csc^3(2a+2bx)\,dx$

Optimal. Leaf size=49

$$\frac{3\sec(a+bx)}{16b}-\frac{3\tanh^{-1}(\cos(a+bx))}{16b}-\frac{\csc^2(a+bx)\sec(a+bx)}{16b}$$

```
[Out] (-3*ArcTanh[Cos[a + b*x]])/(16*b) + (3*Sec[a + b*x])/(16*b) - (Csc[a + b*x]
^2*Sec[a + b*x])/(16*b)
```

Rubi [A] time = 0.0549493, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {4287, 2622, 288, 321, 207}

$$\frac{3\sec(a+bx)}{16b}-\frac{3\tanh^{-1}(\cos(a+bx))}{16b}-\frac{\csc^2(a+bx)\sec(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] (-3*ArcTanh[Cos[a + b*x]])/(16*b) + (3*Sec[a + b*x])/(16*b) - (Csc[a + b*x]
^2*Sec[a + b*x])/(16*b)
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\csc^3(2a+2bx)\,dx &= \frac{1}{8}\int \csc^3(a+bx)\sec^2(a+bx)\,dx\\ &= \frac{\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx,x,\sec(a+bx)\right)}{8b}\\ &= -\frac{\csc^2(a+bx)\sec(a+bx)}{16b}+\frac{3\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,\sec(a+bx)\right)}{16b}\\ &= \frac{3\sec(a+bx)}{16b}-\frac{\csc^2(a+bx)\sec(a+bx)}{16b}+\frac{3\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sec(a+bx)\right)}{16b}\\ &= -\frac{3\tanh^{-1}(\cos(a+bx))}{16b}+\frac{3\sec(a+bx)}{16b}-\frac{\csc^2(a+bx)\sec(a+bx)}{16b}\end{aligned}$$

Mathematica [B] time = 0.254762, size = 143, normalized size = 2.92

$$\frac{\csc^4(a+bx)\left(-6\cos(2(a+bx))+2\cos(3(a+bx))+3\cos(3(a+bx))\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)-3\cos(3(a+bx))\log\left(\sin\left(\frac{1}{2}\right.\right.\right.}{16b\left(\csc^2\left(\frac{1}{2}(a+bx)\right)-\sec^2\left(\frac{1}{2}(a+bx\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] (Csc[a + b*x]^4*(2 - 6*Cos[2*(a + b*x)] + 2*Cos[3*(a + b*x)] + 3*Cos[3*(a +
 b*x)]*Log[Cos[(a + b*x)/2]] - 3*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] + C
os[a + b*x]*(-2 - 3*Log[Cos[(a + b*x)/2]] + 3*Log[Sin[(a + b*x)/2]])))/(16*
b*(Csc[(a + b*x)/2]^2 - Sec[(a + b*x)/2]^2))
```

Maple [A] time = 0.027, size = 57, normalized size = 1.2

$$-\frac{1}{16\,b\left(\sin\left(bx+a\right)\right)^2\cos\left(bx+a\right)}+\frac{3}{16\,b\cos\left(bx+a\right)}+\frac{3\,\ln\left(\csc\left(bx+a\right)-\cot\left(bx+a\right)\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a)^3,x)
```

```
[Out] -1/16/b/sin(b*x+a)^2/cos(b*x+a)+3/16/b/cos(b*x+a)+3/16/b*ln(csc(b*x+a)-cot(
b*x+a))
```

Maxima [B] time = 1.2554, size = 1315, normalized size = 26.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(4*(3*cos(5*b*x + 5*a) - 2*cos(3*b*x + 3*a) + 3*cos(b*x + a))*cos(6*b*
x + 6*a) - 12*(cos(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(5*b*x + 5*a) +
4*(2*cos(3*b*x + 3*a) - 3*cos(b*x + a))*cos(4*b*x + 4*a) + 8*(cos(2*b*x + 2
*a) - 1)*cos(3*b*x + 3*a) - 12*cos(2*b*x + 2*a)*cos(b*x + a) + 3*(2*(cos(4*
b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - cos(6*b*x + 6*a)^2 -
2*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - cos(2*b*x
+ 2*a)^2 + 2*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - sin(6
*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) -
sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*co
s(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 3*(2*(cos(4*
b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - cos(6*b*x + 6*a)^2 -
2*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - cos(2*b*x
+ 2*a)^2 + 2*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - sin(6
*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) -
sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*co
s(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*(3*sin(5*b
*x + 5*a) - 2*sin(3*b*x + 3*a) + 3*sin(b*x + a))*sin(6*b*x + 6*a) - 12*(sin
(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(5*b*x + 5*a) + 4*(2*sin(3*b*x + 3*a)
- 3*sin(b*x + a))*sin(4*b*x + 4*a) + 8*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) -
12*sin(2*b*x + 2*a)*sin(b*x + a) + 12*cos(b*x + a))/(b*cos(6*b*x + 6*a)^2 +
 b*cos(4*b*x + 4*a)^2 + b*cos(2*b*x + 2*a)^2 + b*sin(6*b*x + 6*a)^2 + b*sin
(4*b*x + 4*a)^2 + 2*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b*sin(2*b*x + 2*a
)^2 - 2*(b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a) - b)*cos(6*b*x + 6*a) + 2*
(b*cos(2*b*x + 2*a) - b)*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) - 2*(b*sin
(4*b*x + 4*a) + b*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + b)
```

Fricas [B] time = 0.498507, size = 262, normalized size = 5.35

$$\frac{6\cos(bx+a)^2 - 3\left(\cos(bx+a)^3 - \cos(bx+a)\right)\log\left(\frac{1}{2}\cos(bx+a) + \frac{1}{2}\right) + 3\left(\cos(bx+a)^3 - \cos(bx+a)\right)\log\left(-\frac{1}{2}\,c\right.}{32\left(b\cos(bx+a)^3 - b\cos(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(6*cos(b*x + a)^2 - 3*(cos(b*x + a)^3 - cos(b*x + a))*log(1/2*cos(b*x
+ a) + 1/2) + 3*(cos(b*x + a)^3 - cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2
) - 4)/(b*cos(b*x + a)^3 - b*cos(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27516, size = 189, normalized size = 3.86

$$\frac{\frac{\frac{14\,(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{3\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+1}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}}-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+6\,\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] 1/64*((14*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 3*(cos(b*x + a) - 1)^2/(c
os(b*x + a) + 1)^2 + 1)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + (cos(b*x +
 a) - 1)^2/(cos(b*x + a) + 1)^2) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) +
6*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

3.139 $\int \cos(a+bx)\csc^4(2a+2bx)\,dx$

Optimal. Leaf size=66

$$-\frac{5\csc^3(a+bx)}{96b}-\frac{5\csc(a+bx)}{32b}+\frac{5\tanh^{-1}(\sin(a+bx))}{32b}+\frac{\csc^3(a+bx)\sec^2(a+bx)}{32b}$$

```
[Out] (5*ArcTanh[Sin[a + b*x]])/(32*b) - (5*Csc[a + b*x])/(32*b) - (5*Csc[a + b*x
]^3)/(96*b) + (Csc[a + b*x]^3*Sec[a + b*x]^2)/(32*b)
```

Rubi [A] time = 0.0586615, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {4287, 2621, 288, 302, 207}

$$-\frac{5\csc^3(a+bx)}{96b}-\frac{5\csc(a+bx)}{32b}+\frac{5\tanh^{-1}(\sin(a+bx))}{32b}+\frac{\csc^3(a+bx)\sec^2(a+bx)}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] (5*ArcTanh[Sin[a + b*x]])/(32*b) - (5*Csc[a + b*x])/(32*b) - (5*Csc[a + b*x
]^3)/(96*b) + (Csc[a + b*x]^3*Sec[a + b*x]^2)/(32*b)
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos(a+bx)\csc^4(2a+2bx)\,dx &= \frac{1}{16}\int \csc^4(a+bx)\sec^3(a+bx)\,dx \\
&= -\frac{\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^2}\,dx, x, \csc(a+bx)\right)}{16b} \\
&= \frac{\csc^3(a+bx)\sec^2(a+bx)}{32b} - \frac{5\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{32b} \\
&= \frac{\csc^3(a+bx)\sec^2(a+bx)}{32b} - \frac{5\,\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \csc(a+bx)\right)}{32b} \\
&= -\frac{5\csc(a+bx)}{32b} - \frac{5\csc^3(a+bx)}{96b} + \frac{\csc^3(a+bx)\sec^2(a+bx)}{32b} - \frac{5\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x\right.}{32b} \\
&= \frac{5\tanh^{-1}(\sin(a+bx))}{32b} - \frac{5\csc(a+bx)}{32b} - \frac{5\csc^3(a+bx)}{96b} + \frac{\csc^3(a+bx)\sec^2(a+bx)}{32b}
\end{aligned}$$

Mathematica [C] time = 0.0239051, size = 31, normalized size = 0.47

$$-\frac{\csc^3(a+bx)\text{Hypergeometric2F1}\left(-\frac{3}{2},2,-\frac{1}{2},\sin^2(a+bx)\right)}{48b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] -(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 2, -1/2, Sin[a + b*x]^2])/(48*b)
```

Maple [A] time = 0.027, size = 76, normalized size = 1.2

$$-\frac{1}{48\,b\left(\sin(bx+a)\right)^3\left(\cos(bx+a)\right)^2} + \frac{5}{96\,b\sin(bx+a)\left(\cos(bx+a)\right)^2} - \frac{5}{32\,b\sin(bx+a)} + \frac{5\,\ln\left(\sec(bx+a)+\tan(bx\right.}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a)^4,x)
```

```
[Out] -1/48/b/sin(b*x+a)^3/cos(b*x+a)^2+5/96/b/sin(b*x+a)/cos(b*x+a)^2-5/32/b/sin
(b*x+a)+5/32/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [B] time = 2.16879, size = 2403, normalized size = 36.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/192*(4*(15*sin(9*b*x + 9*a) - 20*sin(7*b*x + 7*a) - 22*sin(5*b*x + 5*a) -
 20*sin(3*b*x + 3*a) + 15*sin(b*x + a))*cos(10*b*x + 10*a) + 60*(sin(8*b*x
+ 8*a) + 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(9*
b*x + 9*a) + 4*(20*sin(7*b*x + 7*a) + 22*sin(5*b*x + 5*a) + 20*sin(3*b*x +
3*a) - 15*sin(b*x + a))*cos(8*b*x + 8*a) - 80*(2*sin(6*b*x + 6*a) - 2*sin(4
*b*x + 4*a) - sin(2*b*x + 2*a))*cos(7*b*x + 7*a) + 8*(22*sin(5*b*x + 5*a) +
 20*sin(3*b*x + 3*a) - 15*sin(b*x + a))*cos(6*b*x + 6*a) + 88*(2*sin(4*b*x
+ 4*a) + sin(2*b*x + 2*a))*cos(5*b*x + 5*a) - 40*(4*sin(3*b*x + 3*a) - 3*si
n(b*x + a))*cos(4*b*x + 4*a) + 15*(2*(cos(8*b*x + 8*a) + 2*cos(6*b*x + 6*a)
 - 2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(10*b*x + 10*a) - cos(10*b
*x + 10*a)^2 - 2*(2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a
) + 1)*cos(8*b*x + 8*a) - cos(8*b*x + 8*a)^2 + 4*(2*cos(4*b*x + 4*a) + cos(
2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 4*cos(6*b*x + 6*a)^2 - 4*(cos(2*b*x +
2*a) - 1)*cos(4*b*x + 4*a) - 4*cos(4*b*x + 4*a)^2 - cos(2*b*x + 2*a)^2 + 2*
(sin(8*b*x + 8*a) + 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) - sin(2*b*x + 2
*a))*sin(10*b*x + 10*a) - sin(10*b*x + 10*a)^2 - 2*(2*sin(6*b*x + 6*a) - 2*
sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - sin(8*b*x + 8*a)^2
+ 4*(2*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 4*sin(6*b*x
+ 6*a)^2 - 4*sin(4*b*x + 4*a)^2 - 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - sin
(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log((cos(b*x + 2*a)^2 + cos(a)^2
- 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + si
n(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x +
2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(15*cos(9*b*x + 9*a) - 20
*cos(7*b*x + 7*a) - 22*cos(5*b*x + 5*a) - 20*cos(3*b*x + 3*a) + 15*cos(b*x
+ a))*sin(10*b*x + 10*a) - 60*(cos(8*b*x + 8*a) + 2*cos(6*b*x + 6*a) - 2*co
s(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*sin(9*b*x + 9*a) - 4*(20*cos(7*b*x +
 7*a) + 22*cos(5*b*x + 5*a) + 20*cos(3*b*x + 3*a) - 15*cos(b*x + a))*sin(8*
b*x + 8*a) + 80*(2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a)
 + 1)*sin(7*b*x + 7*a) - 8*(22*cos(5*b*x + 5*a) + 20*cos(3*b*x + 3*a) - 15*
cos(b*x + a))*sin(6*b*x + 6*a) - 88*(2*cos(4*b*x + 4*a) + cos(2*b*x + 2*a)
- 1)*sin(5*b*x + 5*a) + 40*(4*cos(3*b*x + 3*a) - 3*cos(b*x + a))*sin(4*b*x
+ 4*a) - 80*(cos(2*b*x + 2*a) - 1)*sin(3*b*x + 3*a) + 80*cos(3*b*x + 3*a)*s
in(2*b*x + 2*a) - 60*cos(b*x + a)*sin(2*b*x + 2*a) + 60*cos(2*b*x + 2*a)*si
n(b*x + a) - 60*sin(b*x + a))/(b*cos(10*b*x + 10*a)^2 + b*cos(8*b*x + 8*a)^
2 + 4*b*cos(6*b*x + 6*a)^2 + 4*b*cos(4*b*x + 4*a)^2 + b*cos(2*b*x + 2*a)^2
+ b*sin(10*b*x + 10*a)^2 + b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 +
4*b*sin(4*b*x + 4*a)^2 + 4*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b*sin(2*b*
x + 2*a)^2 - 2*(b*cos(8*b*x + 8*a) + 2*b*cos(6*b*x + 6*a) - 2*b*cos(4*b*x +
 4*a) - b*cos(2*b*x + 2*a) + b)*cos(10*b*x + 10*a) + 2*(2*b*cos(6*b*x + 6*a
) - 2*b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) - 4*(2*
b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a) - b)*cos(6*b*x + 6*a) + 4*(b*cos(2*
b*x + 2*a) - b)*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) - 2*(b*sin(8*b*x +
8*a) + 2*b*sin(6*b*x + 6*a) - 2*b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*si
n(10*b*x + 10*a) + 2*(2*b*sin(6*b*x + 6*a) - 2*b*sin(4*b*x + 4*a) - b*sin(2
*b*x + 2*a))*sin(8*b*x + 8*a) - 4*(2*b*sin(4*b*x + 4*a) + b*sin(2*b*x + 2*a
))*sin(6*b*x + 6*a) + b)
```

Fricas [B] time = 0.514639, size = 343, normalized size = 5.2

$$-\frac{30 \cos (bx+a)^4 - 15 \left(\cos (bx+a)^4 - \cos (bx+a)^2\right) \log (\sin (bx+a)+1) \sin (bx+a) + 15 \left(\cos (bx+a)^4 - \cos (bx\right.}{192 \left(b \cos (bx+a)^4 - b \cos (bx+a)^2\right) \sin (bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] -1/192*(30*cos(b*x + a)^4 - 15*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(sin(b*
```

```
x + a) + 1)*sin(b*x + a) + 15*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(-sin(b*
x + a) + 1)*sin(b*x + a) - 40*cos(b*x + a)^2 + 6)/((b*cos(b*x + a)^4 - b*co
s(b*x + a)^2)*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3007, size = 97, normalized size = 1.47

$$-\frac{\frac{6\,\sin(bx+a)}{\sin(bx+a)^2-1}+\frac{4\left(6\,\sin(bx+a)^2+1\right)}{\sin(bx+a)^3}-15\,\log\left(|\sin\left(bx+a\right)+1|\right)+15\,\log\left(|\sin\left(bx+a\right)-1|\right)}{192\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] -1/192*(6*sin(b*x + a)/(sin(b*x + a)^2 - 1) + 4*(6*sin(b*x + a)^2 + 1)/sin(
b*x + a)^3 - 15*log(abs(sin(b*x + a) + 1)) + 15*log(abs(sin(b*x + a) - 1)))
/b
```

3.140 $\int \cos(a+bx)\csc^5(2a+2bx)\,dx$

Optimal. Leaf size=89

$$\frac{35\sec^3(a+bx)}{768b}+\frac{35\sec(a+bx)}{256b}-\frac{35\tanh^{-1}(\cos(a+bx))}{256b}-\frac{\csc^4(a+bx)\sec^3(a+bx)}{128b}-\frac{7\csc^2(a+bx)\sec^3(a+bx)}{256b}$$

```
[Out] (-35*ArcTanh[Cos[a + b*x]])/(256*b) + (35*Sec[a + b*x])/(256*b) + (35*Sec[a
 + b*x]^3)/(768*b) - (7*Csc[a + b*x]^2*Sec[a + b*x]^3)/(256*b) - (Csc[a + b
*x]^4*Sec[a + b*x]^3)/(128*b)
```

Rubi [A] time = 0.0672624, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {4287, 2622, 288, 302, 207}

$$\frac{35\sec^3(a+bx)}{768b}+\frac{35\sec(a+bx)}{256b}-\frac{35\tanh^{-1}(\cos(a+bx))}{256b}-\frac{\csc^4(a+bx)\sec^3(a+bx)}{128b}-\frac{7\csc^2(a+bx)\sec^3(a+bx)}{256b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Csc[2*a + 2*b*x]^5,x]
```

```
[Out] (-35*ArcTanh[Cos[a + b*x]])/(256*b) + (35*Sec[a + b*x])/(256*b) + (35*Sec[a
 + b*x]^3)/(768*b) - (7*Csc[a + b*x]^2*Sec[a + b*x]^3)/(256*b) - (Csc[a + b
*x]^4*Sec[a + b*x]^3)/(128*b)
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos(a+bx)\csc^5(2a+2bx)\,dx = \frac{1}{32}\int \csc^5(a+bx)\sec^4(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^8}{\left(-1+x^2\right)^3}\,dx, x, \sec(a+bx)\right)}{32b}$$
$$= -\frac{\csc^4(a+bx)\sec^3(a+bx)}{128b} + \frac{7\,\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^2}\,dx, x, \sec(a+bx)\right)}{128b}$$
$$= -\frac{7\csc^2(a+bx)\sec^3(a+bx)}{256b} - \frac{\csc^4(a+bx)\sec^3(a+bx)}{128b} + \frac{35\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \right.}{256b}$$
$$= -\frac{7\csc^2(a+bx)\sec^3(a+bx)}{256b} - \frac{\csc^4(a+bx)\sec^3(a+bx)}{128b} + \frac{35\,\text{Subst}\left(\int \left(1+x^2+\frac{}{-1}\right.\right.}{256}$$
$$= \frac{35\sec(a+bx)}{256b} + \frac{35\sec^3(a+bx)}{768b} - \frac{7\csc^2(a+bx)\sec^3(a+bx)}{256b} - \frac{\csc^4(a+bx)\sec^3(a}{128b}$$
$$= -\frac{35\tanh^{-1}(\cos(a+bx))}{256b} + \frac{35\sec(a+bx)}{256b} + \frac{35\sec^3(a+bx)}{768b} - \frac{7\csc^2(a+bx)\sec^3(a}{256b}$$

Mathematica [B] time = 0.48933, size = 268, normalized size = 3.01

$$-\frac{\csc^{10}(a+bx)\left(658\cos(2(a+bx)) - 228\cos(3(a+bx)) + 140\cos(4(a+bx)) - 76\cos(5(a+bx)) - 210\cos(6(a+bx)) + \right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Csc[2*a + 2*b*x]^5,x]
```

```
[Out] -(Csc[a + b*x]^10*(-204 + 658*Cos[2*(a + b*x)] - 228*Cos[3*(a + b*x)] + 140
*Cos[4*(a + b*x)] - 76*Cos[5*(a + b*x)] - 210*Cos[6*(a + b*x)] + 76*Cos[7*(
a + b*x)] - 315*Cos[3*(a + b*x)]*Log[Cos[(a + b*x)/2]] - 105*Cos[5*(a + b*x
)]*Log[Cos[(a + b*x)/2]] + 105*Cos[7*(a + b*x)]*Log[Cos[(a + b*x)/2]] + 3*C
os[a + b*x]*(76 + 105*Log[Cos[(a + b*x)/2]] - 105*Log[Sin[(a + b*x)/2]]) +
315*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] + 105*Cos[5*(a + b*x)]*Log[Sin[(
a + b*x)/2]] - 105*Cos[7*(a + b*x)]*Log[Sin[(a + b*x)/2]]))/(768*b*(Csc[(a
+ b*x)/2]^2 - Sec[(a + b*x)/2]^2)^3)
```

Maple [A] time = 0.032, size = 99, normalized size = 1.1

$$-\frac{1}{128\,b\left(\sin(bx+a)\right)^4\left(\cos(bx+a)\right)^3} + \frac{7}{384\,b\left(\sin(bx+a)\right)^2\left(\cos(bx+a)\right)^3} - \frac{35}{768\,b\left(\sin(bx+a)\right)^2\cos(bx+a)} + \frac{}{256\,b\,\mathrm{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a)^5,x)
```

```
[Out] -1/128/b/sin(b*x+a)^4/cos(b*x+a)^3+7/384/b/sin(b*x+a)^2/cos(b*x+a)^3-35/768
/b/sin(b*x+a)^2/cos(b*x+a)+35/256/b/cos(b*x+a)+35/256/b*ln(csc(b*x+a)-cot(b
*x+a))
```

Maxima [B] time = 1.86288, size = 5192, normalized size = 58.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] 1/1536*(4*(105*cos(13*b*x + 13*a) - 70*cos(11*b*x + 11*a) - 329*cos(9*b*x +
 9*a) + 204*cos(7*b*x + 7*a) - 329*cos(5*b*x + 5*a) - 70*cos(3*b*x + 3*a) +
 105*cos(b*x + a))*cos(14*b*x + 14*a) - 420*(cos(12*b*x + 12*a) + 3*cos(10*
b*x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a)
+ cos(2*b*x + 2*a) - 1)*cos(13*b*x + 13*a) + 4*(70*cos(11*b*x + 11*a) + 329
*cos(9*b*x + 9*a) - 204*cos(7*b*x + 7*a) + 329*cos(5*b*x + 5*a) + 70*cos(3*
b*x + 3*a) - 105*cos(b*x + a))*cos(12*b*x + 12*a) + 280*(3*cos(10*b*x + 10*
a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) + cos(2*b
*x + 2*a) - 1)*cos(11*b*x + 11*a) + 12*(329*cos(9*b*x + 9*a) - 204*cos(7*b*
x + 7*a) + 329*cos(5*b*x + 5*a) + 70*cos(3*b*x + 3*a) - 105*cos(b*x + a))*c
os(10*b*x + 10*a) - 1316*(3*cos(8*b*x + 8*a) + 3*cos(6*b*x + 6*a) - 3*cos(4
*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(9*b*x + 9*a) + 12*(204*cos(7*b*x +
7*a) - 329*cos(5*b*x + 5*a) - 70*cos(3*b*x + 3*a) + 105*cos(b*x + a))*cos(8
*b*x + 8*a) + 816*(3*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*
a) + 1)*cos(7*b*x + 7*a) - 84*(47*cos(5*b*x + 5*a) + 10*cos(3*b*x + 3*a) -
15*cos(b*x + a))*cos(6*b*x + 6*a) + 1316*(3*cos(4*b*x + 4*a) + cos(2*b*x +
2*a) - 1)*cos(5*b*x + 5*a) + 420*(2*cos(3*b*x + 3*a) - 3*cos(b*x + a))*cos(
4*b*x + 4*a) + 280*(cos(2*b*x + 2*a) - 1)*cos(3*b*x + 3*a) - 420*cos(2*b*x
+ 2*a)*cos(b*x + a) + 105*(2*(cos(12*b*x + 12*a) + 3*cos(10*b*x + 10*a) - 3
*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) + cos(2*b*x + 2
*a) - 1)*cos(14*b*x + 14*a) - cos(14*b*x + 14*a)^2 - 2*(3*cos(10*b*x + 10*a
) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) + cos(2*b*
x + 2*a) - 1)*cos(12*b*x + 12*a) - cos(12*b*x + 12*a)^2 + 6*(3*cos(8*b*x +
8*a) + 3*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(
10*b*x + 10*a) - 9*cos(10*b*x + 10*a)^2 - 6*(3*cos(6*b*x + 6*a) - 3*cos(4*b
*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - 9*cos(8*b*x + 8*a)^2 +
 6*(3*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 9*cos(6*b
*x + 6*a)^2 - 6*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 9*cos(4*b*x + 4*a
)^2 - cos(2*b*x + 2*a)^2 + 2*(sin(12*b*x + 12*a) + 3*sin(10*b*x + 10*a) - 3
*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + sin(2*b*x + 2
*a))*sin(14*b*x + 14*a) - sin(14*b*x + 14*a)^2 - 2*(3*sin(10*b*x + 10*a) -
3*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + sin(2*b*x +
2*a))*sin(12*b*x + 12*a) - sin(12*b*x + 12*a)^2 + 6*(3*sin(8*b*x + 8*a) + 3
*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(10*b*x + 10*
a) - 9*sin(10*b*x + 10*a)^2 - 6*(3*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) -
sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 9*sin(8*b*x + 8*a)^2 + 6*(3*sin(4*b*x
+ 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 9*sin(6*b*x + 6*a)^2 - 9*sin(
4*b*x + 4*a)^2 - 6*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - sin(2*b*x + 2*a)^2 +
 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + si
n(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 105*(2*(cos(12*b*x + 12*a) + 3*c
os(10*b*x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x +
 4*a) + cos(2*b*x + 2*a) - 1)*cos(14*b*x + 14*a) - cos(14*b*x + 14*a)^2 - 2
*(3*cos(10*b*x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*
b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(12*b*x + 12*a) - cos(12*b*x + 12*a)^
2 + 6*(3*cos(8*b*x + 8*a) + 3*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - cos(2
```

```
*b*x + 2*a) + 1)*cos(10*b*x + 10*a) - 9*cos(10*b*x + 10*a)^2 - 6*(3*cos(6*b
*x + 6*a) - 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - 9
*cos(8*b*x + 8*a)^2 + 6*(3*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b
*x + 6*a) - 9*cos(6*b*x + 6*a)^2 - 6*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a
) - 9*cos(4*b*x + 4*a)^2 - cos(2*b*x + 2*a)^2 + 2*(sin(12*b*x + 12*a) + 3*s
in(10*b*x + 10*a) - 3*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x +
 4*a) + sin(2*b*x + 2*a))*sin(14*b*x + 14*a) - sin(14*b*x + 14*a)^2 - 2*(3*
sin(10*b*x + 10*a) - 3*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x
+ 4*a) + sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - sin(12*b*x + 12*a)^2 + 6*(3
*sin(8*b*x + 8*a) + 3*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - sin(2*b*x + 2
*a))*sin(10*b*x + 10*a) - 9*sin(10*b*x + 10*a)^2 - 6*(3*sin(6*b*x + 6*a) -
3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 9*sin(8*b*x + 8*a
)^2 + 6*(3*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 9*sin(6*
b*x + 6*a)^2 - 9*sin(4*b*x + 4*a)^2 - 6*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) -
 sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*c
os(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*(105*sin(
13*b*x + 13*a) - 70*sin(11*b*x + 11*a) - 329*sin(9*b*x + 9*a) + 204*sin(7*b
*x + 7*a) - 329*sin(5*b*x + 5*a) - 70*sin(3*b*x + 3*a) + 105*sin(b*x + a))*
sin(14*b*x + 14*a) - 420*(sin(12*b*x + 12*a) + 3*sin(10*b*x + 10*a) - 3*sin
(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))
*sin(13*b*x + 13*a) + 4*(70*sin(11*b*x + 11*a) + 329*sin(9*b*x + 9*a) - 204
*sin(7*b*x + 7*a) + 329*sin(5*b*x + 5*a) + 70*sin(3*b*x + 3*a) - 105*sin(b*
x + a))*sin(12*b*x + 12*a) + 280*(3*sin(10*b*x + 10*a) - 3*sin(8*b*x + 8*a)
 - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(11*b*x +
 11*a) + 12*(329*sin(9*b*x + 9*a) - 204*sin(7*b*x + 7*a) + 329*sin(5*b*x +
5*a) + 70*sin(3*b*x + 3*a) - 105*sin(b*x + a))*sin(10*b*x + 10*a) - 1316*(3
*sin(8*b*x + 8*a) + 3*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - sin(2*b*x + 2
*a))*sin(9*b*x + 9*a) + 12*(204*sin(7*b*x + 7*a) - 329*sin(5*b*x + 5*a) - 7
0*sin(3*b*x + 3*a) + 105*sin(b*x + a))*sin(8*b*x + 8*a) + 816*(3*sin(6*b*x
+ 6*a) - 3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(7*b*x + 7*a) - 84*(47*s
in(5*b*x + 5*a) + 10*sin(3*b*x + 3*a) - 15*sin(b*x + a))*sin(6*b*x + 6*a) +
 1316*(3*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(5*b*x + 5*a) + 420*(2*sin
(3*b*x + 3*a) - 3*sin(b*x + a))*sin(4*b*x + 4*a) + 280*sin(3*b*x + 3*a)*sin
(2*b*x + 2*a) - 420*sin(2*b*x + 2*a)*sin(b*x + a) + 420*cos(b*x + a))/(b*co
s(14*b*x + 14*a)^2 + b*cos(12*b*x + 12*a)^2 + 9*b*cos(10*b*x + 10*a)^2 + 9*
b*cos(8*b*x + 8*a)^2 + 9*b*cos(6*b*x + 6*a)^2 + 9*b*cos(4*b*x + 4*a)^2 + b*
cos(2*b*x + 2*a)^2 + b*sin(14*b*x + 14*a)^2 + b*sin(12*b*x + 12*a)^2 + 9*b*
sin(10*b*x + 10*a)^2 + 9*b*sin(8*b*x + 8*a)^2 + 9*b*sin(6*b*x + 6*a)^2 + 9*
b*sin(4*b*x + 4*a)^2 + 6*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b*sin(2*b*x
+ 2*a)^2 - 2*(b*cos(12*b*x + 12*a) + 3*b*cos(10*b*x + 10*a) - 3*b*cos(8*b*x
 + 8*a) - 3*b*cos(6*b*x + 6*a) + 3*b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a)
- b)*cos(14*b*x + 14*a) + 2*(3*b*cos(10*b*x + 10*a) - 3*b*cos(8*b*x + 8*a)
- 3*b*cos(6*b*x + 6*a) + 3*b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a) - b)*cos
(12*b*x + 12*a) - 6*(3*b*cos(8*b*x + 8*a) + 3*b*cos(6*b*x + 6*a) - 3*b*cos(
4*b*x + 4*a) - b*cos(2*b*x + 2*a) + b)*cos(10*b*x + 10*a) + 6*(3*b*cos(6*b*
x + 6*a) - 3*b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a)
- 6*(3*b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a) - b)*cos(6*b*x + 6*a) + 6*(b
*cos(2*b*x + 2*a) - b)*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) - 2*(b*sin(1
2*b*x + 12*a) + 3*b*sin(10*b*x + 10*a) - 3*b*sin(8*b*x + 8*a) - 3*b*sin(6*b
*x + 6*a) + 3*b*sin(4*b*x + 4*a) + b*sin(2*b*x + 2*a))*sin(14*b*x + 14*a) +
 2*(3*b*sin(10*b*x + 10*a) - 3*b*sin(8*b*x + 8*a) - 3*b*sin(6*b*x + 6*a) +
3*b*sin(4*b*x + 4*a) + b*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - 6*(3*b*sin(
8*b*x + 8*a) + 3*b*sin(6*b*x + 6*a) - 3*b*sin(4*b*x + 4*a) - b*sin(2*b*x +
2*a))*sin(10*b*x + 10*a) + 6*(3*b*sin(6*b*x + 6*a) - 3*b*sin(4*b*x + 4*a) -
 b*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 6*(3*b*sin(4*b*x + 4*a) + b*sin(2*b
*x + 2*a))*sin(6*b*x + 6*a) + b)
```

Fricas [A] time = 0.518971, size = 419, normalized size = 4.71

$$\frac{210 \cos(bx+a)^6 - 350 \cos(bx+a)^4 + 112 \cos(bx+a)^2 - 105 \left(\cos(bx+a)^7 - 2 \cos(bx+a)^5 + \cos(bx+a)^3\right) \log\left(\right.}{1536 \left(b \cos(bx+a)^7 - 2 b \cos(bx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] 1/1536*(210*cos(b*x + a)^6 - 350*cos(b*x + a)^4 + 112*cos(b*x + a)^2 - 105*
(cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3)*log(1/2*cos(b*x + a) +
 1/2) + 105*(cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3)*log(-1/2*c
os(b*x + a) + 1/2) + 16)/(b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*cos(b*x
 + a)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34766, size = 282, normalized size = 3.17

$$\frac{\frac{3\left(\frac{24(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{210(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2}-\frac{72(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{256\left(\frac{9(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{6(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+5\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^3}+420\,\mathrm{l}}{6144\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] 1/6144*(3*(24*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 210*(cos(b*x + a) - 1
)^2/(cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 - 7
2*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 3*(cos(b*x + a) - 1)^2/(cos(b*x +
 a) + 1)^2 + 256*(9*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 6*(cos(b*x + a)
 - 1)^2/(cos(b*x + a) + 1)^2 + 5)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) +
1)^3 + 420*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

3.141 $\int \cos^2(a+bx)\sin^5(2a+2bx)\,dx$

Optimal. Leaf size=44

$$-\frac{8\cos^{12}(a+bx)}{3b}+\frac{32\cos^{10}(a+bx)}{5b}-\frac{4\cos^{8}(a+bx)}{b}$$

```
[Out] (-4*Cos[a + b*x]^8)/b + (32*Cos[a + b*x]^10)/(5*b) - (8*Cos[a + b*x]^12)/(3
*b)
```

Rubi [A] time = 0.066743, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4287, 2565, 266, 43}

$$-\frac{8\cos^{12}(a+bx)}{3b}+\frac{32\cos^{10}(a+bx)}{5b}-\frac{4\cos^{8}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (-4*Cos[a + b*x]^8)/b + (32*Cos[a + b*x]^10)/(5*b) - (8*Cos[a + b*x]^12)/(3
*b)
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\sin^5(2a+2bx)\,dx &= 32\int \cos^7(a+bx)\sin^5(a+bx)\,dx\\ &= -\frac{32\,\text{Subst}\left(\int x^7\left(1-x^2\right)^2\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{16\,\text{Subst}\left(\int(1-x)^2x^3\,dx,x,\cos^2(a+bx)\right)}{b}\\ &= -\frac{16\,\text{Subst}\left(\int\left(x^3-2x^4+x^5\right)\,dx,x,\cos^2(a+bx)\right)}{b}\\ &= -\frac{4\cos^8(a+bx)}{b}+\frac{32\cos^{10}(a+bx)}{5b}-\frac{8\cos^{12}(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.401642, size = 68, normalized size = 1.55

$$-\frac{600\cos(2(a+bx))+75\cos(4(a+bx))-100\cos(6(a+bx))-30\cos(8(a+bx))+12\cos(10(a+bx))+5\cos(12(a+}{3840b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] -(600*Cos[2*(a + b*x)] + 75*Cos[4*(a + b*x)] - 100*Cos[6*(a + b*x)] - 30*Co
s[8*(a + b*x)] + 12*Cos[10*(a + b*x)] + 5*Cos[12*(a + b*x)])/(3840*b)
```

Maple [B] time = 0.021, size = 86, normalized size = 2.

$$-\frac{5\cos(2bx+2a)}{32b}-\frac{5\cos(4bx+4a)}{256b}+\frac{5\cos(6bx+6a)}{192b}+\frac{\cos(8bx+8a)}{128b}-\frac{\cos(10bx+10a)}{320b}-\frac{\cos(12bx+12}{768b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a)^5,x)
```

```
[Out] -5/32*cos(2*b*x+2*a)/b-5/256*cos(4*b*x+4*a)/b+5/192*cos(6*b*x+6*a)/b+1/128*
cos(8*b*x+8*a)/b-1/320*cos(10*b*x+10*a)/b-1/768*cos(12*b*x+12*a)/b
```

Maxima [A] time = 1.15037, size = 97, normalized size = 2.2

$$-\frac{5\cos(12bx+12a)+12\cos(10bx+10a)-30\cos(8bx+8a)-100\cos(6bx+6a)+75\cos(4bx+4a)+600\,c}{3840b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] -1/3840*(5*cos(12*b*x + 12*a) + 12*cos(10*b*x + 10*a) - 30*cos(8*b*x + 8*a)
 - 100*cos(6*b*x + 6*a) + 75*cos(4*b*x + 4*a) + 600*cos(2*b*x + 2*a))/b
```

Fricas [A] time = 0.510557, size = 96, normalized size = 2.18

$$-\frac{4\left(10\cos(bx+a)^{12}-24\cos(bx+a)^{10}+15\cos(bx+a)^8\right)}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] -4/15*(10*cos(b*x + a)^12 - 24*cos(b*x + a)^10 + 15*cos(b*x + a)^8)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.41454, size = 115, normalized size = 2.61

$$-\frac{\cos\left(12\,bx+12\,a\right)}{768\,b}-\frac{\cos\left(10\,bx+10\,a\right)}{320\,b}+\frac{\cos\left(8\,bx+8\,a\right)}{128\,b}+\frac{5\,\cos\left(6\,bx+6\,a\right)}{192\,b}-\frac{5\,\cos\left(4\,bx+4\,a\right)}{256\,b}-\frac{5\,\cos\left(2\,bx+2\,a\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] -1/768*cos(12*b*x + 12*a)/b - 1/320*cos(10*b*x + 10*a)/b + 1/128*cos(8*b*x
+ 8*a)/b + 5/192*cos(6*b*x + 6*a)/b - 5/256*cos(4*b*x + 4*a)/b - 5/32*cos(2
*b*x + 2*a)/b
```

3.142 $\int \cos^2(a+bx)\sin^4(2a+2bx)\,dx$

Optimal. Leaf size=76

$$\frac{\sin^5(2a+2bx)}{20b}-\frac{\sin^3(2a+2bx)\cos(2a+2bx)}{16b}-\frac{3\sin(2a+2bx)\cos(2a+2bx)}{32b}+\frac{3x}{16}$$

```
[Out] (3*x)/16 - (3*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(32*b) - (Cos[2*a + 2*b*x]
*Sin[2*a + 2*b*x]^3)/(16*b) + Sin[2*a + 2*b*x]^5/(20*b)
```

Rubi [A] time = 0.0644401, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4285, 2635, 8, 2564, 30}

$$\frac{\sin^5(2a+2bx)}{20b}-\frac{\sin^3(2a+2bx)\cos(2a+2bx)}{16b}-\frac{3\sin(2a+2bx)\cos(2a+2bx)}{32b}+\frac{3x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
```


```
[Out] (3*x)/16 - (3*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(32*b) - (Cos[2*a + 2*b*x]
*Sin[2*a + 2*b*x]^3)/(16*b) + Sin[2*a + 2*b*x]^5/(20*b)
```


Rule 4285

```
Int[cos[(a_.) + (b_.)*(x_)]^2*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbo
l] :> Dist[1/2, Int[(g*Sin[c + d*x])^p, x], x] + Dist[1/2, Int[Cos[c + d*x]
*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d,
0] && EqQ[d/b, 2] && IGtQ[p/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \cos^2(a+bx)\sin^4(2a+2bx)\,dx = \frac{1}{2}\int \sin^4(2a+2bx)\,dx + \frac{1}{2}\int \cos(2a+2bx)\sin^4(2a+2bx)\,dx$$
$$= -\frac{\cos(2a+2bx)\sin^3(2a+2bx)}{16b} + \frac{3}{8}\int \sin^2(2a+2bx)\,dx + \frac{\text{Subst}\left(\int x^4\,dx, x, \sin(2a$$
$$= -\frac{3\cos(2a+2bx)\sin(2a+2bx)}{32b} - \frac{\cos(2a+2bx)\sin^3(2a+2bx)}{16b} + \frac{\sin^5(2a+2bx)}{20b} +$$
$$= \frac{3x}{16} - \frac{3\cos(2a+2bx)\sin(2a+2bx)}{32b} - \frac{\cos(2a+2bx)\sin^3(2a+2bx)}{16b} + \frac{\sin^5(2a+2b$$

Mathematica [A] time = 0.188991, size = 62, normalized size = 0.82

$$\frac{20\sin(2(a+bx)) - 40\sin(4(a+bx)) - 10\sin(6(a+bx)) + 5\sin(8(a+bx)) + 2\sin(10(a+bx)) + 120bx}{640b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (120*b*x + 20*Sin[2*(a + b*x)] - 40*Sin[4*(a + b*x)] - 10*Sin[6*(a + b*x)]
+ 5*Sin[8*(a + b*x)] + 2*Sin[10*(a + b*x)])/(640*b)
```

Maple [A] time = 0.03, size = 75, normalized size = 1.

$$\frac{3x}{16} + \frac{\sin(2bx+2a)}{32b} - \frac{\sin(4bx+4a)}{16b} - \frac{\sin(6bx+6a)}{64b} + \frac{\sin(8bx+8a)}{128b} + \frac{\sin(10bx+10a)}{320b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a)^4,x)
```

```
[Out] 3/16*x+1/32*sin(2*b*x+2*a)/b-1/16/b*sin(4*b*x+4*a)-1/64/b*sin(6*b*x+6*a)+1/
128/b*sin(8*b*x+8*a)+1/320/b*sin(10*b*x+10*a)
```

Maxima [A] time = 1.15575, size = 88, normalized size = 1.16

$$\frac{120\,bx + 2\sin(10\,bx+10\,a) + 5\sin(8\,bx+8\,a) - 10\sin(6\,bx+6\,a) - 40\sin(4\,bx+4\,a) + 20\sin(2\,bx+2\,a)}{640\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/640*(120*b*x + 2*sin(10*b*x + 10*a) + 5*sin(8*b*x + 8*a) - 10*sin(6*b*x +
 6*a) - 40*sin(4*b*x + 4*a) + 20*sin(2*b*x + 2*a))/b
```

Fricas [A] time = 0.510664, size = 177, normalized size = 2.33

$$\frac{15\,bx + \left(128\cos(bx+a)^9 - 176\cos(bx+a)^7 + 8\cos(bx+a)^5 + 10\cos(bx+a)^3 + 15\cos(bx+a)\right)\sin(bx+a)}{80\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] 1/80*(15*b*x + (128*cos(b*x + a)^9 - 176*cos(b*x + a)^7 + 8*cos(b*x + a)^5
+ 10*cos(b*x + a)^3 + 15*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34078, size = 100, normalized size = 1.32

$$\frac{3}{16}x + \frac{\sin(10\,bx + 10\,a)}{320\,b} + \frac{\sin(8\,bx + 8\,a)}{128\,b} - \frac{\sin(6\,bx + 6\,a)}{64\,b} - \frac{\sin(4\,bx + 4\,a)}{16\,b} + \frac{\sin(2\,bx + 2\,a)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] 3/16*x + 1/320*sin(10*b*x + 10*a)/b + 1/128*sin(8*b*x + 8*a)/b - 1/64*sin(6
*b*x + 6*a)/b - 1/16*sin(4*b*x + 4*a)/b + 1/32*sin(2*b*x + 2*a)/b
```

3.143 $\int \cos^2(a+bx)\sin^3(2a+2bx)\,dx$

Optimal. Leaf size=28

$$\frac{\cos^8(a+bx)}{b}-\frac{4\cos^6(a+bx)}{3b}$$

```
[Out] (-4*Cos[a + b*x]^6)/(3*b) + Cos[a + b*x]^8/b
```

Rubi [A] time = 0.0565491, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4287, 2565, 14}

$$\frac{\cos^8(a+bx)}{b}-\frac{4\cos^6(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (-4*Cos[a + b*x]^6)/(3*b) + Cos[a + b*x]^8/b
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(a+bx)\sin^3(2a+2bx)\,dx &= 8\int \cos^5(a+bx)\sin^3(a+bx)\,dx\\
&= -\frac{8\,\text{Subst}\left(\int x^5\left(1-x^2\right)\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{8\,\text{Subst}\left(\int \left(x^5-x^7\right)\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{4\cos^6(a+bx)}{3b}+\frac{\cos^8(a+bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.122361, size = 48, normalized size = 1.71

$$\frac{-72\cos(2(a+bx))-12\cos(4(a+bx))+8\cos(6(a+bx))+3\cos(8(a+bx))}{384b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (-72*Cos[2*(a + b*x)] - 12*Cos[4*(a + b*x)] + 8*Cos[6*(a + b*x)] + 3*Cos[8*
(a + b*x)])/(384*b)
```

Maple [B] time = 0.017, size = 58, normalized size = 2.1

$$-\frac{3\cos(2bx+2a)}{16b}-\frac{\cos(4bx+4a)}{32b}+\frac{\cos(6bx+6a)}{48b}+\frac{\cos(8bx+8a)}{128b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a)^3,x)
```

```
[Out] -3/16*cos(2*b*x+2*a)/b-1/32*cos(4*b*x+4*a)/b+1/48*cos(6*b*x+6*a)/b+1/128*co
s(8*b*x+8*a)/b
```

Maxima [A] time = 1.11039, size = 68, normalized size = 2.43

$$\frac{3\cos(8bx+8a)+8\cos(6bx+6a)-12\cos(4bx+4a)-72\cos(2bx+2a)}{384b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 1/384*(3*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 12*cos(4*b*x + 4*a) - 72*c
os(2*b*x + 2*a))/b
```

Fricas [A] time = 0.492864, size = 61, normalized size = 2.18

$$\frac{3\cos(bx+a)^8-4\cos(bx+a)^6}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(3*cos(b*x + a)^8 - 4*cos(b*x + a)^6)/b
```

Sympy [A] time = 174.245, size = 359, normalized size = 12.82

$$\begin{cases}-\frac{3x\sin^2(a+bx)\sin^3(2a+2bx)}{16}-\frac{3x\sin^2(a+bx)\sin(2a+2bx)\cos^2(2a+2bx)}{16}-\frac{3x\sin(a+bx)\sin^2(2a+2bx)\cos(a+bx)\cos(2a+2bx)}{8}-\frac{3x\sin(a+bx)\cos\ldots \\ x\sin^3(2a)\cos^2(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**3,x)
```

```
[Out] Piecewise((-3*x*sin(a + b*x)**2*sin(2*a + 2*b*x)**3/16 - 3*x*sin(a + b*x)**
2*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**2/16 - 3*x*sin(a + b*x)*sin(2*a + 2*b*
x)**2*cos(a + b*x)*cos(2*a + 2*b*x)/8 - 3*x*sin(a + b*x)*cos(a + b*x)*cos(2
*a + 2*b*x)**3/8 + 3*x*sin(2*a + 2*b*x)**3*cos(a + b*x)**2/16 + 3*x*sin(2*a
 + 2*b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/16 - sin(a + b*x)**2*cos(2*a
+ 2*b*x)**3/(96*b) - 3*sin(a + b*x)*sin(2*a + 2*b*x)**3*cos(a + b*x)/(16*b)
 - sin(a + b*x)*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**2/(8*b) - s
in(2*a + 2*b*x)**2*cos(a + b*x)**2*cos(2*a + 2*b*x)/(2*b) - 31*cos(a + b*x)
**2*cos(2*a + 2*b*x)**3/(96*b), Ne(b, 0)), (x*sin(2*a)**3*cos(a)**2, True))
```

Giac [B] time = 1.23138, size = 77, normalized size = 2.75

$$\frac{\cos(8\,bx + 8\,a)}{128\,b} + \frac{\cos(6\,bx + 6\,a)}{48\,b} - \frac{\cos(4\,bx + 4\,a)}{32\,b} - \frac{3\,\cos(2\,bx + 2\,a)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] 1/128*cos(8*b*x + 8*a)/b + 1/48*cos(6*b*x + 6*a)/b - 1/32*cos(4*b*x + 4*a)/
b - 3/16*cos(2*b*x + 2*a)/b
```

3.144 $\int \cos^2(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=49

$$\frac{\sin^3(2a+2bx)}{12b}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{8b}+\frac{x}{4}$$

```
[Out] x/4 - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(8*b) + Sin[2*a + 2*b*x]^3/(12*b)
```

Rubi [A] time = 0.0536167, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4285, 2635, 8, 2564, 30}

$$\frac{\sin^3(2a+2bx)}{12b}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{8b}+\frac{x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] x/4 - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(8*b) + Sin[2*a + 2*b*x]^3/(12*b)
```

Rule 4285

```
Int[cos[(a_.) + (b_.)*(x_)]^2*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbo
l] :> Dist[1/2, Int[(g*Sin[c + d*x])^p, x], x] + Dist[1/2, Int[Cos[c + d*x]
*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d,
0] && EqQ[d/b, 2] && IGtQ[p/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \cos^2(a+bx)\sin^2(2a+2bx)\,dx = \frac{1}{2}\int \sin^2(2a+2bx)\,dx + \frac{1}{2}\int \cos(2a+2bx)\sin^2(2a+2bx)\,dx$$
$$= -\frac{\cos(2a+2bx)\sin(2a+2bx)}{8b} + \frac{\int 1\,dx}{4} + \frac{\text{Subst}\left(\int x^2\,dx, x, \sin(2a+2bx)\right)}{4b}$$
$$= \frac{x}{4} - \frac{\cos(2a+2bx)\sin(2a+2bx)}{8b} + \frac{\sin^3(2a+2bx)}{12b}$$

Mathematica [A] time = 0.094312, size = 40, normalized size = 0.82

$$-\frac{-3\sin(2(a+bx)) + 3\sin(4(a+bx)) + \sin(6(a+bx)) - 12bx}{48b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] -(-12*b*x - 3*Sin[2*(a + b*x)] + 3*Sin[4*(a + b*x)] + Sin[6*(a + b*x)])/(48
*b)
```

Maple [A] time = 0.023, size = 47, normalized size = 1.

$$\frac{x}{4} + \frac{\sin(2bx+2a)}{16b} - \frac{\sin(4bx+4a)}{16b} - \frac{\sin(6bx+6a)}{48b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a)^2,x)
```

```
[Out] 1/4*x+1/16*sin(2*b*x+2*a)/b-1/16/b*sin(4*b*x+4*a)-1/48/b*sin(6*b*x+6*a)
```

Maxima [A] time = 1.20071, size = 58, normalized size = 1.18

$$\frac{12\,bx - \sin(6\,bx + 6\,a) - 3\,\sin(4\,bx + 4\,a) + 3\,\sin(2\,bx + 2\,a)}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(12*b*x - sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))/
b
```

Fricas [A] time = 0.491132, size = 116, normalized size = 2.37

$$\frac{3\,bx - \left(8\,\cos(bx+a)^5 - 2\,\cos(bx+a)^3 - 3\,\cos(bx+a)\right)\sin(bx+a)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*x - (8*cos(b*x + a)^5 - 2*cos(b*x + a)^3 - 3*cos(b*x + a))*sin(b*
x + a))/b
```

Sympy [A] time = 28.5597, size = 231, normalized size = 4.71

$$\begin{cases} \frac{x\sin^2(a+bx)\sin^2(2a+2bx)}{4} + \frac{x\sin^2(a+bx)\cos^2(2a+2bx)}{4} + \frac{x\sin^2(2a+2bx)\cos^2(a+bx)}{4} + \frac{x\cos^2(a+bx)\cos^2(2a+2bx)}{4} + \frac{\sin^2(a+bx)\sin(2a+2bx)c}{24b} \\ x\sin^2(2a)\cos^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**2,x)
```

```
[Out] Piecewise((x*sin(a + b*x)**2*sin(2*a + 2*b*x)**2/4 + x*sin(a + b*x)**2*cos(
2*a + 2*b*x)**2/4 + x*sin(2*a + 2*b*x)**2*cos(a + b*x)**2/4 + x*cos(a + b*x
)**2*cos(2*a + 2*b*x)**2/4 + sin(a + b*x)**2*sin(2*a + 2*b*x)*cos(2*a + 2*b
*x)/(24*b) + sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(a + b*x)/(6*b) + sin(a +
b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**2/(3*b) - 7*sin(2*a + 2*b*x)*cos(a + b*
x)**2*cos(2*a + 2*b*x)/(24*b), Ne(b, 0)), (x*sin(2*a)**2*cos(a)**2, True))
```

Giac [A] time = 1.28467, size = 62, normalized size = 1.27

$$\frac{1}{4}x - \frac{\sin(6bx + 6a)}{48b} - \frac{\sin(4bx + 4a)}{16b} + \frac{\sin(2bx + 2a)}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] 1/4*x - 1/48*sin(6*b*x + 6*a)/b - 1/16*sin(4*b*x + 4*a)/b + 1/16*sin(2*b*x
+ 2*a)/b
```

3.145 $\int \cos^2(a+bx)\sin(2a+2bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\cos^4(a+bx)}{2b}$$

```
[Out] -Cos[a + b*x]^4/(2*b)
```

Rubi [A] time = 0.0328988, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4287, 2565, 30}

$$-\frac{\cos^4(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x],x]
```

```
[Out] -Cos[a + b*x]^4/(2*b)
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\sin(2a+2bx)\,dx &= 2\int \cos^3(a+bx)\sin(a+bx)\,dx\\ &= -\frac{2\,\text{Subst}\left(\int x^3\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\cos^4(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0058333, size = 15, normalized size = 1.

$$-\frac{\cos^4(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x],x]
```

```
[Out] -Cos[a + b*x]^4/(2*b)
```

Maple [B] time = 0.013, size = 30, normalized size = 2.

$$-\frac{\cos(2bx+2a)}{4b}-\frac{\cos(4bx+4a)}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a),x)
```

```
[Out] -1/4*cos(2*b*x+2*a)/b-1/16*cos(4*b*x+4*a)/b
```

Maxima [A] time = 1.12094, size = 35, normalized size = 2.33

$$-\frac{\cos(4bx+4a)+4\cos(2bx+2a)}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -1/16*(cos(4*b*x + 4*a) + 4*cos(2*b*x + 2*a))/b
```

Fricas [A] time = 0.475495, size = 31, normalized size = 2.07

$$-\frac{\cos(bx+a)^4}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] -1/2*cos(b*x + a)^4/b
```

Sympy [A] time = 5.98881, size = 131, normalized size = 8.73

$$\begin{cases} -\frac{x\sin^2(a+bx)\sin(2a+2bx)}{4}-\frac{x\sin(a+bx)\cos(a+bx)\cos(2a+2bx)}{2}+\frac{x\sin(2a+2bx)\cos^2(a+bx)}{4}-\frac{\sin(a+bx)\sin(2a+2bx)\cos(a+bx)}{4b}-\frac{\cos^2(a+bx}{} \\ x\sin(2a)\cos^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a),x)
```

```
[Out] Piecewise((-x*sin(a + b*x)**2*sin(2*a + 2*b*x)/4 - x*sin(a + b*x)*cos(a + b
*x)*cos(2*a + 2*b*x)/2 + x*sin(2*a + 2*b*x)*cos(a + b*x)**2/4 - sin(a + b*x
)*sin(2*a + 2*b*x)*cos(a + b*x)/(4*b) - cos(a + b*x)**2*cos(2*a + 2*b*x)/(2
*b), Ne(b, 0)), (x*sin(2*a)*cos(a)**2, True))
```

Giac [B] time = 1.2904, size = 39, normalized size = 2.6

$$-\frac{\cos(4\,bx + 4\,a)}{16\,b} - \frac{\cos(2\,bx + 2\,a)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] -1/16*cos(4*b*x + 4*a)/b - 1/4*cos(2*b*x + 2*a)/b
```

3.146 $\int \cos^2(a+bx)\csc(2a+2bx)\,dx$

Optimal. Leaf size=14

$$\frac{\log(\sin(a+bx))}{2b}$$

```
[Out] Log[Sin[a + b*x]]/(2*b)
```

Rubi [A] time = 0.02515, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4287, 3475}

$$\frac{\log(\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Csc[2*a + 2*b*x],x]
```

```
[Out] Log[Sin[a + b*x]]/(2*b)
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\csc(2a+2bx)\,dx &= \frac{1}{2}\int \cot(a+bx)\,dx\\ &= \frac{\log(\sin(a+bx))}{2b}\end{aligned}$$

Mathematica [A] time = 0.0146544, size = 22, normalized size = 1.57

$$\frac{\log(\tan(a+bx))+\log(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Csc[2*a + 2*b*x],x]
```

```
[Out] (Log[Cos[a + b*x]] + Log[Tan[a + b*x]])/(2*b)
```

Maple [A] time = 0.014, size = 13, normalized size = 0.9

$$\frac{\ln(\sin(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(2*b*x+2*a),x)
```

```
[Out] 1/2*ln(sin(b*x+a))/b
```

Maxima [B] time = 1.1804, size = 111, normalized size = 7.93

$$\frac{\log\left(\cos(bx)^2 + 2\cos(bx)\cos(a) + \cos(a)^2 + \sin(bx)^2 - 2\sin(bx)\sin(a) + \sin(a)^2\right) + \log\left(\cos(bx)^2 - 2\cos(bx)\cos(\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] 1/4*(log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x
)*sin(a) + sin(a)^2) + log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(
b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b
```

Fricas [A] time = 0.491007, size = 39, normalized size = 2.79

$$\frac{\log\left(\frac{1}{2}\sin(bx+a)\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 1/2*log(1/2*sin(b*x + a))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(2*b*x+2*a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28143, size = 76, normalized size = 5.43

$$\frac{\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right) - 2\log\left(\left|-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right|\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] 1/4*(log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) - 2*log(abs(-(cos(b*
x + a) - 1)/(cos(b*x + a) + 1) + 1)))/b
```

3.147 $\int \cos^2(a+bx)\csc^2(2a+2bx)\,dx$

Optimal. Leaf size=13

$$-\frac{\cot(a+bx)}{4b}$$

```
[Out] -Cot[a + b*x]/(4*b)
```

Rubi [A] time = 0.0329684, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4287, 3767, 8}

$$-\frac{\cot(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^2,x]

[Out] -Cot[a + b*x]/(4*b)
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\csc^2(2a+2bx)\,dx &= \frac{1}{4}\int \csc^2(a+bx)\,dx\\ &= -\frac{\text{Subst}\left(\int 1\,dx, x, \cot(a+bx)\right)}{4b}\\ &= -\frac{\cot(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.012905, size = 13, normalized size = 1.

$$-\frac{\cot(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] -Cot[a + b*x]/(4*b)
```

Maple [A] time = 0.02, size = 12, normalized size = 0.9

$$-\frac{\cot(bx+a)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(2*b*x+2*a)^2,x)
```

```
[Out] -1/4*cot(b*x+a)/b
```

Maxima [B] time = 1.1505, size = 72, normalized size = 5.54

$$-\frac{\sin(2\,bx+2\,a)}{2\left(b\cos(2\,bx+2\,a)^2+b\sin(2\,bx+2\,a)^2-2\,b\cos(2\,bx+2\,a)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*sin(2*b*x + 2*a)/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 - 2*b*co
s(2*b*x + 2*a) + b)
```

Fricas [A] time = 0.458278, size = 49, normalized size = 3.77

$$-\frac{\cos(bx+a)}{4\,b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*cos(b*x + a)/(b*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19405, size = 18, normalized size = 1.38

$$-\frac{1}{4\,b\tan\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] -1/4/(b*tan(b*x + a))
```

3.148 $\int \cos^2(a+bx)\csc^3(2a+2bx)\,dx$

Optimal. Leaf size=30

$$\frac{\log(\tan(a+bx))}{8b}-\frac{\cot^2(a+bx)}{16b}$$

```
[Out] -Cot[a + b*x]^2/(16*b) + Log[Tan[a + b*x]]/(8*b)
```

Rubi [A] time = 0.0467969, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4287, 2620, 14}

$$\frac{\log(\tan(a+bx))}{8b}-\frac{\cot^2(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] -Cot[a + b*x]^2/(16*b) + Log[Tan[a + b*x]]/(8*b)
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(a+bx)\csc^3(2a+2bx)\,dx &= \frac{1}{8}\int \csc^3(a+bx)\sec(a+bx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{1+x^2}{x^3}\,dx,x,\tan(a+bx)\right)}{8b}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{1}{x}\right)dx,x,\tan(a+bx)\right)}{8b}\\
&= -\frac{\cot^2(a+bx)}{16b}+\frac{\log(\tan(a+bx))}{8b}
\end{aligned}$$

Mathematica [A] time = 0.0435241, size = 34, normalized size = 1.13

$$-\frac{\csc^2(a+bx)-2\log(\sin(a+bx))+2\log(\cos(a+bx))}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] -(Csc[a + b*x]^2 + 2*Log[Cos[a + b*x]] - 2*Log[Sin[a + b*x]])/(16*b)
```

Maple [A] time = 0.023, size = 27, normalized size = 0.9

$$-\frac{1}{16\,b\,(\sin(bx+a))^2}+\frac{\ln(\tan(bx+a))}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(2*b*x+2*a)^3,x)
```

```
[Out] -1/16/b/sin(b*x+a)^2+1/8*ln(tan(b*x+a))/b
```

Maxima [B] time = 1.20243, size = 886, normalized size = 29.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(4*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) - 8*cos(2*b*x + 2*a)^2 + (2*(2*co
s(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2
*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*
b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(
2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2) - (2
*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b
*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*
sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*co
s(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (2*(2*cos(2*
b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^
2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x
+ 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos
(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*sin(4*b*x + 4*a)*sin
(2*b*x + 2*a) - 8*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a))/(b*cos(4*b*x + 4
*a)^2 + 4*b*cos(2*b*x + 2*a)^2 + b*sin(4*b*x + 4*a)^2 - 4*b*sin(4*b*x + 4*a
)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(2*b*x + 2*a) - b)*
cos(4*b*x + 4*a) - 4*b*cos(2*b*x + 2*a) + b)
```

Fricas [B] time = 0.499714, size = 177, normalized size = 5.9

$$-\frac{\left(\cos(bx+a)^2-1\right)\log\left(\cos(bx+a)^2\right)-\left(\cos(bx+a)^2-1\right)\log\left(-\frac{1}{4}\cos(bx+a)^2+\frac{1}{4}\right)-1}{16\left(b\cos(bx+a)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] -1/16*((cos(b*x + a)^2 - 1)*log(cos(b*x + a)^2) - (cos(b*x + a)^2 - 1)*log(
-1/4*cos(b*x + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^2 - b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28487, size = 161, normalized size = 5.37

$$-\frac{\frac{\left(\frac{4(\cos(bx+a)-1)}{\cos(bx+a)+1}-1\right)(\cos(bx+a)+1)}{\cos(bx+a)-1}-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-4\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)+8\log\left(\left|-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right|\right)}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] -1/64*((4*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)*(cos(b*x + a) + 1)/(co
s(b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 4*log(abs(-cos(b*
x + a) + 1)/abs(cos(b*x + a) + 1)) + 8*log(abs(-(cos(b*x + a) - 1)/(cos(b*x
 + a) + 1) - 1)))/b
```

3.149 $\int \cos^2(a+bx)\csc^4(2a+2bx)\,dx$

Optimal. Leaf size=42

$$\frac{\tan(a+bx)}{16b}-\frac{\cot^3(a+bx)}{48b}-\frac{\cot(a+bx)}{8b}$$

```
[Out] -Cot[a + b*x]/(8*b) - Cot[a + b*x]^3/(48*b) + Tan[a + b*x]/(16*b)
```

Rubi [A] time = 0.0595376, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4287, 2620, 270}

$$\frac{\tan(a+bx)}{16b}-\frac{\cot^3(a+bx)}{48b}-\frac{\cot(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] -Cot[a + b*x]/(8*b) - Cot[a + b*x]^3/(48*b) + Tan[a + b*x]/(16*b)
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(a+bx)\csc^4(2a+2bx)\,dx &= \frac{1}{16}\int \csc^4(a+bx)\sec^2(a+bx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^4}\,dx,x,\tan(a+bx)\right)}{16b}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}+\frac{2}{x^2}\right)dx,x,\tan(a+bx)\right)}{16b}\\
&= -\frac{\cot(a+bx)}{8b}-\frac{\cot^3(a+bx)}{48b}+\frac{\tan(a+bx)}{16b}
\end{aligned}$$

Mathematica [A] time = 0.0520714, size = 48, normalized size = 1.14

$$\frac{\tan(a+bx)}{16b}-\frac{5\cot(a+bx)}{48b}-\frac{\cot(a+bx)\csc^2(a+bx)}{48b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] (-5*Cot[a + b*x])/(48*b) - (Cot[a + b*x]*Csc[a + b*x]^2)/(48*b) + Tan[a + b
*x]/(16*b)
```

Maple [A] time = 0.03, size = 51, normalized size = 1.2

$$\frac{1}{16\,b}\left(-\frac{1}{3\,(\sin(bx+a))^3\cos(bx+a)}+\frac{4}{3\,\cos(bx+a)\sin(bx+a)}-\frac{8\,\cot(bx+a)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(2*b*x+2*a)^4,x)
```

```
[Out] 1/16/b*(-1/3/sin(b*x+a)^3/cos(b*x+a)+4/3/sin(b*x+a)/cos(b*x+a)-8/3*cot(b*x+
a))
```

Maxima [B] time = 1.13744, size = 416, normalized size = 9.9

$$\frac{(2\,\cos(2\,bx+ \cdots}{3\left(b\cos(8\,bx+8\,a)^2+4\,b\cos(6\,bx+6\,a)^2+4\,b\cos(2\,bx+2\,a)^2+b\sin(8\,bx+8\,a)^2+4\,b\sin(6\,bx+6\,a)^2-8\,b\sin(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/3*((2*cos(2*b*x + 2*a) - 1)*sin(8*b*x + 8*a) - 2*(2*cos(2*b*x + 2*a) - 1)
*sin(6*b*x + 6*a) - 2*cos(8*b*x + 8*a)*sin(2*b*x + 2*a) + 4*cos(6*b*x + 6*a
)*sin(2*b*x + 2*a))/(b*cos(8*b*x + 8*a)^2 + 4*b*cos(6*b*x + 6*a)^2 + 4*b*co
s(2*b*x + 2*a)^2 + b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 - 8*b*sin(
6*b*x + 6*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(6*b*x +
 6*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) - 4*(2*b*cos(2*b*x + 2*a
) - b)*cos(6*b*x + 6*a) - 4*b*cos(2*b*x + 2*a) - 4*(b*sin(6*b*x + 6*a) - b*
sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + b)
```

Fricas [A] time = 0.46893, size = 136, normalized size = 3.24

$$-\frac{8\,\cos(bx+a)^4-12\,\cos(bx+a)^2+3}{48\left(b\cos(bx+a)^3-b\cos(bx+a)\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] -1/48*(8*cos(b*x + a)^4 - 12*cos(b*x + a)^2 + 3)/((b*cos(b*x + a)^3 - b*cos
(b*x + a))*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26408, size = 47, normalized size = 1.12

$$-\frac{\frac{6 \tan(bx+a)^2+1}{\tan(bx+a)^3} - 3 \tan(bx+a)}{48 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] -1/48*((6*tan(b*x + a)^2 + 1)/tan(b*x + a)^3 - 3*tan(b*x + a))/b
```

3.150 $\int \cos^2(a+bx)\csc^5(2a+2bx)\,dx$

Optimal. Leaf size=60

$$\frac{\tan^2(a+bx)}{64b}-\frac{\cot^4(a+bx)}{128b}-\frac{3\cot^2(a+bx)}{64b}+\frac{3\log(\tan(a+bx))}{32b}$$

```
[Out] (-3*Cot[a + b*x]^2)/(64*b) - Cot[a + b*x]^4/(128*b) + (3*Log[Tan[a + b*x]])
/(32*b) + Tan[a + b*x]^2/(64*b)
```

Rubi [A] time = 0.067544, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4287, 2620, 266, 43}

$$\frac{\tan^2(a+bx)}{64b}-\frac{\cot^4(a+bx)}{128b}-\frac{3\cot^2(a+bx)}{64b}+\frac{3\log(\tan(a+bx))}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^5,x]
```

```
[Out] (-3*Cot[a + b*x]^2)/(64*b) - Cot[a + b*x]^4/(128*b) + (3*Log[Tan[a + b*x]])
/(32*b) + Tan[a + b*x]^2/(64*b)
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos^2(a+bx)\csc^5(2a+2bx)\,dx &= \frac{1}{32}\int \csc^5(a+bx)\sec^3(a+bx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^5}\,dx, x, \tan(a+bx)\right)}{32b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{x^3}\,dx, x, \tan^2(a+bx)\right)}{64b}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^3}+\frac{3}{x^2}+\frac{3}{x}\right)dx, x, \tan^2(a+bx)\right)}{64b}\\
&= -\frac{3\cot^2(a+bx)}{64b}-\frac{\cot^4(a+bx)}{128b}+\frac{3\log(\tan(a+bx))}{32b}+\frac{\tan^2(a+bx)}{64b}
\end{aligned}$$

Mathematica [A] time = 0.367386, size = 54, normalized size = 0.9

$$-\frac{\csc^4(a+bx)+4\csc^2(a+bx)-2\sec^2(a+bx)-12\log(\sin(a+bx))+12\log(\cos(a+bx))}{128b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^5,x]
```

```
[Out] -(4*Csc[a + b*x]^2 + Csc[a + b*x]^4 + 12*Log[Cos[a + b*x]] - 12*Log[Sin[a +
 b*x]] - 2*Sec[a + b*x]^2)/(128*b)
```

Maple [A] time = 0.027, size = 69, normalized size = 1.2

$$-\frac{1}{128\,b\,(\sin(bx+a))^4(\cos(bx+a))^2}+\frac{3}{128\,b\,(\sin(bx+a))^2(\cos(bx+a))^2}-\frac{3}{64\,b\,(\sin(bx+a))^2}+\frac{3\ln(\tan(bx+a))}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(2*b*x+2*a)^5,x)
```

```
[Out] -1/128/b/sin(b*x+a)^4/cos(b*x+a)^2+3/128/b/sin(b*x+a)^2/cos(b*x+a)^2-3/64/b
/sin(b*x+a)^2+3/32*ln(tan(b*x+a))/b
```

Maxima [B] time = 1.50663, size = 4304, normalized size = 71.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] 1/64*(4*(3*cos(10*b*x + 10*a) - 6*cos(8*b*x + 8*a) - 2*cos(6*b*x + 6*a) - 6
*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a))*cos(12*b*x + 12*a) + 4*(9*cos(8*b*x
 + 8*a) + 16*cos(6*b*x + 6*a) + 9*cos(4*b*x + 4*a) - 12*cos(2*b*x + 2*a) +
3)*cos(10*b*x + 10*a) - 24*cos(10*b*x + 10*a)^2 - 4*(22*cos(6*b*x + 6*a) -
12*cos(4*b*x + 4*a) - 9*cos(2*b*x + 2*a) + 6)*cos(8*b*x + 8*a) + 24*cos(8*b
```

```
*x + 8*a)^2 - 8*(11*cos(4*b*x + 4*a) - 8*cos(2*b*x + 2*a) + 1)*cos(6*b*x +
6*a) - 32*cos(6*b*x + 6*a)^2 + 12*(3*cos(2*b*x + 2*a) - 2)*cos(4*b*x + 4*a)
 + 24*cos(4*b*x + 4*a)^2 - 24*cos(2*b*x + 2*a)^2 + 3*(2*(2*cos(10*b*x + 10*
a) + cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*cos(2*b*x
 + 2*a) - 1)*cos(12*b*x + 12*a) - cos(12*b*x + 12*a)^2 - 4*(cos(8*b*x + 8*a
) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(10*
b*x + 10*a) - 4*cos(10*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*a) - cos(4*b*x +
4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - cos(8*b*x + 8*a)^2 + 8*(c
os(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 16*cos(6*b*x +
 6*a)^2 - 2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2
- 4*cos(2*b*x + 2*a)^2 + 2*(2*sin(10*b*x + 10*a) + sin(8*b*x + 8*a) - 4*sin
(6*b*x + 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) -
 sin(12*b*x + 12*a)^2 - 4*(sin(8*b*x + 8*a) - 4*sin(6*b*x + 6*a) + sin(4*b*
x + 4*a) + 2*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 4*sin(10*b*x + 10*a)^2
+ 2*(4*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(8*b*x
+ 8*a) - sin(8*b*x + 8*a)^2 + 8*(sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin
(6*b*x + 6*a) - 16*sin(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 4*sin(4*b*x +
4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(
cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*
b*x)*sin(2*a) + sin(2*a)^2) - 3*(2*(2*cos(10*b*x + 10*a) + cos(8*b*x + 8*a)
 - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(12*b
*x + 12*a) - cos(12*b*x + 12*a)^2 - 4*(cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a
) + cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(10*b*x + 10*a) - 4*cos(1
0*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*a) - cos(4*b*x + 4*a) - 2*cos(2*b*x +
2*a) + 1)*cos(8*b*x + 8*a) - cos(8*b*x + 8*a)^2 + 8*(cos(4*b*x + 4*a) + 2*c
os(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 16*cos(6*b*x + 6*a)^2 - 2*(2*cos(2*
b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^
2 + 2*(2*sin(10*b*x + 10*a) + sin(8*b*x + 8*a) - 4*sin(6*b*x + 6*a) + sin(4
*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - sin(12*b*x + 12*a)^2
 - 4*(sin(8*b*x + 8*a) - 4*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*
x + 2*a))*sin(10*b*x + 10*a) - 4*sin(10*b*x + 10*a)^2 + 2*(4*sin(6*b*x + 6*
a) - sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - sin(8*b*x +
8*a)^2 + 8*(sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 16*si
n(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a)
 - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*
x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 3*(2*(2
*cos(10*b*x + 10*a) + cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4
*a) + 2*cos(2*b*x + 2*a) - 1)*cos(12*b*x + 12*a) - cos(12*b*x + 12*a)^2 - 4
*(cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*cos(2*b*x +
2*a) - 1)*cos(10*b*x + 10*a) - 4*cos(10*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*
a) - cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - cos(8*b*
x + 8*a)^2 + 8*(cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a)
 - 16*cos(6*b*x + 6*a)^2 - 2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - co
s(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 + 2*(2*sin(10*b*x + 10*a) + sin(8*b
*x + 8*a) - 4*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin
(12*b*x + 12*a) - sin(12*b*x + 12*a)^2 - 4*(sin(8*b*x + 8*a) - 4*sin(6*b*x
+ 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 4*sin(
10*b*x + 10*a)^2 + 2*(4*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - 2*sin(2*b*x +
 2*a))*sin(8*b*x + 8*a) - sin(8*b*x + 8*a)^2 + 8*(sin(4*b*x + 4*a) + 2*sin(
2*b*x + 2*a))*sin(6*b*x + 6*a) - 16*sin(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2
 - 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x
 + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2
*sin(b*x)*sin(a) + sin(a)^2) + 4*(3*sin(10*b*x + 10*a) - 6*sin(8*b*x + 8*a)
 - 2*sin(6*b*x + 6*a) - 6*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(12*b*x
 + 12*a) + 4*(9*sin(8*b*x + 8*a) + 16*sin(6*b*x + 6*a) + 9*sin(4*b*x + 4*a)
 - 12*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 24*sin(10*b*x + 10*a)^2 - 4*(2
2*sin(6*b*x + 6*a) - 12*sin(4*b*x + 4*a) - 9*sin(2*b*x + 2*a))*sin(8*b*x +
8*a) + 24*sin(8*b*x + 8*a)^2 - 8*(11*sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))
*sin(6*b*x + 6*a) - 32*sin(6*b*x + 6*a)^2 + 24*sin(4*b*x + 4*a)^2 + 36*sin(
```

```
4*b*x + 4*a)*sin(2*b*x + 2*a) - 24*sin(2*b*x + 2*a)^2 + 12*cos(2*b*x + 2*a)
)/(b*cos(12*b*x + 12*a)^2 + 4*b*cos(10*b*x + 10*a)^2 + b*cos(8*b*x + 8*a)^2
 + 16*b*cos(6*b*x + 6*a)^2 + b*cos(4*b*x + 4*a)^2 + 4*b*cos(2*b*x + 2*a)^2
+ b*sin(12*b*x + 12*a)^2 + 4*b*sin(10*b*x + 10*a)^2 + b*sin(8*b*x + 8*a)^2
+ 16*b*sin(6*b*x + 6*a)^2 + b*sin(4*b*x + 4*a)^2 + 4*b*sin(4*b*x + 4*a)*sin
(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(10*b*x + 10*a) + b*cos(
8*b*x + 8*a) - 4*b*cos(6*b*x + 6*a) + b*cos(4*b*x + 4*a) + 2*b*cos(2*b*x +
2*a) - b)*cos(12*b*x + 12*a) + 4*(b*cos(8*b*x + 8*a) - 4*b*cos(6*b*x + 6*a)
 + b*cos(4*b*x + 4*a) + 2*b*cos(2*b*x + 2*a) - b)*cos(10*b*x + 10*a) - 2*(4
*b*cos(6*b*x + 6*a) - b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(8*
b*x + 8*a) - 8*(b*cos(4*b*x + 4*a) + 2*b*cos(2*b*x + 2*a) - b)*cos(6*b*x +
6*a) + 2*(2*b*cos(2*b*x + 2*a) - b)*cos(4*b*x + 4*a) - 4*b*cos(2*b*x + 2*a)
 - 2*(2*b*sin(10*b*x + 10*a) + b*sin(8*b*x + 8*a) - 4*b*sin(6*b*x + 6*a) +
b*sin(4*b*x + 4*a) + 2*b*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) + 4*(b*sin(8*
b*x + 8*a) - 4*b*sin(6*b*x + 6*a) + b*sin(4*b*x + 4*a) + 2*b*sin(2*b*x + 2*
a))*sin(10*b*x + 10*a) - 2*(4*b*sin(6*b*x + 6*a) - b*sin(4*b*x + 4*a) - 2*b
*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 8*(b*sin(4*b*x + 4*a) + 2*b*sin(2*b*x
 + 2*a))*sin(6*b*x + 6*a) + b)
```

Fricas [B] time = 0.516827, size = 369, normalized size = 6.15

$$\frac{6\,\cos\left(bx+a\right)^4-9\,\cos\left(bx+a\right)^2-6\left(\cos\left(bx+a\right)^6-2\,\cos\left(bx+a\right)^4+\cos\left(bx+a\right)^2\right)\log\left(\cos\left(bx+a\right)^2\right)+6\left(\cos\left(bx+a\right.\right.}{128\left(b\cos\left(bx+a\right)^6-2\,b\cos\left(bx+a\right)^4+b\cos\left(bx+a\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] 1/128*(6*cos(b*x + a)^4 - 9*cos(b*x + a)^2 - 6*(cos(b*x + a)^6 - 2*cos(b*x
+ a)^4 + cos(b*x + a)^2)*log(cos(b*x + a)^2) + 6*(cos(b*x + a)^6 - 2*cos(b*
x + a)^4 + cos(b*x + a)^2)*log(-1/4*cos(b*x + a)^2 + 1/4) + 2)/(b*cos(b*x +
 a)^6 - 2*b*cos(b*x + a)^4 + b*cos(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29568, size = 313, normalized size = 5.22

$$\frac{\frac{20\,(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{\frac{18\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{111\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{36\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{72\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}-1}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}\right)^2}+96\,\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)-192\,\mathrm{lo}}{2048\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] 1/2048*(20*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - (cos(b*x + a) - 1)^2/(co
s(b*x + a) + 1)^2 + (18*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 111*(cos(b*
x + a) - 1)^2/(cos(b*x + a) + 1)^2 + 36*(cos(b*x + a) - 1)^3/(cos(b*x + a)
+ 1)^3 + 72*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 1)/((cos(b*x + a) -
 1)/(cos(b*x + a) + 1) + (cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2)^2 + 96*
log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) - 192*log(abs(-(cos(b*x +
 a) - 1)/(cos(b*x + a) + 1) - 1)))/b
```

3.151 $\int \cos^3(a+bx)\sin^5(2a+2bx)\,dx$

Optimal. Leaf size=46

$$-\frac{32\cos^{13}(a+bx)}{13b}+\frac{64\cos^{11}(a+bx)}{11b}-\frac{32\cos^9(a+bx)}{9b}$$

```
[Out] (-32*Cos[a + b*x]^9)/(9*b) + (64*Cos[a + b*x]^11)/(11*b) - (32*Cos[a + b*x]
^13)/(13*b)
```

Rubi [A] time = 0.0612923, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4287, 2565, 270}

$$-\frac{32\cos^{13}(a+bx)}{13b}+\frac{64\cos^{11}(a+bx)}{11b}-\frac{32\cos^9(a+bx)}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (-32*Cos[a + b*x]^9)/(9*b) + (64*Cos[a + b*x]^11)/(11*b) - (32*Cos[a + b*x]
^13)/(13*b)
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^3(a+bx)\sin^5(2a+2bx)\,dx &= 32\int \cos^8(a+bx)\sin^5(a+bx)\,dx\\
&= -\frac{32\,\text{Subst}\left(\int x^8\left(1-x^2\right)^2\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{32\,\text{Subst}\left(\int \left(x^8-2x^{10}+x^{12}\right)\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{32\cos^9(a+bx)}{9b}+\frac{64\cos^{11}(a+bx)}{11b}-\frac{32\cos^{13}(a+bx)}{13b}
\end{aligned}$$

Mathematica [A] time = 0.390068, size = 37, normalized size = 0.8

$$\frac{4\cos^9(a+bx)(540\cos(2(a+bx))-99\cos(4(a+bx))-505)}{1287b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (4*Cos[a + b*x]^9*(-505 + 540*Cos[2*(a + b*x)] - 99*Cos[4*(a + b*x)]))/(128
7*b)
```

Maple [B] time = 0.024, size = 97, normalized size = 2.1

$$-\frac{5\cos(bx+a)}{32b}-\frac{25\cos(3bx+3a)}{384b}+\frac{\cos(5bx+5a)}{128b}+\frac{\cos(7bx+7a)}{64b}+\frac{\cos(9bx+9a)}{576b}-\frac{3\cos(11bx+11a)}{1408b}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(2*b*x+2*a)^5,x)
```

```
[Out] -5/32*cos(b*x+a)/b-25/384*cos(3*b*x+3*a)/b+1/128*cos(5*b*x+5*a)/b+1/64*cos(
7*b*x+7*a)/b+1/576*cos(9*b*x+9*a)/b-3/1408*cos(11*b*x+11*a)/b-1/1664*cos(13
*b*x+13*a)/b
```

Maxima [A] time = 1.12607, size = 108, normalized size = 2.35

$$-\frac{99\cos(13bx+13a)+351\cos(11bx+11a)-286\cos(9bx+9a)-2574\cos(7bx+7a)-1287\cos(5bx+5a)}{164736b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] -1/164736*(99*cos(13*b*x + 13*a) + 351*cos(11*b*x + 11*a) - 286*cos(9*b*x +
 9*a) - 2574*cos(7*b*x + 7*a) - 1287*cos(5*b*x + 5*a) + 10725*cos(3*b*x + 3
*a) + 25740*cos(b*x + a))/b
```

Fricas [A] time = 0.518563, size = 103, normalized size = 2.24

$$-\frac{32\left(99\cos(bx+a)^{13}-234\cos(bx+a)^{11}+143\cos(bx+a)^{9}\right)}{1287b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] -32/1287*(99*cos(b*x + a)^13 - 234*cos(b*x + a)^11 + 143*cos(b*x + a)^9)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.40829, size = 130, normalized size = 2.83

$$-\frac{\cos(13\,bx+13\,a)}{1664\,b}-\frac{3\,\cos(11\,bx+11\,a)}{1408\,b}+\frac{\cos(9\,bx+9\,a)}{576\,b}+\frac{\cos(7\,bx+7\,a)}{64\,b}+\frac{\cos(5\,bx+5\,a)}{128\,b}-\frac{25\,\cos(3\,bx+3\,a)}{384\,b}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] -1/1664*cos(13*b*x + 13*a)/b - 3/1408*cos(11*b*x + 11*a)/b + 1/576*cos(9*b*
x + 9*a)/b + 1/64*cos(7*b*x + 7*a)/b + 1/128*cos(5*b*x + 5*a)/b - 25/384*co
s(3*b*x + 3*a)/b - 5/32*cos(b*x + a)/b
```

3.152 $\int \cos^3(a+bx)\sin^4(2a+2bx)\,dx$

Optimal. Leaf size=61

$$-\frac{16\sin^{11}(a+bx)}{11b}+\frac{16\sin^9(a+bx)}{3b}-\frac{48\sin^7(a+bx)}{7b}+\frac{16\sin^5(a+bx)}{5b}$$

```
[Out] (16*Sin[a + b*x]^5)/(5*b) - (48*Sin[a + b*x]^7)/(7*b) + (16*Sin[a + b*x]^9)
/(3*b) - (16*Sin[a + b*x]^11)/(11*b)
```

Rubi [A] time = 0.064816, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4287, 2564, 270}

$$-\frac{16\sin^{11}(a+bx)}{11b}+\frac{16\sin^9(a+bx)}{3b}-\frac{48\sin^7(a+bx)}{7b}+\frac{16\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (16*Sin[a + b*x]^5)/(5*b) - (48*Sin[a + b*x]^7)/(7*b) + (16*Sin[a + b*x]^9)
/(3*b) - (16*Sin[a + b*x]^11)/(11*b)
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\sin^4(2a+2bx)\,dx &= 16\int \cos^7(a+bx)\sin^4(a+bx)\,dx\\ &= \frac{16\,\text{Subst}\left(\int x^4\left(1-x^2\right)^3\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{16\,\text{Subst}\left(\int \left(x^4-3x^6+3x^8-x^{10}\right)dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{16\sin^5(a+bx)}{5b}-\frac{48\sin^7(a+bx)}{7b}+\frac{16\sin^9(a+bx)}{3b}-\frac{16\sin^{11}(a+bx)}{11b}\end{aligned}$$

Mathematica [A] time = 0.212373, size = 47, normalized size = 0.77

$$\frac{\sin^5(a+bx)(3335\cos(2(a+bx))+910\cos(4(a+bx))+105\cos(6(a+bx))+3042)}{2310b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] ((3042 + 3335*Cos[2*(a + b*x)] + 910*Cos[4*(a + b*x)] + 105*Cos[6*(a + b*x)
])*Sin[a + b*x]^5)/(2310*b)
```

Maple [A] time = 0.03, size = 83, normalized size = 1.4

$$\frac{7\sin(bx+a)}{32b}-\frac{\sin(3bx+3a)}{32b}-\frac{11\sin(5bx+5a)}{320b}-\frac{\sin(7bx+7a)}{448b}+\frac{\sin(9bx+9a)}{192b}+\frac{\sin(11bx+11a)}{704b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(2*b*x+2*a)^4,x)
```

```
[Out] 7/32*sin(b*x+a)/b-1/32*sin(3*b*x+3*a)/b-11/320/b*sin(5*b*x+5*a)-1/448/b*sin
(7*b*x+7*a)+1/192/b*sin(9*b*x+9*a)+1/704/b*sin(11*b*x+11*a)
```

Maxima [A] time = 1.16585, size = 93, normalized size = 1.52

$$\frac{105\sin(11bx+11a)+385\sin(9bx+9a)-165\sin(7bx+7a)-2541\sin(5bx+5a)-2310\sin(3bx+3a)+16170}{73920b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/73920*(105*sin(11*b*x + 11*a) + 385*sin(9*b*x + 9*a) - 165*sin(7*b*x + 7*
a) - 2541*sin(5*b*x + 5*a) - 2310*sin(3*b*x + 3*a) + 16170*sin(b*x + a))/b
```

Fricas [A] time = 0.510513, size = 174, normalized size = 2.85

$$\frac{16\left(105\cos(bx+a)^{10}-140\cos(bx+a)^{8}+5\cos(bx+a)^{6}+6\cos(bx+a)^{4}+8\cos(bx+a)^{2}+16\right)\sin(bx+a)}{1155b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] 16/1155*(105*cos(b*x + a)^10 - 140*cos(b*x + a)^8 + 5*cos(b*x + a)^6 + 6*co
s(b*x + a)^4 + 8*cos(b*x + a)^2 + 16)*sin(b*x + a)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21293, size = 111, normalized size = 1.82

$$\frac{\sin(11\,bx+11\,a)}{704\,b}+\frac{\sin(9\,bx+9\,a)}{192\,b}-\frac{\sin(7\,bx+7\,a)}{448\,b}-\frac{11\,\sin(5\,bx+5\,a)}{320\,b}-\frac{\sin(3\,bx+3\,a)}{32\,b}+\frac{7\,\sin(bx+a)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] 1/704*sin(11*b*x + 11*a)/b + 1/192*sin(9*b*x + 9*a)/b - 1/448*sin(7*b*x + 7
*a)/b - 11/320*sin(5*b*x + 5*a)/b - 1/32*sin(3*b*x + 3*a)/b + 7/32*sin(b*x
+ a)/b
```

3.153 $\int \cos^3(a+bx)\sin^3(2a+2bx)\,dx$

Optimal. Leaf size=31

$$\frac{8\cos^9(a+bx)}{9b}-\frac{8\cos^7(a+bx)}{7b}$$

```
[Out] (-8*Cos[a + b*x]^7)/(7*b) + (8*Cos[a + b*x]^9)/(9*b)
```

Rubi [A] time = 0.0560466, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4287, 2565, 14}

$$\frac{8\cos^9(a+bx)}{9b}-\frac{8\cos^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (-8*Cos[a + b*x]^7)/(7*b) + (8*Cos[a + b*x]^9)/(9*b)
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \cos^3(a+bx)\sin^3(2a+2bx)\,dx &= 8\int \cos^6(a+bx)\sin^3(a+bx)\,dx\\
&= -\frac{8\,\text{Subst}\left(\int x^6\left(1-x^2\right)\,dx, x, \cos(a+bx)\right)}{b}\\
&= -\frac{8\,\text{Subst}\left(\int \left(x^6-x^8\right)\,dx, x, \cos(a+bx)\right)}{b}\\
&= -\frac{8\cos^7(a+bx)}{7b}+\frac{8\cos^9(a+bx)}{9b}
\end{aligned}$$

Mathematica [A] time = 0.135808, size = 27, normalized size = 0.87

$$\frac{4\cos^7(a+bx)(7\cos(2(a+bx))-11)}{63b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (4*Cos[a + b*x]^7*(-11 + 7*Cos[2*(a + b*x)]))/(63*b)
```

Maple [A] time = 0.017, size = 55, normalized size = 1.8

$$-\frac{3\cos(bx+a)}{16b}-\frac{\cos(3bx+3a)}{12b}+\frac{3\cos(7bx+7a)}{224b}+\frac{\cos(9bx+9a)}{288b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(2*b*x+2*a)^3,x)
```

```
[Out] -3/16*cos(b*x+a)/b-1/12*cos(3*b*x+3*a)/b+3/224*cos(7*b*x+7*a)/b+1/288*cos(9
*b*x+9*a)/b
```

Maxima [A] time = 1.10552, size = 63, normalized size = 2.03

$$\frac{7\cos(9bx+9a)+27\cos(7bx+7a)-168\cos(3bx+3a)-378\cos(bx+a)}{2016b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 1/2016*(7*cos(9*b*x + 9*a) + 27*cos(7*b*x + 7*a) - 168*cos(3*b*x + 3*a) - 3
78*cos(b*x + a))/b
```

Fricas [A] time = 0.498608, size = 62, normalized size = 2.

$$\frac{8\left(7\cos(bx+a)^9-9\cos(bx+a)^7\right)}{63b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] 8/63*(7*cos(b*x + a)^9 - 9*cos(b*x + a)^7)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29953, size = 73, normalized size = 2.35

$$\frac{\cos(9\,bx+9\,a)}{288\,b}+\frac{3\,\cos(7\,bx+7\,a)}{224\,b}-\frac{\cos(3\,bx+3\,a)}{12\,b}-\frac{3\,\cos(bx+a)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] 1/288*cos(9*b*x + 9*a)/b + 3/224*cos(7*b*x + 7*a)/b - 1/12*cos(3*b*x + 3*a)
/b - 3/16*cos(b*x + a)/b
```

3.154 $\int \cos^3(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=46

$$\frac{4\sin^7(a+bx)}{7b}-\frac{8\sin^5(a+bx)}{5b}+\frac{4\sin^3(a+bx)}{3b}$$

```
[Out] (4*Sin[a + b*x]^3)/(3*b) - (8*Sin[a + b*x]^5)/(5*b) + (4*Sin[a + b*x]^7)/(7
*b)
```

Rubi [A] time = 0.0603713, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4287, 2564, 270}

$$\frac{4\sin^7(a+bx)}{7b}-\frac{8\sin^5(a+bx)}{5b}+\frac{4\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]

[Out] (4*Sin[a + b*x]^3)/(3*b) - (8*Sin[a + b*x]^5)/(5*b) + (4*Sin[a + b*x]^7)/(7
*b)
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\sin^2(2a+2bx)\,dx &= 4\int \cos^5(a+bx)\sin^2(a+bx)\,dx\\ &= \frac{4\,\text{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{4\,\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{4\sin^3(a+bx)}{3b}-\frac{8\sin^5(a+bx)}{5b}+\frac{4\sin^7(a+bx)}{7b}\end{aligned}$$

Mathematica [A] time = 0.0918017, size = 37, normalized size = 0.8

$$\frac{\sin^3(a+bx)(108\cos(2(a+bx))+15\cos(4(a+bx))+157)}{210b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] ((157 + 108*Cos[2*(a + b*x)] + 15*Cos[4*(a + b*x)])*Sin[a + b*x]^3)/(210*b)
```

Maple [A] time = 0.026, size = 55, normalized size = 1.2

$$\frac{5\sin(bx+a)}{16b}-\frac{\sin(3bx+3a)}{48b}-\frac{3\sin(5bx+5a)}{80b}-\frac{\sin(7bx+7a)}{112b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(2*b*x+2*a)^2,x)
```

```
[Out] 5/16*sin(b*x+a)/b-1/48*sin(3*b*x+3*a)/b-3/80/b*sin(5*b*x+5*a)-1/112/b*sin(7
*b*x+7*a)
```

Maxima [A] time = 1.09732, size = 63, normalized size = 1.37

$$-\frac{15\sin(7bx+7a)+63\sin(5bx+5a)+35\sin(3bx+3a)-525\sin(bx+a)}{1680b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] -1/1680*(15*sin(7*b*x + 7*a) + 63*sin(5*b*x + 5*a) + 35*sin(3*b*x + 3*a) -
525*sin(b*x + a))/b
```

Fricas [A] time = 0.480916, size = 115, normalized size = 2.5

$$-\frac{4\left(15\cos(bx+a)^6-3\cos(bx+a)^4-4\cos(bx+a)^2-8\right)\sin(bx+a)}{105b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] -4/105*(15*cos(b*x + a)^6 - 3*cos(b*x + a)^4 - 4*cos(b*x + a)^2 - 8)*sin(b*
x + a)/b
```

Sympy [A] time = 77.6936, size = 202, normalized size = 4.39

$$\begin{cases} \frac{38\sin^3(a+bx)\sin^2(2a+2bx)}{105b}+\frac{32\sin^3(a+bx)\cos^2(2a+2bx)}{105b}+\frac{8\sin^2(a+bx)\sin(2a+2bx)\cos(a+bx)\cos(2a+2bx)}{35b}+\frac{11\sin(a+bx)\sin^2(2a+2bx)\cos^2(a+ \\ x\sin^2(2a)\cos^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**2,x)
```

```
[Out] Piecewise((38*sin(a + b*x)**3*sin(2*a + 2*b*x)**2/(105*b) + 32*sin(a + b*x)
**3*cos(2*a + 2*b*x)**2/(105*b) + 8*sin(a + b*x)**2*sin(2*a + 2*b*x)*cos(a
+ b*x)*cos(2*a + 2*b*x)/(35*b) + 11*sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(a
+ b*x)**2/(35*b) + 24*sin(a + b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/(35*
b) - 12*sin(2*a + 2*b*x)*cos(a + b*x)**3*cos(2*a + 2*b*x)/(35*b), Ne(b, 0))
, (x*sin(2*a)**2*cos(a)**3, True))
```

Giac [A] time = 1.22179, size = 73, normalized size = 1.59

$$-\frac{\sin\left(7\,bx+7\,a\right)}{112\,b}-\frac{3\,\sin\left(5\,bx+5\,a\right)}{80\,b}-\frac{\sin\left(3\,bx+3\,a\right)}{48\,b}+\frac{5\,\sin\left(bx+a\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] -1/112*sin(7*b*x + 7*a)/b - 3/80*sin(5*b*x + 5*a)/b - 1/48*sin(3*b*x + 3*a)
/b + 5/16*sin(b*x + a)/b
```

3.155 $\int \cos^3(a+bx)\sin(2a+2bx)\,dx$

Optimal. Leaf size=15

$$-\frac{2\cos^5(a+bx)}{5b}$$

```
[Out] (-2*Cos[a + b*x]^5)/(5*b)
```

Rubi [A] time = 0.0329056, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4287, 2565, 30}

$$-\frac{2\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x],x]

[Out] (-2*Cos[a + b*x]^5)/(5*b)
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\sin(2a+2bx)\,dx &= 2\int \cos^4(a+bx)\sin(a+bx)\,dx\\ &= -\frac{2\,\text{Subst}\left(\int x^4\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{2\cos^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.0079298, size = 15, normalized size = 1.

$$-\frac{2\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x],x]
```

```
[Out] (-2*Cos[a + b*x]^5)/(5*b)
```

Maple [B] time = 0.014, size = 41, normalized size = 2.7

$$-\frac{\cos(bx+a)}{4b}-\frac{\cos(3bx+3a)}{8b}-\frac{\cos(5bx+5a)}{40b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(2*b*x+2*a),x)
```

```
[Out] -1/4*cos(b*x+a)/b-1/8*cos(3*b*x+3*a)/b-1/40*cos(5*b*x+5*a)/b
```

Maxima [B] time = 1.12088, size = 46, normalized size = 3.07

$$-\frac{\cos(5bx+5a)+5\cos(3bx+3a)+10\cos(bx+a)}{40b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -1/40*(cos(5*b*x + 5*a) + 5*cos(3*b*x + 3*a) + 10*cos(b*x + a))/b
```

Fricas [A] time = 0.491357, size = 31, normalized size = 2.07

$$-\frac{2\cos(bx+a)^5}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] -2/5*cos(b*x + a)^5/b
```

Sympy [A] time = 19.2011, size = 117, normalized size = 7.8

$$\begin{cases} -\frac{2\sin^3(a+bx)\sin(2a+2bx)}{5b}-\frac{4\sin^2(a+bx)\cos(a+bx)\cos(2a+2bx)}{5b}+\frac{\sin(a+bx)\sin(2a+2bx)\cos^2(a+bx)}{5b}-\frac{2\cos^3(a+bx)\cos(2a+2bx)}{5b} & \text{for } b \neq \\ x\sin(2a)\cos^3(a) & \text{other} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(2*b*x+2*a),x)
```

```
[Out] Piecewise((-2*sin(a + b*x)**3*sin(2*a + 2*b*x)/(5*b) - 4*sin(a + b*x)**2*co
s(a + b*x)*cos(2*a + 2*b*x)/(5*b) + sin(a + b*x)*sin(2*a + 2*b*x)*cos(a + b
*x)**2/(5*b) - 2*cos(a + b*x)**3*cos(2*a + 2*b*x)/(5*b), Ne(b, 0)), (x*sin(
2*a)*cos(a)**3, True))
```

Giac [B] time = 1.2158, size = 54, normalized size = 3.6

$$-\frac{\cos(5\,bx+5\,a)}{40\,b}-\frac{\cos(3\,bx+3\,a)}{8\,b}-\frac{\cos(bx+a)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] -1/40*cos(5*b*x + 5*a)/b - 1/8*cos(3*b*x + 3*a)/b - 1/4*cos(b*x + a)/b
```

3.156 $\int \cos^3(a+bx)\csc(2a+2bx)\,dx$

Optimal. Leaf size=28

$$\frac{\cos(a+bx)}{2b}-\frac{\tanh^{-1}(\cos(a+bx))}{2b}$$

```
[Out] -ArcTanh[Cos[a + b*x]]/(2*b) + Cos[a + b*x]/(2*b)
```

Rubi [A] time = 0.0352463, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4287, 2592, 321, 206}

$$\frac{\cos(a+bx)}{2b}-\frac{\tanh^{-1}(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Csc[2*a + 2*b*x],x]
```

```
[Out] -ArcTanh[Cos[a + b*x]]/(2*b) + Cos[a + b*x]/(2*b)
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^3(a+bx)\csc(2a+2bx)\,dx = \frac{1}{2}\int \cos(a+bx)\cot(a+bx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos(a+bx)\right)}{2b}$$
$$= \frac{\cos(a+bx)}{2b} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(a+bx)\right)}{2b}$$
$$= -\frac{\tanh^{-1}(\cos(a+bx))}{2b} + \frac{\cos(a+bx)}{2b}$$

Mathematica [A] time = 0.0192191, size = 46, normalized size = 1.64

$$\frac{1}{2}\left(\frac{\cos(a+bx)}{b} + \frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b} - \frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Csc[2*a + 2*b*x],x]
```

```
[Out] (Cos[a + b*x]/b - Log[Cos[(a + b*x)/2]]/b + Log[Sin[(a + b*x)/2]]/b)/2
```

Maple [A] time = 0.022, size = 34, normalized size = 1.2

$$\frac{\cos(bx+a)}{2b} + \frac{\ln(\csc(bx+a) - \cot(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a),x)
```

```
[Out] 1/2*cos(b*x+a)/b+1/2/b*ln(csc(b*x+a)-cot(b*x+a))
```

Maxima [B] time = 1.12094, size = 124, normalized size = 4.43

$$\frac{2\cos(bx+a) - \log\left(\cos(bx)^2 + 2\cos(bx)\cos(a) + \cos(a)^2 + \sin(bx)^2 - 2\sin(bx)\sin(a) + \sin(a)^2\right) + \log\left(\cos(bx)^2 -\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] 1/4*(2*cos(b*x + a) - log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b
*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + log(cos(b*x)^2 - 2*cos(b*x)*cos(a)
+ cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b
```

Fricas [A] time = 0.500098, size = 115, normalized size = 4.11

$$\frac{2\cos(bx+a) - \log\left(\frac{1}{2}\cos(bx+a) + \frac{1}{2}\right) + \log\left(-\frac{1}{2}\cos(bx+a) + \frac{1}{2}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*cos(b*x + a) - log(1/2*cos(b*x + a) + 1/2) + log(-1/2*cos(b*x + a) +
 1/2))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24362, size = 77, normalized size = 2.75

$$-\frac{\frac{4}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1}-\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] -1/4*(4/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1) - log(abs(-cos(b*x + a)
 + 1)/abs(cos(b*x + a) + 1)))/b
```

3.157 $\int \cos^3(a+bx)\csc^2(2a+2bx)\,dx$

Optimal. Leaf size=13

$$-\frac{\csc(a+bx)}{4b}$$

```
[Out] -Csc[a + b*x]/(4*b)
```

Rubi [A] time = 0.0345041, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4287, 2606, 8}

$$-\frac{\csc(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] -Csc[a + b*x]/(4*b)
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\csc^2(2a+2bx)\,dx &= \frac{1}{4}\int \cot(a+bx)\csc(a+bx)\,dx\\ &= -\frac{\text{Subst}\left(\int 1\,dx, x, \csc(a+bx)\right)}{4b}\\ &= -\frac{\csc(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0122181, size = 13, normalized size = 1.

$$-\frac{\csc(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^2,x]

[Out] -Csc[a + b*x]/(4*b)
```

Maple [A] time = 0.017, size = 14, normalized size = 1.1

$$-\frac{1}{4\,b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^2,x)

[Out] -1/4/b/sin(b*x+a)
```

Maxima [B] time = 1.06123, size = 113, normalized size = 8.69

$$-\frac{\cos\left(bx+a\right)\sin\left(2\,bx+2\,a\right)-\cos\left(2\,bx+2\,a\right)\sin\left(bx+a\right)+\sin\left(bx+a\right)}{2\left(b\cos\left(2\,bx+2\,a\right)^{2}+b\sin\left(2\,bx+2\,a\right)^{2}-2\,b\cos\left(2\,bx+2\,a\right)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^2,x, algorithm="maxima")

[Out] -1/2*(cos(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)*sin(b*x + a) + sin(b
*x + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 - 2*b*cos(2*b*x + 2*a
) + b)
```

Fricas [A] time = 0.458795, size = 31, normalized size = 2.38

$$-\frac{1}{4\,b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^2,x, algorithm="fricas")

[Out] -1/4/(b*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**2,x)

[Out] Timed out
```

Giac [A] time = 1.23168, size = 18, normalized size = 1.38

$$-\frac{1}{4 b \sin(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] -1/4/(b*sin(b*x + a))
```

3.158 $\int \cos^3(a+bx)\csc^3(2a+2bx)\,dx$

Optimal. Leaf size=34

$$-\frac{\tanh^{-1}(\cos(a+bx))}{16b}-\frac{\cot(a+bx)\csc(a+bx)}{16b}$$

```
[Out] -ArcTanh[Cos[a + b*x]]/(16*b) - (Cot[a + b*x]*Csc[a + b*x])/(16*b)
```

Rubi [A] time = 0.040758, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4287, 3768, 3770}

$$-\frac{\tanh^{-1}(\cos(a+bx))}{16b}-\frac{\cot(a+bx)\csc(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^3,x]

[Out] -ArcTanh[Cos[a + b*x]]/(16*b) - (Cot[a + b*x]*Csc[a + b*x])/(16*b)
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\csc^3(2a+2bx)\,dx &= \frac{1}{8}\int \csc^3(a+bx)\,dx\\ &= -\frac{\cot(a+bx)\csc(a+bx)}{16b}+\frac{1}{16}\int \csc(a+bx)\,dx\\ &= -\frac{\tanh^{-1}(\cos(a+bx))}{16b}-\frac{\cot(a+bx)\csc(a+bx)}{16b}\end{aligned}$$

Mathematica [B] time = 0.0151075, size = 79, normalized size = 2.32

$$\frac{1}{8}\left(-\frac{\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b}+\frac{\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{2b}-\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{2b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] (-Csc[(a + b*x)/2]^2/(8*b) - Log[Cos[(a + b*x)/2]]/(2*b) + Log[Sin[(a + b*x
)/2]]/(2*b) + Sec[(a + b*x)/2]^2/(8*b))/8
```

Maple [A] time = 0.048, size = 40, normalized size = 1.2

$$-\frac{\cot(bx+a)\csc(bx+a)}{16b}+\frac{\ln(\csc(bx+a)-\cot(bx+a))}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^3,x)
```

```
[Out] -1/16*cot(b*x+a)*csc(b*x+a)/b+1/16/b*ln(csc(b*x+a)-cot(b*x+a))
```

Maxima [B] time = 1.20577, size = 753, normalized size = 22.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(4*(cos(3*b*x + 3*a) + cos(b*x + a))*cos(4*b*x + 4*a) - 4*(2*cos(2*b*x
 + 2*a) - 1)*cos(3*b*x + 3*a) - 8*cos(2*b*x + 2*a)*cos(b*x + a) + (2*(2*cos
(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*
a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b
*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) +
cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (2*(2*cos(2*b*x + 2
*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin
(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^
2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 +
 sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*(sin(3*b*x + 3*a) + sin(b*x
 + a))*sin(4*b*x + 4*a) - 8*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) - 8*sin(2*b*x
 + 2*a)*sin(b*x + a) + 4*cos(b*x + a))/(b*cos(4*b*x + 4*a)^2 + 4*b*cos(2*b*
x + 2*a)^2 + b*sin(4*b*x + 4*a)^2 - 4*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) +
 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(2*b*x + 2*a) - b)*cos(4*b*x + 4*a) - 4
*b*cos(2*b*x + 2*a) + b)
```

Fricas [B] time = 0.493301, size = 203, normalized size = 5.97

$$-\frac{\left(\cos(bx+a)^2-1\right)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-\left(\cos(bx+a)^2-1\right)\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-2\cos(bx+a)}{32\left(b\cos(bx+a)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*((cos(b*x + a)^2 - 1)*log(1/2*cos(b*x + a) + 1/2) - (cos(b*x + a)^2 -
 1)*log(-1/2*cos(b*x + a) + 1/2) - 2*cos(b*x + a))/(b*cos(b*x + a)^2 - b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31529, size = 124, normalized size = 3.65

$$-\frac{\frac{\left(\frac{2(\cos(bx+a)-1)}{\cos(bx+a)+1}-1\right)(\cos(bx+a)+1)}{\cos(bx+a)-1}+\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-2\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] -1/64*((2*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)*(cos(b*x + a) + 1)/(co
s(b*x + a) - 1) + (cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 2*log(abs(-cos(b*
x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

3.159 $\int \cos^3(a+bx)\csc^4(2a+2bx)\,dx$

Optimal. Leaf size=43

$$-\frac{\csc^3(a+bx)}{48b}-\frac{\csc(a+bx)}{16b}+\frac{\tanh^{-1}(\sin(a+bx))}{16b}$$

```
[Out] ArcTanh[Sin[a + b*x]]/(16*b) - Csc[a + b*x]/(16*b) - Csc[a + b*x]^3/(48*b)
```

Rubi [A] time = 0.0513725, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4287, 2621, 302, 207}

$$-\frac{\csc^3(a+bx)}{48b}-\frac{\csc(a+bx)}{16b}+\frac{\tanh^{-1}(\sin(a+bx))}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(16*b) - Csc[a + b*x]/(16*b) - Csc[a + b*x]^3/(48*b)
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^3(a+bx)\csc^4(2a+2bx)\,dx = \frac{1}{16}\int \csc^4(a+bx)\sec(a+bx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{16b}$$
$$= -\frac{\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \csc(a+bx)\right)}{16b}$$
$$= -\frac{\csc(a+bx)}{16b} - \frac{\csc^3(a+bx)}{48b} - \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{16b}$$
$$= \frac{\tanh^{-1}(\sin(a+bx))}{16b} - \frac{\csc(a+bx)}{16b} - \frac{\csc^3(a+bx)}{48b}$$

Mathematica [C] time = 0.0181735, size = 31, normalized size = 0.72

$$-\frac{\csc^3(a+bx)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\sin^2(a+bx)\right)}{48b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] -(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[a + b*x]^2])/(48*b)
```

Maple [A] time = 0.027, size = 47, normalized size = 1.1

$$-\frac{1}{48\,b\,(\sin(bx+a))^3} - \frac{1}{16\,b\sin(bx+a)} + \frac{\ln(\sec(bx+a)+\tan(bx+a))}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^4,x)
```

```
[Out] -1/48/b/sin(b*x+a)^3-1/16/b/sin(b*x+a)+1/16/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [B] time = 2.41644, size = 1126, normalized size = 26.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/96*(4*(3*sin(5*b*x + 5*a) - 10*sin(3*b*x + 3*a) + 3*sin(b*x + a))*cos(6*b
*x + 6*a) + 36*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(5*b*x + 5*a) + 12*
(10*sin(3*b*x + 3*a) - 3*sin(b*x + a))*cos(4*b*x + 4*a) + 3*(2*(3*cos(4*b*x
 + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) - cos(6*b*x + 6*a)^2 + 6
*(3*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 9*cos(4*b*x + 4*a)^2 - 9*cos(2
*b*x + 2*a)^2 + 6*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(6*b*x + 6*a) -
sin(6*b*x + 6*a)^2 - 9*sin(4*b*x + 4*a)^2 + 18*sin(4*b*x + 4*a)*sin(2*b*x +
```

```
 2*a) - 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2*a) - 1)*log((cos(b*x + 2*a)^
2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a
)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a)
 + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(3*cos(5*b*x
 + 5*a) - 10*cos(3*b*x + 3*a) + 3*cos(b*x + a))*sin(6*b*x + 6*a) - 12*(3*co
s(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*sin(5*b*x + 5*a) - 12*(10*cos(3*b*
x + 3*a) - 3*cos(b*x + a))*sin(4*b*x + 4*a) - 40*(3*cos(2*b*x + 2*a) - 1)*s
in(3*b*x + 3*a) + 120*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) - 36*cos(b*x + a)*s
in(2*b*x + 2*a) + 36*cos(2*b*x + 2*a)*sin(b*x + a) - 12*sin(b*x + a))/(b*co
s(6*b*x + 6*a)^2 + 9*b*cos(4*b*x + 4*a)^2 + 9*b*cos(2*b*x + 2*a)^2 + b*sin(
6*b*x + 6*a)^2 + 9*b*sin(4*b*x + 4*a)^2 - 18*b*sin(4*b*x + 4*a)*sin(2*b*x +
 2*a) + 9*b*sin(2*b*x + 2*a)^2 - 2*(3*b*cos(4*b*x + 4*a) - 3*b*cos(2*b*x +
2*a) + b)*cos(6*b*x + 6*a) - 6*(3*b*cos(2*b*x + 2*a) - b)*cos(4*b*x + 4*a)
- 6*b*cos(2*b*x + 2*a) - 6*(b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*sin(6*
b*x + 6*a) + b)
```

Fricas [B] time = 0.501965, size = 254, normalized size = 5.91

$$\frac{3\left(\cos\left(bx+a\right)^2-1\right)\log\left(\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-3\left(\cos\left(bx+a\right)^2-1\right)\log\left(-\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-6\cos}{96\left(b\cos\left(bx+a\right)^2-b\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] 1/96*(3*(cos(b*x + a)^2 - 1)*log(sin(b*x + a) + 1)*sin(b*x + a) - 3*(cos(b*
x + a)^2 - 1)*log(-sin(b*x + a) + 1)*sin(b*x + a) - 6*cos(b*x + a)^2 + 8)/(
(b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23286, size = 70, normalized size = 1.63

$$-\frac{\frac{2\left(3\sin\left(bx+a\right)^2+1\right)}{\sin\left(bx+a\right)^3}-3\log\left(\left|\sin\left(bx+a\right)+1\right|\right)+3\log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] -1/96*(2*(3*sin(b*x + a)^2 + 1)/sin(b*x + a)^3 - 3*log(abs(sin(b*x + a) + 1
)) + 3*log(abs(sin(b*x + a) - 1)))/b
```

3.160 $\int \cos^3(a+bx)\csc^5(2a+2bx)\,dx$

Optimal. Leaf size=70

$$\frac{15\sec(a+bx)}{256b}-\frac{15\tanh^{-1}(\cos(a+bx))}{256b}-\frac{\csc^4(a+bx)\sec(a+bx)}{128b}-\frac{5\csc^2(a+bx)\sec(a+bx)}{256b}$$

```
[Out] (-15*ArcTanh[Cos[a + b*x]])/(256*b) + (15*Sec[a + b*x])/(256*b) - (5*Csc[a
+ b*x]^2*Sec[a + b*x])/(256*b) - (Csc[a + b*x]^4*Sec[a + b*x])/(128*b)
```

Rubi [A] time = 0.069867, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4287, 2622, 288, 321, 207}

$$\frac{15\sec(a+bx)}{256b}-\frac{15\tanh^{-1}(\cos(a+bx))}{256b}-\frac{\csc^4(a+bx)\sec(a+bx)}{128b}-\frac{5\csc^2(a+bx)\sec(a+bx)}{256b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^5,x]
```

```
[Out] (-15*ArcTanh[Cos[a + b*x]])/(256*b) + (15*Sec[a + b*x])/(256*b) - (5*Csc[a
+ b*x]^2*Sec[a + b*x])/(256*b) - (Csc[a + b*x]^4*Sec[a + b*x])/(128*b)
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^3(a+bx)\csc^5(2a+2bx)\,dx = \frac{1}{32}\int \csc^5(a+bx)\sec^2(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^3}\,dx, x, \sec(a+bx)\right)}{32b}$$
$$= -\frac{\csc^4(a+bx)\sec(a+bx)}{128b} + \frac{5\,\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, \sec(a+bx)\right)}{128b}$$
$$= -\frac{5\csc^2(a+bx)\sec(a+bx)}{256b} - \frac{\csc^4(a+bx)\sec(a+bx)}{128b} + \frac{15\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \text{s}\right.}{256b}$$
$$= \frac{15\sec(a+bx)}{256b} - \frac{5\csc^2(a+bx)\sec(a+bx)}{256b} - \frac{\csc^4(a+bx)\sec(a+bx)}{128b} + \frac{15\,\text{Subst}\left(\right.}{}$$
$$= -\frac{15\tanh^{-1}(\cos(a+bx))}{256b} + \frac{15\sec(a+bx)}{256b} - \frac{5\csc^2(a+bx)\sec(a+bx)}{256b} - \frac{\csc^4(a+}{}$$

Mathematica [B] time = 0.338197, size = 195, normalized size = 2.79

$$-\frac{\csc^4\left(\frac{1}{2}(a+bx)\right)}{2048b} - \frac{7\csc^2\left(\frac{1}{2}(a+bx)\right)}{1024b} + \frac{\sec^4\left(\frac{1}{2}(a+bx)\right)}{2048b} + \frac{7\sec^2\left(\frac{1}{2}(a+bx)\right)}{1024b} + \frac{15\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{256b} - \frac{15\log\left(\text{co}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^5,x]
```

```
[Out] (-7*Csc[(a + b*x)/2]^2)/(1024*b) - Csc[(a + b*x)/2]^4/(2048*b) - (15*Log[Co
s[(a + b*x)/2]])/(256*b) + (15*Log[Sin[(a + b*x)/2]])/(256*b) + (7*Sec[(a +
 b*x)/2]^2)/(1024*b) + Sec[(a + b*x)/2]^4/(2048*b) + Sin[(a + b*x)/2]/(32*b
*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])) - Sin[(a + b*x)/2]/(32*b*(Cos[(a +
b*x)/2] + Sin[(a + b*x)/2]))
```

Maple [A] time = 0.03, size = 78, normalized size = 1.1

$$-\frac{1}{128\,b\left(\sin(bx+a)\right)^4\cos(bx+a)} - \frac{5}{256\,b\left(\sin(bx+a)\right)^2\cos(bx+a)} + \frac{15}{256\,b\cos(bx+a)} + \frac{15\ln(\csc(bx+a)-\cot(b}{256\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^5,x)
```

```
[Out] -1/128/b/sin(b*x+a)^4/cos(b*x+a)-5/256/b/sin(b*x+a)^2/cos(b*x+a)+15/256/b/c
os(b*x+a)+15/256/b*ln(csc(b*x+a)-cot(b*x+a))
```

Maxima [B] time = 1.49514, size = 3020, normalized size = 43.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] 1/512*(4*(15*cos(9*b*x + 9*a) - 40*cos(7*b*x + 7*a) + 18*cos(5*b*x + 5*a) -
 40*cos(3*b*x + 3*a) + 15*cos(b*x + a))*cos(10*b*x + 10*a) - 60*(3*cos(8*b*
x + 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) - 1
)*cos(9*b*x + 9*a) + 12*(40*cos(7*b*x + 7*a) - 18*cos(5*b*x + 5*a) + 40*cos
(3*b*x + 3*a) - 15*cos(b*x + a))*cos(8*b*x + 8*a) - 160*(2*cos(6*b*x + 6*a)
 + 2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(7*b*x + 7*a) + 8*(18*co
s(5*b*x + 5*a) - 40*cos(3*b*x + 3*a) + 15*cos(b*x + a))*cos(6*b*x + 6*a) +
72*(2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(5*b*x + 5*a) - 40*(8*c
os(3*b*x + 3*a) - 3*cos(b*x + a))*cos(4*b*x + 4*a) + 160*(3*cos(2*b*x + 2*a
) - 1)*cos(3*b*x + 3*a) - 180*cos(2*b*x + 2*a)*cos(b*x + a) + 15*(2*(3*cos(
8*b*x + 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a)
 - 1)*cos(10*b*x + 10*a) - cos(10*b*x + 10*a)^2 + 6*(2*cos(6*b*x + 6*a) + 2
*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - 9*cos(8*b*x
+ 8*a)^2 - 4*(2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a)
 - 4*cos(6*b*x + 6*a)^2 + 4*(3*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 4*c
os(4*b*x + 4*a)^2 - 9*cos(2*b*x + 2*a)^2 + 2*(3*sin(8*b*x + 8*a) - 2*sin(6*
b*x + 6*a) - 2*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) -
sin(10*b*x + 10*a)^2 + 6*(2*sin(6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - 3*sin(2
*b*x + 2*a))*sin(8*b*x + 8*a) - 9*sin(8*b*x + 8*a)^2 - 4*(2*sin(4*b*x + 4*a
) - 3*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 4*sin(6*b*x + 6*a)^2 - 4*sin(4*b
*x + 4*a)^2 + 12*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 9*sin(2*b*x + 2*a)^2 +
 6*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + si
n(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 15*(2*(3*cos(8*b*x + 8*a) - 2*co
s(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) - 1)*cos(10*b*x +
10*a) - cos(10*b*x + 10*a)^2 + 6*(2*cos(6*b*x + 6*a) + 2*cos(4*b*x + 4*a) -
 3*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - 9*cos(8*b*x + 8*a)^2 - 4*(2*cos
(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) - 4*cos(6*b*x + 6*
a)^2 + 4*(3*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 4*cos(4*b*x + 4*a)^2 -
 9*cos(2*b*x + 2*a)^2 + 2*(3*sin(8*b*x + 8*a) - 2*sin(6*b*x + 6*a) - 2*sin(
4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - sin(10*b*x + 10*a)^
2 + 6*(2*sin(6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(8*
b*x + 8*a) - 9*sin(8*b*x + 8*a)^2 - 4*(2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2
*a))*sin(6*b*x + 6*a) - 4*sin(6*b*x + 6*a)^2 - 4*sin(4*b*x + 4*a)^2 + 12*si
n(4*b*x + 4*a)*sin(2*b*x + 2*a) - 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2*a)
 - 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*
x)*sin(a) + sin(a)^2) + 4*(15*sin(9*b*x + 9*a) - 40*sin(7*b*x + 7*a) + 18*s
in(5*b*x + 5*a) - 40*sin(3*b*x + 3*a) + 15*sin(b*x + a))*sin(10*b*x + 10*a)
 - 60*(3*sin(8*b*x + 8*a) - 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) + 3*sin
(2*b*x + 2*a))*sin(9*b*x + 9*a) + 12*(40*sin(7*b*x + 7*a) - 18*sin(5*b*x +
5*a) + 40*sin(3*b*x + 3*a) - 15*sin(b*x + a))*sin(8*b*x + 8*a) - 160*(2*sin
(6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(7*b*x + 7*a) +
 8*(18*sin(5*b*x + 5*a) - 40*sin(3*b*x + 3*a) + 15*sin(b*x + a))*sin(6*b*x
+ 6*a) + 72*(2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(5*b*x + 5*a) - 40
*(8*sin(3*b*x + 3*a) - 3*sin(b*x + a))*sin(4*b*x + 4*a) + 480*sin(3*b*x + 3
*a)*sin(2*b*x + 2*a) - 180*sin(2*b*x + 2*a)*sin(b*x + a) + 60*cos(b*x + a))
/(b*cos(10*b*x + 10*a)^2 + 9*b*cos(8*b*x + 8*a)^2 + 4*b*cos(6*b*x + 6*a)^2
+ 4*b*cos(4*b*x + 4*a)^2 + 9*b*cos(2*b*x + 2*a)^2 + b*sin(10*b*x + 10*a)^2
+ 9*b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 + 4*b*sin(4*b*x + 4*a)^2
- 12*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 9*b*sin(2*b*x + 2*a)^2 - 2*(3*b*
cos(8*b*x + 8*a) - 2*b*cos(6*b*x + 6*a) - 2*b*cos(4*b*x + 4*a) + 3*b*cos(2*
b*x + 2*a) - b)*cos(10*b*x + 10*a) - 6*(2*b*cos(6*b*x + 6*a) + 2*b*cos(4*b*
```

```
x + 4*a) - 3*b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) + 4*(2*b*cos(4*b*x +
4*a) - 3*b*cos(2*b*x + 2*a) + b)*cos(6*b*x + 6*a) - 4*(3*b*cos(2*b*x + 2*a)
 - b)*cos(4*b*x + 4*a) - 6*b*cos(2*b*x + 2*a) - 2*(3*b*sin(8*b*x + 8*a) - 2
*b*sin(6*b*x + 6*a) - 2*b*sin(4*b*x + 4*a) + 3*b*sin(2*b*x + 2*a))*sin(10*b
*x + 10*a) - 6*(2*b*sin(6*b*x + 6*a) + 2*b*sin(4*b*x + 4*a) - 3*b*sin(2*b*x
 + 2*a))*sin(8*b*x + 8*a) + 4*(2*b*sin(4*b*x + 4*a) - 3*b*sin(2*b*x + 2*a))
*sin(6*b*x + 6*a) + b)
```

Fricas [B] time = 0.506712, size = 375, normalized size = 5.36

$$\frac{30\cos(bx+a)^4 - 50\cos(bx+a)^2 - 15\left(\cos(bx+a)^5 - 2\cos(bx+a)^3 + \cos(bx+a)\right)\log\left(\frac{1}{2}\cos(bx+a) + \frac{1}{2}\right) + 15\,(c\cdots}{512\left(b\cos(bx+a)^5 - 2\,b\cos(bx+a)^3 + b\cos(bx\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] 1/512*(30*cos(b*x + a)^4 - 50*cos(b*x + a)^2 - 15*(cos(b*x + a)^5 - 2*cos(b
*x + a)^3 + cos(b*x + a))*log(1/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^5
- 2*cos(b*x + a)^3 + cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2) + 16)/(b*co
s(b*x + a)^5 - 2*b*cos(b*x + a)^3 + b*cos(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.3116, size = 220, normalized size = 3.14

$$\frac{\frac{\left(\frac{16(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{90(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} - 1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2} - \frac{16(\cos(bx+a)-1)}{\cos(bx+a)+1} + \frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{128}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1} + 60\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{2048\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] 1/2048*((16*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 90*(cos(b*x + a) - 1)^2
/(cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 - 16*(
cos(b*x + a) - 1)/(cos(b*x + a) + 1) + (cos(b*x + a) - 1)^2/(cos(b*x + a) +
 1)^2 + 128/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1) + 60*log(abs(-cos(b
*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

3.161 $\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=136

$$\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}+\frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{16b}-\frac{5\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{24b}-\frac{5\sin^{-1}(\cos(a+bx)-\text{si}}{32b}$$

```
[Out] (-5*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(32*b) - (5*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(32*b) + (5*Sin[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(16*b) - (5*Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(24*b) + (Sin[a
 + b*x]*Sin[2*a + 2*b*x]^(5/2))/(6*b)
```

Rubi [A] time = 0.0913343, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4301, 4302, 4306}

$$\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}+\frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{16b}-\frac{5\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{24b}-\frac{5\sin^{-1}(\cos(a+bx)-\text{si}}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (-5*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(32*b) - (5*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(32*b) + (5*Sin[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(16*b) - (5*Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(24*b) + (Sin[a
 + b*x]*Sin[2*a + 2*b*x]^(5/2))/(6*b)
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b} + \frac{5}{6}\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$= -\frac{5\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{24b} + \frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b} + \frac{5}{8}\int \cos(a+bx)\sqrt{\sin}$$
$$= \frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{16b} - \frac{5\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{24b} + \frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+}{6b}$$
$$= -\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{32b} - \frac{5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+}\right.}{32b}$$

Mathematica [A] time = 0.357032, size = 98, normalized size = 0.72

$$\frac{\frac{2}{3}\sqrt{\sin(2(a+bx))}(14\sin(a+bx)-3\sin(3(a+bx))-2\sin(5(a+bx)))-5\left(\sin^{-1}(\cos(a+bx)-\sin(a+bx))+\log\left(\sin(a\right.\right.}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (-5*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]) + (2*Sqrt[Sin[2*(a + b*x)]]*(14*Sin[a + b*x] - 3
*Sin[3*(a + b*x)] - 2*Sin[5*(a + b*x)]))/3)/(32*b)
```

Maple [B] time = 61.386, size = 199880296, normalized size = 1469708.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)\sin(2bx+2a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)*sin(2*b*x + 2*a)^(5/2), x)
```

Fricas [B] time = 0.572508, size = 806, normalized size = 5.93

$$-\frac{8\sqrt{2}\left(32\cos(bx+a)^4-12\cos(bx+a)^2-15\right)\sqrt{\cos(bx+a)\sin(bx+a)}\sin(bx+a)-30\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+}}{\cos(bx}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/384*(8*sqrt(2)*(32*cos(b*x + a)^4 - 12*cos(b*x + a)^2 - 15)*sqrt(cos(b*x
 + a)*sin(b*x + a))*sin(b*x + a) - 30*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*si
n(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos
(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) + 30*arctan(-(2*sqrt(2)*sqr
t(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) -
 sin(b*x + a))) - 15*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 -
 (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*si
n(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx + a) \sin(2\,bx + 2\,a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*sin(2*b*x + 2*a)^(5/2), x)
```

3.162 $\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=110

$$\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}-\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{16b}-\frac{3\sqrt{\sin(2a+2bx)}\cos(a+bx)}{8b}+\frac{3\log\left(\sin(a+bx)+\right.}{}$$

```
[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(16*b) + (3*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(16*b) - (3*Cos[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(8*b) + (Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(4*b)
```

Rubi [A] time = 0.0666506, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4301, 4302, 4305}

$$\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}-\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{16b}-\frac{3\sqrt{\sin(2a+2bx)}\cos(a+bx)}{8b}+\frac{3\log\left(\sin(a+bx)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(16*b) + (3*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(16*b) - (3*Cos[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(8*b) + (Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(4*b)
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b} + \frac{3}{4}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$
$$= -\frac{3\cos(a+bx)\sqrt{\sin(2a+2bx)}}{8b} + \frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b} + \frac{3}{8}\int \frac{\cos(a+bx}{\sqrt{\sin(2a+2}}$$
$$= -\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{16b} + \frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a}\right.}{16b}$$

Mathematica [A] time = 0.185962, size = 86, normalized size = 0.78

$$\frac{3\left(\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(a+bx)\right)-\sin^{-1}(\cos(a+bx)-\sin(a+bx))\right)-2\sqrt{\sin(2(a+bx))}(2\cos(a}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (3*(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]) - 2*(2*Cos[a + b*x] + Cos[3*(a + b*x)])*Sqrt[Sin
[2*(a + b*x)]])/(16*b)
```

Maple [B] time = 18.977, size = 74316378, normalized size = 675603.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)\sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)*sin(2*b*x + 2*a)^(3/2), x)
```

Fricas [B] time = 0.550913, size = 768, normalized size = 6.98

$$-\frac{8\sqrt{2}\left(4\cos(bx+a)^3-\cos(bx+a)\right)\sqrt{\cos(bx+a)\sin(bx+a)}-6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(8*sqrt(2)*(4*cos(b*x + a)^3 - cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*
x + a)) - 6*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a)
- sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x +
a)*sin(b*x + a) - 1)) + 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)
) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + 3*log(-32
*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(
b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a
)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)\sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*sin(2*b*x + 2*a)^(3/2), x)
```

3.163 $\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal. Leaf size=84

$$\frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{2b}-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{4b}$$

```
[Out] -ArcSin[Cos[a + b*x] - Sin[a + b*x]]/(4*b) - Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*a + 2*b*x]]]/(4*b) + (Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(
2*b)
```

Rubi [A] time = 0.0432653, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4301, 4306}

$$\frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{2b}-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -ArcSin[Cos[a + b*x] - Sin[a + b*x]]/(4*b) - Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*a + 2*b*x]]]/(4*b) + (Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(
2*b)
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{2b}+\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$$

$$= -\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b}-\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+\cdots}\right.}{4b}$$

Mathematica [A] time = 0.0969553, size = 70, normalized size = 0.83

$$-\frac{-2\sin(a+bx)\sqrt{\sin(2(a+bx))}+\sin^{-1}(\cos(a+bx)-\sin(a+bx))+\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(a+b\cdots\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x] + S
qrt[Sin[2*(a + b*x)]]] - 2*Sin[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/(4*b)
```

Maple [B] time = 1.177, size = 5339854, normalized size = 63569.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)\sqrt{\sin(2\,bx+2\,a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)*sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [B] time = 0.533313, size = 736, normalized size = 8.76

$$8\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\sin(bx+a) + 2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right) - 2\,\mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*sin(b*x + a) + 2*arctan(-(s
qrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(
b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1))
- 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin
(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + log(-32*cos(b*x + a)^4 + 4*sqrt
(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a
))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*si
n(b*x + a) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)\sqrt{\sin(2\,bx+2\,a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*sqrt(sin(2*b*x + 2*a)), x)
```

3.164 $\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal. Leaf size=58

$$\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{2b}$$

```
[Out] -ArcSin[Cos[a + b*x] - Sin[a + b*x]]/(2*b) + Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b)
```

Rubi [A] time = 0.0212411, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {4305}

$$\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -ArcSin[Cos[a + b*x] - Sin[a + b*x]]/(2*b) + Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b)
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx=-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{2b}+\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{2b}$$

Mathematica [A] time = 0.0470183, size = 52, normalized size = 0.9

$$\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(a+bx)\right)-\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x] + S
qrt[Sin[2*(a + b*x)]]])/(2*b)
```

Maple [B] time = 0.355, size = 622763, normalized size = 10737.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)}{\sqrt{\sin(2\,bx+2\,a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)/sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [B] time = 0.530315, size = 657, normalized size = 11.33

$$2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)-2\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}-\cos(bx+a)-\sin(b}{\cos(bx+a)-\sin(bx+a)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin
(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*si
n(b*x + a) - 1)) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - c
os(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - log(-32*cos(b*
x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a
) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 1
6*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)}{\sqrt{\sin(2\,bx+2\,a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/sqrt(sin(2*b*x + 2*a)), x)
```

3.165 $\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\, dx$

Optimal. Leaf size=24

$$-\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

```
[Out] -(Cos[a + b*x]/(b*Sqrt[Sin[2*a + 2*b*x]]))
```

Rubi [A] time = 0.0178053, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {4291}

$$-\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] -(Cos[a + b*x]/(b*Sqrt[Sin[2*a + 2*b*x]]))
```

Rule 4291

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> -Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*
m), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/
b, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\, dx = -\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A] time = 0.0199098, size = 23, normalized size = 0.96

$$-\frac{\cos(a+bx)}{b\sqrt{\sin(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] -(Cos[a + b*x]/(b*Sqrt[Sin[2*(a + b*x)]]))
```

Maple [B] time = 6.793, size = 55926091, normalized size = 2330253.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (bx + a)}{\sin (2\,bx + 2\,a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)/sin(2*b*x + 2*a)^(3/2), x)
```

Fricas [A] time = 0.490641, size = 108, normalized size = 4.5

$$-\frac{\sqrt{2}\sqrt{\cos (bx + a)\sin (bx + a)} + \sin (bx + a)}{2\,b\sin (bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) + sin(b*x + a))/(b*sin(b*x +
a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (bx + a)}{\sin (2\,bx + 2\,a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/sin(2*b*x + 2*a)^(3/2), x)
```

3.166 $\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=53

$$\frac{2\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

```
[Out] -Cos[a + b*x]/(3*b*Sin[2*a + 2*b*x]^(3/2)) + (2*Sin[a + b*x])/(3*b*Sqrt[Sin
[2*a + 2*b*x]])
```

Rubi [A] time = 0.0376203, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4303, 4292}

$$\frac{2\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] -Cos[a + b*x]/(3*b*Sin[2*a + 2*b*x]^(3/2)) + (2*Sin[a + b*x])/(3*b*Sqrt[Sin
[2*a + 2*b*x]])
```

Rule 4303

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx &= -\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{2}{3}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx\\ &= -\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{2\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [A] time = 0.102384, size = 43, normalized size = 0.81

$$\frac{\sqrt{\sin(2(a+bx))}\left(\frac{1}{4}\sec(a+bx)-\frac{1}{12}\cot(a+bx)\csc(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] ((-(Cot[a + b*x]*Csc[a + b*x])/12 + Sec[a + b*x]/4)*Sqrt[Sin[2*(a + b*x)]])
/b
```

Maple [C] time = 6.571, size = 194, normalized size = 3.7

$$-\frac{1}{24b}\sqrt{-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2-1\right)^{-1}}\left(\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2-1\right)\left(2\sqrt{\tan(1/2\,bx+a/2)+1}\sqrt{-2\tan(1/2\,bx+a/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] -1/24/b*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1
/2*a)^2-1)/tan(1/2*b*x+1/2*a)*(2*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b
*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a
)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)-tan(1/2*b*x+1/2*a)^4+1)/(tan(1/2
*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b
*x+1/2*a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)/sin(2*b*x + 2*a)^(5/2), x)
```

Fricas [A] time = 0.503775, size = 193, normalized size = 3.64

$$\frac{4\cos(bx+a)^3+\sqrt{2}\left(4\cos(bx+a)^2-3\right)\sqrt{\cos(bx+a)\sin(bx+a)}-4\cos(bx+a)}{12\left(b\cos(bx+a)^3-b\cos(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(4*cos(b*x + a)^3 + sqrt(2)*(4*cos(b*x + a)^2 - 3)*sqrt(cos(b*x + a)*s
in(b*x + a)) - 4*cos(b*x + a))/(b*cos(b*x + a)^3 - b*cos(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/sin(2*b*x + 2*a)^(5/2), x)
```

3.167 $\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=79

$$\frac{4\sin(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{8\cos(a+bx)}{15b\sqrt{\sin(2a+2bx)}}$$

```
[Out] -Cos[a + b*x]/(5*b*Sin[2*a + 2*b*x]^(5/2)) + (4*Sin[a + b*x])/(15*b*Sin[2*a
 + 2*b*x]^(3/2)) - (8*Cos[a + b*x])/(15*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.0589234, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4303, 4304, 4291}

$$\frac{4\sin(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{8\cos(a+bx)}{15b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
```


```
[Out] -Cos[a + b*x]/(5*b*Sin[2*a + 2*b*x]^(5/2)) + (4*Sin[a + b*x])/(15*b*Sin[2*a
 + 2*b*x]^(3/2)) - (8*Cos[a + b*x])/(15*b*Sqrt[Sin[2*a + 2*b*x]])
```


Rule 4303

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4304

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> -Simp[(Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dis
t[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4291

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> -Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*
m), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/
b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}dx = -\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{4}{5}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}dx$$
$$= -\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{4\sin(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{8}{15}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}dx$$
$$= -\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{4\sin(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)} - \frac{8\cos(a+bx)}{15b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A] time = 0.120671, size = 52, normalized size = 0.66

$$-\frac{\sqrt{\sin(2(a+bx))}\left(3\csc^3(a+bx)+27\csc(a+bx)-5\tan(a+bx)\sec(a+bx)\right)}{120b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] -(Sqrt[Sin[2*(a + b*x)]]*(27*Csc[a + b*x] + 3*Csc[a + b*x]^3 - 5*Sec[a + b*
x]*Tan[a + b*x]))/(120*b)
```

Maple [C] time = 89.464, size = 481, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a)^(7/2),x)
```

```
[Out] -1/160/b*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)/tan(1/2*b*x+1
/2*a)^3*(24*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*
x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/
2)*EllipticE((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^2
-12*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1/2*a)
+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*Ellip
ticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^2+(tan(1/
2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*tan(1/2*b*x+1/2*a)^6+12*(tan(1
/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)*tan(1/2*b*x+1/2*a)^4-(tan(1/2*b*x
+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*tan(1/2*b*x+1/2*a)^4-12*(tan(1/2*b*
x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)*tan(1/2*b*x+1/2*a)^2-(tan(1/2*b*x+1/2*
a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*tan(1/2*b*x+1/2*a)^2+(tan(1/2*b*x+1/2*a)
*(tan(1/2*b*x+1/2*a)^2-1))^(1/2))/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)}{\sin(2\,bx+2\,a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)/sin(2*b*x + 2*a)^(7/2), x)
```

Fricas [A] time = 0.514449, size = 269, normalized size = 3.41

$$-\frac{\sqrt{2}\left(32\cos\left(bx+a\right)^{4}-40\cos\left(bx+a\right)^{2}+5\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+32\left(\cos\left(bx+a\right)^{4}-\cos\left(bx+a\right)^{2}\right)\sin\left(bx+a\right)}{120\left(b\cos\left(bx+a\right)^{4}-b\cos\left(bx+a\right)^{2}\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/120*(sqrt(2)*(32*cos(b*x + a)^4 - 40*cos(b*x + a)^2 + 5)*sqrt(cos(b*x +
a)*sin(b*x + a)) + 32*(cos(b*x + a)^4 - cos(b*x + a)^2)*sin(b*x + a))/((b*c
os(b*x + a)^4 - b*cos(b*x + a)^2)*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(bx+a\right)}{\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/sin(2*b*x + 2*a)^(7/2), x)
```

3.168 $\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=105

$$\frac{6\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{16\sin(a+bx)}{35b\sqrt{\sin(2a+2bx)}}-\frac{8\cos(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

```
[Out] -Cos[a + b*x]/(7*b*Sin[2*a + 2*b*x]^(7/2)) + (6*Sin[a + b*x])/(35*b*Sin[2*a
 + 2*b*x]^(5/2)) - (8*Cos[a + b*x])/(35*b*Sin[2*a + 2*b*x]^(3/2)) + (16*Sin
[a + b*x])/(35*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.0799988, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4303, 4304, 4292}

$$\frac{6\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{16\sin(a+bx)}{35b\sqrt{\sin(2a+2bx)}}-\frac{8\cos(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] -Cos[a + b*x]/(7*b*Sin[2*a + 2*b*x]^(7/2)) + (6*Sin[a + b*x])/(35*b*Sin[2*a
 + 2*b*x]^(5/2)) - (8*Cos[a + b*x])/(35*b*Sin[2*a + 2*b*x]^(3/2)) + (16*Sin
[a + b*x])/(35*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4303

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4304

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> -Simp[(Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dis
t[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = -\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{6}{7}\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$$
$$= -\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{6\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{24}{35}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$$
$$= -\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{6\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{8\cos(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{16}{35}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$$
$$= -\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{6\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{8\cos(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{16\sin(a+bx)}{35b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A] time = 0.14338, size = 67, normalized size = 0.64

$$\frac{\sqrt{\sin(2(a+bx))}(-10\cos(2(a+bx)) - 4\cos(4(a+bx)) + 4\cos(6(a+bx)) + 5)\csc^4(a+bx)\sec^3(a+bx)}{560b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] ((5 - 10*Cos[2*(a + b*x)] - 4*Cos[4*(a + b*x)] + 4*Cos[6*(a + b*x)])*Csc[a
 + b*x]^4*Sec[a + b*x]^3*Sqrt[Sin[2*(a + b*x)]])/(560*b)
```

Maple [C] time = 183.381, size = 222, normalized size = 2.1

$$\frac{1}{2688\,b}\sqrt{-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2-1\right)^{-1}}\left(\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2-1\right)\left(3\,(\tan(1/2\,bx+a/2))^8+40\sqrt{\tan(1/2\,bx+a/}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a)^(9/2),x)
```

```
[Out] 1/2688/b*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+
1/2*a)^2-1)/tan(1/2*b*x+1/2*a)^3*(3*tan(1/2*b*x+1/2*a)^8+40*(tan(1/2*b*x+1/
2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*E
llipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^3-26*
tan(1/2*b*x+1/2*a)^6+26*tan(1/2*b*x+1/2*a)^2-3)/(tan(1/2*b*x+1/2*a)*(tan(1/
2*b*x+1/2*a)^2-1))^(1/2)/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)}{\sin(2\,bx+2\,a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)/sin(2*b*x + 2*a)^(9/2), x)
```

Fricas [A] time = 0.534884, size = 320, normalized size = 3.05

$$\frac{128\cos(bx+a)^7 - 256\cos(bx+a)^5 + 128\cos(bx+a)^3 + \sqrt{2}\left(128\cos(bx+a)^6 - 224\cos(bx+a)^4 + 84\cos(bx+\right.}{560\left(b\cos(bx+a)^7 - 2b\cos(bx+a)^5 + b\cos(bx+a)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/560*(128*cos(b*x + a)^7 - 256*cos(b*x + a)^5 + 128*cos(b*x + a)^3 + sqrt(
2)*(128*cos(b*x + a)^6 - 224*cos(b*x + a)^4 + 84*cos(b*x + a)^2 + 7)*sqrt(c
os(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*cos(b
*x + a)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)}{\sin(2\,bx+2\,a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/sin(2*b*x + 2*a)^(9/2), x)
```

3.169 $\int \cos^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=98

$$\frac{5\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{42b}+\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{14b}-\frac{5\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{42b}$$

```
[Out] (5*EllipticF[a - Pi/4 + b*x, 2])/(42*b) - (5*Cos[2*a + 2*b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(42*b) - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(5/2))/(14*b) + Sin[
2*a + 2*b*x]^(9/2)/(18*b)
```

Rubi [A] time = 0.0573313, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4297, 2635, 2641}

$$\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}+\frac{5F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{42b}-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{14b}-\frac{5\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{42b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (5*EllipticF[a - Pi/4 + b*x, 2])/(42*b) - (5*Cos[2*a + 2*b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(42*b) - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(5/2))/(14*b) + Sin[
2*a + 2*b*x]^(9/2)/(18*b)
```

Rule 4297

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e^2*(e*Cos[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 1
))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Cos[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ[
m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b} + \frac{1}{2}\int \sin^{\frac{7}{2}}(2a+2bx)\,dx$$
$$= -\frac{\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2bx)}{14b} + \frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b} + \frac{5}{14}\int \sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$= -\frac{5\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{42b} - \frac{\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2bx)}{14b} + \frac{\sin^{\frac{9}{2}}(2a+2}{18b}$$
$$= \frac{5F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{42b} - \frac{5\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{42b} - \frac{\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+}{14b}$$

Mathematica [A] time = 0.384852, size = 96, normalized size = 0.98

$$\frac{240\sqrt{\sin(2(a+bx))}\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)+70\sin(2(a+bx))-156\sin(4(a+bx))-35\sin(6(a+bx))+18\sin(8(a}{2016b\sqrt{\sin(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (240*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*(a + b*x)]] + 70*Sin[2*(a + b*
x)] - 156*Sin[4*(a + b*x)] - 35*Sin[6*(a + b*x)] + 18*Sin[8*(a + b*x)] + 7*
Sin[10*(a + b*x)])/(2016*b*Sqrt[Sin[2*(a + b*x)]])
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (\cos(bx+a))^2 (\sin(2bx+2a))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x)
```

```
[Out] int(cos(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)^2 \sin(2bx+2a)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^2*sin(2*b*x + 2*a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos(2bx+2a)^2\cos(bx+a)^2-\cos(bx+a)^2\right)\sin(2bx+2a)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(2*b*x + 2*a)^2*cos(b*x + a)^2 - cos(b*x + a)^2)*sin(2*b*x +
2*a)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.170 $\int \cos^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=69

$$\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}+\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{10b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{10b}$$

```
[Out] (3*EllipticE[a - Pi/4 + b*x, 2])/(10*b) - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x
]^(3/2))/(10*b) + Sin[2*a + 2*b*x]^(7/2)/(14*b)
```

Rubi [A] time = 0.0453035, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4297, 2635, 2639}

$$\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}+\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{10b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (3*EllipticE[a - Pi/4 + b*x, 2])/(10*b) - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x
]^(3/2))/(10*b) + Sin[2*a + 2*b*x]^(7/2)/(14*b)
```

Rule 4297

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e^2*(e*Cos[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 1
))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Cos[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ[
m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx&=\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}+\frac{1}{2}\int \sin^{\frac{5}{2}}(2a+2bx)\,dx\\&=-\frac{\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{10b}+\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}+\frac{3}{10}\int \sqrt{\sin(2a+2bx)}\,dx\\&=\frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{10b}-\frac{\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{10b}+\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}\end{aligned}$$

Mathematica [A] time = 0.208149, size = 66, normalized size = 0.96

$$\frac{\sqrt{\sin(2(a+bx))}(15\sin(2(a+bx))-14\sin(4(a+bx))-5\sin(6(a+bx)))+84E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{280b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (84*EllipticE[a - Pi/4 + b*x, 2] + Sqrt[Sin[2*(a + b*x)]]*(15*Sin[2*(a + b*
x)] - 14*Sin[4*(a + b*x)] - 5*Sin[6*(a + b*x)]))/(280*b)
```

Maple [B] time = 110.597, size = 336654858, normalized size = 4879055.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)^2 \sin(2\,bx+2\,a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^2*sin(2*b*x + 2*a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos(2\,bx+2\,a)^2\cos(bx+a)^2-\cos(bx+a)^2\right)\sqrt{\sin(2\,bx+2\,a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(2*b*x + 2*a)^2*cos(b*x + a)^2 - cos(b*x + a)^2)*sqrt(sin(2*b
*x + 2*a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.171 $\int \cos^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=69

$$\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{6b}+\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{6b}$$

```
[Out] EllipticF[a - Pi/4 + b*x, 2]/(6*b) - (Cos[2*a + 2*b*x]*Sqrt[Sin[2*a + 2*b*x
]])/(6*b) + Sin[2*a + 2*b*x]^(5/2)/(10*b)
```

Rubi [A] time = 0.045327, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4297, 2635, 2641}

$$\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}+\frac{F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{6b}-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] EllipticF[a - Pi/4 + b*x, 2]/(6*b) - (Cos[2*a + 2*b*x]*Sqrt[Sin[2*a + 2*b*x
]])/(6*b) + Sin[2*a + 2*b*x]^(5/2)/(10*b)
```

Rule 4297

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e^2*(e*Cos[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 1
))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Cos[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ[
m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx &= \frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}+\frac{1}{2}\int \sin^{\frac{3}{2}}(2a+2bx)\,dx\\
&= -\frac{\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{6b}+\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}+\frac{1}{6}\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx\\
&= \frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{6b}-\frac{\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{6b}+\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}
\end{aligned}$$

Mathematica [A] time = 0.347827, size = 76, normalized size = 1.1

$$\frac{20\sqrt{\sin(2(a+bx))}\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)+9\sin(2(a+bx))-10\sin(4(a+bx))-3\sin(6(a+bx))}{120b\sqrt{\sin(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (20*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*(a + b*x)]] + 9*Sin[2*(a + b*x)
] - 10*Sin[4*(a + b*x)] - 3*Sin[6*(a + b*x)])/(120*b*Sqrt[Sin[2*(a + b*x)]]
)
```

Maple [B] time = 33.345, size = 174364218, normalized size = 2527017.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)^2 \sin(2\,bx+2\,a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cos(bx+a)^2 \sin(2\,bx+2\,a)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)^2 \sin(2bx+2a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

3.172 $\int \cos^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal. Leaf size=40

$$\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}+\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}$$

```
[Out] EllipticE[a - Pi/4 + b*x, 2]/(2*b) + Sin[2*a + 2*b*x]^(3/2)/(6*b)
```

Rubi [A] time = 0.0345386, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4297, 2639}

$$\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}+\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]

[Out] EllipticE[a - Pi/4 + b*x, 2]/(2*b) + Sin[2*a + 2*b*x]^(3/2)/(6*b)
```

Rule 4297

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e^2*(e*Cos[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 1
))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Cos[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ[
m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx &= \frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}+\frac{1}{2}\int \sqrt{\sin(2a+2bx)}\,dx\\ &= \frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b}+\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}\end{aligned}$$

Mathematica [A] time = 0.0577439, size = 34, normalized size = 0.85

$$\frac{\sin^{\frac{3}{2}}(2(a+bx))+3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (3*EllipticE[a - Pi/4 + b*x, 2] + Sin[2*(a + b*x)]^(3/2))/(6*b)
```

Maple [B] time = 3.975, size = 24847123, normalized size = 621178.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)^2 \sqrt{\sin(2\,bx+2\,a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\cos(bx+a)^2 \sqrt{\sin(2\,bx+2\,a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)^2 \sqrt{\sin(2\,bx+2\,a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x)
```

3.173 $\int \frac{\cos^2(a+bx)}{\sqrt{\sin(2a+2bx)}} dx$

Optimal. Leaf size=40

$$\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{2b}+\frac{\sqrt{\sin(2a+2bx)}}{2b}$$

```
[Out] EllipticF[a - Pi/4 + b*x, 2]/(2*b) + Sqrt[Sin[2*a + 2*b*x]]/(2*b)
```

Rubi [A] time = 0.0350123, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4297, 2641}

$$\frac{\sqrt{\sin(2a+2bx)}}{2b}+\frac{F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] EllipticF[a - Pi/4 + b*x, 2]/(2*b) + Sqrt[Sin[2*a + 2*b*x]]/(2*b)
```

Rule 4297

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e^2*(e*Cos[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 1
))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Cos[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ[
m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx &= \frac{\sqrt{\sin(2a+2bx)}}{2b}+\frac{1}{2}\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= \frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b}+\frac{\sqrt{\sin(2a+2bx)}}{2b}\end{aligned}$$

Mathematica [A] time = 0.863694, size = 76, normalized size = 1.9

$$\frac{2\sqrt{\sin(2(a+bx))}-\frac{\sqrt{2}(\sin(a+bx)+\cos(a+bx))\text{EllipticF}\left(\sin^{-1}(\cos(a+bx)-\sin(a+bx)),\frac{1}{2}\right)}{\sqrt{\sin(2(a+bx))+1}}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (2*Sqrt[Sin[2*(a + b*x)]] - (Sqrt[2]*EllipticF[ArcSin[Cos[a + b*x] - Sin[a
+ b*x]], 1/2]*(Cos[a + b*x] + Sin[a + b*x]))/Sqrt[1 + Sin[2*(a + b*x)]])/(4
*b)
```

Maple [B] time = 8.96, size = 59635246, normalized size = 1490881.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^2}{\sqrt{\sin(2\,bx+2\,a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(bx+a)^2}{\sqrt{\sin(2\,bx+2\,a)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (bx+a)^2}{\sqrt{\sin (2\,bx+2\,a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

3.174 $\int \frac{\cos^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=46

$$-\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}-\frac{\cos^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

```
[Out] -EllipticE[a - Pi/4 + b*x, 2]/(2*b) - Cos[a + b*x]^2/(b*Sqrt[Sin[2*a + 2*b*
x]])
```

Rubi [A] time = 0.035921, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4295, 2639}

$$-\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}-\frac{\cos^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] -EllipticE[a - Pi/4 + b*x, 2]/(2*b) - Cos[a + b*x]^2/(b*Sqrt[Sin[2*a + 2*b*
x]])
```

Rule 4295

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(p + 1)), x] + Dist[(e^2*(m + 2*p + 2))/(4*g^2*(p + 1)), Int[(e*Cos[a + b*x
])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x]
 && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[p
, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m,
 2*p]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx &= -\frac{\cos^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}-\frac{1}{2}\int \sqrt{\sin(2a+2bx)}\,dx\\ &= -\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b}-\frac{\cos^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [A] time = 0.113925, size = 39, normalized size = 0.85

$$-\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)+\sqrt{\sin(2(a+bx))}\cot(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] -(EllipticE[a - Pi/4 + b*x, 2] + Cot[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/(2*b)
```

Maple [B] time = 9.786, size = 94273592, normalized size = 2049425.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\cos(bx+a)^2\sqrt{\sin(2\,bx+2\,a)}}{\cos(2\,bx+2\,a)^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-cos(b*x + a)^2*sqrt(sin(2*b*x + 2*a))/(cos(2*b*x + 2*a)^2 - 1), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(3/2), x)
```

3.175 $\int \frac{\cos^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=48

$$\frac{\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{6b}-\frac{\cos^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

```
[Out] EllipticF[a - Pi/4 + b*x, 2]/(6*b) - Cos[a + b*x]^2/(3*b*Sin[2*a + 2*b*x]^(
3/2))
```

Rubi [A] time = 0.0357419, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4295, 2641}

$$\frac{F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{6b}-\frac{\cos^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] EllipticF[a - Pi/4 + b*x, 2]/(6*b) - Cos[a + b*x]^2/(3*b*Sin[2*a + 2*b*x]^(
3/2))
```

Rule 4295

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(p + 1)), x] + Dist[(e^2*(m + 2*p + 2))/(4*g^2*(p + 1)), Int[(e*Cos[a + b*x
])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x]
 && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[p
, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m,
 2*p]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx &= -\frac{\cos^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{1}{6}\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= \frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{6b}-\frac{\cos^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}\end{aligned}$$

Mathematica [A] time = 1.00654, size = 82, normalized size = 1.71

$$-\frac{\frac{\sqrt{2}(\sin(a+bx)+\cos(a+bx))\text{EllipticF}\left(\sin^{-1}(\cos(a+bx)-\sin(a+bx)),\frac{1}{2}\right)}{\sqrt{\sin(2(a+bx))+1}}+\sqrt{\sin(2(a+bx))}\csc^2(a+bx)}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] -(Csc[a + b*x]^2*Sqrt[Sin[2*(a + b*x)]] + (Sqrt[2]*EllipticF[ArcSin[Cos[a +
 b*x] - Sin[a + b*x]], 1/2]*(Cos[a + b*x] + Sin[a + b*x]))/Sqrt[1 + Sin[2*(
a + b*x)]])/(12*b)
```

Maple [A] time = 11.904, size = 123, normalized size = 2.6

$$\frac{1}{12\cos(2bx+2a)b}\left(\sqrt{\sin(2bx+2a)+1}\sqrt{-2\sin(2bx+2a)+2}\sqrt{-\sin(2bx+2a)}\mathrm{EllipticF}\left(\sqrt{\sin(2bx+2a)+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] 1/12/sin(2*b*x+2*a)^(3/2)/cos(2*b*x+2*a)*((sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(
2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticF((sin(2*b*x+2*a)+1)^(1
/2),1/2*2^(1/2))*sin(2*b*x+2*a)-2*cos(2*b*x+2*a)^2-2*cos(2*b*x+2*a))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^2}{\sin(2bx+2a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\cos(bx+a)^2}{\left(\cos(2bx+2a)^2-1\right)\sqrt{\sin(2bx+2a)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-cos(b*x + a)^2/((cos(2*b*x + 2*a)^2 - 1)*sqrt(sin(2*b*x + 2*a))),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^2}{\sin(2bx+2a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(5/2), x)
```

3.176 $\int \frac{\cos^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=77

$$-\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{10b}-\frac{\cos^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{3\cos(2a+2bx)}{10b\sqrt{\sin(2a+2bx)}}$$

```
[Out] (-3*EllipticE[a - Pi/4 + b*x, 2])/(10*b) - Cos[a + b*x]^2/(5*b*Sin[2*a + 2*
b*x]^(5/2)) - (3*Cos[2*a + 2*b*x])/(10*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.0463152, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4295, 2636, 2639}

$$-\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{10b}-\frac{\cos^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{3\cos(2a+2bx)}{10b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (-3*EllipticE[a - Pi/4 + b*x, 2])/(10*b) - Cos[a + b*x]^2/(5*b*Sin[2*a + 2*
b*x]^(5/2)) - (3*Cos[2*a + 2*b*x])/(10*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4295

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(p + 1)), x] + Dist[(e^2*(m + 2*p + 2))/(4*g^2*(p + 1)), Int[(e*Cos[a + b*x
])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x]
 && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[p
, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m,
 2*p]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = -\frac{\cos^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{3}{10}\int \frac{1}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$$
$$= -\frac{\cos^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{3\cos(2a+2bx)}{10b\sqrt{\sin(2a+2bx)}} - \frac{3}{10}\int \sqrt{\sin(2a+2bx)}\,dx$$
$$= -\frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{10b} - \frac{\cos^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{3\cos(2a+2bx)}{10b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A] time = 0.561703, size = 64, normalized size = 0.83

$$\frac{\frac{2(-6\cos(2(a+bx))+3\cos(4(a+bx))+1)\cot(a+bx)}{\sin^{\frac{3}{2}}(2(a+bx))} - 12E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{40b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (-12*EllipticE[a - Pi/4 + b*x, 2] + (2*(1 - 6*Cos[2*(a + b*x)] + 3*Cos[4*(a
 + b*x)])*Cot[a + b*x])/Sin[2*(a + b*x)]^(3/2))/(40*b)
```

Maple [B] time = 71.1, size = 227, normalized size = 3.

$$\frac{\sqrt{2}}{32\,b}\left(-\frac{8\sqrt{2}}{5}(\sin(2\,bx+2\,a))^{-\frac{5}{2}} + \frac{4\sqrt{2}}{5\cos(2\,bx+2\,a)}\left(6\sqrt{\sin(2\,bx+2\,a)+1}\sqrt{-2\sin(2\,bx+2\,a)+2}\sqrt{-\sin(2\,bx+2\,a)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x)
```

```
[Out] 1/32*2^(1/2)*(-8/5*2^(1/2)/sin(2*b*x+2*a)^(5/2)+4/5*2^(1/2)/sin(2*b*x+2*a)^
(5/2)*(6*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2
*a))^(1/2)*sin(2*b*x+2*a)^2*EllipticE((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))
-3*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(
1/2)*sin(2*b*x+2*a)^2*EllipticF((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))+6*sin
(2*b*x+2*a)^4-4*sin(2*b*x+2*a)^2-2)/cos(2*b*x+2*a))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^2}{\sin(2\,bx+2\,a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(bx+a\right)^{2}\sqrt{\sin\left(2\,bx+2\,a\right)}}{\cos\left(2\,bx+2\,a\right)^{4}-2\,\cos\left(2\,bx+2\,a\right)^{2}+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^2*sqrt(sin(2*b*x + 2*a))/(cos(2*b*x + 2*a)^4 - 2*cos(
2*b*x + 2*a)^2 + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(bx+a\right)^{2}}{\sin\left(2\,bx+2\,a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(7/2), x)
```

3.177 $\int \cos^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=136

$$\frac{7\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{48b}+\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{12b}-\frac{7\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{64b}-\frac{7\sqrt{\sin(2a+2bx)}\cos(}{32b}$$

```
[Out] (-7*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(64*b) + (7*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(64*b) - (7*Cos[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(32*b) + (7*Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(48*b) + (Cos[a
 + b*x]*Sin[2*a + 2*b*x]^(5/2))/(12*b)
```

Rubi [A] time = 0.0987861, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4297, 4301, 4302, 4305}

$$\frac{7\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{48b}+\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{12b}-\frac{7\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{64b}-\frac{7\sqrt{\sin(2a+2bx)}\cos(}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```


```
[Out] (-7*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(64*b) + (7*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(64*b) - (7*Cos[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(32*b) + (7*Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(48*b) + (Cos[a
 + b*x]*Sin[2*a + 2*b*x]^(5/2))/(12*b)
```


Rule 4297

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e^2*(e*Cos[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 1
))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Cos[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ[
m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \cos^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \frac{\cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b} + \frac{7}{12}\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$

$$= \frac{7\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{48b} + \frac{\cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b} + \frac{7}{16}\int \sin(a+bx)\sqrt{}$$

$$= -\frac{7\cos(a+bx)\sqrt{\sin(2a+2bx)}}{32b} + \frac{7\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{48b} + \frac{\cos(a+bx)\sin}{12b}$$

$$= -\frac{7\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{64b} + \frac{7\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2}\right.}{64b}$$

Mathematica [A] time = 0.327734, size = 99, normalized size = 0.73

$$\frac{-7\sin^{-1}(\cos(a+bx)-\sin(a+bx)) - \frac{2}{3}\sqrt{\sin(2(a+bx))}(10\cos(a+bx)+9\cos(3(a+bx))+2\cos(5(a+bx))) + 7\log}{64b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-7*ArcSin[Cos[a + b*x] - Sin[a + b*x]] + 7*Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]] - (2*(10*Cos[a + b*x] + 9*Cos[3*(a + b*x)] + 2*C
os[5*(a + b*x)])*Sqrt[Sin[2*(a + b*x)]])/3)/(64*b)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (\cos(bx+a))^3 (\sin(2bx+2a))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] int(cos(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)^3 \sin(2bx+2a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^3*sin(2*b*x + 2*a)^(3/2), x)
```

Fricas [B] time = 0.561961, size = 803, normalized size = 5.9

$$-\frac{8\sqrt{2}\left(32\cos\left(bx+a\right)^{5}-4\cos\left(bx+a\right)^{3}-7\cos\left(bx+a\right)\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}-42\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}}{\cos(bx+a}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/768*(8*sqrt(2)*(32*cos(b*x + a)^5 - 4*cos(b*x + a)^3 - 7*cos(b*x + a))*s
qrt(cos(b*x + a)*sin(b*x + a)) - 42*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(
b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b
*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) + 42*arctan(-(2*sqrt(2)*sqrt(
cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - s
in(b*x + a))) + 21*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (
4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(
b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(bx+a\right)^{3}\sin\left(2\,bx+2\,a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^3*sin(2*b*x + 2*a)^(3/2), x)
```

3.178 $\int \cos^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal. Leaf size=110

$$\frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{16b}+\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{8b}-\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{32b}-\frac{5\log\left(\sin(a+bx)\right.}{}$$

```
[Out] (-5*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(32*b) - (5*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(32*b) + (5*Sin[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(16*b) + (Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(8*b)
```

Rubi [A] time = 0.075635, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4297, 4301, 4306}

$$\frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{16b}+\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{8b}-\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{32b}-\frac{5\log\left(\sin(a+bx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-5*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(32*b) - (5*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(32*b) + (5*Sin[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(16*b) + (Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(8*b)
```

Rule 4297

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e^2*(e*Cos[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 1
))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Cos[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ[
m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \cos^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = \frac{\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b} + \frac{5}{8}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$
$$= \frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{16b} + \frac{\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b} + \frac{5}{16}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}$$
$$= -\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{32b} - \frac{5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+\right.}}{32b}$$

Mathematica [A] time = 0.181851, size = 84, normalized size = 0.76

$$\frac{2\sqrt{\sin(2(a+bx))}(6\sin(a+bx)+\sin(3(a+bx))) - 5\left(\sin^{-1}(\cos(a+bx)-\sin(a+bx)) + \log\left(\sin(a+bx)+\sqrt{\sin(2(a+\right.\right.}}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-5*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]) + 2*Sqrt[Sin[2*(a + b*x)]]*(6*Sin[a + b*x] + Sin
[3*(a + b*x)]))/(32*b)
```

Maple [B] time = 19.511, size = 88762396, normalized size = 806930.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)^3 \sqrt{\sin(2\,bx+2\,a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^3*sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [B] time = 0.550257, size = 774, normalized size = 7.04

$$8\sqrt{2}\left(4\cos(bx+a)^2+5\right)\sqrt{\cos(bx+a)\sin(bx+a)}\sin(bx+a) + 10\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/128*(8*sqrt(2)*(4*cos(b*x + a)^2 + 5)*sqrt(cos(b*x + a)*sin(b*x + a))*sin
(b*x + a) + 10*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x +
a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x
 + a)*sin(b*x + a) - 1)) - 10*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x
+ a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + 5*log
(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*
sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x
 + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.179 $\int \frac{\cos^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal. Leaf size=84

$$-\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{8b}+\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{4b}+\frac{3\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{8b}$$

```
[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(8*b) + (3*Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(8*b) + (Cos[a + b*x]*Sqrt[Sin[2*a + 2*
b*x]])/(4*b)
```

Rubi [A] time = 0.0524824, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4297, 4305}

$$-\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{8b}+\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{4b}+\frac{3\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(8*b) + (3*Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(8*b) + (Cos[a + b*x]*Sqrt[Sin[2*a + 2*
b*x]])/(4*b)
```

Rule 4297

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e^2*(e*Cos[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 1
))/(2*b*g*(m + 2*p)), x] + Dist[(e^2*(m + p - 1))/(m + 2*p), Int[(e*Cos[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ[
m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx &= \frac{\cos(a+bx)\sqrt{\sin(2a+2bx)}}{4b}+\frac{3}{4}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= -\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{8b}+\frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{8b}+\frac{\cos(a+bx)\sqrt{\sin(2a+2bx)}}{4b}\end{aligned}$$

Mathematica [A] time = 0.11038, size = 73, normalized size = 0.87

$$\frac{-3\sin^{-1}(\cos(a+bx)-\sin(a+bx))+\sin^{\frac{3}{2}}(2(a+bx))\csc(a+bx)+3\log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(a+bx)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]] + 3*Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]] + Csc[a + b*x]*Sin[2*(a + b*x)]^(3/2))/(8*b)
```

Maple [B] time = 38.924, size = 190984194, normalized size = 2273621.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(bx+a\right)^{3}}{\sqrt{\sin\left(2\,bx+2\,a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^3/sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [B] time = 0.545128, size = 738, normalized size = 8.79

$$8\,\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}\cos\left(bx+a\right) + 6\,\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^{2}+2\,\cos(bx+a)\sin(bx+a)-1}\right) - 6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/32*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*cos(b*x + a) + 6*arctan(-(s
qrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(
b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1))
- 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin
(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - 3*log(-32*cos(b*x + a)^4 + 4*sq
rt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x +
 a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*
sin(b*x + a) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^3}{\sqrt{\sin(2bx+2a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^3/sqrt(sin(2*b*x + 2*a)), x)
```

3.180 $\int \frac{\cos^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=82

$$\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b}-\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}+\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{4b}$$

```
[Out] ArcSin[Cos[a + b*x] - Sin[a + b*x]]/(4*b) + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*a + 2*b*x]]]/(4*b) - Cos[a + b*x]/(b*Sqrt[Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.0760366, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4293, 4307, 4306}

$$\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b}-\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}+\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] ArcSin[Cos[a + b*x] - Sin[a + b*x]]/(4*b) + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*a + 2*b*x]]]/(4*b) - Cos[a + b*x]/(b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4293

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e^2*(e*Cos[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 1
))/(2*b*g*(p + 1)), x] + Dist[(e^4*(m + p - 1))/(4*g^2*(p + 1)), Int[(e*Cos
[a + b*x])^(m - 4)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e
, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 2]
&& LtQ[p, -1] && (GtQ[m, 3] || EqQ[p, -3/2]) && IntegersQ[2*m, 2*p]
```

Rule 4307

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/cos[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}} - \frac{1}{4}\int \sec(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$
$$= -\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}} - \frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$$
$$= \frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b} + \frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{4b} - \frac{\cos(a}{b\sqrt{\sin(2}}$$

Mathematica [A] time = 0.0885056, size = 70, normalized size = 0.85

$$\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx)) - 2\sqrt{\sin(2(a+bx))}\csc(a+bx) + \log\left(\sin(a+bx)+\sqrt{\sin(2(a+bx))}+\cos(a+bx)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x] + Sq
rt[Sin[2*(a + b*x)]]] - 2*Csc[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/(4*b)
```

Maple [B] time = 29.262, size = 179366588, normalized size = 2187397.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(3/2), x)
```

Fricas [B] time = 0.54088, size = 815, normalized size = 9.94

$$-\frac{2\,\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\,\cos(bx+a)\sin(bx+a)-1}\right)\sin(bx+a) - 2\,\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}-\cos}{\cos(bx+a)-\sin(bx}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/16*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - s
in(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*
sin(b*x + a) - 1))*sin(b*x + a) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*si
n(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a)))*s
in(b*x + a) + log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos
(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x +
 a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1)*sin(b*x + a) +
 8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) + 8*sin(b*x + a))/(b*sin(b*x + a
))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(3/2), x)
```

3.181 $\int \frac{\cos^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=28

$$-\frac{\cos^3(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

```
[Out] -Cos[a + b*x]^3/(3*b*Sin[2*a + 2*b*x]^(3/2))
```

Rubi [A] time = 0.027477, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {4291}

$$-\frac{\cos^3(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] -Cos[a + b*x]^3/(3*b*Sin[2*a + 2*b*x]^(3/2))
```

Rule 4291

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> -Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*
m), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/
b, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{\cos^3(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

Mathematica [A] time = 0.0539586, size = 27, normalized size = 0.96

$$-\frac{\sin^{\frac{3}{2}}(2(a+bx))\csc^3(a+bx)}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] -(Csc[a + b*x]^3*Sin[2*(a + b*x)]^(3/2))/(24*b)
```

Maple [C] time = 52.142, size = 192, normalized size = 6.9

$$\frac{1}{24b}\sqrt{-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2-1\right)^{-1}}\left(\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2-1\right)\left(4\sqrt{\tan(1/2\,bx+a/2)+1}\sqrt{-2\tan(1/2\,bx+a/2)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] 1/24*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1/2*
a)^2-1)/tan(1/2*b*x+1/2*a)*(4*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+
1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1
)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)+tan(1/2*b*x+1/2*a)^4-1)/(tan(1/2*b*
x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+
1/2*a))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(5/2), x)
```

Fricas [B] time = 0.489825, size = 142, normalized size = 5.07

$$\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\cos(bx+a)+\cos(bx+a)^2-1}{12\left(b\cos(bx+a)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*cos(b*x + a) + cos(b*x + a)^2
 - 1)/(b*cos(b*x + a)^2 - b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(5/2), x)
```

3.182 $\int \frac{\cos^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=55

$$-\frac{\cos^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

```
[Out] -Cos[a + b*x]^3/(5*b*Sin[2*a + 2*b*x]^(5/2)) - Cos[a + b*x]/(5*b*Sqrt[Sin[2
*a + 2*b*x]])
```

Rubi [A] time = 0.0471591, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4295, 4291}

$$-\frac{\cos^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] -Cos[a + b*x]^3/(5*b*Sin[2*a + 2*b*x]^(5/2)) - Cos[a + b*x]/(5*b*Sqrt[Sin[2
*a + 2*b*x]])
```

Rule 4295

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(p + 1)), x] + Dist[(e^2*(m + 2*p + 2))/(4*g^2*(p + 1)), Int[(e*Cos[a + b*x
])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x]
 && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[p
, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m,
 2*p]
```

Rule 4291

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> -Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*
m), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/
b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx &= -\frac{\cos^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{1}{5}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx\\ &= -\frac{\cos^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{5b\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [A] time = 0.0934569, size = 35, normalized size = 0.64

$$-\frac{\sqrt{\sin(2(a+bx))}\csc(a+bx)\left(\csc^2(a+bx)+4\right)}{40b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] -(Csc[a + b*x]*(4 + Csc[a + b*x]^2)*Sqrt[Sin[2*(a + b*x)]])/(40*b)
```

Maple [C] time = 213.665, size = 482, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x)
```

```
[Out] 1/160*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)/tan(1/2*b*x+1/2*
a)^3*(16*(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1
/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*
EllipticE((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^2-8*
(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1/2*a)+1)^
(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF
((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)^2-(tan(1/2*b*
x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*tan(1/2*b*x+1/2*a)^6+(tan(1/2*b*x+
1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*tan(1/2*b*x+1/2*a)^4+8*(tan(1/2*b*x+
1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)*tan(1/2*b*x+1/2*a)^4+(tan(1/2*b*x+1/2*a)
*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*tan(1/2*b*x+1/2*a)^2-8*(tan(1/2*b*x+1/2*a)
^3-tan(1/2*b*x+1/2*a))^(1/2)*tan(1/2*b*x+1/2*a)^2-(tan(1/2*b*x+1/2*a)*(tan(
1/2*b*x+1/2*a)^2-1))^(1/2))/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)
/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(7/2), x)
```

Fricas [A] time = 0.50141, size = 200, normalized size = 3.64

$$-\frac{\sqrt{2}\left(4\,\cos(bx+a)^2-5\right)\sqrt{\cos(bx+a)\sin(bx+a)}+4\left(\cos(bx+a)^2-1\right)\sin(bx+a)}{40\left(b\cos(bx+a)^2-b\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/40*(sqrt(2)*(4*cos(b*x + a)^2 - 5)*sqrt(cos(b*x + a)*sin(b*x + a)) + 4*(
cos(b*x + a)^2 - 1)*sin(b*x + a))/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(7/2), x)
```

3.183 $\int \frac{\cos^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=81

$$\frac{4\sin(a+bx)}{21b\sqrt{\sin(2a+2bx)}}-\frac{\cos^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}$$

```
[Out] -Cos[a + b*x]^3/(7*b*Sin[2*a + 2*b*x]^(7/2)) - (2*Cos[a + b*x])/(21*b*Sin[2
*a + 2*b*x]^(3/2)) + (4*Sin[a + b*x])/(21*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.0678956, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4295, 4303, 4292}

$$\frac{4\sin(a+bx)}{21b\sqrt{\sin(2a+2bx)}}-\frac{\cos^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] -Cos[a + b*x]^3/(7*b*Sin[2*a + 2*b*x]^(7/2)) - (2*Cos[a + b*x])/(21*b*Sin[2
*a + 2*b*x]^(3/2)) + (4*Sin[a + b*x])/(21*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4295

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(p + 1)), x] + Dist[(e^2*(m + 2*p + 2))/(4*g^2*(p + 1)), Int[(e*Cos[a + b*x
])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x]
 && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[p
, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m,
 2*p]
```

Rule 4303

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx = -\frac{\cos^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} + \frac{2}{7}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$$
$$= -\frac{\cos^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} - \frac{2\cos(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{4}{21}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$$
$$= -\frac{\cos^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)} - \frac{2\cos(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{4\sin(a+bx)}{21b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A] time = 0.114477, size = 55, normalized size = 0.68

$$\frac{\sqrt{\sin(2(a+bx))}(-12\cos(2(a+bx))+4\cos(4(a+bx))+5)\csc^4(a+bx)\sec(a+bx)}{336b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(9/2),x]

[Out] ((5 - 12*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Csc[a + b*x]^4*Sec[a + b*x]
*Sqrt[Sin[2*(a + b*x)]])/(336*b)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (\cos(bx+a))^3 (\sin(2bx+2a))^{-\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x)

[Out] int(cos(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^3}{\sin(2bx+2a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")

[Out] integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(9/2), x)
```

Fricas [A] time = 0.505902, size = 282, normalized size = 3.48

$$\frac{32\cos(bx+a)^5 - 64\cos(bx+a)^3 + \sqrt{2}\left(32\cos(bx+a)^4 - 56\cos(bx+a)^2 + 21\right)\sqrt{\cos(bx+a)\sin(bx+a)} + 32\cos}{336\left(b\cos(bx+a)^5 - 2b\cos(bx+a)^3 + b\cos(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/336*(32*cos(b*x + a)^5 - 64*cos(b*x + a)^3 + sqrt(2)*(32*cos(b*x + a)^4 -
 56*cos(b*x + a)^2 + 21)*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a))
/(b*cos(b*x + a)^5 - 2*b*cos(b*x + a)^3 + b*cos(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^3}{\sin(2\,bx+2\,a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(9/2), x)
```

3.184 $\int \frac{\cos^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=107

$$\frac{4\sin(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{8\cos(a+bx)}{45b\sqrt{\sin(2a+2bx)}}$$

```
[Out] -Cos[a + b*x]^3/(9*b*Sin[2*a + 2*b*x]^(9/2)) - Cos[a + b*x]/(15*b*Sin[2*a +
 2*b*x]^(5/2)) + (4*Sin[a + b*x])/(45*b*Sin[2*a + 2*b*x]^(3/2)) - (8*Cos[a
+ b*x])/(45*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.0904905, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4295, 4303, 4304, 4291}

$$\frac{4\sin(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{8\cos(a+bx)}{45b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(11/2),x]
```

```
[Out] -Cos[a + b*x]^3/(9*b*Sin[2*a + 2*b*x]^(9/2)) - Cos[a + b*x]/(15*b*Sin[2*a +
 2*b*x]^(5/2)) + (4*Sin[a + b*x])/(45*b*Sin[2*a + 2*b*x]^(3/2)) - (8*Cos[a
+ b*x])/(45*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4295

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g*
(p + 1)), x] + Dist[(e^2*(m + 2*p + 2))/(4*g^2*(p + 1)), Int[(e*Cos[a + b*x
])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x]
 && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[p
, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m,
 2*p]
```

Rule 4303

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4304

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> -Simp[(Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dis
t[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4291

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> -Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*
m), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/
b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx &= -\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}+\frac{1}{3}\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx\\ &= -\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{4}{15}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx\\ &= -\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{4\sin(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{8}{45}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx\\ &= -\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{4\sin(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{8\cos(a+bx)}{45b\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [A] time = 0.0935069, size = 62, normalized size = 0.58

$$-\frac{\sqrt{\sin(2(a+bx))}\left(5\csc^5(a+bx)+17\csc^3(a+bx)+113\csc(a+bx)-15\tan(a+bx)\sec(a+bx)\right)}{1440b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(11/2),x]
```

```
[Out] -(Sqrt[Sin[2*(a + b*x)]]*(113*Csc[a + b*x] + 17*Csc[a + b*x]^3 + 5*Csc[a +
b*x]^5 - 15*Sec[a + b*x]*Tan[a + b*x]))/(1440*b)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (\cos(bx+a))^3 (\sin(2bx+2a))^{-\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x)
```

```
[Out] int(cos(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^3}{\sin(2bx+2a)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(11/2), x)
```

Fricas [A] time = 0.525805, size = 358, normalized size = 3.35

$$-\frac{\sqrt{2}\left(128\,\cos\left(bx+a\right)^{6}-288\,\cos\left(bx+a\right)^{4}+180\,\cos\left(bx+a\right)^{2}-15\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+128\,\left(\cos\left(bx+a\right)^{6}-\right.}{1440\left(b\cos\left(bx+a\right)^{6}-2\,b\cos\left(bx+a\right)^{4}+b\cos\left(bx+a\right)^{2}\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="fricas")
```

```
[Out] -1/1440*(sqrt(2)*(128*cos(b*x + a)^6 - 288*cos(b*x + a)^4 + 180*cos(b*x + a
)^2 - 15)*sqrt(cos(b*x + a)*sin(b*x + a)) + 128*(cos(b*x + a)^6 - 2*cos(b*x
 + a)^4 + cos(b*x + a)^2)*sin(b*x + a))/((b*cos(b*x + a)^6 - 2*b*cos(b*x +
a)^4 + b*cos(b*x + a)^2)*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(bx+a\right)^{3}}{\sin\left(2\,bx+2\,a\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(11/2), x)
```

3.185 $\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\frac{1}{2}\sin^{-1}(\cos(x)-\sin(x))$$

```
[Out] -ArcSin[Cos[x] - Sin[x]]/2 + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2
```

Rubi [A] time = 0.0129827, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4305}

$$\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\frac{1}{2}\sin^{-1}(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/Sqrt[Sin[2*x]],x]
```

```
[Out] -ArcSin[Cos[x] - Sin[x]]/2 + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx=-\frac{1}{2}\sin^{-1}(\cos(x)-\sin(x))+\frac{1}{2}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)$$

Mathematica [A] time = 0.0248818, size = 29, normalized size = 0.94

$$\frac{1}{2}\left(\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\sin^{-1}(\cos(x)-\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/Sqrt[Sin[2*x]],x]
```

```
[Out] (-ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]])/2
```

Maple [C] time = 0.046, size = 98, normalized size = 3.2

$$\sqrt{-\tan\left(\frac{x}{2}\right)\left(\left(\tan\left(\frac{x}{2}\right)\right)^2-1\right)^{-1}}\left(\left(\tan\left(\frac{x}{2}\right)\right)^2-1\right)\sqrt{\tan\left(\frac{x}{2}\right)+1}\sqrt{-2\tan(x/2)+2}\sqrt{-\tan\left(\frac{x}{2}\right)}\mathrm{EllipticF}\left(\sqrt{\tan\left(\frac{x}{2}\right)+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(2*x)^(1/2),x)
```

```
[Out] (-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)/(tan(1/2*x)*(tan(1/2*
x)^2-1))^(1/2)*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(
1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)*EllipticF((tan(1/2*x)+1)^(1/2),1/2*2^(
1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(x)/sqrt(sin(2*x)), x)
```

Fricas [B] time = 0.507912, size = 455, normalized size = 14.68

$$\frac{1}{4}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right)-\frac{1}{4}\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)-\sin(x)}{\cos(x)-\sin(x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) + cos(x)*sin(x))
/(cos(x)^2 + 2*cos(x)*sin(x) - 1)) - 1/4*arctan(-(2*sqrt(2)*sqrt(cos(x)*sin
(x)) - cos(x) - sin(x))/(cos(x) - sin(x))) - 1/8*log(-32*cos(x)^4 + 4*sqrt(
2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)) +
32*cos(x)^2 + 16*cos(x)*sin(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(2*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(x)/sqrt(sin(2*x)), x)
```

3.186 $\int \csc(x)\sqrt{\sin(2x)}\, dx$

Optimal. Leaf size=25

$$\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\sin^{-1}(\cos(x)-\sin(x))$$

```
[Out] -ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]
```

Rubi [A] time = 0.0303279, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4308, 4305}

$$\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\sin^{-1}(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]*Sqrt[Sin[2*x]],x]
```

```
[Out] -ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\begin{aligned}\int \csc(x)\sqrt{\sin(2x)}\, dx &= 2\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\, dx\\ &= -\sin^{-1}(\cos(x)-\sin(x))+\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)\end{aligned}$$

Mathematica [A] time = 0.0164929, size = 25, normalized size = 1.

$$\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\sin^{-1}(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]*Sqrt[Sin[2*x]],x]
```

```
[Out] -ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]
```

Maple [C] time = 0.046, size = 99, normalized size = 4.

$$2\frac{\left((\tan(x/2))^2-1\right)\sqrt{\tan(x/2)+1}\sqrt{-2\tan(x/2)+2}\sqrt{-\tan(x/2)}\operatorname{EllipticF}\left(\sqrt{\tan(x/2)+1},1/2\sqrt{2}\right)}{\sqrt{\tan(x/2)\left((\tan(x/2))^2-1\right)}\sqrt{(\tan(x/2))^3-\tan(x/2)}}\sqrt{-\frac{\tan(x/2)}{(\tan(x/2))^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)*sin(2*x)^(1/2),x)
```

```
[Out] 2*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)/(tan(1/2*x)*(tan(1/
2*x)^2-1))^(1/2)*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))
^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)*EllipticF((tan(1/2*x)+1)^(1/2),1/2*2
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(x)\sqrt{\sin(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(x)*sqrt(sin(2*x)), x)
```

Fricas [B] time = 0.508127, size = 455, normalized size = 18.2

$$\frac{1}{2}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right)-\frac{1}{2}\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)-\sin(x)}{\cos(x)-\sin(x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) + cos(x)*sin(x))
/(cos(x)^2 + 2*cos(x)*sin(x) - 1)) - 1/2*arctan(-(2*sqrt(2)*sqrt(cos(x)*sin
(x)) - cos(x) - sin(x))/(cos(x) - sin(x))) - 1/4*log(-32*cos(x)^4 + 4*sqrt(
2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)) +
32*cos(x)^2 + 16*cos(x)*sin(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(2*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \csc(x)\sqrt{\sin(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(2*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(x)*sqrt(sin(2*x)), x)
```

3.187 $\int \cos^3(a+bx)\sin^m(2a+2bx)\,dx$

Optimal. Leaf size=85

$$-\frac{\cos^3(a+bx)\cot(a+bx)\sin^2(a+bx)^{\frac{1-m}{2}}\sin^m(2a+2bx)\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{m+4}{2},\frac{m+6}{2},\cos^2(a+bx)\right)}{b(m+4)}$$

```
[Out] -((Cos[a + b*x]^3*Cot[a + b*x]*Hypergeometric2F1[(1 - m)/2, (4 + m)/2, (6 +
 m)/2, Cos[a + b*x]^2]*(Sin[a + b*x]^2)^((1 - m)/2)*Sin[2*a + 2*b*x]^m)/(b*
(4 + m)))
```

Rubi [A] time = 0.0712569, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4309, 2576}

$$-\frac{\cos^3(a+bx)\cot(a+bx)\sin^2(a+bx)^{\frac{1-m}{2}}\sin^m(2a+2bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{m+4}{2};\frac{m+6}{2};\cos^2(a+bx)\right)}{b(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] -((Cos[a + b*x]^3*Cot[a + b*x]*Hypergeometric2F1[(1 - m)/2, (4 + m)/2, (6 +
 m)/2, Cos[a + b*x]^2]*(Sin[a + b*x]^2)^((1 - m)/2)*Sin[2*a + 2*b*x]^m)/(b*
(4 + m)))
```

Rule 4309

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/((e*Cos[a + b*x])^p*Sin[a + b*x]^
p), Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x] /; FreeQ[{a, b, c,
d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int \cos^3(a+bx)\sin^m(2a+2bx)\,dx = \left(\cos^{-m}(a+bx)\sin^{-m}(a+bx)\sin^m(2a+2bx)\right)\int \cos^{3+m}(a+bx)\sin^m(a+bx)\,dx$$
$$= -\frac{\cos^3(a+bx)\cot(a+bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{4+m}{2};\frac{6+m}{2};\cos^2(a+bx)\right)\sin^2(a+bx)^{\frac{1-m}{2}}\sin^m(2}{b(4+m)}$$

Mathematica [C] time = 13.303, size = 2472, normalized size = 29.08

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] (2^(1 + m)*(6*AppellF1[(1 + m)/2, -m, 2*(1 + m), (3 + m)/2, Tan[(a + b*x)/2
]^2, -Tan[(a + b*x)/2]^2] + 8*AppellF1[(1 + m)/2, -m, 2*(2 + m), (3 + m)/2,
 Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - AppellF1[(1 + m)/2, -m, 1 + 2*m
, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 12*AppellF1[(1 + m)
/2, -m, 3 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*Cos[a
 + b*x]^3*(Sec[(a + b*x)/2]^2)^(2*m)*(Cos[(a + b*x)/2]*(-Sin[(a + b*x)/2] +
 Sin[(3*(a + b*x))/2]))^m*Sin[2*(a + b*x)]^m*Tan[(a + b*x)/2])/(b*(1 + m)*(
Cos[a + b*x]*Sec[(a + b*x)/2]^2)^m*((2^m*(6*AppellF1[(1 + m)/2, -m, 2*(1 +
m), (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 8*AppellF1[(1 + m
)/2, -m, 2*(2 + m), (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - A
ppellF1[(1 + m)/2, -m, 1 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*
x)/2]^2] - 12*AppellF1[(1 + m)/2, -m, 3 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^
2, -Tan[(a + b*x)/2]^2])*(Sec[(a + b*x)/2]^2)^(1 + 2*m)*(Cos[(a + b*x)/2]*(
-Sin[(a + b*x)/2] + Sin[(3*(a + b*x))/2]))^m)/((1 + m)*(Cos[a + b*x]*Sec[(a
 + b*x)/2]^2)^m) + (2^(1 + m)*m*(6*AppellF1[(1 + m)/2, -m, 2*(1 + m), (3 +
m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 8*AppellF1[(1 + m)/2, -m,
2*(2 + m), (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - AppellF1[(
1 + m)/2, -m, 1 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]
- 12*AppellF1[(1 + m)/2, -m, 3 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(
a + b*x)/2]^2])*(Sec[(a + b*x)/2]^2)^(2*m)*(Cos[(a + b*x)/2]*(-Sin[(a + b*x
)/2] + Sin[(3*(a + b*x))/2]))^(-1 + m)*(Cos[(a + b*x)/2]*(-Cos[(a + b*x)/2]
/2 + (3*Cos[(3*(a + b*x))/2])/2) - (Sin[(a + b*x)/2]*(-Sin[(a + b*x)/2] + S
in[(3*(a + b*x))/2]))/2)*Tan[(a + b*x)/2])/((1 + m)*(Cos[a + b*x]*Sec[(a +
b*x)/2]^2)^m) + (2^(2 + m)*m*(6*AppellF1[(1 + m)/2, -m, 2*(1 + m), (3 + m)/
2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 8*AppellF1[(1 + m)/2, -m, 2*(
2 + m), (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - AppellF1[(1 +
 m)/2, -m, 1 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 1
2*AppellF1[(1 + m)/2, -m, 3 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a +
 b*x)/2]^2])*(Sec[(a + b*x)/2]^2)^(2*m)*(Cos[(a + b*x)/2]*(-Sin[(a + b*x)/2
] + Sin[(3*(a + b*x))/2]))^m*Tan[(a + b*x)/2]^2)/((1 + m)*(Cos[a + b*x]*Sec
[(a + b*x)/2]^2)^m) - (2^(1 + m)*m*(6*AppellF1[(1 + m)/2, -m, 2*(1 + m), (3
 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 8*AppellF1[(1 + m)/2, -
m, 2*(2 + m), (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - AppellF
1[(1 + m)/2, -m, 1 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^
2] - 12*AppellF1[(1 + m)/2, -m, 3 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Ta
n[(a + b*x)/2]^2])*(Sec[(a + b*x)/2]^2)^(2*m)*(Cos[a + b*x]*Sec[(a + b*x)/2
]^2)^(-1 - m)*(Cos[(a + b*x)/2]*(-Sin[(a + b*x)/2] + Sin[(3*(a + b*x))/2]))
^m*Tan[(a + b*x)/2]*(-(Sec[(a + b*x)/2]^2*Sin[a + b*x]) + Cos[a + b*x]*Sec[
(a + b*x)/2]^2*Tan[(a + b*x)/2]))/(1 + m) + (2^(1 + m)*(Sec[(a + b*x)/2]^2)
^(2*m)*(Cos[(a + b*x)/2]*(-Sin[(a + b*x)/2] + Sin[(3*(a + b*x))/2]))^m*Tan[
(a + b*x)/2]*((m*(1 + m)*AppellF1[1 + (1 + m)/2, 1 - m, 1 + 2*m, 1 + (3 + m
)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[(a + b
*x)/2])/(3 + m) + ((1 + m)*(1 + 2*m)*AppellF1[1 + (1 + m)/2, -m, 2 + 2*m, 1
 + (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2*T
an[(a + b*x)/2])/(3 + m) - 12*(-((m*(1 + m)*AppellF1[1 + (1 + m)/2, 1 - m,
3 + 2*m, 1 + (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[(a + b
*x)/2]^2*Tan[(a + b*x)/2])/(3 + m)) - ((1 + m)*(3 + 2*m)*AppellF1[1 + (1 +
m)/2, -m, 4 + 2*m, 1 + (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*
Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2])/(3 + m)) + 6*(-((m*(1 + m)*AppellF1[1
+ (1 + m)/2, 1 - m, 2*(1 + m), 1 + (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a +
 b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2])/(3 + m)) - (2*(1 + m)^2*Ap
pellF1[1 + (1 + m)/2, -m, 1 + 2*(1 + m), 1 + (3 + m)/2, Tan[(a + b*x)/2]^2,
 -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2])/(3 + m)) + 8*(-(
(m*(1 + m)*AppellF1[1 + (1 + m)/2, 1 - m, 2*(2 + m), 1 + (3 + m)/2, Tan[(a
+ b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2])/(3 +
```

```
m)) - (2*(1 + m)*(2 + m)*AppellF1[1 + (1 + m)/2, -m, 1 + 2*(2 + m), 1 + (3
+ m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[(a
+ b*x)/2])/(3 + m))))/((1 + m)*(Cos[a + b*x]*Sec[(a + b*x)/2]^2)^m)))
```

Maple [F] time = 0.992, size = 0, normalized size = 0.

$$\int (\cos (bx+a))^{3} (\sin (2 bx+2 a))^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(2*b*x+2*a)^m,x)
```

```
[Out] int(cos(b*x+a)^3*sin(2*b*x+2*a)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin (2 bx+2 a)^{m} \cos (bx+a)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*cos(b*x + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sin (2 bx+2 a)^{m} \cos (bx+a)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

```
[Out] integral(sin(2*b*x + 2*a)^m*cos(b*x + a)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin (2 bx+2 a)^{m} \cos (bx+a)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*cos(b*x + a)^3, x)
```

3.188 $\int \cos^2(a+bx)\sin^m(2a+2bx)\,dx$

Optimal. Leaf size=85

$$-\frac{\cos^2(a+bx)\cot(a+bx)\sin^2(a+bx)^{\frac{1-m}{2}}\sin^m(2a+2bx)\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{m+3}{2},\frac{m+5}{2},\cos^2(a+bx)\right)}{b(m+3)}$$

```
[Out] -((Cos[a + b*x]^2*Cot[a + b*x]*Hypergeometric2F1[(1 - m)/2, (3 + m)/2, (5 +
 m)/2, Cos[a + b*x]^2]*(Sin[a + b*x]^2)^((1 - m)/2)*Sin[2*a + 2*b*x]^m)/(b*
(3 + m)))
```

Rubi [A] time = 0.0707474, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4309, 2576}

$$-\frac{\cos^2(a+bx)\cot(a+bx)\sin^2(a+bx)^{\frac{1-m}{2}}\sin^m(2a+2bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{m+3}{2};\frac{m+5}{2};\cos^2(a+bx)\right)}{b(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] -((Cos[a + b*x]^2*Cot[a + b*x]*Hypergeometric2F1[(1 - m)/2, (3 + m)/2, (5 +
 m)/2, Cos[a + b*x]^2]*(Sin[a + b*x]^2)^((1 - m)/2)*Sin[2*a + 2*b*x]^m)/(b*
(3 + m)))
```

Rule 4309

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/((e*Cos[a + b*x])^p*Sin[a + b*x]^
p), Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x] /; FreeQ[{a, b, c,
d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int \cos^2(a+bx)\sin^m(2a+2bx)\,dx = \left(\cos^{-m}(a+bx)\sin^{-m}(a+bx)\sin^m(2a+2bx)\right)\int \cos^{2+m}(a+bx)\sin^m(a+bx)\,dx$$
$$= -\frac{\cos^2(a+bx)\cot(a+bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{3+m}{2};\frac{5+m}{2};\cos^2(a+bx)\right)\sin^2(a+bx)^{\frac{1-m}{2}}\sin^m(2}{b(3+m)}$$

Mathematica [C] time = 7.96872, size = 890, normalized size = 10.47

$$\frac{}{b(m+1)\left(8(m+3)F_1\left(\frac{m+1}{2};-m,2(m+1);\frac{m+3}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)\cos^2\left(\frac{1}{2}(a+bx)\right)-2(m+3)F_1\left(\frac{m+}{2}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] (4*(3 + m)*(4*AppellF1[(1 + m)/2, -m, 2*(1 + m), (3 + m)/2, Tan[(a + b*x)/2
]^2, -Tan[(a + b*x)/2]^2] - AppellF1[(1 + m)/2, -m, 1 + 2*m, (3 + m)/2, Tan
[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 4*AppellF1[(1 + m)/2, -m, 3 + 2*m,
(3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*Cos[(a + b*x)/2]^3*Cos
[a + b*x]^2*Sin[(a + b*x)/2]*Sin[2*(a + b*x)]^m)/(b*(1 + m)*(8*(3 + m)*Appe
llF1[(1 + m)/2, -m, 2*(1 + m), (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x
)/2]^2]*Cos[(a + b*x)/2]^2 - 2*(3 + m)*AppellF1[(1 + m)/2, -m, 1 + 2*m, (3
+ m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^2 - 8*(3
+ m)*AppellF1[(1 + m)/2, -m, 3 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(
a + b*x)/2]^2]*Cos[(a + b*x)/2]^2 + 2*(4*m*AppellF1[(3 + m)/2, 1 - m, 2*(1
+ m), (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - m*AppellF1[(3 +
 m)/2, 1 - m, 1 + 2*m, (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]
- 4*m*AppellF1[(3 + m)/2, 1 - m, 3 + 2*m, (5 + m)/2, Tan[(a + b*x)/2]^2, -T
an[(a + b*x)/2]^2] - AppellF1[(3 + m)/2, -m, 2*(1 + m), (5 + m)/2, Tan[(a +
 b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 2*m*AppellF1[(3 + m)/2, -m, 2*(1 + m), (
5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 12*AppellF1[(3 + m)/2,
 -m, 2*(2 + m), (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 8*m*A
ppellF1[(3 + m)/2, -m, 2*(2 + m), (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a +
b*x)/2]^2] + 8*AppellF1[(3 + m)/2, -m, 3 + 2*m, (5 + m)/2, Tan[(a + b*x)/2]
^2, -Tan[(a + b*x)/2]^2] + 8*m*AppellF1[(3 + m)/2, -m, 3 + 2*m, (5 + m)/2,
Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*(-1 + Cos[a + b*x])))
```

Maple [F] time = 0.875, size = 0, normalized size = 0.

$$\int (\cos (bx+a))^2 (\sin (2bx+2a))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a)^m,x)
```

```
[Out] int(cos(b*x+a)^2*sin(2*b*x+2*a)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin (2bx+2a)^m \cos (bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*cos(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sin (2bx+2a)^m \cos (bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

```
[Out] integral(sin(2*b*x + 2*a)^m*cos(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(2\,bx + 2\,a)^m \cos(bx + a)^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*cos(b*x + a)^2, x)
```

3.189 $\int \cos(a+bx)\sin^m(2a+2bx)\,dx$

Optimal. Leaf size=83

$$-\frac{\cos(a+bx)\cot(a+bx)\sin^2(a+bx)^{\frac{1-m}{2}}\sin^m(2a+2bx)\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{m+2}{2},\frac{m+4}{2},\cos^2(a+bx)\right)}{b(m+2)}$$

```
[Out] -((Cos[a + b*x]*Cot[a + b*x]*Hypergeometric2F1[(1 - m)/2, (2 + m)/2, (4 + m
)/2, Cos[a + b*x]^2]*(Sin[a + b*x]^2)^((1 - m)/2)*Sin[2*a + 2*b*x]^m)/(b*(2
 + m)))
```

Rubi [A] time = 0.0608929, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4309, 2576}

$$-\frac{\cos(a+bx)\cot(a+bx)\sin^2(a+bx)^{\frac{1-m}{2}}\sin^m(2a+2bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{m+2}{2};\frac{m+4}{2};\cos^2(a+bx)\right)}{b(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] -((Cos[a + b*x]*Cot[a + b*x]*Hypergeometric2F1[(1 - m)/2, (2 + m)/2, (4 + m
)/2, Cos[a + b*x]^2]*(Sin[a + b*x]^2)^((1 - m)/2)*Sin[2*a + 2*b*x]^m)/(b*(2
 + m)))
```

Rule 4309

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/((e*Cos[a + b*x])^p*Sin[a + b*x]^
p), Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x] /; FreeQ[{a, b, c,
d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int \cos(a+bx)\sin^m(2a+2bx)\,dx = \left(\cos^{-m}(a+bx)\sin^{-m}(a+bx)\sin^m(2a+2bx)\right)\int \cos^{1+m}(a+bx)\sin^m(a+bx)\,dx$$

$$= -\frac{\cos(a+bx)\cot(a+bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{2+m}{2};\frac{4+m}{2};\cos^2(a+bx)\right)\sin^2(a+bx)^{\frac{1-m}{2}}\sin^m(}{b(2+m)}$$

Mathematica [C] time = 0.244453, size = 149, normalized size = 1.8

$$\frac{2^{-m-1}e^{i(a+bx)}\left(-ie^{-2i(a+bx)}\left(-1+e^{4i(a+bx)}\right)\right)^{m+1}\left((2m-1)\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2m+3),\frac{1}{4}(3-2m),e^{4i(a+bx)}\right)+(2\right.}{b\left(4m^2-1\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] (2^(-1 - m)*E^(I*(a + b*x))*(((-I)*(-1 + E^((4*I)*(a + b*x))))/E^((2*I)*(a
+ b*x)))^(1 + m)*((-1 + 2*m)*Hypergeometric2F1[1, (3 + 2*m)/4, (3 - 2*m)/4,
 E^((4*I)*(a + b*x))] + E^((2*I)*(a + b*x))*(1 + 2*m)*Hypergeometric2F1[1,
(5 + 2*m)/4, (5 - 2*m)/4, E^((4*I)*(a + b*x))]))/(b*(-1 + 4*m^2))
```

Maple [F] time = 0.914, size = 0, normalized size = 0.

$$\int \cos \left(bx+a\right) \left(\sin \left(2\,bx+2\,a\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^m,x)
```

```
[Out] int(cos(b*x+a)*sin(2*b*x+2*a)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin \left(2\,bx+2\,a\right)^{m}\cos \left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*cos(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sin \left(2\,bx+2\,a\right)^{m}\cos \left(bx+a\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

```
[Out] integral(sin(2*b*x + 2*a)^m*cos(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin (2\, bx + 2\, a)^{m} \cos (bx + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*cos(b*x + a), x)
```

3.190 $\int \cos^2(a+bx)\sin^3(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=46

$$-\frac{4\cos^9(a+bx)}{9b}+\frac{8\cos^7(a+bx)}{7b}-\frac{4\cos^5(a+bx)}{5b}$$

```
[Out] (-4*Cos[a + b*x]^5)/(5*b) + (8*Cos[a + b*x]^7)/(7*b) - (4*Cos[a + b*x]^9)/(
9*b)
```

Rubi [A] time = 0.0966612, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {4312, 2565, 270}

$$-\frac{4\cos^9(a+bx)}{9b}+\frac{8\cos^7(a+bx)}{7b}-\frac{4\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (-4*Cos[a + b*x]^5)/(5*b) + (8*Cos[a + b*x]^7)/(7*b) - (4*Cos[a + b*x]^9)/(
9*b)
```

Rule 4312

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((f_.)*sin[(a_.) + (b_.)*(x_)])^(
n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_Symbol] :> Dist[2^p/(e^p*f^p), Int[(e
*Cos[a + b*x])^(m + p)*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && IntegerQ[p]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\sin^3(a+bx)\sin^2(2a+2bx)\,dx &= 4\int \cos^4(a+bx)\sin^5(a+bx)\,dx\\ &= -\frac{4\,\text{Subst}\left(\int x^4\left(1-x^2\right)^2\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{4\,\text{Subst}\left(\int \left(x^4-2x^6+x^8\right)\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{4\cos^5(a+bx)}{5b}+\frac{8\cos^7(a+bx)}{7b}-\frac{4\cos^9(a+bx)}{9b}\end{aligned}$$

Mathematica [A] time = 0.163096, size = 37, normalized size = 0.8

$$\frac{\cos^5(a+bx)(220\cos(2(a+bx))-35\cos(4(a+bx))-249)}{630b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (Cos[a + b*x]^5*(-249 + 220*Cos[2*(a + b*x)] - 35*Cos[4*(a + b*x)]))/(630*b
)
```

Maple [A] time = 0.023, size = 69, normalized size = 1.5

$$-\frac{3\cos(bx+a)}{32b}-\frac{\cos(3bx+3a)}{48b}+\frac{\cos(5bx+5a)}{80b}+\frac{\cos(7bx+7a)}{448b}-\frac{\cos(9bx+9a)}{576b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^3*sin(2*b*x+2*a)^2,x)
```

```
[Out] -3/32*cos(b*x+a)/b-1/48*cos(3*b*x+3*a)/b+1/80*cos(5*b*x+5*a)/b+1/448*cos(7*
b*x+7*a)/b-1/576*cos(9*b*x+9*a)/b
```

Maxima [A] time = 1.2228, size = 78, normalized size = 1.7

$$-\frac{35\cos(9bx+9a)-45\cos(7bx+7a)-252\cos(5bx+5a)+420\cos(3bx+3a)+1890\cos(bx+a)}{20160b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] -1/20160*(35*cos(9*b*x + 9*a) - 45*cos(7*b*x + 7*a) - 252*cos(5*b*x + 5*a)
+ 420*cos(3*b*x + 3*a) + 1890*cos(b*x + a))/b
```

Fricas [A] time = 0.490691, size = 95, normalized size = 2.07

$$-\frac{4\left(35\cos(bx+a)^9-90\cos(bx+a)^7+63\cos(bx+a)^5\right)}{315b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] -4/315*(35*cos(b*x + a)^9 - 90*cos(b*x + a)^7 + 63*cos(b*x + a)^5)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**3*sin(2*b*x+2*a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.85772, size = 92, normalized size = 2.

$$-\frac{\cos(9\,bx+9\,a)}{576\,b}+\frac{\cos(7\,bx+7\,a)}{448\,b}+\frac{\cos(5\,bx+5\,a)}{80\,b}-\frac{\cos(3\,bx+3\,a)}{48\,b}-\frac{3\,\cos(bx+a)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] -1/576*cos(9*b*x + 9*a)/b + 1/448*cos(7*b*x + 7*a)/b + 1/80*cos(5*b*x + 5*a
)/b - 1/48*cos(3*b*x + 3*a)/b - 3/32*cos(b*x + a)/b
```

3.191 $\int \sin(a+bx)\sin^n(c+dx)\,dx$

Optimal. Leaf size=293

$$-\frac{2^{-n-1}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\left(1-e^{2ic+2idx}\right)^{-n}\text{Hypergeometric2F1}\left(-n,\frac{b-dn}{2d},\frac{1}{2}\left(\frac{b}{d}-n+2\right),e^{2i(c+dx)}\right)\exp(i(a-cn)+}{b-dn}$$

```
[Out] -((2^(-1 - n)*E^(I*(a - c*n) + I*(b - d*n)*x + I*n*(c + d*x))*(I/E^(I*(c +
d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, (b - d*n)/(2*d), (2 + b/
d - n)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x))^n*(b - d*n))
) - (2^(-1 - n)*E^((-I)*(a + c*n) - I*(b + d*n)*x + I*n*(c + d*x))*(I/E^(I*
(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, -(b + d*n)/(2*d), 1
 - (b + d*n)/(2*d), E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x))^n*
(b + d*n))
```

Rubi [A] time = 0.84838, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4553, 2285, 2253, 2252, 2251}

$$-\frac{2^{-n-1}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\left(1-e^{2ic+2idx}\right)^{-n}\,_2F_1\left(-n,\frac{b-dn}{2d};\frac{1}{2}\left(\frac{b}{d}-n+2\right);e^{2i(c+dx)}\right)\exp(i(a-cn)+ix(b-dn)+in(c}{b-dn}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[c + d*x]^n,x]
```


```
[Out] -((2^(-1 - n)*E^(I*(a - c*n) + I*(b - d*n)*x + I*n*(c + d*x))*(I/E^(I*(c +
d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, (b - d*n)/(2*d), (2 + b/
d - n)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x))^n*(b - d*n))
) - (2^(-1 - n)*E^((-I)*(a + c*n) - I*(b + d*n)*x + I*n*(c + d*x))*(I/E^(I*
(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, -(b + d*n)/(2*d), 1
 - (b + d*n)/(2*d), E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x))^n*
(b + d*n))
```


Rule 4553

```
Int[Sin[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Dist[1/2^(p + q), Int[ExpandIntegrand[(I/E^(I*(c + d*x)) - I*E^(I*(c + d
*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
 b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Rule 2285

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Dist[(a*F^
v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n), Int[u*F^(n*v)*(a
+ b*F^ExpandToSum[w - v, x])^n, x], x] /; FreeQ[{F, a, b, n}, x] &&  !Integ
erQ[n] && LinearQ[{v, w}, x]
```

Rule 2253

```
Int[((a_) + (b_.)*(F_)^((e_.)*(v_)))^(p_)*(G_)^((h_.)*(u_)), x_Symbol] :> I
nt[G^(h*ExpandToSum[u, x])*(a + b*F^(e*ExpandToSum[v, x]))^p, x] /; FreeQ[{
F, G, a, b, e, h, p}, x] && LinearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rule 2252

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Dist[(a + b*F^(e*(c + d*x)))^p/(1 + (b/a)*F^
(e*(c + d*x)))^p, Int[G^(h*(f + g*x))*(1 + (b*F^(e*(c + d*x)))/a)^p, x], x]
 /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x] &&  !(ILtQ[p, 0] || GtQ[a,
0])
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \sin(a+bx)\sin^n(c+dx)\,dx = 2^{-1-n}\int \left(ie^{-ia-ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n - ie^{ia+ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\right)dx$$
$$= \left(i2^{-1-n}\right)\int e^{-ia-ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n dx - \left(i2^{-1-n}\right)\int e^{ia+ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n dx$$
$$= \left(i2^{-1-n}e^{in(c+dx)}\left(i-ie^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\right)\int e^{-ia-ibx-in(c+dx)}\left(i-ie^{2ic+2idx}\right)^n$$
$$= -\left(\left(i2^{-1-n}e^{in(c+dx)}\left(i-ie^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\right)\int e^{i(a-cn)+i(b-dn)x}\left(i-ie^{2ic+2idx}\right)\right.$$
$$= -\left(\left(i2^{-1-n}e^{in(c+dx)}\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\right)\int e^{i(a-cn)+i(b-dn)x}\left(1-e^{2ic+2idx}\right)\right.$$
$$= -\frac{2^{-1-n}\exp(i(a-cn)+i(b-dn)x+in(c+dx))\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n {}_2F}{b-dn}$$

Mathematica [A] time = 0.857461, size = 209, normalized size = 0.71

$$\frac{2^{-n-1}e^{-ix(b+d)}\left(-1+e^{2i(c+dx)}\right)\left(-ie^{-i(c+dx)}\left(-1+e^{2i(c+dx)}\right)\right)^n\left(e^{idx}(\cos(a)-i\sin(a))(b-dn)\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(-\frac{b}{d}\right.\right.\right.}{(b-d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[a + b*x]*Sin[c + d*x]^n,x]
```

```
[Out] (2^(-1 - n)*(-1 + E^((2*I)*(c + d*x)))*(((-I)*(-1 + E^((2*I)*(c + d*x))))/E
^(I*(c + d*x)))^n*(E^(I*d*x)*(b - d*n)*Hypergeometric2F1[1, (2 - b/d + n)/2
, -(b + d*(-2 + n))/(2*d), E^((2*I)*(c + d*x))]*(Cos[a] - I*Sin[a]) + E^(I*
(2*b + d)*x)*(b + d*n)*Hypergeometric2F1[1, (b + d*(2 + n))/(2*d), (2 + b/d
 - n)/2, E^((2*I)*(c + d*x))]*(Cos[a] + I*Sin[a])))/(E^(I*(b + d)*x)*(b - d
*n)*(b + d*n))
```

Maple [F] time = 1.304, size = 0, normalized size = 0.

$$\int \sin(bx+a)\left(\sin(dx+c)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(d*x+c)^n,x)
```

```
[Out] int(sin(b*x+a)*sin(d*x+c)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(dx+c)^n \sin(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^n*sin(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sin(dx+c)^n \sin(bx+a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] integral(sin(d*x + c)^n*sin(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(dx+c)^n \sin(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*sin(b*x + a), x)
```

3.192 $\int \sin(a+bx)\sin^3(c+dx)\,dx$

Optimal. Leaf size=91

$$-\frac{\sin(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\sin(a+x(b-d)-c)}{8(b-d)}-\frac{3\sin(a+x(b+d)+c)}{8(b+d)}+\frac{\sin(a+x(b+3d)+3c)}{8(b+3d)}$$

```
[Out] -Sin[a - 3*c + (b - 3*d)*x]/(8*(b - 3*d)) + (3*Sin[a - c + (b - d)*x])/(8*(
b - d)) - (3*Sin[a + c + (b + d)*x])/(8*(b + d)) + Sin[a + 3*c + (b + 3*d)*
x]/(8*(b + 3*d))
```

Rubi [A] time = 0.0764363, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4569, 2637}

$$-\frac{\sin(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\sin(a+x(b-d)-c)}{8(b-d)}-\frac{3\sin(a+x(b+d)+c)}{8(b+d)}+\frac{\sin(a+x(b+3d)+3c)}{8(b+3d)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[c + d*x]^3,x]
```

```
[Out] -Sin[a - 3*c + (b - 3*d)*x]/(8*(b - 3*d)) + (3*Sin[a - c + (b - d)*x])/(8*(
b - d)) - (3*Sin[a + c + (b + d)*x])/(8*(b + d)) + Sin[a + 3*c + (b + 3*d)*
x]/(8*(b + 3*d))
```

Rule 4569

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\sin^3(c+dx)\,dx &= \int \left(-\frac{1}{8}\cos(a-3c+(b-3d)x)+\frac{3}{8}\cos(a-c+(b-d)x)-\frac{3}{8}\cos(a+c+(b+d)x)+\frac{1}{8}\text{c}\right.\\ &= -\left(\frac{1}{8}\int \cos(a-3c+(b-3d)x)\,dx\right)+\frac{1}{8}\int \cos(a+3c+(b+3d)x)\,dx+\frac{3}{8}\int \cos(a-c+\\ &= -\frac{\sin(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\sin(a-c+(b-d)x)}{8(b-d)}-\frac{3\sin(a+c+(b+d)x)}{8(b+d)}+\frac{\sin(a+3}{8(}\end{aligned}$$

Mathematica [A] time = 0.511428, size = 86, normalized size = 0.95

$$\frac{1}{8}\left(-\frac{\sin(a+bx-3c-3dx)}{b-3d}+\frac{3\sin(a+bx-c-dx)}{b-d}+\frac{\sin(a+bx+3c+3dx)}{b+3d}-\frac{3\sin(a+x(b+d)+c)}{b+d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[c + d*x]^3,x]
```

```
[Out] (-(Sin[a - 3*c + b*x - 3*d*x]/(b - 3*d)) + (3*Sin[a - c + b*x - d*x])/(b -
d) + Sin[a + 3*c + b*x + 3*d*x]/(b + 3*d) - (3*Sin[a + c + (b + d)*x])/(b +
 d))/8
```

Maple [A] time = 0.027, size = 84, normalized size = 0.9

$$-\frac{\sin\left(a-3\,c+\left(b-3\,d\right)x\right)}{8\,b-24\,d}+\frac{3\,\sin\left(a-c+\left(b-d\right)x\right)}{8\,b-8\,d}-\frac{3\,\sin\left(a+c+\left(b+d\right)x\right)}{8\,b+8\,d}+\frac{\sin\left(a+3\,c+\left(b+3\,d\right)x\right)}{8\,b+24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(d*x+c)^3,x)
```

```
[Out] -1/8*sin(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sin(a-c+(b-d)*x)/(b-d)-3/8*sin(a+c+(b
+d)*x)/(b+d)+1/8*sin(a+3*c+(b+3*d)*x)/(b+3*d)
```

Maxima [B] time = 1.60639, size = 1237, normalized size = 13.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((b^3*sin(3*c) - 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) + 3*d^3*sin(3*c))*
cos((b + 3*d)*x + a + 6*c) - (b^3*sin(3*c) - 3*b^2*d*sin(3*c) - b*d^2*sin(3
*c) + 3*d^3*sin(3*c))*cos((b + 3*d)*x + a) - 3*(b^3*sin(3*c) - b^2*d*sin(3*
c) - 9*b*d^2*sin(3*c) + 9*d^3*sin(3*c))*cos((b + d)*x + a + 4*c) + 3*(b^3*s
in(3*c) - b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) + 9*d^3*sin(3*c))*cos((b + d)*x
 + a - 2*c) - 3*(b^3*sin(3*c) + b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) - 9*d^3*s
in(3*c))*cos(-(b - d)*x - a + 4*c) + 3*(b^3*sin(3*c) + b^2*d*sin(3*c) - 9*b
*d^2*sin(3*c) - 9*d^3*sin(3*c))*cos(-(b - d)*x - a - 2*c) + (b^3*sin(3*c) +
 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) - 3*d^3*sin(3*c))*cos(-(b - 3*d)*x - a +
 6*c) - (b^3*sin(3*c) + 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) - 3*d^3*sin(3*c))
*cos(-(b - 3*d)*x - a) - (b^3*cos(3*c) - 3*b^2*d*cos(3*c) - b*d^2*cos(3*c)
+ 3*d^3*cos(3*c))*sin((b + 3*d)*x + a + 6*c) - (b^3*cos(3*c) - 3*b^2*d*cos(
3*c) - b*d^2*cos(3*c) + 3*d^3*cos(3*c))*sin((b + 3*d)*x + a) + 3*(b^3*cos(3
*c) - b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) + 9*d^3*cos(3*c))*sin((b + d)*x + a
 + 4*c) + 3*(b^3*cos(3*c) - b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) + 9*d^3*cos(3
*c))*sin((b + d)*x + a - 2*c) + 3*(b^3*cos(3*c) + b^2*d*cos(3*c) - 9*b*d^2*
cos(3*c) - 9*d^3*cos(3*c))*sin(-(b - d)*x - a + 4*c) + 3*(b^3*cos(3*c) + b^
2*d*cos(3*c) - 9*b*d^2*cos(3*c) - 9*d^3*cos(3*c))*sin(-(b - d)*x - a - 2*c)
 - (b^3*cos(3*c) + 3*b^2*d*cos(3*c) - b*d^2*cos(3*c) - 3*d^3*cos(3*c))*sin(
-(b - 3*d)*x - a + 6*c) - (b^3*cos(3*c) + 3*b^2*d*cos(3*c) - b*d^2*cos(3*c)
 - 3*d^3*cos(3*c))*sin(-(b - 3*d)*x - a))/(b^4*cos(3*c)^2 + b^4*sin(3*c)^2
+ 9*(cos(3*c)^2 + sin(3*c)^2)*d^4 - 10*(b^2*cos(3*c)^2 + b^2*sin(3*c)^2)*d^
2)
```

Fricas [A] time = 0.507553, size = 267, normalized size = 2.93

$$-\frac{3\left(\left(b^2d-d^3\right)\cos\left(dx+c\right)^3-\left(b^2d-3\,d^3\right)\cos\left(dx+c\right)\right)\sin\left(bx+a\right)-\left(\left(b^3-bd^2\right)\cos\left(bx+a\right)\cos\left(dx+c\right)^2-\left(b^3-7\right.\right.}{b^4-10\,b^2d^2+9\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -(3*((b^2*d - d^3)*cos(d*x + c)^3 - (b^2*d - 3*d^3)*cos(d*x + c))*sin(b*x +
 a) - ((b^3 - b*d^2)*cos(b*x + a)*cos(d*x + c)^2 - (b^3 - 7*b*d^2)*cos(b*x
+ a))*sin(d*x + c))/(b^4 - 10*b^2*d^2 + 9*d^4)
```

Sympy [A] time = 133.383, size = 921, normalized size = 10.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)**3,x)
```

```
[Out] Piecewise((x*sin(a)*sin(c)**3, Eq(b, 0) & Eq(d, 0)), (x*sin(a - 3*d*x)*sin(
c + d*x)**3/8 - 3*x*sin(a - 3*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 - 3*x*sin
(c + d*x)**2*cos(a - 3*d*x)*cos(c + d*x)/8 + x*cos(a - 3*d*x)*cos(c + d*x)*
*3/8 + sin(a - 3*d*x)*cos(c + d*x)**3/(8*d) + 7*sin(c + d*x)**3*cos(a - 3*d
*x)/(24*d) + sin(c + d*x)*cos(a - 3*d*x)*cos(c + d*x)**2/(4*d), Eq(b, -3*d)
), (3*x*sin(a - d*x)*sin(c + d*x)**3/8 + 3*x*sin(a - d*x)*sin(c + d*x)*cos(
c + d*x)**2/8 - 3*x*sin(c + d*x)**2*cos(a - d*x)*cos(c + d*x)/8 - 3*x*cos(a
 - d*x)*cos(c + d*x)**3/8 + 3*sin(a - d*x)*cos(c + d*x)**3/(8*d) + 5*sin(c
+ d*x)**3*cos(a - d*x)/(8*d) + 3*sin(c + d*x)*cos(a - d*x)*cos(c + d*x)**2/
(4*d), Eq(b, -d)), (3*x*sin(a + d*x)*sin(c + d*x)**3/8 + 3*x*sin(a + d*x)*s
in(c + d*x)*cos(c + d*x)**2/8 + 3*x*sin(c + d*x)**2*cos(a + d*x)*cos(c + d*
x)/8 + 3*x*cos(a + d*x)*cos(c + d*x)**3/8 + 3*sin(a + d*x)*cos(c + d*x)**3/
(8*d) - 5*sin(c + d*x)**3*cos(a + d*x)/(8*d) - 3*sin(c + d*x)*cos(a + d*x)*
cos(c + d*x)**2/(4*d), Eq(b, d)), (x*sin(a + 3*d*x)*sin(c + d*x)**3/8 - 3*x
*sin(a + 3*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 + 3*x*sin(c + d*x)**2*cos(a
+ 3*d*x)*cos(c + d*x)/8 - x*cos(a + 3*d*x)*cos(c + d*x)**3/8 + sin(a + 3*d*
x)*cos(c + d*x)**3/(8*d) - 7*sin(c + d*x)**3*cos(a + 3*d*x)/(24*d) - sin(c
+ d*x)*cos(a + 3*d*x)*cos(c + d*x)**2/(4*d), Eq(b, 3*d)), (-b**3*sin(c + d*
x)**3*cos(a + b*x)/(b**4 - 10*b**2*d**2 + 9*d**4) + 3*b**2*d*sin(a + b*x)*s
in(c + d*x)**2*cos(c + d*x)/(b**4 - 10*b**2*d**2 + 9*d**4) + 7*b*d**2*sin(c
 + d*x)**3*cos(a + b*x)/(b**4 - 10*b**2*d**2 + 9*d**4) + 6*b*d**2*sin(c + d
*x)*cos(a + b*x)*cos(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4) - 9*d**3*si
n(a + b*x)*sin(c + d*x)**2*cos(c + d*x)/(b**4 - 10*b**2*d**2 + 9*d**4) - 6*
d**3*sin(a + b*x)*cos(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4), True))
```

Giac [A] time = 1.13697, size = 113, normalized size = 1.24

$$\frac{\sin\left(bx + 3\,dx + a + 3\,c\right)}{8\,(b + 3\,d)} - \frac{3\,\sin\left(bx + dx + a + c\right)}{8\,(b + d)} + \frac{3\,\sin\left(bx - dx + a - c\right)}{8\,(b - d)} - \frac{\sin\left(bx - 3\,dx + a - 3\,c\right)}{8\,(b - 3\,d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*sin(b*x + 3*d*x + a + 3*c)/(b + 3*d) - 3/8*sin(b*x + d*x + a + c)/(b +
d) + 3/8*sin(b*x - d*x + a - c)/(b - d) - 1/8*sin(b*x - 3*d*x + a - 3*c)/(b
 - 3*d)
```

3.193 $\int \sin(a+bx)\sin^2(c+dx)\,dx$

Optimal. Leaf size=62

$$\frac{\cos(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\cos(a+x(b+2d)+2c)}{4(b+2d)}-\frac{\cos(a+bx)}{2b}$$

```
[Out] -Cos[a + b*x]/(2*b) + Cos[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) + Cos[a + 2*
c + (b + 2*d)*x]/(4*(b + 2*d))
```

Rubi [A] time = 0.0533329, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4569, 2638}

$$\frac{\cos(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\cos(a+x(b+2d)+2c)}{4(b+2d)}-\frac{\cos(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[c + d*x]^2,x]
```


```
[Out] -Cos[a + b*x]/(2*b) + Cos[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) + Cos[a + 2*
c + (b + 2*d)*x]/(4*(b + 2*d))
```


Rule 4569

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\sin^2(c+dx)\,dx &= \int \left(\frac{1}{2}\sin(a+bx)-\frac{1}{4}\sin(a-2c+(b-2d)x)-\frac{1}{4}\sin(a+2c+(b+2d)x)\right)dx\\ &= -\left(\frac{1}{4}\int \sin(a-2c+(b-2d)x)\,dx\right)-\frac{1}{4}\int \sin(a+2c+(b+2d)x)\,dx+\frac{1}{2}\int \sin(a+\\ &= -\frac{\cos(a+bx)}{2b}+\frac{\cos(a-2c+(b-2d)x)}{4(b-2d)}+\frac{\cos(a+2c+(b+2d)x)}{4(b+2d)}\end{aligned}$$

Mathematica [A] time = 0.714395, size = 69, normalized size = 1.11

$$\frac{1}{4}\left(\frac{\cos(a+bx-2c-2dx)}{b-2d}+\frac{\cos(a+bx+2c+2dx)}{b+2d}+\frac{2\sin(a)\sin(bx)}{b}-\frac{2\cos(a)\cos(bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[c + d*x]^2,x]
```

```
[Out] ((-2*Cos[a]*Cos[b*x])/b + Cos[a - 2*c + b*x - 2*d*x]/(b - 2*d) + Cos[a + 2*
c + b*x + 2*d*x]/(b + 2*d) + (2*Sin[a]*Sin[b*x])/b)/4
```

Maple [A] time = 0.016, size = 57, normalized size = 0.9

$$-\frac{\cos(bx+a)}{2b}+\frac{\cos(a-2c+(b-2d)x)}{4b-8d}+\frac{\cos(a+2c+(b+2d)x)}{4b+8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(d*x+c)^2,x)
```

```
[Out] -1/2*cos(b*x+a)/b+1/4*cos(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*cos(a+2*c+(b+2*d)*x)
/(b+2*d)
```

Maxima [B] time = 1.33922, size = 559, normalized size = 9.02

$$\frac{\left(b^2\cos(2c)-2bd\cos(2c)\right)\cos((b+2d)x+a+4c)+\left(b^2\cos(2c)-2bd\cos(2c)\right)\cos((b+2d)x+a)+\left(b^2\cos(2c)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/8*((b^2*cos(2*c) - 2*b*d*cos(2*c))*cos((b + 2*d)*x + a + 4*c) + (b^2*cos(
2*c) - 2*b*d*cos(2*c))*cos((b + 2*d)*x + a) + (b^2*cos(2*c) + 2*b*d*cos(2*c
))*cos(-(b - 2*d)*x - a + 4*c) + (b^2*cos(2*c) + 2*b*d*cos(2*c))*cos(-(b -
2*d)*x - a) - 2*(b^2*cos(2*c) - 4*d^2*cos(2*c))*cos(b*x + a + 2*c) - 2*(b^2
*cos(2*c) - 4*d^2*cos(2*c))*cos(b*x + a - 2*c) + (b^2*sin(2*c) - 2*b*d*sin(
2*c))*sin((b + 2*d)*x + a + 4*c) - (b^2*sin(2*c) - 2*b*d*sin(2*c))*sin((b +
 2*d)*x + a) + (b^2*sin(2*c) + 2*b*d*sin(2*c))*sin(-(b - 2*d)*x - a + 4*c)
- (b^2*sin(2*c) + 2*b*d*sin(2*c))*sin(-(b - 2*d)*x - a) - 2*(b^2*sin(2*c) -
 4*d^2*sin(2*c))*sin(b*x + a + 2*c) + 2*(b^2*sin(2*c) - 4*d^2*sin(2*c))*sin
(b*x + a - 2*c))/(b^3*cos(2*c)^2 + b^3*sin(2*c)^2 - 4*(b*cos(2*c)^2 + b*sin
(2*c)^2)*d^2)
```

Fricas [A] time = 0.494425, size = 171, normalized size = 2.76

$$\frac{b^2\cos(bx+a)\cos(dx+c)^2+2bd\cos(dx+c)\sin(bx+a)\sin(dx+c)-\left(b^2-2d^2\right)\cos(bx+a)}{b^3-4bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (b^2*cos(b*x + a)*cos(d*x + c)^2 + 2*b*d*cos(d*x + c)*sin(b*x + a)*sin(d*x
+ c) - (b^2 - 2*d^2)*cos(b*x + a))/(b^3 - 4*b*d^2)
```

Sympy [A] time = 10.7247, size = 401, normalized size = 6.47

$$\begin{cases} x\sin(a)\sin^2(c) \\ \left(\frac{x\sin^2(c+dx)}{2}+\frac{x\cos^2(c+dx)}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)\sin(a) \\ \frac{x\sin(a-2dx)\sin^2(c+dx)}{4}-\frac{x\sin(a-2dx)\cos^2(c+dx)}{4}-\frac{x\sin(c+dx)\cos(a-2dx)\cos(c+dx)}{2}+\frac{\sin(a-2dx)\sin(c+dx)\cos(c+dx)}{4d}+\frac{\sin^2(c+dx)\cos(a-}{2d} \\ \frac{x\sin(a+2dx)\sin^2(c+dx)}{4}-\frac{x\sin(a+2dx)\cos^2(c+dx)}{4}+\frac{x\sin(c+dx)\cos(a+2dx)\cos(c+dx)}{2}-\frac{3\sin^2(c+dx)\cos(a+2dx)}{8d}-\frac{\cos(a+2dx)\cos^2(c+dx)}{8d} \\ -\frac{b^2\sin^2(c+dx)\cos(a+bx)}{b^3-4bd^2}+\frac{2bd\sin(a+bx)\sin(c+dx)\cos(c+dx)}{b^3-4bd^2}+\frac{2d^2\sin^2(c+dx)\cos(a+bx)}{b^3-4bd^2}+\frac{2d^2\cos(a+bx)\cos^2(c+dx)}{b^3-4bd^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)**2,x)
```

```
[Out] Piecewise((x*sin(a)*sin(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**2/2
+ x*cos(c + d*x)**2/2 - sin(c + d*x)*cos(c + d*x)/(2*d))*sin(a), Eq(b, 0)),
 (x*sin(a - 2*d*x)*sin(c + d*x)**2/4 - x*sin(a - 2*d*x)*cos(c + d*x)**2/4 -
 x*sin(c + d*x)*cos(a - 2*d*x)*cos(c + d*x)/2 + sin(a - 2*d*x)*sin(c + d*x)
*cos(c + d*x)/(4*d) + sin(c + d*x)**2*cos(a - 2*d*x)/(2*d), Eq(b, -2*d)), (
x*sin(a + 2*d*x)*sin(c + d*x)**2/4 - x*sin(a + 2*d*x)*cos(c + d*x)**2/4 + x
*sin(c + d*x)*cos(a + 2*d*x)*cos(c + d*x)/2 - 3*sin(c + d*x)**2*cos(a + 2*d
*x)/(8*d) - cos(a + 2*d*x)*cos(c + d*x)**2/(8*d), Eq(b, 2*d)), (-b**2*sin(c
 + d*x)**2*cos(a + b*x)/(b**3 - 4*b*d**2) + 2*b*d*sin(a + b*x)*sin(c + d*x)
*cos(c + d*x)/(b**3 - 4*b*d**2) + 2*d**2*sin(c + d*x)**2*cos(a + b*x)/(b**3
 - 4*b*d**2) + 2*d**2*cos(a + b*x)*cos(c + d*x)**2/(b**3 - 4*b*d**2), True)
)
```

Giac [A] time = 1.11569, size = 76, normalized size = 1.23

$$\frac{\cos(bx+2\,dx+a+2\,c)}{4\,(b+2\,d)}+\frac{\cos(bx-2\,dx+a-2\,c)}{4\,(b-2\,d)}-\frac{\cos(bx+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*cos(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/4*cos(b*x - 2*d*x + a - 2*c)/(
b - 2*d) - 1/2*cos(b*x + a)/b
```

3.194 $\int \sin(a+bx)\sin(c+dx)\,dx$

Optimal. Leaf size=43

$$\frac{\sin(a+x(b-d)-c)}{2(b-d)}-\frac{\sin(a+x(b+d)+c)}{2(b+d)}$$

```
[Out] Sin[a - c + (b - d)*x]/(2*(b - d)) - Sin[a + c + (b + d)*x]/(2*(b + d))
```

Rubi [A] time = 0.0352663, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4569, 2637}

$$\frac{\sin(a+x(b-d)-c)}{2(b-d)}-\frac{\sin(a+x(b+d)+c)}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[c + d*x],x]
```

```
[Out] Sin[a - c + (b - d)*x]/(2*(b - d)) - Sin[a + c + (b + d)*x]/(2*(b + d))
```

Rule 4569

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\sin(c+dx)\,dx &= \int \left(\frac{1}{2}\cos(a-c+(b-d)x)-\frac{1}{2}\cos(a+c+(b+d)x)\right)dx\\ &= \frac{1}{2}\int \cos(a-c+(b-d)x)\,dx-\frac{1}{2}\int \cos(a+c+(b+d)x)\,dx\\ &= \frac{\sin(a-c+(b-d)x)}{2(b-d)}-\frac{\sin(a+c+(b+d)x)}{2(b+d)}\end{aligned}$$

Mathematica [A] time = 0.187207, size = 43, normalized size = 1.

$$\frac{\sin(a+x(b-d)-c)}{2(b-d)}-\frac{\sin(a+x(b+d)+c)}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[c + d*x],x]
```

```
[Out] Sin[a - c + (b - d)*x]/(2*(b - d)) - Sin[a + c + (b + d)*x]/(2*(b + d))
```

Maple [A] time = 0.016, size = 40, normalized size = 0.9

$$\frac{\sin\left(a-c+\left(b-d\right)x\right)}{2b-2d}-\frac{\sin\left(a+c+\left(b+d\right)x\right)}{2b+2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(d*x+c),x)
```

```
[Out] 1/2*sin(a-c+(b-d)*x)/(b-d)-1/2*sin(a+c+(b+d)*x)/(b+d)
```

Maxima [A] time = 1.24925, size = 54, normalized size = 1.26

$$-\frac{\sin\left(bx+dx+a+c\right)}{2\left(b+d\right)}-\frac{\sin\left(-bx+dx-a+c\right)}{2\left(b-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*sin(b*x + d*x + a + c)/(b + d) - 1/2*sin(-b*x + d*x - a + c)/(b - d)
```

Fricas [A] time = 0.475341, size = 99, normalized size = 2.3

$$\frac{d\cos\left(dx+c\right)\sin\left(bx+a\right)-b\cos\left(bx+a\right)\sin\left(dx+c\right)}{b^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] (d*cos(d*x + c)*sin(b*x + a) - b*cos(b*x + a)*sin(d*x + c))/(b^2 - d^2)
```

Sympy [A] time = 3.6131, size = 153, normalized size = 3.56

$$\begin{cases} x\sin\left(a\right)\sin\left(c\right) & \text{for}\ b=0\wedge d=0 \\ \frac{x\sin\left(a-dx\right)\sin\left(c+dx\right)}{2}-\frac{x\cos\left(a-dx\right)\cos\left(c+dx\right)}{2}+\frac{\sin\left(c+dx\right)\cos\left(a-dx\right)}{2d} & \text{for}\ b=-d \\ \frac{x\sin\left(a+dx\right)\sin\left(c+dx\right)}{2}+\frac{x\cos\left(a+dx\right)\cos\left(c+dx\right)}{2}-\frac{\sin\left(a+dx\right)\cos\left(c+dx\right)}{2d} & \text{for}\ b=d \\ -\frac{b\sin\left(c+dx\right)\cos\left(a+bx\right)}{b^2-d^2}+\frac{d\sin\left(a+bx\right)\cos\left(c+dx\right)}{b^2-d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((x*sin(a)*sin(c), Eq(b, 0) & Eq(d, 0)), (x*sin(a - d*x)*sin(c + d
*x)/2 - x*cos(a - d*x)*cos(c + d*x)/2 + sin(c + d*x)*cos(a - d*x)/(2*d), Eq
(b, -d)), (x*sin(a + d*x)*sin(c + d*x)/2 + x*cos(a + d*x)*cos(c + d*x)/2 -
sin(a + d*x)*cos(c + d*x)/(2*d), Eq(b, d)), (-b*sin(c + d*x)*cos(a + b*x)/(
b**2 - d**2) + d*sin(a + b*x)*cos(c + d*x)/(b**2 - d**2), True))
```

Giac [A] time = 1.14063, size = 54, normalized size = 1.26

$$-\frac{\sin\left(bx+dx+a+c\right)}{2\left(b+d\right)}+\frac{\sin\left(bx-dx+a-c\right)}{2\left(b-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -1/2*sin(b*x + d*x + a + c)/(b + d) + 1/2*sin(b*x - d*x + a - c)/(b - d)
```

3.195 $\int \csc(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\sin(a-c)\log(\sin(bx+c))}{b}+x\cos(a-c)$$

```
[Out] x*Cos[a - c] + (Log[Sin[c + b*x]]*Sin[a - c])/b
```

Rubi [A] time = 0.0274765, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4582, 3475, 8}

$$\frac{\sin(a-c)\log(\sin(bx+c))}{b}+x\cos(a-c)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + b*x]*Sin[a + b*x],x]
```

```
[Out] x*Cos[a - c] + (Log[Sin[c + b*x]]*Sin[a - c])/b
```

Rule 4582

```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Sin[v - w], Int[Cot[w]*Csc[w]^
(n - 1), x], x] + Dist[Cos[v - w], Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \csc(c+bx)\sin(a+bx)\,dx &= \cos(a-c)\int 1\,dx+\sin(a-c)\int \cot(c+bx)\,dx\\ &= x\cos(a-c)+\frac{\log(\sin(c+bx))\sin(a-c)}{b}\end{aligned}$$

Mathematica [A] time = 0.140974, size = 26, normalized size = 1.

$$\frac{\sin(a-c)\log(\sin(bx+c))}{b}+x\cos(a-c)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + b*x]*Sin[a + b*x],x]
```

```
[Out] x*Cos[a - c] + (Log[Sin[c + b*x]]*Sin[a - c])/b
```

Maple [B] time = 0.201, size = 325, normalized size = 12.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+c)*sin(b*x+a),x)
```

```
[Out] 1/2/b/(cos(c)^2+sin(c)^2)/(cos(a)^2+sin(a)^2)*ln(1+tan(b*x+a)^2)*cos(a)*sin
(c)-1/2/b/(cos(c)^2+sin(c)^2)/(cos(a)^2+sin(a)^2)*ln(1+tan(b*x+a)^2)*sin(a)
*cos(c)+1/b/(cos(c)^2+sin(c)^2)/(cos(a)^2+sin(a)^2)*cos(a)*cos(c)*arctan(ta
n(b*x+a))+1/b/(cos(c)^2+sin(c)^2)/(cos(a)^2+sin(a)^2)*sin(a)*sin(c)*arctan(
tan(b*x+a))-1/b/(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^2+cos(c)^2*sin(a)^2+sin(
a)^2*sin(c)^2)*ln(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*sin(a)*sin(c)+cos(a)*
sin(c)-sin(a)*cos(c))*cos(a)*sin(c)+1/b/(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^
2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)*ln(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+
a)*sin(a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c))*sin(a)*cos(c)
```

Maxima [B] time = 1.3371, size = 146, normalized size = 5.62

$$\frac{2\,bx\cos\left(-a+c\right)-\log\left(\cos\left(bx\right)^{2}+2\,\cos\left(bx\right)\cos\left(c\right)+\cos\left(c\right)^{2}+\sin\left(bx\right)^{2}-2\,\sin\left(bx\right)\sin\left(c\right)+\sin\left(c\right)^{2}\right)\sin\left(-a+c\right)-\mathrm{l}\cdots}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x*cos(-a + c) - log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + si
n(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2)*sin(-a + c) - log(cos(b*x)^2 - 2*c
os(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2)*sin(
-a + c))/b
```

Fricas [A] time = 0.495983, size = 77, normalized size = 2.96

$$\frac{bx\cos\left(-a+c\right)-\log\left(\frac{1}{2}\,\sin\left(bx+c\right)\right)\sin\left(-a+c\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*x*cos(-a + c) - log(1/2*sin(b*x + c))*sin(-a + c))/b
```

Sympy [B] time = 18.4057, size = 335, normalized size = 12.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)*sin(b*x+a),x)
```

```
[Out] Piecewise((0, Eq(b, 0) & (Eq(b, 0) | Eq(c, 0))), (x, Eq(c, 0)), (-b*x*tan(c
/2)**2/(b*tan(c/2)**2 + b) + b*x/(b*tan(c/2)**2 + b) - 2*log(tan(c/2) + tan
(b*x/2))*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b*x/2) - 1/tan(c/2))*tan(
c/2)/(b*tan(c/2)**2 + b) + 2*log(tan(b*x/2)**2 + 1)*tan(c/2)/(b*tan(c/2)**2
 + b), True))*cos(a) + Piecewise((zoo*x, Eq(b, 0) & Eq(c, 0)), (x/sin(c), E
q(b, 0)), (log(sin(b*x))/b, Eq(c, 0)), (2*b*x*tan(c/2)/(b*tan(c/2)**2 + b)
- log(tan(c/2) + tan(b*x/2))*tan(c/2)**2/(b*tan(c/2)**2 + b) + log(tan(c/2)
 + tan(b*x/2))/(b*tan(c/2)**2 + b) - log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)*
*2/(b*tan(c/2)**2 + b) + log(tan(b*x/2) - 1/tan(c/2))/(b*tan(c/2)**2 + b) +
 log(tan(b*x/2)**2 + 1)*tan(c/2)**2/(b*tan(c/2)**2 + b) - log(tan(b*x/2)**2
 + 1)/(b*tan(c/2)**2 + b), True))*sin(a)
```

Giac [B] time = 1.2109, size = 319, normalized size = 12.27

$$\frac{\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}a\right)^2+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}c\right)^2+1\right)(bx+c)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1}-\frac{2\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}c\right)\right)\log\left(\tan\cdots\right.}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] ((tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c) - tan(
1/2*c)^2 + 1)*(b*x + c)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2
*c)^2 + 1) - 2*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2
*a) - tan(1/2*c))*log(tan(1/2*b*x + 1/2*c)^2 + 1)/(tan(1/2*a)^2*tan(1/2*c)^
2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + 2*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2
*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2*c)))/
(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1))/b
```

3.196 $\int \csc^2(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=36

$$-\frac{\cos(a-c)\tanh^{-1}(\cos(bx+c))}{b}-\frac{\sin(a-c)\csc(bx+c)}{b}$$

```
[Out] -((ArcTanh[Cos[c + b*x]]*Cos[a - c])/b) - (Csc[c + b*x]*Sin[a - c])/b
```

Rubi [A] time = 0.0327012, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4582, 2606, 8, 3770}

$$-\frac{\cos(a-c)\tanh^{-1}(\cos(bx+c))}{b}-\frac{\sin(a-c)\csc(bx+c)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -((ArcTanh[Cos[c + b*x]]*Cos[a - c])/b) - (Csc[c + b*x]*Sin[a - c])/b
```

Rule 4582

```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Sin[v - w], Int[Cot[w]*Csc[w]^
(n - 1), x], x] + Dist[Cos[v - w], Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \csc^2(c+bx)\sin(a+bx)\,dx &= \cos(a-c)\int \csc(c+bx)\,dx+\sin(a-c)\int \cot(c+bx)\csc(c+bx)\,dx\\
&= -\frac{\tanh^{-1}(\cos(c+bx))\cos(a-c)}{b}-\frac{\sin(a-c)\,\text{Subst}\left(\int 1\,dx,x,\csc(c+bx)\right)}{b}\\
&= -\frac{\tanh^{-1}(\cos(c+bx))\cos(a-c)}{b}-\frac{\csc(c+bx)\sin(a-c)}{b}
\end{aligned}$$

Mathematica [C] time = 0.0968568, size = 90, normalized size = 2.5

$$-\frac{\sin(a-c)\csc(bx+c)}{b}-\frac{2i\cos(a-c)\tan^{-1}\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{\sin(c)\cos\left(\frac{bx}{2}\right)+i\cos(c)\cos\left(\frac{bx}{2}\right)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + b*x]^2*Sin[a + b*x],x]
```

```
[Out] ((-2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c]*Cos[(b*x)/2] - Sin[c]*Sin[(b*x)
/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b - (Csc[c
 + b*x]*Sin[a - c])/b
```

Maple [B] time = 0.446, size = 890, normalized size = 24.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+c)^2*sin(b*x+a),x)
```

```
[Out] -8/b/(-4*cos(a)^2*cos(c)^2-4*cos(a)^2*sin(c)^2-4*cos(c)^2*sin(a)^2-4*sin(a)
^2*sin(c)^2)/(-cos(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+cos(c)*sin(a)*tan(1/2*b*x
+1/2*a)^2+2*tan(1/2*b*x+1/2*a)*cos(a)*cos(c)+2*tan(1/2*b*x+1/2*a)*sin(a)*si
n(c)+cos(a)*sin(c)-sin(a)*cos(c))*tan(1/2*b*x+1/2*a)*cos(a)*cos(c)-8/b/(-4*
cos(a)^2*cos(c)^2-4*cos(a)^2*sin(c)^2-4*cos(c)^2*sin(a)^2-4*sin(a)^2*sin(c)
^2)/(-cos(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+cos(c)*sin(a)*tan(1/2*b*x+1/2*a)^2
+2*tan(1/2*b*x+1/2*a)*cos(a)*cos(c)+2*tan(1/2*b*x+1/2*a)*sin(a)*sin(c)+cos(
a)*sin(c)-sin(a)*cos(c))*tan(1/2*b*x+1/2*a)*sin(a)*sin(c)-8/b/(-4*cos(a)^2*
cos(c)^2-4*cos(a)^2*sin(c)^2-4*cos(c)^2*sin(a)^2-4*sin(a)^2*sin(c)^2)/(-cos
(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+cos(c)*sin(a)*tan(1/2*b*x+1/2*a)^2+2*tan(1/
2*b*x+1/2*a)*cos(a)*cos(c)+2*tan(1/2*b*x+1/2*a)*sin(a)*sin(c)+cos(a)*sin(c)
-sin(a)*cos(c))*cos(a)*sin(c)+8/b/(-4*cos(a)^2*cos(c)^2-4*cos(a)^2*sin(c)^2
-4*cos(c)^2*sin(a)^2-4*sin(a)^2*sin(c)^2)/(-cos(a)*sin(c)*tan(1/2*b*x+1/2*a
)^2+cos(c)*sin(a)*tan(1/2*b*x+1/2*a)^2+2*tan(1/2*b*x+1/2*a)*cos(a)*cos(c)+2
*tan(1/2*b*x+1/2*a)*sin(a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c))*sin(a)*cos(c
)-8/b/(-4*cos(a)^2*cos(c)^2-4*cos(a)^2*sin(c)^2-4*cos(c)^2*sin(a)^2-4*sin(a
)^2*sin(c)^2)/(-cos(a)^2*cos(c)^2-cos(a)^2*sin(c)^2-cos(c)^2*sin(a)^2-sin(a
)^2*sin(c)^2)^(1/2)*arctan(1/2*(2*(sin(a)*cos(c)-cos(a)*sin(c))*tan(1/2*b*x
+1/2*a)+2*cos(a)*cos(c)+2*sin(a)*sin(c))/(-cos(a)^2*cos(c)^2-cos(a)^2*sin(c
)^2-cos(c)^2*sin(a)^2-sin(a)^2*sin(c)^2)^(1/2))*cos(a)*cos(c)-8/b/(-4*cos(a
)^2*cos(c)^2-4*cos(a)^2*sin(c)^2-4*cos(c)^2*sin(a)^2-4*sin(a)^2*sin(c)^2)/(
-cos(a)^2*cos(c)^2-cos(a)^2*sin(c)^2-cos(c)^2*sin(a)^2-sin(a)^2*sin(c)^2)^(
1/2)*arctan(1/2*(2*(sin(a)*cos(c)-cos(a)*sin(c))*tan(1/2*b*x+1/2*a)+2*cos(a
)*cos(c)+2*sin(a)*sin(c))/(-cos(a)^2*cos(c)^2-cos(a)^2*sin(c)^2-cos(c)^2*si
n(a)^2-sin(a)^2*sin(c)^2)^(1/2))*sin(a)*sin(c)
```

Maxima [B] time = 1.28253, size = 613, normalized size = 17.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(cos(b*x + 2*a) - cos(b*x + 2*c))*cos(2*b*x + a + 2*c) - 2*cos(b*x
+ 2*a)*cos(a) + 2*cos(b*x + 2*c)*cos(a) + (cos(2*b*x + a + 2*c)^2*cos(-a +
c) - 2*cos(2*b*x + a + 2*c)*cos(a)*cos(-a + c) + cos(-a + c)*sin(2*b*x + a
+ 2*c)^2 - 2*cos(-a + c)*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2
)*cos(-a + c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 -
 2*sin(b*x)*sin(c) + sin(c)^2) - (cos(2*b*x + a + 2*c)^2*cos(-a + c) - 2*co
s(2*b*x + a + 2*c)*cos(a)*cos(-a + c) + cos(-a + c)*sin(2*b*x + a + 2*c)^2
- 2*cos(-a + c)*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*cos(-a
+ c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*
x)*sin(c) + sin(c)^2) + 2*(sin(b*x + 2*a) - sin(b*x + 2*c))*sin(2*b*x + a +
 2*c) - 2*sin(b*x + 2*a)*sin(a) + 2*sin(b*x + 2*c)*sin(a))/(b*cos(2*b*x + a
 + 2*c)^2 - 2*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(2*b*x + a + 2*c)^2 - 2*
b*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b)
```

Fricas [A] time = 0.502203, size = 203, normalized size = 5.64

$$-\frac{\cos\left(-a+c\right)\log\left(\frac{1}{2}\,\cos\left(bx+c\right)+\frac{1}{2}\right)\sin\left(bx+c\right)-\cos\left(-a+c\right)\log\left(-\frac{1}{2}\,\cos\left(bx+c\right)+\frac{1}{2}\right)\sin\left(bx+c\right)-2\,\sin\left(-a+c\right)}{2\,b\sin\left(bx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(cos(-a + c)*log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c) - cos(-a + c)*lo
g(-1/2*cos(b*x + c) + 1/2)*sin(b*x + c) - 2*sin(-a + c))/(b*sin(b*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)**2*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.13628, size = 471, normalized size = 13.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] ((tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c) - tan(
1/2*c)^2 + 1)*log(abs(tan(1/2*b*x + 1/2*c)))/(tan(1/2*a)^2*tan(1/2*c)^2 + t
an(1/2*a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2*tan(1/
2*c) - tan(1/2*b*x + 1/2*c)*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*b*x + 1/2*c)*
tan(1/2*a) - tan(1/2*b*x + 1/2*c)*tan(1/2*c))/(tan(1/2*a)^2*tan(1/2*c)^2 +
```

```
tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2*tan(1
/2*c)^2 - tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2 + 4*tan(1/2*b*x + 1/2*c)*tan(1/
2*a)*tan(1/2*c) + tan(1/2*a)^2*tan(1/2*c) - tan(1/2*b*x + 1/2*c)*tan(1/2*c)
^2 - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*b*x + 1/2*c) + tan(1/2*a) - tan(1/2*
c))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*tan(1/2*
b*x + 1/2*c)))/b
```

3.197 $\int \csc^3(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=39

$$-\frac{\cos(a-c)\cot(bx+c)}{b}-\frac{\sin(a-c)\csc^2(bx+c)}{2b}$$

```
[Out] -((Cos[a - c]*Cot[c + b*x])/b) - (Csc[c + b*x]^2*Sin[a - c])/(2*b)
```

Rubi [A] time = 0.0437446, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4582, 2606, 30, 3767, 8}

$$-\frac{\cos(a-c)\cot(bx+c)}{b}-\frac{\sin(a-c)\csc^2(bx+c)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -((Cos[a - c]*Cot[c + b*x])/b) - (Csc[c + b*x]^2*Sin[a - c])/(2*b)
```

Rule 4582

```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Sin[v - w], Int[Cot[w]*Csc[w]^
(n - 1), x], x] + Dist[Cos[v - w], Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \csc^3(c+bx)\sin(a+bx)\,dx = \cos(a-c)\int \csc^2(c+bx)\,dx + \sin(a-c)\int \cot(c+bx)\csc^2(c+bx)\,dx$$
$$= -\frac{\cos(a-c)\,\mathrm{Subst}(\int 1\,dx, x, \cot(c+bx))}{b} - \frac{\sin(a-c)\,\mathrm{Subst}(\int x\,dx, x, \csc(c+bx))}{b}$$
$$= -\frac{\cos(a-c)\cot(c+bx)}{b} - \frac{\csc^2(c+bx)\sin(a-c)}{2b}$$

Mathematica [A] time = 0.190225, size = 35, normalized size = 0.9

$$\frac{\csc(c)\csc^2(bx+c)(\cos(a)-\cos(a-c)\cos(2bx+c))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + b*x]^3*Sin[a + b*x],x]
```

```
[Out] ((Cos[a] - Cos[a - c]*Cos[c + 2*b*x])*Csc[c]*Csc[c + b*x]^2)/(2*b)
```

Maple [B] time = 0.602, size = 120, normalized size = 3.1

$$\frac{1}{b}\left(-\frac{1}{(\cos(a)\cos(c)+\sin(a)\sin(c))^2(\tan(bx+a)\cos(a)\cos(c)+\tan(bx+a)\sin(a)\sin(c)+\cos(a)\sin(c)-\sin(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+c)^3*sin(b*x+a),x)
```

```
[Out] 1/b*(-1/(cos(a)*cos(c)+sin(a)*sin(c))^2/(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a
)*sin(a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c))-1/2*(sin(a)*cos(c)-cos(a)*sin(
c))/(cos(a)*cos(c)+sin(a)*sin(c))^2/(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*si
n(a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c))^2)
```

Maxima [B] time = 1.19331, size = 539, normalized size = 13.82

$$\frac{(2\sin(2bx+2a+2c)-\sin(2a)-\sin(2c))\cos(4bx+a+5c)-2(2\sin(2bx+2a+2c)-\sin(2a)-\sin(2c))\cos}{b\cos(4bx+a+5c)^2+4b\cos(2bx+a+3c)^2-4b\cos(2bx+a+3c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] ((2*sin(2*b*x + 2*a + 2*c) - sin(2*a) - sin(2*c))*cos(4*b*x + a + 5*c) - 2*
(2*sin(2*b*x + 2*a + 2*c) - sin(2*a) - sin(2*c))*cos(2*b*x + a + 3*c) - (si
n(2*a) + sin(2*c))*cos(a + c) - (2*cos(2*b*x + 2*a + 2*c) - cos(2*a) - cos(
2*c))*sin(4*b*x + a + 5*c) + 2*cos(a + c)*sin(2*b*x + 2*a + 2*c) + 2*(2*cos
(2*b*x + 2*a + 2*c) - cos(2*a) - cos(2*c))*sin(2*b*x + a + 3*c) + (cos(2*a)
 + cos(2*c))*sin(a + c) - 2*cos(2*b*x + 2*a + 2*c)*sin(a + c))/(b*cos(4*b*x
 + a + 5*c)^2 + 4*b*cos(2*b*x + a + 3*c)^2 - 4*b*cos(2*b*x + a + 3*c)*cos(a
 + c) + b*cos(a + c)^2 + b*sin(4*b*x + a + 5*c)^2 + 4*b*sin(2*b*x + a + 3*c
)^2 - 4*b*sin(2*b*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 - 2*(2*b*cos(2*b
*x + a + 3*c) - b*cos(a + c))*cos(4*b*x + a + 5*c) - 2*(2*b*sin(2*b*x + a +
```

```
3*c) - b*sin(a + c))*sin(4*b*x + a + 5*c))
```

Fricas [A] time = 0.469974, size = 113, normalized size = 2.9

$$\frac{2 \cos(bx+c)\cos(-a+c)\sin(bx+c) - \sin(-a+c)}{2\left(b\cos(bx+c)^2 - b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c) - sin(-a + c))/(b*cos(b*x + c)
^2 - b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)**3*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17347, size = 196, normalized size = 5.03

$$-\frac{\tan(bx+c)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2 - \tan(bx+c)\tan\left(\frac{1}{2}a\right)^2 + 4\tan(bx+c)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right) + \tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right) \cdots}{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}a\right)^2 + \tan\left(\frac{1}{2}c\right)^2 + \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -(tan(b*x + c)*tan(1/2*a)^2*tan(1/2*c)^2 - tan(b*x + c)*tan(1/2*a)^2 + 4*ta
n(b*x + c)*tan(1/2*a)*tan(1/2*c) + tan(1/2*a)^2*tan(1/2*c) - tan(b*x + c)*t
an(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^2 + tan(b*x + c) + tan(1/2*a) - tan(1/2
*c))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*b*tan(b
*x + c)^2)
```

3.198 $\int \csc^4(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=67

$$-\frac{\cos(a-c)\tanh^{-1}(\cos(bx+c))}{2b}-\frac{\sin(a-c)\csc^3(bx+c)}{3b}-\frac{\cos(a-c)\cot(bx+c)\csc(bx+c)}{2b}$$

```
[Out] -(ArcTanh[Cos[c + b*x]]*Cos[a - c])/(2*b) - (Cos[a - c]*Cot[c + b*x]*Csc[c
+ b*x])/(2*b) - (Csc[c + b*x]^3*Sin[a - c])/(3*b)
```

Rubi [A] time = 0.0464963, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4582, 2606, 30, 3768, 3770}

$$-\frac{\cos(a-c)\tanh^{-1}(\cos(bx+c))}{2b}-\frac{\sin(a-c)\csc^3(bx+c)}{3b}-\frac{\cos(a-c)\cot(bx+c)\csc(bx+c)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + b*x]^4*Sin[a + b*x],x]
```

```
[Out] -(ArcTanh[Cos[c + b*x]]*Cos[a - c])/(2*b) - (Cos[a - c]*Cot[c + b*x]*Csc[c
+ b*x])/(2*b) - (Csc[c + b*x]^3*Sin[a - c])/(3*b)
```

Rule 4582

```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Sin[v - w], Int[Cot[w]*Csc[w]^
(n - 1), x], x] + Dist[Cos[v - w], Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^4(c+bx)\sin(a+bx)\,dx = \cos(a-c)\int \csc^3(c+bx)\,dx + \sin(a-c)\int \cot(c+bx)\csc^3(c+bx)\,dx$$
$$= -\frac{\cos(a-c)\cot(c+bx)\csc(c+bx)}{2b} + \frac{1}{2}\cos(a-c)\int \csc(c+bx)\,dx - \frac{\sin(a-c)\,\text{Subst}\,(}{}$$
$$= -\frac{\tanh^{-1}(\cos(c+bx))\cos(a-c)}{2b} - \frac{\cos(a-c)\cot(c+bx)\csc(c+bx)}{2b} - \frac{\csc^3(c+bx)\sin}{3b}$$

Mathematica [A] time = 0.553062, size = 67, normalized size = 1.

$$-\frac{2\sin(a-c)\csc^3(bx+c) + 3\cos(a-c)\cot(bx+c)\csc(bx+c) + 6\cos(a-c)\tanh^{-1}\left(\cos(c) - \sin(c)\tan\left(\frac{bx}{2}\right)\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + b*x]^4*Sin[a + b*x],x]
```

```
[Out] -(6*ArcTanh[Cos[c] - Sin[c]*Tan[(b*x)/2]]*Cos[a - c] + 3*Cos[a - c]*Cot[c +
 b*x]*Csc[c + b*x] + 2*Csc[c + b*x]^3*Sin[a - c])/(6*b)
```

Maple [B] time = 2.466, size = 14880, normalized size = 222.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+c)^4*sin(b*x+a),x)
```

```
[Out] result too large to display
```

Maxima [B] time = 1.65498, size = 2394, normalized size = 35.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^4*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/12*(2*(3*cos(5*b*x + 2*a + 4*c) + 3*cos(5*b*x + 6*c) + 8*cos(3*b*x + 2*a
+ 2*c) - 8*cos(3*b*x + 4*c) - 3*cos(b*x + 2*a) - 3*cos(b*x + 2*c))*cos(6*b*
x + a + 6*c) - 6*(3*cos(4*b*x + a + 4*c) - 3*cos(2*b*x + a + 2*c) + cos(a))
*cos(5*b*x + 2*a + 4*c) - 6*(3*cos(4*b*x + a + 4*c) - 3*cos(2*b*x + a + 2*c
) + cos(a))*cos(5*b*x + 6*c) - 6*(8*cos(3*b*x + 2*a + 2*c) - 8*cos(3*b*x +
4*c) - 3*cos(b*x + 2*a) - 3*cos(b*x + 2*c))*cos(4*b*x + a + 4*c) + 16*(3*co
s(2*b*x + a + 2*c) - cos(a))*cos(3*b*x + 2*a + 2*c) - 16*(3*cos(2*b*x + a +
 2*c) - cos(a))*cos(3*b*x + 4*c) - 18*(cos(b*x + 2*a) + cos(b*x + 2*c))*cos
(2*b*x + a + 2*c) + 6*cos(b*x + 2*a)*cos(a) + 6*cos(b*x + 2*c)*cos(a) - 3*(
cos(6*b*x + a + 6*c)^2*cos(-a + c) + 9*cos(4*b*x + a + 4*c)^2*cos(-a + c) +
 9*cos(2*b*x + a + 2*c)^2*cos(-a + c) - 6*cos(2*b*x + a + 2*c)*cos(a)*cos(-
a + c) + cos(-a + c)*sin(6*b*x + a + 6*c)^2 + 9*cos(-a + c)*sin(4*b*x + a +
 4*c)^2 + 9*cos(-a + c)*sin(2*b*x + a + 2*c)^2 - 6*cos(-a + c)*sin(2*b*x +
```

```
a + 2*c)*sin(a) - 2*(3*cos(4*b*x + a + 4*c)*cos(-a + c) - 3*cos(2*b*x + a +
 2*c)*cos(-a + c) + cos(a)*cos(-a + c))*cos(6*b*x + a + 6*c) - 6*(3*cos(2*b
*x + a + 2*c)*cos(-a + c) - cos(a)*cos(-a + c))*cos(4*b*x + a + 4*c) + (cos
(a)^2 + sin(a)^2)*cos(-a + c) - 2*(3*cos(-a + c)*sin(4*b*x + a + 4*c) - 3*c
os(-a + c)*sin(2*b*x + a + 2*c) + cos(-a + c)*sin(a))*sin(6*b*x + a + 6*c)
- 6*(3*cos(-a + c)*sin(2*b*x + a + 2*c) - cos(-a + c)*sin(a))*sin(4*b*x + a
 + 4*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin
(b*x)*sin(c) + sin(c)^2) + 3*(cos(6*b*x + a + 6*c)^2*cos(-a + c) + 9*cos(4*
b*x + a + 4*c)^2*cos(-a + c) + 9*cos(2*b*x + a + 2*c)^2*cos(-a + c) - 6*cos
(2*b*x + a + 2*c)*cos(a)*cos(-a + c) + cos(-a + c)*sin(6*b*x + a + 6*c)^2 +
 9*cos(-a + c)*sin(4*b*x + a + 4*c)^2 + 9*cos(-a + c)*sin(2*b*x + a + 2*c)^
2 - 6*cos(-a + c)*sin(2*b*x + a + 2*c)*sin(a) - 2*(3*cos(4*b*x + a + 4*c)*c
os(-a + c) - 3*cos(2*b*x + a + 2*c)*cos(-a + c) + cos(a)*cos(-a + c))*cos(6
*b*x + a + 6*c) - 6*(3*cos(2*b*x + a + 2*c)*cos(-a + c) - cos(a)*cos(-a + c
))*cos(4*b*x + a + 4*c) + (cos(a)^2 + sin(a)^2)*cos(-a + c) - 2*(3*cos(-a +
 c)*sin(4*b*x + a + 4*c) - 3*cos(-a + c)*sin(2*b*x + a + 2*c) + cos(-a + c)
*sin(a))*sin(6*b*x + a + 6*c) - 6*(3*cos(-a + c)*sin(2*b*x + a + 2*c) - cos
(-a + c)*sin(a))*sin(4*b*x + a + 4*c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) +
 cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) + 2*(3*sin(5*b*x + 2
*a + 4*c) + 3*sin(5*b*x + 6*c) + 8*sin(3*b*x + 2*a + 2*c) - 8*sin(3*b*x + 4
*c) - 3*sin(b*x + 2*a) - 3*sin(b*x + 2*c))*sin(6*b*x + a + 6*c) - 6*(3*sin(
4*b*x + a + 4*c) - 3*sin(2*b*x + a + 2*c) + sin(a))*sin(5*b*x + 2*a + 4*c)
- 6*(3*sin(4*b*x + a + 4*c) - 3*sin(2*b*x + a + 2*c) + sin(a))*sin(5*b*x +
6*c) - 6*(8*sin(3*b*x + 2*a + 2*c) - 8*sin(3*b*x + 4*c) - 3*sin(b*x + 2*a)
- 3*sin(b*x + 2*c))*sin(4*b*x + a + 4*c) + 16*(3*sin(2*b*x + a + 2*c) - sin
(a))*sin(3*b*x + 2*a + 2*c) - 16*(3*sin(2*b*x + a + 2*c) - sin(a))*sin(3*b*
x + 4*c) - 18*(sin(b*x + 2*a) + sin(b*x + 2*c))*sin(2*b*x + a + 2*c) + 6*si
n(b*x + 2*a)*sin(a) + 6*sin(b*x + 2*c)*sin(a))/(b*cos(6*b*x + a + 6*c)^2 +
9*b*cos(4*b*x + a + 4*c)^2 + 9*b*cos(2*b*x + a + 2*c)^2 - 6*b*cos(2*b*x + a
 + 2*c)*cos(a) + b*sin(6*b*x + a + 6*c)^2 + 9*b*sin(4*b*x + a + 4*c)^2 + 9*
b*sin(2*b*x + a + 2*c)^2 - 6*b*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + si
n(a)^2)*b - 2*(3*b*cos(4*b*x + a + 4*c) - 3*b*cos(2*b*x + a + 2*c) + b*cos(
a))*cos(6*b*x + a + 6*c) - 6*(3*b*cos(2*b*x + a + 2*c) - b*cos(a))*cos(4*b*
x + a + 4*c) - 2*(3*b*sin(4*b*x + a + 4*c) - 3*b*sin(2*b*x + a + 2*c) + b*s
in(a))*sin(6*b*x + a + 6*c) - 6*(3*b*sin(2*b*x + a + 2*c) - b*sin(a))*sin(4
*b*x + a + 4*c))
```

Fricas [B] time = 0.51614, size = 377, normalized size = 5.63

$$\frac{6 \cos(bx+c)\cos(-a+c)\sin(bx+c) - 3\left(\cos(bx+c)^2\cos(-a+c) - \cos(-a+c)\right)\log\left(\frac{1}{2}\cos(bx+c) + \frac{1}{2}\right)\sin(bx \cdots}{12\left(b\cos(bx+c)^2 - b\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^4*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(6*cos(b*x + c)*cos(-a + c)*sin(b*x + c) - 3*(cos(b*x + c)^2*cos(-a +
c) - cos(-a + c))*log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c) + 3*(cos(b*x + c
)^2*cos(-a + c) - cos(-a + c))*log(-1/2*cos(b*x + c) + 1/2)*sin(b*x + c) -
4*sin(-a + c))/((b*cos(b*x + c)^2 - b)*sin(b*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)**4*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24204, size = 2998, normalized size = 44.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^4*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/24*(12*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c
) - tan(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x + 1/2*c)))/(tan(1/2*a)^2*tan(1/2*
c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - (2*tan(1/2*b*x + 1/2*c)^3*tan(1/2
*a)^6*tan(1/2*c)^5 - 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^5*tan(1/2*c)^6 - 3
*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^6*tan(1/2*c)^6 + 4*tan(1/2*b*x + 1/2*c)^
3*tan(1/2*a)^6*tan(1/2*c)^3 - 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^5*tan(1/2
*c)^4 - 3*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^6*tan(1/2*c)^4 + 2*tan(1/2*b*x
+ 1/2*c)^3*tan(1/2*a)^4*tan(1/2*c)^5 - 12*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)
^5*tan(1/2*c)^5 + 6*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^6*tan(1/2*c)^5 - 4*tan(
1/2*b*x + 1/2*c)^3*tan(1/2*a)^3*tan(1/2*c)^6 - 3*tan(1/2*b*x + 1/2*c)^2*tan
(1/2*a)^4*tan(1/2*c)^6 - 6*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^5*tan(1/2*c)^6 +
 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^6*tan(1/2*c) + 2*tan(1/2*b*x + 1/2*c)^
3*tan(1/2*a)^5*tan(1/2*c)^2 + 3*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^6*tan(1/2
*c)^2 + 4*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^4*tan(1/2*c)^3 - 24*tan(1/2*b*x
 + 1/2*c)^2*tan(1/2*a)^5*tan(1/2*c)^3 + 12*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^
6*tan(1/2*c)^3 - 4*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^3*tan(1/2*c)^4 - 3*tan
(1/2*b*x + 1/2*c)^2*tan(1/2*a)^4*tan(1/2*c)^4 - 6*tan(1/2*b*x + 1/2*c)*tan(
1/2*a)^5*tan(1/2*c)^4 - 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^2*tan(1/2*c)^5
- 24*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^3*tan(1/2*c)^5 + 6*tan(1/2*b*x + 1/2
*c)*tan(1/2*a)^4*tan(1/2*c)^5 - 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)*tan(1/2
*c)^6 + 3*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^2*tan(1/2*c)^6 - 12*tan(1/2*b*x
 + 1/2*c)*tan(1/2*a)^3*tan(1/2*c)^6 + 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^5
 + 3*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^6 + 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2
*a)^4*tan(1/2*c) - 12*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^5*tan(1/2*c) + 6*ta
n(1/2*b*x + 1/2*c)*tan(1/2*a)^6*tan(1/2*c) + 4*tan(1/2*b*x + 1/2*c)^3*tan(1
/2*a)^3*tan(1/2*c)^2 + 3*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^4*tan(1/2*c)^2 +
 6*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^5*tan(1/2*c)^2 - 4*tan(1/2*b*x + 1/2*c)^
3*tan(1/2*a)^2*tan(1/2*c)^3 - 48*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^3*tan(1/
2*c)^3 + 12*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^4*tan(1/2*c)^3 - 2*tan(1/2*b*x
+ 1/2*c)^3*tan(1/2*a)*tan(1/2*c)^4 + 3*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^2*
tan(1/2*c)^4 - 12*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^3*tan(1/2*c)^4 - 2*tan(1/
2*b*x + 1/2*c)^3*tan(1/2*c)^5 - 12*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)*tan(1/
2*c)^5 - 6*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2*tan(1/2*c)^5 + 3*tan(1/2*b*x +
 1/2*c)^2*tan(1/2*c)^6 - 6*tan(1/2*b*x + 1/2*c)*tan(1/2*a)*tan(1/2*c)^6 + 4
*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^3 + 3*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^
4 + 6*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^5 - 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*
a)^2*tan(1/2*c) - 24*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^3*tan(1/2*c) + 6*tan
(1/2*b*x + 1/2*c)*tan(1/2*a)^4*tan(1/2*c) + 2*tan(1/2*b*x + 1/2*c)^3*tan(1/
2*a)*tan(1/2*c)^2 - 3*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^2*tan(1/2*c)^2 + 12
*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^3*tan(1/2*c)^2 - 4*tan(1/2*b*x + 1/2*c)^3*
tan(1/2*c)^3 - 24*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)*tan(1/2*c)^3 - 12*tan(1
/2*b*x + 1/2*c)*tan(1/2*a)^2*tan(1/2*c)^3 + 3*tan(1/2*b*x + 1/2*c)^2*tan(1/
2*c)^4 - 6*tan(1/2*b*x + 1/2*c)*tan(1/2*a)*tan(1/2*c)^4 - 6*tan(1/2*b*x + 1
```

```
/2*c)*tan(1/2*c)^5 + 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a) - 3*tan(1/2*b*x +
1/2*c)^2*tan(1/2*a)^2 + 12*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^3 - 2*tan(1/2*b*
x + 1/2*c)^3*tan(1/2*c) - 12*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)*tan(1/2*c) -
 6*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2*tan(1/2*c) - 3*tan(1/2*b*x + 1/2*c)^2*
tan(1/2*c)^2 + 6*tan(1/2*b*x + 1/2*c)*tan(1/2*a)*tan(1/2*c)^2 - 12*tan(1/2*
b*x + 1/2*c)*tan(1/2*c)^3 - 3*tan(1/2*b*x + 1/2*c)^2 + 6*tan(1/2*b*x + 1/2*
c)*tan(1/2*a) - 6*tan(1/2*b*x + 1/2*c)*tan(1/2*c))/(tan(1/2*a)^6*tan(1/2*c)
^6 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 3*tan(1/2*a)^4*tan(1/2*c)^6 + 3*tan(1/2*
a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^4*tan(1/2*c)^4 + 3*tan(1/2*a)^2*tan(1/2*c)
^6 + tan(1/2*a)^6 + 9*tan(1/2*a)^4*tan(1/2*c)^2 + 9*tan(1/2*a)^2*tan(1/2*c)
^4 + tan(1/2*c)^6 + 3*tan(1/2*a)^4 + 9*tan(1/2*a)^2*tan(1/2*c)^2 + 3*tan(1/
2*c)^4 + 3*tan(1/2*a)^2 + 3*tan(1/2*c)^2 + 1) - (22*tan(1/2*b*x + 1/2*c)^3*
tan(1/2*a)^2*tan(1/2*c)^2 - 22*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^2 + 88*tan
(1/2*b*x + 1/2*c)^3*tan(1/2*a)*tan(1/2*c) + 6*tan(1/2*b*x + 1/2*c)^2*tan(1/
2*a)^2*tan(1/2*c) - 22*tan(1/2*b*x + 1/2*c)^3*tan(1/2*c)^2 - 6*tan(1/2*b*x
+ 1/2*c)^2*tan(1/2*a)*tan(1/2*c)^2 + 3*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2*ta
n(1/2*c)^2 + 22*tan(1/2*b*x + 1/2*c)^3 + 6*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a
) - 3*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2 - 6*tan(1/2*b*x + 1/2*c)^2*tan(1/2*
c) + 12*tan(1/2*b*x + 1/2*c)*tan(1/2*a)*tan(1/2*c) + 2*tan(1/2*a)^2*tan(1/2
*c) - 3*tan(1/2*b*x + 1/2*c)*tan(1/2*c)^2 - 2*tan(1/2*a)*tan(1/2*c)^2 + 3*t
an(1/2*b*x + 1/2*c) + 2*tan(1/2*a) - 2*tan(1/2*c))/((tan(1/2*a)^2*tan(1/2*c
)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*tan(1/2*b*x + 1/2*c)^3))/b
```

3.199 $\int \csc^5(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=60

$$-\frac{\cos(a-c)\cot^3(bx+c)}{3b}-\frac{\cos(a-c)\cot(bx+c)}{b}-\frac{\sin(a-c)\csc^4(bx+c)}{4b}$$

```
[Out] -((Cos[a - c]*Cot[c + b*x])/b) - (Cos[a - c]*Cot[c + b*x]^3)/(3*b) - (Csc[c
 + b*x]^4*Sin[a - c])/(4*b)
```

Rubi [A] time = 0.0466843, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4582, 2606, 30, 3767}

$$-\frac{\cos(a-c)\cot^3(bx+c)}{3b}-\frac{\cos(a-c)\cot(bx+c)}{b}-\frac{\sin(a-c)\csc^4(bx+c)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + b*x]^5*Sin[a + b*x],x]
```

```
[Out] -((Cos[a - c]*Cot[c + b*x])/b) - (Cos[a - c]*Cot[c + b*x]^3)/(3*b) - (Csc[c
 + b*x]^4*Sin[a - c])/(4*b)
```

Rule 4582

```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Sin[v - w], Int[Cot[w]*Csc[w]^
(n - 1), x], x] + Dist[Cos[v - w], Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^5(c+bx)\sin(a+bx)\,dx &= \cos(a-c)\int \csc^4(c+bx)\,dx+\sin(a-c)\int \cot(c+bx)\csc^4(c+bx)\,dx\\ &= -\frac{\cos(a-c)\operatorname{Subst}\left(\int \left(1+x^2\right)dx,x,\cot(c+bx)\right)}{b}-\frac{\sin(a-c)\operatorname{Subst}\left(\int x^3\,dx,x,\csc(c+\right.}{b}\\ &= -\frac{\cos(a-c)\cot(c+bx)}{b}-\frac{\cos(a-c)\cot^3(c+bx)}{3b}-\frac{\csc^4(c+bx)\sin(a-c)}{4b}\end{aligned}$$

Mathematica [A] time = 0.374245, size = 58, normalized size = 0.97

$$\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\csc^4(bx+c)(\cos(a-c)(\cos(4bx+3c)-4\cos(2bx+c))+3\cos(a))}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + b*x]^5*Sin[a + b*x],x]
```

```
[Out] ((3*Cos[a] + Cos[a - c]*(-4*Cos[c + 2*b*x] + Cos[3*c + 4*b*x]))*Csc[c/2]*Cs
c[c + b*x]^4*Sec[c/2])/(24*b)
```

Maple [B] time = 1.888, size = 321, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+c)^5*sin(b*x+a),x)
```

```
[Out] 1/b*(1/4*(cos(a)*sin(c)-sin(a)*cos(c))*(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^2
+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)/(cos(a)*cos(c)+sin(a)*sin(c))^4/(tan(
b*x+a)*cos(a)*cos(c)+tan(b*x+a)*sin(a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c))^
4-1/3*(cos(a)^2*cos(c)^2+3*cos(a)^2*sin(c)^2-4*cos(a)*cos(c)*sin(a)*sin(c)+
3*cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)/(cos(a)*cos(c)+sin(a)*sin(c))^4/(tan
(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*sin(a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c))
^3-1/(cos(a)*cos(c)+sin(a)*sin(c))^4/(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*s
in(a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c))-1/2*(-3*cos(a)*sin(c)+3*sin(a)*co
s(c))/(cos(a)*cos(c)+sin(a)*sin(c))^4/(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*
sin(a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c))^2)
```

Maxima [B] time = 1.57422, size = 1453, normalized size = 24.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^5*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -2/3*((6*sin(4*b*x + 2*a + 4*c) - 4*sin(2*b*x + 2*a + 2*c) - 4*sin(2*b*x +
4*c) + sin(2*a) + sin(2*c))*cos(8*b*x + a + 9*c) - 4*(6*sin(4*b*x + 2*a + 4
*c) - 4*sin(2*b*x + 2*a + 2*c) - 4*sin(2*b*x + 4*c) + sin(2*a) + sin(2*c))*
cos(6*b*x + a + 7*c) + 6*(4*sin(2*b*x + a + 3*c) - sin(a + c))*cos(4*b*x +
2*a + 4*c) + 6*(6*sin(4*b*x + 2*a + 4*c) - 4*sin(2*b*x + 2*a + 2*c) - 4*sin
(2*b*x + 4*c) + sin(2*a) + sin(2*c))*cos(4*b*x + a + 5*c) + 4*(4*sin(2*b*x
+ 2*a + 2*c) - sin(2*a) - sin(2*c))*cos(2*b*x + a + 3*c) - 4*(4*sin(2*b*x +
 a + 3*c) - sin(a + c))*cos(2*b*x + 4*c) + (sin(2*a) + sin(2*c))*cos(a + c)
 - (6*cos(4*b*x + 2*a + 4*c) - 4*cos(2*b*x + 2*a + 2*c) - 4*cos(2*b*x + 4*c
) + cos(2*a) + cos(2*c))*sin(8*b*x + a + 9*c) + 4*(6*cos(4*b*x + 2*a + 4*c)
 - 4*cos(2*b*x + 2*a + 2*c) - 4*cos(2*b*x + 4*c) + cos(2*a) + cos(2*c))*sin
(6*b*x + a + 7*c) - 6*(4*cos(2*b*x + a + 3*c) - cos(a + c))*sin(4*b*x + 2*a
 + 4*c) - 6*(6*cos(4*b*x + 2*a + 4*c) - 4*cos(2*b*x + 2*a + 2*c) - 4*cos(2*
b*x + 4*c) + cos(2*a) + cos(2*c))*sin(4*b*x + a + 5*c) - 4*cos(a + c)*sin(2
*b*x + 2*a + 2*c) - 4*(4*cos(2*b*x + 2*a + 2*c) - cos(2*a) - cos(2*c))*sin(
```

```
2*b*x + a + 3*c) + 4*(4*cos(2*b*x + a + 3*c) - cos(a + c))*sin(2*b*x + 4*c)
 - (cos(2*a) + cos(2*c))*sin(a + c) + 4*cos(2*b*x + 2*a + 2*c)*sin(a + c))/
(b*cos(8*b*x + a + 9*c)^2 + 16*b*cos(6*b*x + a + 7*c)^2 + 36*b*cos(4*b*x +
a + 5*c)^2 + 16*b*cos(2*b*x + a + 3*c)^2 - 8*b*cos(2*b*x + a + 3*c)*cos(a +
 c) + b*cos(a + c)^2 + b*sin(8*b*x + a + 9*c)^2 + 16*b*sin(6*b*x + a + 7*c)
^2 + 36*b*sin(4*b*x + a + 5*c)^2 + 16*b*sin(2*b*x + a + 3*c)^2 - 8*b*sin(2*
b*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 - 2*(4*b*cos(6*b*x + a + 7*c) -
6*b*cos(4*b*x + a + 5*c) + 4*b*cos(2*b*x + a + 3*c) - b*cos(a + c))*cos(8*b
*x + a + 9*c) - 8*(6*b*cos(4*b*x + a + 5*c) - 4*b*cos(2*b*x + a + 3*c) + b*
cos(a + c))*cos(6*b*x + a + 7*c) - 12*(4*b*cos(2*b*x + a + 3*c) - b*cos(a +
 c))*cos(4*b*x + a + 5*c) - 2*(4*b*sin(6*b*x + a + 7*c) - 6*b*sin(4*b*x + a
 + 5*c) + 4*b*sin(2*b*x + a + 3*c) - b*sin(a + c))*sin(8*b*x + a + 9*c) - 8
*(6*b*sin(4*b*x + a + 5*c) - 4*b*sin(2*b*x + a + 3*c) + b*sin(a + c))*sin(6
*b*x + a + 7*c) - 12*(4*b*sin(2*b*x + a + 3*c) - b*sin(a + c))*sin(4*b*x +
a + 5*c))
```

Fricas [A] time = 0.481041, size = 193, normalized size = 3.22

$$\frac{4\left(2\cos\left(bx+c\right)^{3}\cos\left(-a+c\right)-3\cos\left(bx+c\right)\cos\left(-a+c\right)\right)\sin\left(bx+c\right)+3\sin\left(-a+c\right)}{12\left(b\cos\left(bx+c\right)^{4}-2b\cos\left(bx+c\right)^{2}+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^5*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(4*(2*cos(b*x + c)^3*cos(-a + c) - 3*cos(b*x + c)*cos(-a + c))*sin(b*x
 + c) + 3*sin(-a + c))/(b*cos(b*x + c)^4 - 2*b*cos(b*x + c)^2 + b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)**5*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15743, size = 406, normalized size = 6.77

$$-\frac{6\tan\left(bx+c\right)^{3}\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}-6\tan\left(bx+c\right)^{3}\tan\left(\frac{1}{2}a\right)^{2}+24\tan\left(bx+c\right)^{3}\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)+6\tan\left(bx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^5*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/6*(6*tan(b*x + c)^3*tan(1/2*a)^2*tan(1/2*c)^2 - 6*tan(b*x + c)^3*tan(1/2
*a)^2 + 24*tan(b*x + c)^3*tan(1/2*a)*tan(1/2*c) + 6*tan(b*x + c)^2*tan(1/2*
```

```
a)^2*tan(1/2*c) - 6*tan(b*x + c)^3*tan(1/2*c)^2 - 6*tan(b*x + c)^2*tan(1/2*
a)*tan(1/2*c)^2 + 2*tan(b*x + c)*tan(1/2*a)^2*tan(1/2*c)^2 + 6*tan(b*x + c)
^3 + 6*tan(b*x + c)^2*tan(1/2*a) - 2*tan(b*x + c)*tan(1/2*a)^2 - 6*tan(b*x
+ c)^2*tan(1/2*c) + 8*tan(b*x + c)*tan(1/2*a)*tan(1/2*c) + 3*tan(1/2*a)^2*t
an(1/2*c) - 2*tan(b*x + c)*tan(1/2*c)^2 - 3*tan(1/2*a)*tan(1/2*c)^2 + 2*tan
(b*x + c) + 3*tan(1/2*a) - 3*tan(1/2*c))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(
1/2*a)^2 + tan(1/2*c)^2 + 1)*b*tan(b*x + c)^4)
```

3.200 $\int \csc^6(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=94

$$-\frac{3\cos(a-c)\tanh^{-1}(\cos(bx+c))}{8b}-\frac{\sin(a-c)\csc^5(bx+c)}{5b}-\frac{\cos(a-c)\cot(bx+c)\csc^3(bx+c)}{4b}-\frac{3\cos(a-c)\cot(bx+c)}{8b}$$

```
[Out] (-3*ArcTanh[Cos[c + b*x]]*Cos[a - c])/(8*b) - (3*Cos[a - c]*Cot[c + b*x]*Cs
c[c + b*x])/(8*b) - (Cos[a - c]*Cot[c + b*x]*Csc[c + b*x]^3)/(4*b) - (Csc[c
 + b*x]^5*Sin[a - c])/(5*b)
```

Rubi [A] time = 0.0582293, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4582, 2606, 30, 3768, 3770}

$$-\frac{3\cos(a-c)\tanh^{-1}(\cos(bx+c))}{8b}-\frac{\sin(a-c)\csc^5(bx+c)}{5b}-\frac{\cos(a-c)\cot(bx+c)\csc^3(bx+c)}{4b}-\frac{3\cos(a-c)\cot(bx+c)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + b*x]^6*Sin[a + b*x],x]
```

```
[Out] (-3*ArcTanh[Cos[c + b*x]]*Cos[a - c])/(8*b) - (3*Cos[a - c]*Cot[c + b*x]*Cs
c[c + b*x])/(8*b) - (Cos[a - c]*Cot[c + b*x]*Csc[c + b*x]^3)/(4*b) - (Csc[c
 + b*x]^5*Sin[a - c])/(5*b)
```

Rule 4582

```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Sin[v - w], Int[Cot[w]*Csc[w]^
(n - 1), x], x] + Dist[Cos[v - w], Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^6(c+bx)\sin(a+bx)\,dx = \cos(a-c)\int \csc^5(c+bx)\,dx + \sin(a-c)\int \cot(c+bx)\csc^5(c+bx)\,dx$$
$$= -\frac{\cos(a-c)\cot(c+bx)\csc^3(c+bx)}{4b} + \frac{1}{4}(3\cos(a-c))\int \csc^3(c+bx)\,dx - \frac{\sin(a-c)}{}$$
$$= -\frac{3\cos(a-c)\cot(c+bx)\csc(c+bx)}{8b} - \frac{\cos(a-c)\cot(c+bx)\csc^3(c+bx)}{4b} - \frac{\csc^5(c}{}$$
$$= -\frac{3\tanh^{-1}(\cos(c+bx))\cos(a-c)}{8b} - \frac{3\cos(a-c)\cot(c+bx)\csc(c+bx)}{8b} - \frac{\cos(a-c)}{}$$

Mathematica [A] time = 1.16511, size = 79, normalized size = 0.84

$$-\frac{2\csc^5(bx+c)(5\cos(a-c)(14\sin(2(bx+c))-3\sin(4(bx+c)))+64\sin(a-c))+480\cos(a-c)\tanh^{-1}\left(\cos(c)-\mathrm{si}\right.}{640b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + b*x]^6*Sin[a + b*x],x]
```

```
[Out] -(480*ArcTanh[Cos[c] - Sin[c]*Tan[(b*x)/2]]*Cos[a - c] + 2*Csc[c + b*x]^5*(
64*Sin[a - c] + 5*Cos[a - c]*(14*Sin[2*(c + b*x)] - 3*Sin[4*(c + b*x)])))/(
640*b)
```

Maple [B] time = 10.075, size = 97954, normalized size = 1042.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+c)^6*sin(b*x+a),x)
```

```
[Out] result too large to display
```

Maxima [B] time = 2.23054, size = 5237, normalized size = 55.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^6*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/80*(2*(15*cos(9*b*x + 2*a + 8*c) + 15*cos(9*b*x + 10*c) - 70*cos(7*b*x +
2*a + 6*c) - 70*cos(7*b*x + 8*c) - 128*cos(5*b*x + 2*a + 4*c) + 128*cos(5*b
*x + 6*c) + 70*cos(3*b*x + 2*a + 2*c) + 70*cos(3*b*x + 4*c) - 15*cos(b*x +
2*a) - 15*cos(b*x + 2*c))*cos(10*b*x + a + 10*c) - 30*(5*cos(8*b*x + a + 8*
c) - 10*cos(6*b*x + a + 6*c) + 10*cos(4*b*x + a + 4*c) - 5*cos(2*b*x + a +
2*c) + cos(a))*cos(9*b*x + 2*a + 8*c) - 30*(5*cos(8*b*x + a + 8*c) - 10*cos
(6*b*x + a + 6*c) + 10*cos(4*b*x + a + 4*c) - 5*cos(2*b*x + a + 2*c) + cos(
a))*cos(9*b*x + 10*c) + 10*(70*cos(7*b*x + 2*a + 6*c) + 70*cos(7*b*x + 8*c)
 + 128*cos(5*b*x + 2*a + 4*c) - 128*cos(5*b*x + 6*c) - 70*cos(3*b*x + 2*a +
 2*c) - 70*cos(3*b*x + 4*c) + 15*cos(b*x + 2*a) + 15*cos(b*x + 2*c))*cos(8*
```

b*x + a + 8*c) - 140*(10*cos(6*b*x + a + 6*c) - 10*cos(4*b*x + a + 4*c) + 5
*cos(2*b*x + a + 2*c) - cos(a))*cos(7*b*x + 2*a + 6*c) - 140*(10*cos(6*b*x
+ a + 6*c) - 10*cos(4*b*x + a + 4*c) + 5*cos(2*b*x + a + 2*c) - cos(a))*cos
(7*b*x + 8*c) - 20*(128*cos(5*b*x + 2*a + 4*c) - 128*cos(5*b*x + 6*c) - 70*
cos(3*b*x + 2*a + 2*c) - 70*cos(3*b*x + 4*c) + 15*cos(b*x + 2*a) + 15*cos(b
*x + 2*c))*cos(6*b*x + a + 6*c) + 256*(10*cos(4*b*x + a + 4*c) - 5*cos(2*b*
x + a + 2*c) + cos(a))*cos(5*b*x + 2*a + 4*c) - 256*(10*cos(4*b*x + a + 4*c
) - 5*cos(2*b*x + a + 2*c) + cos(a))*cos(5*b*x + 6*c) - 100*(14*cos(3*b*x +
 2*a + 2*c) + 14*cos(3*b*x + 4*c) - 3*cos(b*x + 2*a) - 3*cos(b*x + 2*c))*co
s(4*b*x + a + 4*c) + 140*(5*cos(2*b*x + a + 2*c) - cos(a))*cos(3*b*x + 2*a
+ 2*c) + 140*(5*cos(2*b*x + a + 2*c) - cos(a))*cos(3*b*x + 4*c) - 150*(cos(
b*x + 2*a) + cos(b*x + 2*c))*cos(2*b*x + a + 2*c) + 30*cos(b*x + 2*a)*cos(a
) + 30*cos(b*x + 2*c)*cos(a) - 15*(cos(10*b*x + a + 10*c)^2*cos(-a + c) + 2
5*cos(8*b*x + a + 8*c)^2*cos(-a + c) + 100*cos(6*b*x + a + 6*c)^2*cos(-a +
c) + 100*cos(4*b*x + a + 4*c)^2*cos(-a + c) + 25*cos(2*b*x + a + 2*c)^2*cos
(-a + c) - 10*cos(2*b*x + a + 2*c)*cos(a)*cos(-a + c) + cos(-a + c)*sin(10*
b*x + a + 10*c)^2 + 25*cos(-a + c)*sin(8*b*x + a + 8*c)^2 + 100*cos(-a + c)
*sin(6*b*x + a + 6*c)^2 + 100*cos(-a + c)*sin(4*b*x + a + 4*c)^2 + 25*cos(-
a + c)*sin(2*b*x + a + 2*c)^2 - 10*cos(-a + c)*sin(2*b*x + a + 2*c)*sin(a)
- 2*(5*cos(8*b*x + a + 8*c)*cos(-a + c) - 10*cos(6*b*x + a + 6*c)*cos(-a +
c) + 10*cos(4*b*x + a + 4*c)*cos(-a + c) - 5*cos(2*b*x + a + 2*c)*cos(-a +
c) + cos(a)*cos(-a + c))*cos(10*b*x + a + 10*c) - 10*(10*cos(6*b*x + a + 6*
c)*cos(-a + c) - 10*cos(4*b*x + a + 4*c)*cos(-a + c) + 5*cos(2*b*x + a + 2*
c)*cos(-a + c) - cos(a)*cos(-a + c))*cos(8*b*x + a + 8*c) - 20*(10*cos(4*b*
x + a + 4*c)*cos(-a + c) - 5*cos(2*b*x + a + 2*c)*cos(-a + c) + cos(a)*cos(
-a + c))*cos(6*b*x + a + 6*c) - 20*(5*cos(2*b*x + a + 2*c)*cos(-a + c) - co
s(a)*cos(-a + c))*cos(4*b*x + a + 4*c) + (cos(a)^2 + sin(a)^2)*cos(-a + c)
- 2*(5*cos(-a + c)*sin(8*b*x + a + 8*c) - 10*cos(-a + c)*sin(6*b*x + a + 6*
c) + 10*cos(-a + c)*sin(4*b*x + a + 4*c) - 5*cos(-a + c)*sin(2*b*x + a + 2*
c) + cos(-a + c)*sin(a))*sin(10*b*x + a + 10*c) - 10*(10*cos(-a + c)*sin(6*
b*x + a + 6*c) - 10*cos(-a + c)*sin(4*b*x + a + 4*c) + 5*cos(-a + c)*sin(2*
b*x + a + 2*c) - cos(-a + c)*sin(a))*sin(8*b*x + a + 8*c) - 20*(10*cos(-a +
 c)*sin(4*b*x + a + 4*c) - 5*cos(-a + c)*sin(2*b*x + a + 2*c) + cos(-a + c)
*sin(a))*sin(6*b*x + a + 6*c) - 20*(5*cos(-a + c)*sin(2*b*x + a + 2*c) - co
s(-a + c)*sin(a))*sin(4*b*x + a + 4*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c)
+ cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) + 15*(cos(10*b*x +
a + 10*c)^2*cos(-a + c) + 25*cos(8*b*x + a + 8*c)^2*cos(-a + c) + 100*cos(6
*b*x + a + 6*c)^2*cos(-a + c) + 100*cos(4*b*x + a + 4*c)^2*cos(-a + c) + 25
*cos(2*b*x + a + 2*c)^2*cos(-a + c) - 10*cos(2*b*x + a + 2*c)*cos(a)*cos(-a
 + c) + cos(-a + c)*sin(10*b*x + a + 10*c)^2 + 25*cos(-a + c)*sin(8*b*x + a
 + 8*c)^2 + 100*cos(-a + c)*sin(6*b*x + a + 6*c)^2 + 100*cos(-a + c)*sin(4*
b*x + a + 4*c)^2 + 25*cos(-a + c)*sin(2*b*x + a + 2*c)^2 - 10*cos(-a + c)*s
in(2*b*x + a + 2*c)*sin(a) - 2*(5*cos(8*b*x + a + 8*c)*cos(-a + c) - 10*cos
(6*b*x + a + 6*c)*cos(-a + c) + 10*cos(4*b*x + a + 4*c)*cos(-a + c) - 5*cos
(2*b*x + a + 2*c)*cos(-a + c) + cos(a)*cos(-a + c))*cos(10*b*x + a + 10*c)
- 10*(10*cos(6*b*x + a + 6*c)*cos(-a + c) - 10*cos(4*b*x + a + 4*c)*cos(-a
+ c) + 5*cos(2*b*x + a + 2*c)*cos(-a + c) - cos(a)*cos(-a + c))*cos(8*b*x +
 a + 8*c) - 20*(10*cos(4*b*x + a + 4*c)*cos(-a + c) - 5*cos(2*b*x + a + 2*c
)*cos(-a + c) + cos(a)*cos(-a + c))*cos(6*b*x + a + 6*c) - 20*(5*cos(2*b*x
+ a + 2*c)*cos(-a + c) - cos(a)*cos(-a + c))*cos(4*b*x + a + 4*c) + (cos(a)
^2 + sin(a)^2)*cos(-a + c) - 2*(5*cos(-a + c)*sin(8*b*x + a + 8*c) - 10*cos
(-a + c)*sin(6*b*x + a + 6*c) + 10*cos(-a + c)*sin(4*b*x + a + 4*c) - 5*cos
(-a + c)*sin(2*b*x + a + 2*c) + cos(-a + c)*sin(a))*sin(10*b*x + a + 10*c)
- 10*(10*cos(-a + c)*sin(6*b*x + a + 6*c) - 10*cos(-a + c)*sin(4*b*x + a +
4*c) + 5*cos(-a + c)*sin(2*b*x + a + 2*c) - cos(-a + c)*sin(a))*sin(8*b*x +
 a + 8*c) - 20*(10*cos(-a + c)*sin(4*b*x + a + 4*c) - 5*cos(-a + c)*sin(2*b
*x + a + 2*c) + cos(-a + c)*sin(a))*sin(6*b*x + a + 6*c) - 20*(5*cos(-a + c
)*sin(2*b*x + a + 2*c) - cos(-a + c)*sin(a))*sin(4*b*x + a + 4*c))*log(cos(
b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + si

```
n(c)^2) + 2*(15*sin(9*b*x + 2*a + 8*c) + 15*sin(9*b*x + 10*c) - 70*sin(7*b*
x + 2*a + 6*c) - 70*sin(7*b*x + 8*c) - 128*sin(5*b*x + 2*a + 4*c) + 128*sin
(5*b*x + 6*c) + 70*sin(3*b*x + 2*a + 2*c) + 70*sin(3*b*x + 4*c) - 15*sin(b*
x + 2*a) - 15*sin(b*x + 2*c))*sin(10*b*x + a + 10*c) - 30*(5*sin(8*b*x + a
+ 8*c) - 10*sin(6*b*x + a + 6*c) + 10*sin(4*b*x + a + 4*c) - 5*sin(2*b*x +
a + 2*c) + sin(a))*sin(9*b*x + 2*a + 8*c) - 30*(5*sin(8*b*x + a + 8*c) - 10
*sin(6*b*x + a + 6*c) + 10*sin(4*b*x + a + 4*c) - 5*sin(2*b*x + a + 2*c) +
sin(a))*sin(9*b*x + 10*c) + 10*(70*sin(7*b*x + 2*a + 6*c) + 70*sin(7*b*x +
8*c) + 128*sin(5*b*x + 2*a + 4*c) - 128*sin(5*b*x + 6*c) - 70*sin(3*b*x + 2
*a + 2*c) - 70*sin(3*b*x + 4*c) + 15*sin(b*x + 2*a) + 15*sin(b*x + 2*c))*si
n(8*b*x + a + 8*c) - 140*(10*sin(6*b*x + a + 6*c) - 10*sin(4*b*x + a + 4*c)
 + 5*sin(2*b*x + a + 2*c) - sin(a))*sin(7*b*x + 2*a + 6*c) - 140*(10*sin(6*
b*x + a + 6*c) - 10*sin(4*b*x + a + 4*c) + 5*sin(2*b*x + a + 2*c) - sin(a))
*sin(7*b*x + 8*c) - 20*(128*sin(5*b*x + 2*a + 4*c) - 128*sin(5*b*x + 6*c) -
 70*sin(3*b*x + 2*a + 2*c) - 70*sin(3*b*x + 4*c) + 15*sin(b*x + 2*a) + 15*s
in(b*x + 2*c))*sin(6*b*x + a + 6*c) + 256*(10*sin(4*b*x + a + 4*c) - 5*sin(
2*b*x + a + 2*c) + sin(a))*sin(5*b*x + 2*a + 4*c) - 256*(10*sin(4*b*x + a +
 4*c) - 5*sin(2*b*x + a + 2*c) + sin(a))*sin(5*b*x + 6*c) - 100*(14*sin(3*b
*x + 2*a + 2*c) + 14*sin(3*b*x + 4*c) - 3*sin(b*x + 2*a) - 3*sin(b*x + 2*c)
)*sin(4*b*x + a + 4*c) + 140*(5*sin(2*b*x + a + 2*c) - sin(a))*sin(3*b*x +
2*a + 2*c) + 140*(5*sin(2*b*x + a + 2*c) - sin(a))*sin(3*b*x + 4*c) - 150*(
sin(b*x + 2*a) + sin(b*x + 2*c))*sin(2*b*x + a + 2*c) + 30*sin(b*x + 2*a)*s
in(a) + 30*sin(b*x + 2*c)*sin(a))/(b*cos(10*b*x + a + 10*c)^2 + 25*b*cos(8*
b*x + a + 8*c)^2 + 100*b*cos(6*b*x + a + 6*c)^2 + 100*b*cos(4*b*x + a + 4*c
)^2 + 25*b*cos(2*b*x + a + 2*c)^2 - 10*b*cos(2*b*x + a + 2*c)*cos(a) + b*si
n(10*b*x + a + 10*c)^2 + 25*b*sin(8*b*x + a + 8*c)^2 + 100*b*sin(6*b*x + a
+ 6*c)^2 + 100*b*sin(4*b*x + a + 4*c)^2 + 25*b*sin(2*b*x + a + 2*c)^2 - 10*
b*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b - 2*(5*b*cos(8*b*x
+ a + 8*c) - 10*b*cos(6*b*x + a + 6*c) + 10*b*cos(4*b*x + a + 4*c) - 5*b*co
s(2*b*x + a + 2*c) + b*cos(a))*cos(10*b*x + a + 10*c) - 10*(10*b*cos(6*b*x
+ a + 6*c) - 10*b*cos(4*b*x + a + 4*c) + 5*b*cos(2*b*x + a + 2*c) - b*cos(a
))*cos(8*b*x + a + 8*c) - 20*(10*b*cos(4*b*x + a + 4*c) - 5*b*cos(2*b*x + a
 + 2*c) + b*cos(a))*cos(6*b*x + a + 6*c) - 20*(5*b*cos(2*b*x + a + 2*c) - b
*cos(a))*cos(4*b*x + a + 4*c) - 2*(5*b*sin(8*b*x + a + 8*c) - 10*b*sin(6*b*
x + a + 6*c) + 10*b*sin(4*b*x + a + 4*c) - 5*b*sin(2*b*x + a + 2*c) + b*sin
(a))*sin(10*b*x + a + 10*c) - 10*(10*b*sin(6*b*x + a + 6*c) - 10*b*sin(4*b*
x + a + 4*c) + 5*b*sin(2*b*x + a + 2*c) - b*sin(a))*sin(8*b*x + a + 8*c) -
20*(10*b*sin(4*b*x + a + 4*c) - 5*b*sin(2*b*x + a + 2*c) + b*sin(a))*sin(6*
b*x + a + 6*c) - 20*(5*b*sin(2*b*x + a + 2*c) - b*sin(a))*sin(4*b*x + a + 4
*c))
```

Fricas [B] time = 0.532326, size = 543, normalized size = 5.78

$$-\frac{15\left(\cos(bx+c)^4\cos(-a+c) - 2\cos(bx+c)^2\cos(-a+c) + \cos(-a+c)\right)\log\left(\frac{1}{2}\cos(bx+c) + \frac{1}{2}\right)\sin(bx+c) - 15}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^6*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/80*(15*(cos(b*x + c)^4*cos(-a + c) - 2*cos(b*x + c)^2*cos(-a + c) + cos(
-a + c))*log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c) - 15*(cos(b*x + c)^4*cos(
-a + c) - 2*cos(b*x + c)^2*cos(-a + c) + cos(-a + c))*log(-1/2*cos(b*x + c)
 + 1/2)*sin(b*x + c) - 10*(3*cos(b*x + c)^3*cos(-a + c) - 5*cos(b*x + c)*co
s(-a + c))*sin(b*x + c) - 16*sin(-a + c))/((b*cos(b*x + c)^4 - 2*b*cos(b*x
+ c)^2 + b)*sin(b*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)**6*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31999, size = 10847, normalized size = 115.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^6*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/320*(120*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2
*c) - tan(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x + 1/2*c)))/(tan(1/2*a)^2*tan(1/
2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - (4*tan(1/2*b*x + 1/2*c)^5*tan(1
/2*a)^10*tan(1/2*c)^9 - 4*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^9*tan(1/2*c)^10
 - 5*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^10*tan(1/2*c)^10 + 16*tan(1/2*b*x +
1/2*c)^5*tan(1/2*a)^10*tan(1/2*c)^7 - 12*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^
9*tan(1/2*c)^8 - 15*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^10*tan(1/2*c)^8 + 12*
tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^8*tan(1/2*c)^9 - 20*tan(1/2*b*x + 1/2*c)^
4*tan(1/2*a)^9*tan(1/2*c)^9 + 20*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^10*tan(1
/2*c)^9 - 16*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^7*tan(1/2*c)^10 - 15*tan(1/2
*b*x + 1/2*c)^4*tan(1/2*a)^8*tan(1/2*c)^10 - 20*tan(1/2*b*x + 1/2*c)^3*tan(
1/2*a)^9*tan(1/2*c)^10 - 40*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^10*tan(1/2*c)
^10 + 24*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^10*tan(1/2*c)^5 - 8*tan(1/2*b*x
+ 1/2*c)^5*tan(1/2*a)^9*tan(1/2*c)^6 - 10*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)
^10*tan(1/2*c)^6 + 48*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^8*tan(1/2*c)^7 - 80
*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^9*tan(1/2*c)^7 + 80*tan(1/2*b*x + 1/2*c)
^3*tan(1/2*a)^10*tan(1/2*c)^7 - 48*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^7*tan(
1/2*c)^8 - 45*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^8*tan(1/2*c)^8 - 60*tan(1/2
*b*x + 1/2*c)^3*tan(1/2*a)^9*tan(1/2*c)^8 - 120*tan(1/2*b*x + 1/2*c)^2*tan(
1/2*a)^10*tan(1/2*c)^8 + 8*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^6*tan(1/2*c)^9
 - 80*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^7*tan(1/2*c)^9 + 60*tan(1/2*b*x + 1
/2*c)^3*tan(1/2*a)^8*tan(1/2*c)^9 - 160*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^9
*tan(1/2*c)^9 + 40*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^10*tan(1/2*c)^9 - 24*tan
(1/2*b*x + 1/2*c)^5*tan(1/2*a)^5*tan(1/2*c)^10 - 10*tan(1/2*b*x + 1/2*c)^4*
tan(1/2*a)^6*tan(1/2*c)^10 - 80*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^7*tan(1/2
*c)^10 - 120*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^8*tan(1/2*c)^10 - 40*tan(1/2
*b*x + 1/2*c)*tan(1/2*a)^9*tan(1/2*c)^10 + 16*tan(1/2*b*x + 1/2*c)^5*tan(1/
2*a)^10*tan(1/2*c)^3 + 8*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^9*tan(1/2*c)^4 +
 10*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^10*tan(1/2*c)^4 + 72*tan(1/2*b*x + 1/
2*c)^5*tan(1/2*a)^8*tan(1/2*c)^5 - 120*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^9*
tan(1/2*c)^5 + 120*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^10*tan(1/2*c)^5 - 32*t
an(1/2*b*x + 1/2*c)^5*tan(1/2*a)^7*tan(1/2*c)^6 - 30*tan(1/2*b*x + 1/2*c)^4
*tan(1/2*a)^8*tan(1/2*c)^6 - 40*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^9*tan(1/2
*c)^6 - 80*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^10*tan(1/2*c)^6 + 32*tan(1/2*b
*x + 1/2*c)^5*tan(1/2*a)^6*tan(1/2*c)^7 - 320*tan(1/2*b*x + 1/2*c)^4*tan(1/
2*a)^7*tan(1/2*c)^7 + 240*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^8*tan(1/2*c)^7
- 640*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^9*tan(1/2*c)^7 + 160*tan(1/2*b*x +
```

```
1/2*c)*tan(1/2*a)^10*tan(1/2*c)^7 - 72*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^5*
tan(1/2*c)^8 - 30*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^6*tan(1/2*c)^8 - 240*ta
n(1/2*b*x + 1/2*c)^3*tan(1/2*a)^7*tan(1/2*c)^8 - 360*tan(1/2*b*x + 1/2*c)^2
*tan(1/2*a)^8*tan(1/2*c)^8 - 120*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^9*tan(1/2*
c)^8 - 8*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^4*tan(1/2*c)^9 - 120*tan(1/2*b*x
 + 1/2*c)^4*tan(1/2*a)^5*tan(1/2*c)^9 + 40*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a
)^6*tan(1/2*c)^9 - 640*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^7*tan(1/2*c)^9 + 1
20*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^8*tan(1/2*c)^9 - 16*tan(1/2*b*x + 1/2*c)
^5*tan(1/2*a)^3*tan(1/2*c)^10 + 10*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^4*tan(
1/2*c)^10 - 120*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^5*tan(1/2*c)^10 - 80*tan(
1/2*b*x + 1/2*c)^2*tan(1/2*a)^6*tan(1/2*c)^10 - 160*tan(1/2*b*x + 1/2*c)*ta
n(1/2*a)^7*tan(1/2*c)^10 + 4*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^10*tan(1/2*c
) + 12*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^9*tan(1/2*c)^2 + 15*tan(1/2*b*x +
1/2*c)^4*tan(1/2*a)^10*tan(1/2*c)^2 + 48*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^
8*tan(1/2*c)^3 - 80*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^9*tan(1/2*c)^3 + 80*t
an(1/2*b*x + 1/2*c)^3*tan(1/2*a)^10*tan(1/2*c)^3 + 32*tan(1/2*b*x + 1/2*c)^
5*tan(1/2*a)^7*tan(1/2*c)^4 + 30*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^8*tan(1/
2*c)^4 + 40*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^9*tan(1/2*c)^4 + 80*tan(1/2*b
*x + 1/2*c)^2*tan(1/2*a)^10*tan(1/2*c)^4 + 48*tan(1/2*b*x + 1/2*c)^5*tan(1/
2*a)^6*tan(1/2*c)^5 - 480*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^7*tan(1/2*c)^5
+ 360*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^8*tan(1/2*c)^5 - 960*tan(1/2*b*x +
1/2*c)^2*tan(1/2*a)^9*tan(1/2*c)^5 + 240*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^10
*tan(1/2*c)^5 - 48*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^5*tan(1/2*c)^6 - 20*ta
n(1/2*b*x + 1/2*c)^4*tan(1/2*a)^6*tan(1/2*c)^6 - 160*tan(1/2*b*x + 1/2*c)^3
*tan(1/2*a)^7*tan(1/2*c)^6 - 240*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^8*tan(1/
2*c)^6 - 80*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^9*tan(1/2*c)^6 - 32*tan(1/2*b*x
 + 1/2*c)^5*tan(1/2*a)^4*tan(1/2*c)^7 - 480*tan(1/2*b*x + 1/2*c)^4*tan(1/2*
a)^5*tan(1/2*c)^7 + 160*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^6*tan(1/2*c)^7 -
2560*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^7*tan(1/2*c)^7 + 480*tan(1/2*b*x + 1
/2*c)*tan(1/2*a)^8*tan(1/2*c)^7 - 48*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^3*ta
n(1/2*c)^8 + 30*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^4*tan(1/2*c)^8 - 360*tan(
1/2*b*x + 1/2*c)^3*tan(1/2*a)^5*tan(1/2*c)^8 - 240*tan(1/2*b*x + 1/2*c)^2*t
an(1/2*a)^6*tan(1/2*c)^8 - 480*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^7*tan(1/2*c)
^8 - 12*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^2*tan(1/2*c)^9 - 80*tan(1/2*b*x +
 1/2*c)^4*tan(1/2*a)^3*tan(1/2*c)^9 - 40*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^
4*tan(1/2*c)^9 - 960*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^5*tan(1/2*c)^9 + 80*
tan(1/2*b*x + 1/2*c)*tan(1/2*a)^6*tan(1/2*c)^9 - 4*tan(1/2*b*x + 1/2*c)^5*t
an(1/2*a)*tan(1/2*c)^10 + 15*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^2*tan(1/2*c)
^10 - 80*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^3*tan(1/2*c)^10 + 80*tan(1/2*b*x
 + 1/2*c)^2*tan(1/2*a)^4*tan(1/2*c)^10 - 240*tan(1/2*b*x + 1/2*c)*tan(1/2*a
)^5*tan(1/2*c)^10 + 4*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^9 + 5*tan(1/2*b*x +
 1/2*c)^4*tan(1/2*a)^10 + 12*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^8*tan(1/2*c)
 - 20*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^9*tan(1/2*c) + 20*tan(1/2*b*x + 1/2
*c)^3*tan(1/2*a)^10*tan(1/2*c) + 48*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^7*tan
(1/2*c)^2 + 45*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^8*tan(1/2*c)^2 + 60*tan(1/
2*b*x + 1/2*c)^3*tan(1/2*a)^9*tan(1/2*c)^2 + 120*tan(1/2*b*x + 1/2*c)^2*tan
(1/2*a)^10*tan(1/2*c)^2 + 32*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^6*tan(1/2*c)
^3 - 320*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^7*tan(1/2*c)^3 + 240*tan(1/2*b*x
 + 1/2*c)^3*tan(1/2*a)^8*tan(1/2*c)^3 - 640*tan(1/2*b*x + 1/2*c)^2*tan(1/2*
a)^9*tan(1/2*c)^3 + 160*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^10*tan(1/2*c)^3 + 4
8*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^5*tan(1/2*c)^4 + 20*tan(1/2*b*x + 1/2*c
)^4*tan(1/2*a)^6*tan(1/2*c)^4 + 160*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^7*tan
(1/2*c)^4 + 240*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^8*tan(1/2*c)^4 + 80*tan(1
/2*b*x + 1/2*c)*tan(1/2*a)^9*tan(1/2*c)^4 - 48*tan(1/2*b*x + 1/2*c)^5*tan(1
/2*a)^4*tan(1/2*c)^5 - 720*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^5*tan(1/2*c)^5
 + 240*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^6*tan(1/2*c)^5 - 3840*tan(1/2*b*x
+ 1/2*c)^2*tan(1/2*a)^7*tan(1/2*c)^5 + 720*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^
8*tan(1/2*c)^5 - 32*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^3*tan(1/2*c)^6 + 20*t
an(1/2*b*x + 1/2*c)^4*tan(1/2*a)^4*tan(1/2*c)^6 - 240*tan(1/2*b*x + 1/2*c)^
```

3*tan(1/2*a)^5*tan(1/2*c)^6 - 160*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^6*tan(1
/2*c)^6 - 320*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^7*tan(1/2*c)^6 - 48*tan(1/2*b
*x + 1/2*c)^5*tan(1/2*a)^2*tan(1/2*c)^7 - 320*tan(1/2*b*x + 1/2*c)^4*tan(1/
2*a)^3*tan(1/2*c)^7 - 160*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^4*tan(1/2*c)^7
- 3840*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^5*tan(1/2*c)^7 + 320*tan(1/2*b*x +
1/2*c)*tan(1/2*a)^6*tan(1/2*c)^7 - 12*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)*ta
n(1/2*c)^8 + 45*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^2*tan(1/2*c)^8 - 240*tan(
1/2*b*x + 1/2*c)^3*tan(1/2*a)^3*tan(1/2*c)^8 + 240*tan(1/2*b*x + 1/2*c)^2*t
an(1/2*a)^4*tan(1/2*c)^8 - 720*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^5*tan(1/2*c)
^8 - 4*tan(1/2*b*x + 1/2*c)^5*tan(1/2*c)^9 - 20*tan(1/2*b*x + 1/2*c)^4*tan(
1/2*a)*tan(1/2*c)^9 - 60*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^2*tan(1/2*c)^9 -
640*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^3*tan(1/2*c)^9 - 80*tan(1/2*b*x + 1/
2*c)*tan(1/2*a)^4*tan(1/2*c)^9 + 5*tan(1/2*b*x + 1/2*c)^4*tan(1/2*c)^10 - 2
0*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)*tan(1/2*c)^10 + 120*tan(1/2*b*x + 1/2*c
)^2*tan(1/2*a)^2*tan(1/2*c)^10 - 160*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^3*tan(
1/2*c)^10 + 16*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^7 + 15*tan(1/2*b*x + 1/2*c
)^4*tan(1/2*a)^8 + 20*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^9 + 40*tan(1/2*b*x
+ 1/2*c)^2*tan(1/2*a)^10 + 8*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^6*tan(1/2*c)
- 80*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^7*tan(1/2*c) + 60*tan(1/2*b*x + 1/2
*c)^3*tan(1/2*a)^8*tan(1/2*c) - 160*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^9*tan
(1/2*c) + 40*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^10*tan(1/2*c) + 72*tan(1/2*b*x
+ 1/2*c)^5*tan(1/2*a)^5*tan(1/2*c)^2 + 30*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a
)^6*tan(1/2*c)^2 + 240*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^7*tan(1/2*c)^2 + 3
60*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^8*tan(1/2*c)^2 + 120*tan(1/2*b*x + 1/2
*c)*tan(1/2*a)^9*tan(1/2*c)^2 - 32*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^4*tan(
1/2*c)^3 - 480*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^5*tan(1/2*c)^3 + 160*tan(1
/2*b*x + 1/2*c)^3*tan(1/2*a)^6*tan(1/2*c)^3 - 2560*tan(1/2*b*x + 1/2*c)^2*t
an(1/2*a)^7*tan(1/2*c)^3 + 480*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^8*tan(1/2*c)
^3 + 32*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^3*tan(1/2*c)^4 - 20*tan(1/2*b*x +
1/2*c)^4*tan(1/2*a)^4*tan(1/2*c)^4 + 240*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)
^5*tan(1/2*c)^4 + 160*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^6*tan(1/2*c)^4 + 32
0*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^7*tan(1/2*c)^4 - 72*tan(1/2*b*x + 1/2*c)^
5*tan(1/2*a)^2*tan(1/2*c)^5 - 480*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^3*tan(1
/2*c)^5 - 240*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^4*tan(1/2*c)^5 - 5760*tan(1
/2*b*x + 1/2*c)^2*tan(1/2*a)^5*tan(1/2*c)^5 + 480*tan(1/2*b*x + 1/2*c)*tan(
1/2*a)^6*tan(1/2*c)^5 - 8*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)*tan(1/2*c)^6 +
30*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^2*tan(1/2*c)^6 - 160*tan(1/2*b*x + 1/2
*c)^3*tan(1/2*a)^3*tan(1/2*c)^6 + 160*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^4*t
an(1/2*c)^6 - 480*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^5*tan(1/2*c)^6 - 16*tan(1
/2*b*x + 1/2*c)^5*tan(1/2*c)^7 - 80*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)*tan(1
/2*c)^7 - 240*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^2*tan(1/2*c)^7 - 2560*tan(1
/2*b*x + 1/2*c)^2*tan(1/2*a)^3*tan(1/2*c)^7 - 320*tan(1/2*b*x + 1/2*c)*tan(
1/2*a)^4*tan(1/2*c)^7 + 15*tan(1/2*b*x + 1/2*c)^4*tan(1/2*c)^8 - 60*tan(1/2
*b*x + 1/2*c)^3*tan(1/2*a)*tan(1/2*c)^8 + 360*tan(1/2*b*x + 1/2*c)^2*tan(1/
2*a)^2*tan(1/2*c)^8 - 480*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^3*tan(1/2*c)^8 -
20*tan(1/2*b*x + 1/2*c)^3*tan(1/2*c)^9 - 160*tan(1/2*b*x + 1/2*c)^2*tan(1/2
*a)*tan(1/2*c)^9 - 120*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2*tan(1/2*c)^9 + 40*
tan(1/2*b*x + 1/2*c)^2*tan(1/2*c)^10 - 40*tan(1/2*b*x + 1/2*c)*tan(1/2*a)*t
an(1/2*c)^10 + 24*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^5 + 10*tan(1/2*b*x + 1/
2*c)^4*tan(1/2*a)^6 + 80*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^7 + 120*tan(1/2*
b*x + 1/2*c)^2*tan(1/2*a)^8 + 40*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^9 - 8*tan(
1/2*b*x + 1/2*c)^5*tan(1/2*a)^4*tan(1/2*c) - 120*tan(1/2*b*x + 1/2*c)^4*tan
(1/2*a)^5*tan(1/2*c) + 40*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^6*tan(1/2*c) -
640*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^7*tan(1/2*c) + 120*tan(1/2*b*x + 1/2*
c)*tan(1/2*a)^8*tan(1/2*c) + 48*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^3*tan(1/2
*c)^2 - 30*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^4*tan(1/2*c)^2 + 360*tan(1/2*b
*x + 1/2*c)^3*tan(1/2*a)^5*tan(1/2*c)^2 + 240*tan(1/2*b*x + 1/2*c)^2*tan(1/
2*a)^6*tan(1/2*c)^2 + 480*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^7*tan(1/2*c)^2 -
48*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^2*tan(1/2*c)^3 - 320*tan(1/2*b*x + 1/2

*c)^4*tan(1/2*a)^3*tan(1/2*c)^3 - 160*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^4*t
an(1/2*c)^3 - 3840*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^5*tan(1/2*c)^3 + 320*t
an(1/2*b*x + 1/2*c)*tan(1/2*a)^6*tan(1/2*c)^3 + 8*tan(1/2*b*x + 1/2*c)^5*ta
n(1/2*a)*tan(1/2*c)^4 - 30*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^2*tan(1/2*c)^4
 + 160*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^3*tan(1/2*c)^4 - 160*tan(1/2*b*x +
 1/2*c)^2*tan(1/2*a)^4*tan(1/2*c)^4 + 480*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^5
*tan(1/2*c)^4 - 24*tan(1/2*b*x + 1/2*c)^5*tan(1/2*c)^5 - 120*tan(1/2*b*x +
1/2*c)^4*tan(1/2*a)*tan(1/2*c)^5 - 360*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^2*
tan(1/2*c)^5 - 3840*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^3*tan(1/2*c)^5 - 480*
tan(1/2*b*x + 1/2*c)*tan(1/2*a)^4*tan(1/2*c)^5 + 10*tan(1/2*b*x + 1/2*c)^4*
tan(1/2*c)^6 - 40*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)*tan(1/2*c)^6 + 240*tan(
1/2*b*x + 1/2*c)^2*tan(1/2*a)^2*tan(1/2*c)^6 - 320*tan(1/2*b*x + 1/2*c)*tan
(1/2*a)^3*tan(1/2*c)^6 - 80*tan(1/2*b*x + 1/2*c)^3*tan(1/2*c)^7 - 640*tan(1
/2*b*x + 1/2*c)^2*tan(1/2*a)*tan(1/2*c)^7 - 480*tan(1/2*b*x + 1/2*c)*tan(1/
2*a)^2*tan(1/2*c)^7 + 120*tan(1/2*b*x + 1/2*c)^2*tan(1/2*c)^8 - 120*tan(1/2
*b*x + 1/2*c)*tan(1/2*a)*tan(1/2*c)^8 - 40*tan(1/2*b*x + 1/2*c)*tan(1/2*c)^
9 + 16*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^3 - 10*tan(1/2*b*x + 1/2*c)^4*tan(
1/2*a)^4 + 120*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^5 + 80*tan(1/2*b*x + 1/2*c
)^2*tan(1/2*a)^6 + 160*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^7 - 12*tan(1/2*b*x +
 1/2*c)^5*tan(1/2*a)^2*tan(1/2*c) - 80*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^3*
tan(1/2*c) - 40*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^4*tan(1/2*c) - 960*tan(1/
2*b*x + 1/2*c)^2*tan(1/2*a)^5*tan(1/2*c) + 80*tan(1/2*b*x + 1/2*c)*tan(1/2*
a)^6*tan(1/2*c) + 12*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)*tan(1/2*c)^2 - 45*ta
n(1/2*b*x + 1/2*c)^4*tan(1/2*a)^2*tan(1/2*c)^2 + 240*tan(1/2*b*x + 1/2*c)^3
*tan(1/2*a)^3*tan(1/2*c)^2 - 240*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^4*tan(1/
2*c)^2 + 720*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^5*tan(1/2*c)^2 - 16*tan(1/2*b*
x + 1/2*c)^5*tan(1/2*c)^3 - 80*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)*tan(1/2*c)
^3 - 240*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^2*tan(1/2*c)^3 - 2560*tan(1/2*b*
x + 1/2*c)^2*tan(1/2*a)^3*tan(1/2*c)^3 - 320*tan(1/2*b*x + 1/2*c)*tan(1/2*a
)^4*tan(1/2*c)^3 - 10*tan(1/2*b*x + 1/2*c)^4*tan(1/2*c)^4 + 40*tan(1/2*b*x
+ 1/2*c)^3*tan(1/2*a)*tan(1/2*c)^4 - 240*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^
2*tan(1/2*c)^4 + 320*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^3*tan(1/2*c)^4 - 120*t
an(1/2*b*x + 1/2*c)^3*tan(1/2*c)^5 - 960*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)*
tan(1/2*c)^5 - 720*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2*tan(1/2*c)^5 + 80*tan(
1/2*b*x + 1/2*c)^2*tan(1/2*c)^6 - 80*tan(1/2*b*x + 1/2*c)*tan(1/2*a)*tan(1/
2*c)^6 - 160*tan(1/2*b*x + 1/2*c)*tan(1/2*c)^7 + 4*tan(1/2*b*x + 1/2*c)^5*t
an(1/2*a) - 15*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^2 + 80*tan(1/2*b*x + 1/2*c
)^3*tan(1/2*a)^3 - 80*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^4 + 240*tan(1/2*b*x
 + 1/2*c)*tan(1/2*a)^5 - 4*tan(1/2*b*x + 1/2*c)^5*tan(1/2*c) - 20*tan(1/2*b
*x + 1/2*c)^4*tan(1/2*a)*tan(1/2*c) - 60*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^
2*tan(1/2*c) - 640*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^3*tan(1/2*c) - 80*tan(
1/2*b*x + 1/2*c)*tan(1/2*a)^4*tan(1/2*c) - 15*tan(1/2*b*x + 1/2*c)^4*tan(1/
2*c)^2 + 60*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)*tan(1/2*c)^2 - 360*tan(1/2*b*
x + 1/2*c)^2*tan(1/2*a)^2*tan(1/2*c)^2 + 480*tan(1/2*b*x + 1/2*c)*tan(1/2*a
)^3*tan(1/2*c)^2 - 80*tan(1/2*b*x + 1/2*c)^3*tan(1/2*c)^3 - 640*tan(1/2*b*x
 + 1/2*c)^2*tan(1/2*a)*tan(1/2*c)^3 - 480*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2
*tan(1/2*c)^3 - 80*tan(1/2*b*x + 1/2*c)^2*tan(1/2*c)^4 + 80*tan(1/2*b*x + 1
/2*c)*tan(1/2*a)*tan(1/2*c)^4 - 240*tan(1/2*b*x + 1/2*c)*tan(1/2*c)^5 - 5*t
an(1/2*b*x + 1/2*c)^4 + 20*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a) - 120*tan(1/2*
b*x + 1/2*c)^2*tan(1/2*a)^2 + 160*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^3 - 20*ta
n(1/2*b*x + 1/2*c)^3*tan(1/2*c) - 160*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)*tan
(1/2*c) - 120*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2*tan(1/2*c) - 120*tan(1/2*b*
x + 1/2*c)^2*tan(1/2*c)^2 + 120*tan(1/2*b*x + 1/2*c)*tan(1/2*a)*tan(1/2*c)^
2 - 160*tan(1/2*b*x + 1/2*c)*tan(1/2*c)^3 - 40*tan(1/2*b*x + 1/2*c)^2 + 40*
tan(1/2*b*x + 1/2*c)*tan(1/2*a) - 40*tan(1/2*b*x + 1/2*c)*tan(1/2*c))/(tan(
1/2*a)^10*tan(1/2*c)^10 + 5*tan(1/2*a)^10*tan(1/2*c)^8 + 5*tan(1/2*a)^8*tan
(1/2*c)^10 + 10*tan(1/2*a)^10*tan(1/2*c)^6 + 25*tan(1/2*a)^8*tan(1/2*c)^8 +
 10*tan(1/2*a)^6*tan(1/2*c)^10 + 10*tan(1/2*a)^10*tan(1/2*c)^4 + 50*tan(1/2
*a)^8*tan(1/2*c)^6 + 50*tan(1/2*a)^6*tan(1/2*c)^8 + 10*tan(1/2*a)^4*tan(1/2

```
*c)^10 + 5*tan(1/2*a)^10*tan(1/2*c)^2 + 50*tan(1/2*a)^8*tan(1/2*c)^4 + 100*
tan(1/2*a)^6*tan(1/2*c)^6 + 50*tan(1/2*a)^4*tan(1/2*c)^8 + 5*tan(1/2*a)^2*t
an(1/2*c)^10 + tan(1/2*a)^10 + 25*tan(1/2*a)^8*tan(1/2*c)^2 + 100*tan(1/2*a
)^6*tan(1/2*c)^4 + 100*tan(1/2*a)^4*tan(1/2*c)^6 + 25*tan(1/2*a)^2*tan(1/2*
c)^8 + tan(1/2*c)^10 + 5*tan(1/2*a)^8 + 50*tan(1/2*a)^6*tan(1/2*c)^2 + 100*
tan(1/2*a)^4*tan(1/2*c)^4 + 50*tan(1/2*a)^2*tan(1/2*c)^6 + 5*tan(1/2*c)^8 +
 10*tan(1/2*a)^6 + 50*tan(1/2*a)^4*tan(1/2*c)^2 + 50*tan(1/2*a)^2*tan(1/2*c
)^4 + 10*tan(1/2*c)^6 + 10*tan(1/2*a)^4 + 25*tan(1/2*a)^2*tan(1/2*c)^2 + 10
*tan(1/2*c)^4 + 5*tan(1/2*a)^2 + 5*tan(1/2*c)^2 + 1) - (274*tan(1/2*b*x + 1
/2*c)^5*tan(1/2*a)^2*tan(1/2*c)^2 - 274*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^2
 + 1096*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)*tan(1/2*c) + 40*tan(1/2*b*x + 1/2
*c)^4*tan(1/2*a)^2*tan(1/2*c) - 274*tan(1/2*b*x + 1/2*c)^5*tan(1/2*c)^2 - 4
0*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)*tan(1/2*c)^2 + 40*tan(1/2*b*x + 1/2*c)^
3*tan(1/2*a)^2*tan(1/2*c)^2 + 274*tan(1/2*b*x + 1/2*c)^5 + 40*tan(1/2*b*x +
 1/2*c)^4*tan(1/2*a) - 40*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^2 - 40*tan(1/2*
b*x + 1/2*c)^4*tan(1/2*c) + 160*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)*tan(1/2*c
) + 20*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^2*tan(1/2*c) - 40*tan(1/2*b*x + 1/
2*c)^3*tan(1/2*c)^2 - 20*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)*tan(1/2*c)^2 + 5
*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2*tan(1/2*c)^2 + 40*tan(1/2*b*x + 1/2*c)^3
 + 20*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a) - 5*tan(1/2*b*x + 1/2*c)*tan(1/2*a)
^2 - 20*tan(1/2*b*x + 1/2*c)^2*tan(1/2*c) + 20*tan(1/2*b*x + 1/2*c)*tan(1/2
*a)*tan(1/2*c) + 4*tan(1/2*a)^2*tan(1/2*c) - 5*tan(1/2*b*x + 1/2*c)*tan(1/2
*c)^2 - 4*tan(1/2*a)*tan(1/2*c)^2 + 5*tan(1/2*b*x + 1/2*c) + 4*tan(1/2*a) -
 4*tan(1/2*c))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 +
1)*tan(1/2*b*x + 1/2*c)^5))/b
```

3.201 $\int \sin^2(a+bx)\sin^n(c+dx)\,dx$

Optimal. Leaf size=410

$$-\frac{i2^{-n-2}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\left(1-e^{2ic+2idx}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(-\frac{2b}{d}-n\right),-n,\frac{1}{2}\left(-\frac{2b}{d}-n+2\right),e^{2i(c+dx)}\right)\exp}{2b+dn}$$

```
[Out] ((-I)*2^(-2 - n)*E^((-I)*(2*a + c*n) - I*(2*b + d*n)*x + I*n*(c + d*x))*(I/
E^(I*(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[((-2*b)/d - n)/2,
-n, (2 - (2*b)/d - n)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x
))^n*(2*b + d*n)) + (I*2^(-2 - n)*E^(I*(2*a - c*n) + I*(2*b - d*n)*x + I*n*
(c + d*x))*(I/E^(I*(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[((2*
b)/d - n)/2, -n, (2 + (2*b)/d - n)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*
c + (2*I)*d*x))^n*(2*b - d*n)) + (I*2^(-1 - n)*(I/E^(I*(c + d*x)) - I*E^(I*
(c + d*x)))^n*Hypergeometric2F1[-n, -n/2, 1 - n/2, E^((2*I)*(c + d*x))])/(d
*(1 - E^((2*I)*(c + d*x)))^n*n)
```

Rubi [A] time = 0.972974, antiderivative size = 410, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {4553, 2282, 1980, 2032, 365, 364, 2285, 2253, 2252, 2251}

$$-\frac{i2^{-n-2}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\left(1-e^{2ic+2idx}\right)^{-n}{}_2F_1\left(\frac{1}{2}\left(-\frac{2b}{d}-n\right),-n;\frac{1}{2}\left(-\frac{2b}{d}-n+2\right);e^{2i(c+dx)}\right)\exp(-i(2a+cn)-ix(2}{2b+dn}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[c + d*x]^n,x]
```


```
[Out] ((-I)*2^(-2 - n)*E^((-I)*(2*a + c*n) - I*(2*b + d*n)*x + I*n*(c + d*x))*(I/
E^(I*(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[((-2*b)/d - n)/2,
-n, (2 - (2*b)/d - n)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x
))^n*(2*b + d*n)) + (I*2^(-2 - n)*E^(I*(2*a - c*n) + I*(2*b - d*n)*x + I*n*
(c + d*x))*(I/E^(I*(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[((2*
b)/d - n)/2, -n, (2 + (2*b)/d - n)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*
c + (2*I)*d*x))^n*(2*b - d*n)) + (I*2^(-1 - n)*(I/E^(I*(c + d*x)) - I*E^(I*
(c + d*x)))^n*Hypergeometric2F1[-n, -n/2, 1 - n/2, E^((2*I)*(c + d*x))])/(d
*(1 - E^((2*I)*(c + d*x)))^n*n)
```


Rule 4553

```
Int[Sin[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Dist[1/2^(p + q), Int[ExpandIntegrand[(I/E^(I*(c + d*x)) - I*E^(I*(c + d
*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
 b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 1980

```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && GeneralizedBinomialQ[u, x] && !Generali
zedBinomialMatchQ[u, x]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2285

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Dist[(a*F^
v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n), Int[u*F^(n*v)*(a
+ b*F^ExpandToSum[w - v, x])^n, x], x] /; FreeQ[{F, a, b, n}, x] && !Integ
erQ[n] && LinearQ[{v, w}, x]
```

Rule 2253

```
Int[((a_) + (b_.)*(F_)^((e_.)*(v_)))^(p_)*(G_)^((h_.)*(u_)), x_Symbol] :> I
nt[G^(h*ExpandToSum[u, x])*(a + b*F^(e*ExpandToSum[v, x]))^p, x] /; FreeQ[{
F, G, a, b, e, h, p}, x] && LinearQ[{u, v}, x] && !LinearMatchQ[{u, v}, x]
```

Rule 2252

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Dist[(a + b*F^(e*(c + d*x)))^p/(1 + (b/a)*F^
(e*(c + d*x)))^p, Int[G^(h*(f + g*x))*(1 + (b*F^(e*(c + d*x)))/a)^p, x], x]
 /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x] && !(ILtQ[p, 0] || GtQ[a,
0])
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \sin^2(a+bx)\sin^n(c+dx)\,dx = 2^{-2-n}\int \left(2\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n - e^{-2ia-2ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n - e^{2ia+2ibx}\left(ie^{-i(c+\cdots\right.\right.$$

$$= -\left(2^{-2-n}\int e^{-2ia-2ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n dx\right) - 2^{-2-n}\int e^{2ia+2ibx}\left(ie^{-i(c+dx)}-ie^{i(c+d\cdots\right.$$

$$= -\frac{\left(i2^{-1-n}\right)\text{Subst}\left(\int \frac{\left(-\frac{i(-1+x^2)}{x}\right)^n}{x}dx, x, e^{i(c+dx)}\right)}{d} - \left(2^{-2-n}e^{in(c+dx)}\left(i-ie^{2ic+2idx}\right)^{-n}\left(ie^{-i(c\cdots\right.\right.$$

$$= -\frac{\left(i2^{-1-n}\right)\text{Subst}\left(\int \frac{\left(\frac{i}{x}-ix\right)^n}{x}dx, x, e^{i(c+dx)}\right)}{d} - \left(2^{-2-n}e^{in(c+dx)}\left(i-ie^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}\cdots\right.\right.$$

$$= -\left(\left(2^{-2-n}e^{in(c+dx)}\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\right)\int e^{i(2a-cn)+i(2b-dn)x}\left(1-e^{2ic\cdots}\right.\right.$$

$$= -\frac{i2^{-2-n}\exp(-i(2a+cn)-i(2b+dn)x+in(c+dx))\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(\cdots}\right.}{2b+dn}$$

$$= -\frac{i2^{-2-n}\exp(-i(2a+cn)-i(2b+dn)x+in(c+dx))\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(\cdots}\right.}{2b+dn}$$

Mathematica [F] time = 0.375229, size = 0, normalized size = 0.

$$\int \sin^2(a+bx)\sin^n(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[a + b*x]^2*Sin[c + d*x]^n,x]
```

```
[Out] Integrate[Sin[a + b*x]^2*Sin[c + d*x]^n, x]
```

Maple [F] time = 1.504, size = 0, normalized size = 0.

$$\int (\sin(bx+a))^2(\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(d*x+c)^n,x)
```

```
[Out] int(sin(b*x+a)^2*sin(d*x+c)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(dx+c)^n\sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^n*sin(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos\left(bx+a\right)^{2}-1\right)\sin\left(dx+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sin(d*x + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(d*x+c)**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(dx+c\right)^{n}\sin\left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*sin(b*x + a)^2, x)
```

3.202 $\int \sin^2(a+bx)\sin(c+dx)\,dx$

Optimal. Leaf size=68

$$-\frac{\cos(2a+x(2b-d)-c)}{4(2b-d)}+\frac{\cos(2a+x(2b+d)+c)}{4(2b+d)}-\frac{\cos(c+dx)}{2d}$$

```
[Out] -Cos[2*a - c + (2*b - d)*x]/(4*(2*b - d)) - Cos[c + d*x]/(2*d) + Cos[2*a +
c + (2*b + d)*x]/(4*(2*b + d))
```

Rubi [A] time = 0.0536243, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4569, 2638}

$$-\frac{\cos(2a+x(2b-d)-c)}{4(2b-d)}+\frac{\cos(2a+x(2b+d)+c)}{4(2b+d)}-\frac{\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[c + d*x],x]
```

```
[Out] -Cos[2*a - c + (2*b - d)*x]/(4*(2*b - d)) - Cos[c + d*x]/(2*d) + Cos[2*a +
c + (2*b + d)*x]/(4*(2*b + d))
```

Rule 4569

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+bx)\sin(c+dx)\,dx &= \int \left(\frac{1}{4}\sin(2a-c+(2b-d)x)+\frac{1}{2}\sin(c+dx)-\frac{1}{4}\sin(2a+c+(2b+d)x)\right)dx\\ &= \frac{1}{4}\int \sin(2a-c+(2b-d)x)\,dx-\frac{1}{4}\int \sin(2a+c+(2b+d)x)\,dx+\frac{1}{2}\int \sin(c+dx)\,dx\\ &= -\frac{\cos(2a-c+(2b-d)x)}{4(2b-d)}-\frac{\cos(c+dx)}{2d}+\frac{\cos(2a+c+(2b+d)x)}{4(2b+d)}\end{aligned}$$

Mathematica [A] time = 0.35091, size = 80, normalized size = 1.18

$$-\frac{\cos(2a+2bx-c-dx)}{4(2b-d)}+\frac{\cos(2a+x(2b+d)+c)}{4(2b+d)}+\frac{1}{2}\left(\frac{\sin(c)\sin(dx)}{d}-\frac{\cos(c)\cos(dx)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[c + d*x],x]
```

```
[Out] -Cos[2*a - c + 2*b*x - d*x]/(4*(2*b - d)) + Cos[2*a + c + (2*b + d)*x]/(4*(
2*b + d)) + (-((Cos[c]*Cos[d*x])/d) + (Sin[c]*Sin[d*x])/d)/2
```

Maple [A] time = 0.017, size = 63, normalized size = 0.9

$$-\frac{\cos(2a-c+(2b-d)x)}{8b-4d}-\frac{\cos(dx+c)}{2d}+\frac{\cos(2a+c+(2b+d)x)}{8b+4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(d*x+c),x)
```

```
[Out] -1/4*cos(2*a-c+(2*b-d)*x)/(2*b-d)-1/2*cos(d*x+c)/d+1/4*cos(2*a+c+(2*b+d)*x)
/(2*b+d)
```

Maxima [B] time = 1.149, size = 501, normalized size = 7.37

$$-\frac{\left(2\,bd\cos(c)-d^2\cos(c)\right)\cos((2\,b+d)x+2\,a+2\,c)+\left(2\,bd\cos(c)-d^2\cos(c)\right)\cos((2\,b+d)x+2\,a)-\left(2\,bd\cos(c)+d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*((2*b*d*cos(c) - d^2*cos(c))*cos((2*b + d)*x + 2*a + 2*c) + (2*b*d*cos
(c) - d^2*cos(c))*cos((2*b + d)*x + 2*a) - (2*b*d*cos(c) + d^2*cos(c))*cos(
-(2*b - d)*x - 2*a + 2*c) - (2*b*d*cos(c) + d^2*cos(c))*cos(-(2*b - d)*x -
2*a) - 2*(4*b^2*cos(c) - d^2*cos(c))*cos(d*x + 2*c) - 2*(4*b^2*cos(c) - d^2
*cos(c))*cos(d*x) + (2*b*d*sin(c) - d^2*sin(c))*sin((2*b + d)*x + 2*a + 2*c
) - (2*b*d*sin(c) - d^2*sin(c))*sin((2*b + d)*x + 2*a) - (2*b*d*sin(c) + d^
2*sin(c))*sin(-(2*b - d)*x - 2*a + 2*c) + (2*b*d*sin(c) + d^2*sin(c))*sin(-
(2*b - d)*x - 2*a) - 2*(4*b^2*sin(c) - d^2*sin(c))*sin(d*x + 2*c) + 2*(4*b^
2*sin(c) - d^2*sin(c))*sin(d*x))/((cos(c)^2 + sin(c)^2)*d^3 - 4*(b^2*cos(c)
^2 + b^2*sin(c)^2)*d)
```

Fricas [A] time = 0.477438, size = 155, normalized size = 2.28

$$-\frac{2\,bd\cos(bx+a)\sin(bx+a)\sin(dx+c)+\left(d^2\cos(bx+a)^2+2\,b^2-d^2\right)\cos(dx+c)}{4\,b^2d-d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -(2*b*d*cos(b*x + a)*sin(b*x + a)*sin(d*x + c) + (d^2*cos(b*x + a)^2 + 2*b^
2 - d^2)*cos(d*x + c))/(4*b^2*d - d^3)
```

Sympy [A] time = 18.2682, size = 410, normalized size = 6.03

$$\begin{cases} x\sin^2(a)\sin(c) \\ \frac{x\sin^2\left(a-\frac{dx}{2}\right)\sin(c+dx)}{4} - \frac{x\sin\left(a-\frac{dx}{2}\right)\cos\left(a-\frac{dx}{2}\right)\cos(c+dx)}{2} - \frac{x\sin(c+dx)\cos^2\left(a-\frac{dx}{2}\right)}{4} - \frac{\sin^2\left(a-\frac{dx}{2}\right)\cos(c+dx)}{d} - \frac{\sin\left(a-\frac{dx}{2}\right)\sin(c+dx)\cos\left(a-\frac{dx}{2}\right)}{2d} \\ \frac{x\sin^2\left(a+\frac{dx}{2}\right)\sin(c+dx)}{4} + \frac{x\sin\left(a+\frac{dx}{2}\right)\cos\left(a+\frac{dx}{2}\right)\cos(c+dx)}{2} - \frac{x\sin(c+dx)\cos^2\left(a+\frac{dx}{2}\right)}{4} - \frac{3\sin\left(a+\frac{dx}{2}\right)\sin(c+dx)\cos\left(a+\frac{dx}{2}\right)}{2d} - \frac{\cos^2\left(a+\frac{dx}{2}\right)\cos(c+dx)}{d} \\ \left(\frac{x\sin^2(a+bx)}{2} + \frac{x\cos^2(a+bx)}{2} - \frac{\sin(a+bx)\cos(a+bx)}{2b}\right)\sin(c) \\ -\frac{2b^2\sin^2(a+bx)\cos(c+dx)}{4b^2d-d^3} - \frac{2b^2\cos^2(a+bx)\cos(c+dx)}{4b^2d-d^3} - \frac{2bd\sin(a+bx)\sin(c+dx)\cos(a+bx)}{4b^2d-d^3} + \frac{d^2\sin^2(a+bx)\cos(c+dx)}{4b^2d-d^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(d*x+c),x)
```

```
[Out] Piecewise((x*sin(a)**2*sin(c), Eq(b, 0) & Eq(d, 0)), (x*sin(a - d*x/2)**2*s
in(c + d*x)/4 - x*sin(a - d*x/2)*cos(a - d*x/2)*cos(c + d*x)/2 - x*sin(c +
d*x)*cos(a - d*x/2)**2/4 - sin(a - d*x/2)**2*cos(c + d*x)/d - sin(a - d*x/2
)*sin(c + d*x)*cos(a - d*x/2)/(2*d), Eq(b, -d/2)), (x*sin(a + d*x/2)**2*sin
(c + d*x)/4 + x*sin(a + d*x/2)*cos(a + d*x/2)*cos(c + d*x)/2 - x*sin(c + d*
x)*cos(a + d*x/2)**2/4 - 3*sin(a + d*x/2)*sin(c + d*x)*cos(a + d*x/2)/(2*d)
 - cos(a + d*x/2)**2*cos(c + d*x)/d, Eq(b, d/2)), ((x*sin(a + b*x)**2/2 + x
*cos(a + b*x)**2/2 - sin(a + b*x)*cos(a + b*x)/(2*b))*sin(c), Eq(d, 0)), (-
2*b**2*sin(a + b*x)**2*cos(c + d*x)/(4*b**2*d - d**3) - 2*b**2*cos(a + b*x)
**2*cos(c + d*x)/(4*b**2*d - d**3) - 2*b*d*sin(a + b*x)*sin(c + d*x)*cos(a
+ b*x)/(4*b**2*d - d**3) + d**2*sin(a + b*x)**2*cos(c + d*x)/(4*b**2*d - d*
*3), True))
```

Giac [A] time = 1.13754, size = 82, normalized size = 1.21

$$\frac{\cos(2bx + dx + 2a + c)}{4(2b + d)} - \frac{\cos(2bx - dx + 2a - c)}{4(2b - d)} - \frac{\cos(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*cos(2*b*x + d*x + 2*a + c)/(2*b + d) - 1/4*cos(2*b*x - d*x + 2*a - c)/(
2*b - d) - 1/2*cos(d*x + c)/d
```

3.203 $\int \sin^2(a+bx)\sin^2(c+dx)\,dx$

Optimal. Leaf size=88

$$\frac{\sin(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sin(2(a+c)+2x(b+d))}{16(b+d)}-\frac{\sin(2a+2bx)}{8b}-\frac{\sin(2c+2dx)}{8d}+\frac{x}{4}$$

```
[Out] x/4 - Sin[2*a + 2*b*x]/(8*b) + Sin[2*(a - c) + 2*(b - d)*x]/(16*(b - d)) -
Sin[2*c + 2*d*x]/(8*d) + Sin[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

Rubi [A] time = 0.0650691, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4569, 2637}

$$\frac{\sin(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sin(2(a+c)+2x(b+d))}{16(b+d)}-\frac{\sin(2a+2bx)}{8b}-\frac{\sin(2c+2dx)}{8d}+\frac{x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[c + d*x]^2,x]
```

```
[Out] x/4 - Sin[2*a + 2*b*x]/(8*b) + Sin[2*(a - c) + 2*(b - d)*x]/(16*(b - d)) -
Sin[2*c + 2*d*x]/(8*d) + Sin[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

Rule 4569

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+bx)\sin^2(c+dx)\,dx &= \int \left(\frac{1}{4}-\frac{1}{4}\cos(2a+2bx)+\frac{1}{8}\cos(2(a-c)+2(b-d)x)-\frac{1}{4}\cos(2c+2dx)+\frac{1}{8}\cos(2(a+\right.\\ &= \frac{x}{4}+\frac{1}{8}\int \cos(2(a-c)+2(b-d)x)\,dx+\frac{1}{8}\int \cos(2(a+c)+2(b+d)x)\,dx-\frac{1}{4}\int \cos(2a\\ &= \frac{x}{4}-\frac{\sin(2a+2bx)}{8b}+\frac{\sin(2(a-c)+2(b-d)x)}{16(b-d)}-\frac{\sin(2c+2dx)}{8d}+\frac{\sin(2(a+c)+2(b+}{16(b+d)}\end{aligned}$$

Mathematica [A] time = 0.792074, size = 106, normalized size = 1.2

$$\frac{\left(2d^3-2b^2d\right)\sin(2(a+bx))+bd(b+d)\sin(2(a+x(b-d)-c))+b(b-d)(d(\sin(2(a+x(b+d)+c))+4x(b+d))-2(b+}{16bd(b-d)(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[c + d*x]^2,x]
```

```
[Out] ((-2*b^2*d + 2*d^3)*Sin[2*(a + b*x)] + b*d*(b + d)*Sin[2*(a - c + (b - d)*x
)] + b*(b - d)*(-2*(b + d)*Sin[2*(c + d*x)] + d*(4*(b + d)*x + Sin[2*(a + c
 + (b + d)*x)])))/(16*b*(b - d)*d*(b + d))
```

Maple [A] time = 0.027, size = 83, normalized size = 0.9

$$\frac{x}{4}-\frac{\sin(2bx+2a)}{8b}-\frac{\sin(2dx+2c)}{8d}+\frac{\sin((2b-2d)x-2c+2a)}{16b-16d}+\frac{\sin((2b+2d)x+2a+2c)}{16b+16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(d*x+c)^2,x)
```

```
[Out] 1/4*x-1/8*sin(2*b*x+2*a)/b-1/8*sin(2*d*x+2*c)/d+1/16/(b-d)*sin((2*b-2*d)*x-
2*c+2*a)+1/16/(b+d)*sin((2*b+2*d)*x+2*a+2*c)
```

Maxima [B] time = 1.24694, size = 837, normalized size = 9.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(8*((b*cos(2*c)^2 + b*sin(2*c)^2)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c)
^2)*d)*x + (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a + 4*c) -
 (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a) - (b^2*d*sin(2*c)
 + b*d^2*sin(2*c))*cos(-2*(b - d)*x - 2*a + 4*c) + (b^2*d*sin(2*c) + b*d^2*
sin(2*c))*cos(-2*(b - d)*x - 2*a) - 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*cos(2
*b*x + 2*a + 2*c) + 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*cos(2*b*x + 2*a - 2*c
) + 2*(b^3*sin(2*c) - b*d^2*sin(2*c))*cos(2*d*x) - 2*(b^3*sin(2*c) - b*d^2*
sin(2*c))*cos(2*d*x + 4*c) - (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(2*(b + d
)*x + 2*a + 4*c) - (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(2*(b + d)*x + 2*a)
 + (b^2*d*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a + 4*c) + (b^2*d
*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a) + 2*(b^2*d*cos(2*c) - d
^3*cos(2*c))*sin(2*b*x + 2*a + 2*c) + 2*(b^2*d*cos(2*c) - d^3*cos(2*c))*sin
(2*b*x + 2*a - 2*c) + 2*(b^3*cos(2*c) - b*d^2*cos(2*c))*sin(2*d*x) + 2*(b^3
*cos(2*c) - b*d^2*cos(2*c))*sin(2*d*x + 4*c))/((b*cos(2*c)^2 + b*sin(2*c)^2
)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c)^2)*d)
```

Fricas [A] time = 0.512301, size = 263, normalized size = 2.99

$$-\frac{\left(2\,bd^2\cos(bx+a)^2+b^3-2\,bd^2\right)\cos(dx+c)\sin(dx+c)-\left(b^3d-bd^3\right)x-\left(2\,b^2d\cos(bx+a)\cos(dx+c)^2-\left(2\,b^2d\right.\right.}{4\left(b^3d-bd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*((2*b*d^2*cos(b*x + a)^2 + b^3 - 2*b*d^2)*cos(d*x + c)*sin(d*x + c) -
(b^3*d - b*d^3)*x - (2*b^2*d*cos(b*x + a)*cos(d*x + c)^2 - (2*b^2*d - d^3)*
```

```
cos(b*x + a))*sin(b*x + a))/(b^3*d - b*d^3)
```

Sympy [A] time = 40.9371, size = 1027, normalized size = 11.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(d*x+c)**2,x)
```

```
[Out] Piecewise((x*sin(a)**2*sin(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**2
/2 + x*cos(c + d*x)**2/2 - sin(c + d*x)*cos(c + d*x)/(2*d))*sin(a)**2, Eq(b
, 0)), (3*x*sin(a - d*x)**2*sin(c + d*x)**2/8 + x*sin(a - d*x)**2*cos(c + d
*x)**2/8 - x*sin(a - d*x)*sin(c + d*x)*cos(a - d*x)*cos(c + d*x)/2 + x*sin(
c + d*x)**2*cos(a - d*x)**2/8 + 3*x*cos(a - d*x)**2*cos(c + d*x)**2/8 - 5*s
in(a - d*x)**2*sin(c + d*x)*cos(c + d*x)/(8*d) + sin(a - d*x)*cos(a - d*x)*
cos(c + d*x)**2/(2*d) + sin(c + d*x)*cos(a - d*x)**2*cos(c + d*x)/(8*d), Eq
(b, -d)), (3*x*sin(a + d*x)**2*sin(c + d*x)**2/8 + x*sin(a + d*x)**2*cos(c
+ d*x)**2/8 + x*sin(a + d*x)*sin(c + d*x)*cos(a + d*x)*cos(c + d*x)/2 + x*s
in(c + d*x)**2*cos(a + d*x)**2/8 + 3*x*cos(a + d*x)**2*cos(c + d*x)**2/8 -
5*sin(a + d*x)**2*sin(c + d*x)*cos(c + d*x)/(8*d) - sin(a + d*x)*cos(a + d*
x)*cos(c + d*x)**2/(2*d) + sin(c + d*x)*cos(a + d*x)**2*cos(c + d*x)/(8*d),
 Eq(b, d)), ((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 - sin(a + b*x)*cos(
a + b*x)/(2*b))*sin(c)**2, Eq(d, 0)), (b**3*d*x*sin(a + b*x)**2*sin(c + d*x
)**2/(4*b**3*d - 4*b*d**3) + b**3*d*x*sin(a + b*x)**2*cos(c + d*x)**2/(4*b*
*3*d - 4*b*d**3) + b**3*d*x*sin(c + d*x)**2*cos(a + b*x)**2/(4*b**3*d - 4*b
*d**3) + b**3*d*x*cos(a + b*x)**2*cos(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b
**3*sin(a + b*x)**2*sin(c + d*x)*cos(c + d*x)/(4*b**3*d - 4*b*d**3) - b**3*
sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x)/(4*b**3*d - 4*b*d**3) - 2*b**2*d*
sin(a + b*x)*sin(c + d*x)**2*cos(a + b*x)/(4*b**3*d - 4*b*d**3) - b*d**3*x*
sin(a + b*x)**2*sin(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*sin(a + b*
x)**2*cos(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*sin(c + d*x)**2*cos(
a + b*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*cos(a + b*x)**2*cos(c + d*x)**
2/(4*b**3*d - 4*b*d**3) + 2*b*d**2*sin(a + b*x)**2*sin(c + d*x)*cos(c + d*x
)/(4*b**3*d - 4*b*d**3) + d**3*sin(a + b*x)*sin(c + d*x)**2*cos(a + b*x)/(4
*b**3*d - 4*b*d**3) + d**3*sin(a + b*x)*cos(a + b*x)*cos(c + d*x)**2/(4*b**
3*d - 4*b*d**3), True))
```

Giac [A] time = 1.1683, size = 108, normalized size = 1.23

$$\frac{1}{4}x + \frac{\sin\left(2\,bx + 2\,dx + 2\,a + 2\,c\right)}{16\,(b + d)} + \frac{\sin\left(2\,bx - 2\,dx + 2\,a - 2\,c\right)}{16\,(b - d)} - \frac{\sin\left(2\,bx + 2\,a\right)}{8\,b} - \frac{\sin\left(2\,dx + 2\,c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*x + 1/16*sin(2*b*x + 2*d*x + 2*a + 2*c)/(b + d) + 1/16*sin(2*b*x - 2*d*
x + 2*a - 2*c)/(b - d) - 1/8*sin(2*b*x + 2*a)/b - 1/8*sin(2*d*x + 2*c)/d
```

3.204 $\int \sin^2(a+bx)\sin^3(c+dx)\,dx$

Optimal. Leaf size=144

$$\frac{\cos(2a+x(2b-3d)-3c)}{16(2b-3d)}-\frac{3\cos(2a+x(2b-d)-c)}{16(2b-d)}+\frac{3\cos(2a+x(2b+d)+c)}{16(2b+d)}-\frac{\cos(2a+x(2b+3d)+3c)}{16(2b+3d)}-3\text{c}$$

```
[Out] Cos[2*a - 3*c + (2*b - 3*d)*x]/(16*(2*b - 3*d)) - (3*Cos[2*a - c + (2*b - d
)*x])/(16*(2*b - d)) - (3*Cos[c + d*x])/(8*d) + Cos[3*c + 3*d*x]/(24*d) + (
3*Cos[2*a + c + (2*b + d)*x])/(16*(2*b + d)) - Cos[2*a + 3*c + (2*b + 3*d)*
x]/(16*(2*b + 3*d))
```

Rubi [A] time = 0.0972936, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4569, 2638}

$$\frac{\cos(2a+x(2b-3d)-3c)}{16(2b-3d)}-\frac{3\cos(2a+x(2b-d)-c)}{16(2b-d)}+\frac{3\cos(2a+x(2b+d)+c)}{16(2b+d)}-\frac{\cos(2a+x(2b+3d)+3c)}{16(2b+3d)}-3\text{c}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[c + d*x]^3,x]
```

```
[Out] Cos[2*a - 3*c + (2*b - 3*d)*x]/(16*(2*b - 3*d)) - (3*Cos[2*a - c + (2*b - d
)*x])/(16*(2*b - d)) - (3*Cos[c + d*x])/(8*d) + Cos[3*c + 3*d*x]/(24*d) + (
3*Cos[2*a + c + (2*b + d)*x])/(16*(2*b + d)) - Cos[2*a + 3*c + (2*b + 3*d)*
x]/(16*(2*b + 3*d))
```

Rule 4569

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+bx)\sin^3(c+dx)\,dx &= \int \left(-\frac{1}{16}\sin(2a-3c+(2b-3d)x)+\frac{3}{16}\sin(2a-c+(2b-d)x)+\frac{3}{8}\sin(c+dx)-\frac{1}{8}\right.\\ &= -\left(\frac{1}{16}\int \sin(2a-3c+(2b-3d)x)\,dx\right)+\frac{1}{16}\int \sin(2a+3c+(2b+3d)x)\,dx-\frac{1}{8}\int\\ &= \frac{\cos(2a-3c+(2b-3d)x)}{16(2b-3d)}-\frac{3\cos(2a-c+(2b-d)x)}{16(2b-d)}-\frac{3\cos(c+dx)}{8d}+\frac{\cos(3c+3}{24d}\end{aligned}$$

Mathematica [A] time = 1.64462, size = 158, normalized size = 1.1

$$\frac{1}{48}\left(\frac{3\cos(2a+2bx-3c-3dx)}{2b-3d}-\frac{9\cos(2a+2bx-c-dx)}{2b-d}+\frac{9\cos(2a+2bx+c+dx)}{2b+d}-\frac{3\cos(2a+2bx+3c+3dx)}{2b+3d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[c + d*x]^3,x]
```

```
[Out] ((-18*Cos[c]*Cos[d*x])/d + (2*Cos[3*c]*Cos[3*d*x])/d + (3*Cos[2*a - 3*c + 2
*b*x - 3*d*x])/(2*b - 3*d) - (9*Cos[2*a - c + 2*b*x - d*x])/(2*b - d) + (9*
Cos[2*a + c + 2*b*x + d*x])/(2*b + d) - (3*Cos[2*a + 3*c + 2*b*x + 3*d*x])/
(2*b + 3*d) + (18*Sin[c]*Sin[d*x])/d - (2*Sin[3*c]*Sin[3*d*x])/d)/48
```

Maple [A] time = 0.023, size = 133, normalized size = 0.9

$$\frac{\cos\left(2a-3c+\left(2b-3d\right)x\right)}{32b-48d}-\frac{3\cos\left(2a-c+\left(2b-d\right)x\right)}{32b-16d}-\frac{3\cos\left(dx+c\right)}{8d}+\frac{\cos\left(3dx+3c\right)}{24d}+\frac{3\cos\left(2a+c+\left(2b+d\right)x\right)}{32b+16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(d*x+c)^3,x)
```

```
[Out] 1/16*cos(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)-3/16*cos(2*a-c+(2*b-d)*x)/(2*b-d)-3
/8*cos(d*x+c)/d+1/24*cos(3*d*x+3*c)/d+3/16*cos(2*a+c+(2*b+d)*x)/(2*b+d)-1/1
6*cos(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

Maxima [B] time = 1.46137, size = 1839, normalized size = 12.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/96*(3*(8*b^3*d*cos(3*c) - 12*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) + 3*d^4
*cos(3*c))*cos((2*b + 3*d)*x + 2*a + 6*c) + 3*(8*b^3*d*cos(3*c) - 12*b^2*d^
2*cos(3*c) - 2*b*d^3*cos(3*c) + 3*d^4*cos(3*c))*cos((2*b + 3*d)*x + 2*a) -
9*(8*b^3*d*cos(3*c) - 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c) + 9*d^4*cos(3*
c))*cos((2*b + d)*x + 2*a + 4*c) - 9*(8*b^3*d*cos(3*c) - 4*b^2*d^2*cos(3*c)
 - 18*b*d^3*cos(3*c) + 9*d^4*cos(3*c))*cos((2*b + d)*x + 2*a - 2*c) + 9*(8*
b^3*d*cos(3*c) + 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c) - 9*d^4*cos(3*c))*c
os(-(2*b - d)*x - 2*a + 4*c) + 9*(8*b^3*d*cos(3*c) + 4*b^2*d^2*cos(3*c) - 1
8*b*d^3*cos(3*c) - 9*d^4*cos(3*c))*cos(-(2*b - d)*x - 2*a - 2*c) - 3*(8*b^3
*d*cos(3*c) + 12*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) - 3*d^4*cos(3*c))*cos(
-(2*b - 3*d)*x - 2*a + 6*c) - 3*(8*b^3*d*cos(3*c) + 12*b^2*d^2*cos(3*c) - 2
*b*d^3*cos(3*c) - 3*d^4*cos(3*c))*cos(-(2*b - 3*d)*x - 2*a) - 2*(16*b^4*cos
(3*c) - 40*b^2*d^2*cos(3*c) + 9*d^4*cos(3*c))*cos(3*d*x) - 2*(16*b^4*cos(3*
c) - 40*b^2*d^2*cos(3*c) + 9*d^4*cos(3*c))*cos(3*d*x + 6*c) + 18*(16*b^4*co
s(3*c) - 40*b^2*d^2*cos(3*c) + 9*d^4*cos(3*c))*cos(d*x + 4*c) + 18*(16*b^4*
cos(3*c) - 40*b^2*d^2*cos(3*c) + 9*d^4*cos(3*c))*cos(d*x - 2*c) + 3*(8*b^3*
d*sin(3*c) - 12*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c) + 3*d^4*sin(3*c))*sin((
2*b + 3*d)*x + 2*a + 6*c) - 3*(8*b^3*d*sin(3*c) - 12*b^2*d^2*sin(3*c) - 2*b
*d^3*sin(3*c) + 3*d^4*sin(3*c))*sin((2*b + 3*d)*x + 2*a) - 9*(8*b^3*d*sin(3
*c) - 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c) + 9*d^4*sin(3*c))*sin((2*b + d
)*x + 2*a + 4*c) + 9*(8*b^3*d*sin(3*c) - 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(
3*c) + 9*d^4*sin(3*c))*sin((2*b + d)*x + 2*a - 2*c) + 9*(8*b^3*d*sin(3*c) +
 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c) - 9*d^4*sin(3*c))*sin(-(2*b - d)*x
- 2*a + 4*c) - 9*(8*b^3*d*sin(3*c) + 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c)
 - 9*d^4*sin(3*c))*sin(-(2*b - d)*x - 2*a - 2*c) - 3*(8*b^3*d*sin(3*c) + 12
*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c) - 3*d^4*sin(3*c))*sin(-(2*b - 3*d)*x -
```

```
2*a + 6*c) + 3*(8*b^3*d*sin(3*c) + 12*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c)
- 3*d^4*sin(3*c))*sin(-(2*b - 3*d)*x - 2*a) + 2*(16*b^4*sin(3*c) - 40*b^2*d
^2*sin(3*c) + 9*d^4*sin(3*c))*sin(3*d*x) - 2*(16*b^4*sin(3*c) - 40*b^2*d^2*
sin(3*c) + 9*d^4*sin(3*c))*sin(3*d*x + 6*c) + 18*(16*b^4*sin(3*c) - 40*b^2*
d^2*sin(3*c) + 9*d^4*sin(3*c))*sin(d*x + 4*c) - 18*(16*b^4*sin(3*c) - 40*b^
2*d^2*sin(3*c) + 9*d^4*sin(3*c))*sin(d*x - 2*c))/(9*(cos(3*c)^2 + sin(3*c)^
2)*d^5 - 40*(b^2*cos(3*c)^2 + b^2*sin(3*c)^2)*d^3 + 16*(b^4*cos(3*c)^2 + b^
4*sin(3*c)^2)*d)
```

Fricas [A] time = 0.545475, size = 429, normalized size = 2.98

$$\frac{\left(8\,b^4-38\,b^2d^2+9\,d^4+9\left(4\,b^2d^2-d^4\right)\cos\left(bx+a\right)^2\right)\cos\left(dx+c\right)^3+6\left(\left(4\,b^3d-bd^3\right)\cos\left(bx+a\right)\cos\left(dx+c\right)^2-\left(4\,b\cdots\right.\right.}{3\left(16\,b^4d-4\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/3*((8*b^4 - 38*b^2*d^2 + 9*d^4 + 9*(4*b^2*d^2 - d^4)*cos(b*x + a)^2)*cos(
d*x + c)^3 + 6*((4*b^3*d - b*d^3)*cos(b*x + a)*cos(d*x + c)^2 - (4*b^3*d -
7*b*d^3)*cos(b*x + a))*sin(b*x + a)*sin(d*x + c) - 3*(8*b^4 - 26*b^2*d^2 +
9*d^4 + 3*(4*b^2*d^2 - 3*d^4)*cos(b*x + a)^2)*cos(d*x + c))/(16*b^4*d - 40*
b^2*d^3 + 9*d^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16644, size = 174, normalized size = 1.21

$$-\frac{\cos\left(2\,bx+3\,dx+2\,a+3\,c\right)}{16\left(2\,b+3\,d\right)}+\frac{3\,\cos\left(2\,bx+dx+2\,a+c\right)}{16\left(2\,b+d\right)}-\frac{3\,\cos\left(2\,bx-dx+2\,a-c\right)}{16\left(2\,b-d\right)}+\frac{\cos\left(2\,bx-3\,dx+2\,a-3\,c\cdots\right.}{16\left(2\,b-3\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/16*cos(2*b*x + 3*d*x + 2*a + 3*c)/(2*b + 3*d) + 3/16*cos(2*b*x + d*x + 2
*a + c)/(2*b + d) - 3/16*cos(2*b*x - d*x + 2*a - c)/(2*b - d) + 1/16*cos(2*
b*x - 3*d*x + 2*a - 3*c)/(2*b - 3*d) + 1/24*cos(3*d*x + 3*c)/d - 3/8*cos(d*
x + c)/d
```

3.205 $\int \sin^3(a+bx)\sin^n(c+dx)\,dx$

Optimal. Leaf size=600

$$\frac{2^{-n-3}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\left(1-e^{2ic+2idx}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(\frac{3b}{d}-n\right),-n,\frac{1}{2}\left(\frac{3b}{d}-n+2\right),e^{2i(c+dx)}\right)\exp(i(3a-c}{3b-dn}$$

```
[Out] (2^(-3 - n)*E^(I*(3*a - c*n) + I*(3*b - d*n)*x + I*n*(c + d*x))*(I/E^(I*(c
+ d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[((3*b)/d - n)/2, -n, (2 +
(3*b)/d - n)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x))^n*(3*b
 - d*n)) - (3*2^(-3 - n)*E^(I*(a - c*n) + I*(b - d*n)*x + I*n*(c + d*x))*(I
/E^(I*(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, (b - d*n)/(2*
d), (2 + b/d - n)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x))^n
*(b - d*n)) - (3*2^(-3 - n)*E^((-I)*(a + c*n) - I*(b + d*n)*x + I*n*(c + d*
x))*(I/E^(I*(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, -(b + d
*n)/(2*d), 1 - (b + d*n)/(2*d), E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2
*I)*d*x))^n*(b + d*n)) + (2^(-3 - n)*E^((-I)*(3*a + c*n) - I*(3*b + d*n)*x
+ I*n*(c + d*x))*(I/E^(I*(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F
1[-n, -(3*b + d*n)/(2*d), (2 - (3*b)/d - n)/2, E^((2*I)*(c + d*x))])/((1 -
E^((2*I)*c + (2*I)*d*x))^n*(3*b + d*n))
```

Rubi [A] time = 1.72159, antiderivative size = 600, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {4553, 2285, 2253, 2252, 2251}

$$\frac{2^{-n-3}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\left(1-e^{2ic+2idx}\right)^{-n}\ {}_2F_1\left(\frac{1}{2}\left(\frac{3b}{d}-n\right),-n;\frac{1}{2}\left(\frac{3b}{d}-n+2\right);e^{2i(c+dx)}\right)\exp(i(3a-cn)+ix(3b-dn)+}{3b-dn}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[c + d*x]^n,x]
```


```
[Out] (2^(-3 - n)*E^(I*(3*a - c*n) + I*(3*b - d*n)*x + I*n*(c + d*x))*(I/E^(I*(c
+ d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[((3*b)/d - n)/2, -n, (2 +
(3*b)/d - n)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x))^n*(3*b
 - d*n)) - (3*2^(-3 - n)*E^(I*(a - c*n) + I*(b - d*n)*x + I*n*(c + d*x))*(I
/E^(I*(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, (b - d*n)/(2*
d), (2 + b/d - n)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x))^n
*(b - d*n)) - (3*2^(-3 - n)*E^((-I)*(a + c*n) - I*(b + d*n)*x + I*n*(c + d*
x))*(I/E^(I*(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, -(b + d
*n)/(2*d), 1 - (b + d*n)/(2*d), E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2
*I)*d*x))^n*(b + d*n)) + (2^(-3 - n)*E^((-I)*(3*a + c*n) - I*(3*b + d*n)*x
+ I*n*(c + d*x))*(I/E^(I*(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F
1[-n, -(3*b + d*n)/(2*d), (2 - (3*b)/d - n)/2, E^((2*I)*(c + d*x))])/((1 -
E^((2*I)*c + (2*I)*d*x))^n*(3*b + d*n))
```


Rule 4553

```
Int[Sin[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Dist[1/2^(p + q), Int[ExpandIntegrand[(I/E^(I*(c + d*x)) - I*E^(I*(c + d
*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
 b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Rule 2285

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Dist[(a*F^
v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n), Int[u*F^(n*v)*(a
```

```
+ b*F^ExpandToSum[w - v, x])^n, x], x] /; FreeQ[{F, a, b, n}, x] &&  !Integ
erQ[n] && LinearQ[{v, w}, x]
```

Rule 2253

```
Int[((a_) + (b_.)*(F_)^((e_.)*(v_)))^(p_)*(G_)^((h_.)*(u_)), x_Symbol] :> I
nt[G^(h*ExpandToSum[u, x])*(a + b*F^(e*ExpandToSum[v, x]))^p, x] /; FreeQ[{
F, G, a, b, e, h, p}, x] && LinearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rule 2252

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Dist[(a + b*F^(e*(c + d*x)))^p/(1 + (b/a)*F^
(e*(c + d*x)))^p, Int[G^(h*(f + g*x))*(1 + (b*F^(e*(c + d*x)))/a)^p, x], x]
 /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x] &&  !(ILtQ[p, 0] || GtQ[a,
0])
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \sin^3(a+bx)\sin^n(c+dx)\,dx = 2^{-3-n}\int \left(3ie^{-ia-ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n - 3ie^{ia+ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n - ie^{-3ia-3ib}\right.$$

$$= -\left(\left(i2^{-3-n}\right)\int e^{-3ia-3ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n dx\right) + \left(i2^{-3-n}\right)\int e^{3ia+3ibx}\left(ie^{-i(c+dx)} - \right.$$

$$= -\left(\left(i2^{-3-n}e^{in(c+dx)}\left(i-ie^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\right)\int e^{-3ia-3ibx-in(c+dx)}\left(i-ie^{2}\right.\right.$$

$$= \left(i2^{-3-n}e^{in(c+dx)}\left(i-ie^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\right)\int e^{i(3a-cn)+i(3b-dn)x}\left(i-ie^{2ic+2}\right.$$

$$= \left(i2^{-3-n}e^{in(c+dx)}\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\right)\int e^{i(3a-cn)+i(3b-dn)x}\left(1-e^{2ic+2i}\right.$$

$$= \frac{2^{-3-n}\exp(i(3a-cn)+i(3b-dn)x+in(c+dx))\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right.}{3b-dn}$$

Mathematica [F] time = 0.542276, size = 0, normalized size = 0.

$$\int \sin^3(a+bx)\sin^n(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[a + b*x]^3*Sin[c + d*x]^n,x]
```

```
[Out] Integrate[Sin[a + b*x]^3*Sin[c + d*x]^n, x]
```

Maple [F] time = 1.488, size = 0, normalized size = 0.

$$\int (\sin(bx+a))^3(\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(d*x+c)^n,x)
```

```
[Out] int(sin(b*x+a)^3*sin(d*x+c)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(dx+c)^n \sin(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^n*sin(b*x + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos(bx+a)^2-1\right)\sin(dx+c)^n \sin(bx+a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sin(d*x + c)^n*sin(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(d*x+c)**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(dx+c)^n \sin(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*sin(b*x + a)^3, x)
```

3.206 $\int \sin^3(a+bx)\sin(c+dx)\,dx$

Optimal. Leaf size=97

$$\frac{3\sin(a+x(b-d)-c)}{8(b-d)}-\frac{\sin(3a+x(3b-d)-c)}{8(3b-d)}-\frac{3\sin(a+x(b+d)+c)}{8(b+d)}+\frac{\sin(3a+x(3b+d)+c)}{8(3b+d)}$$

```
[Out] (3*Sin[a - c + (b - d)*x])/(8*(b - d)) - Sin[3*a - c + (3*b - d)*x]/(8*(3*b
 - d)) - (3*Sin[a + c + (b + d)*x])/(8*(b + d)) + Sin[3*a + c + (3*b + d)*x
]/(8*(3*b + d))
```

Rubi [A] time = 0.074767, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4569, 2637}

$$\frac{3\sin(a+x(b-d)-c)}{8(b-d)}-\frac{\sin(3a+x(3b-d)-c)}{8(3b-d)}-\frac{3\sin(a+x(b+d)+c)}{8(b+d)}+\frac{\sin(3a+x(3b+d)+c)}{8(3b+d)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[c + d*x],x]
```

```
[Out] (3*Sin[a - c + (b - d)*x])/(8*(b - d)) - Sin[3*a - c + (3*b - d)*x]/(8*(3*b
 - d)) - (3*Sin[a + c + (b + d)*x])/(8*(b + d)) + Sin[3*a + c + (3*b + d)*x
]/(8*(3*b + d))
```

Rule 4569

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin^3(a+bx)\sin(c+dx)\,dx &= \int \left(\frac{3}{8}\cos(a-c+(b-d)x)-\frac{1}{8}\cos(3a-c+(3b-d)x)-\frac{3}{8}\cos(a+c+(b+d)x)+\frac{1}{8}\right.\\ &= -\left(\frac{1}{8}\int \cos(3a-c+(3b-d)x)\,dx\right)+\frac{1}{8}\int \cos(3a+c+(3b+d)x)\,dx+\frac{3}{8}\int \cos(a-\\ &= \frac{3\sin(a-c+(b-d)x)}{8(b-d)}-\frac{\sin(3a-c+(3b-d)x)}{8(3b-d)}-\frac{3\sin(a+c+(b+d)x)}{8(b+d)}+\frac{\sin(3a-}{8}\end{aligned}$$

Mathematica [A] time = 0.51821, size = 91, normalized size = 0.94

$$\frac{1}{8}\left(\frac{3\sin(a+bx-c-dx)}{b-d}-\frac{\sin(3a+3bx-c-dx)}{3b-d}+\frac{\sin(3a+3bx+c+dx)}{3b+d}-\frac{3\sin(a+x(b+d)+c)}{b+d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sin[c + d*x],x]
```

```
[Out] ((3*Sin[a - c + b*x - d*x])/(b - d) - Sin[3*a - c + 3*b*x - d*x]/(3*b - d)
+ Sin[3*a + c + 3*b*x + d*x]/(3*b + d) - (3*Sin[a + c + (b + d)*x])/(b + d)
)/8
```

Maple [A] time = 0.024, size = 90, normalized size = 0.9

$$\frac{3\sin(a-c+(b-d)x)}{8b-8d}-\frac{\sin(3a-c+(3b-d)x)}{24b-8d}-\frac{3\sin(a+c+(b+d)x)}{8b+8d}+\frac{\sin(3a+c+(3b+d)x)}{24b+8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(d*x+c),x)
```

```
[Out] 3/8*sin(a-c+(b-d)*x)/(b-d)-1/8*sin(3*a-c+(3*b-d)*x)/(3*b-d)-3/8*sin(a+c+(b+
d)*x)/(b+d)+1/8*sin(3*a+c+(3*b+d)*x)/(3*b+d)
```

Maxima [B] time = 1.38801, size = 1065, normalized size = 10.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/16*((3*b^3*sin(c) - b^2*d*sin(c) - 3*b*d^2*sin(c) + d^3*sin(c))*cos((3*b
 + d)*x + 3*a + 2*c) - (3*b^3*sin(c) - b^2*d*sin(c) - 3*b*d^2*sin(c) + d^3*
sin(c))*cos((3*b + d)*x + 3*a) + (3*b^3*sin(c) + b^2*d*sin(c) - 3*b*d^2*sin
(c) - d^3*sin(c))*cos(-(3*b - d)*x - 3*a + 2*c) - (3*b^3*sin(c) + b^2*d*sin
(c) - 3*b*d^2*sin(c) - d^3*sin(c))*cos(-(3*b - d)*x - 3*a) - 3*(9*b^3*sin(c
) - 9*b^2*d*sin(c) - b*d^2*sin(c) + d^3*sin(c))*cos((b + d)*x + a + 2*c) +
3*(9*b^3*sin(c) - 9*b^2*d*sin(c) - b*d^2*sin(c) + d^3*sin(c))*cos((b + d)*x
 + a) - 3*(9*b^3*sin(c) + 9*b^2*d*sin(c) - b*d^2*sin(c) - d^3*sin(c))*cos(-
(b - d)*x - a + 2*c) + 3*(9*b^3*sin(c) + 9*b^2*d*sin(c) - b*d^2*sin(c) - d^
3*sin(c))*cos(-(b - d)*x - a) - (3*b^3*cos(c) - b^2*d*cos(c) - 3*b*d^2*cos(
c) + d^3*cos(c))*sin((3*b + d)*x + 3*a + 2*c) - (3*b^3*cos(c) - b^2*d*cos(c
) - 3*b*d^2*cos(c) + d^3*cos(c))*sin((3*b + d)*x + 3*a) - (3*b^3*cos(c) + b
^2*d*cos(c) - 3*b*d^2*cos(c) - d^3*cos(c))*sin(-(3*b - d)*x - 3*a + 2*c) -
(3*b^3*cos(c) + b^2*d*cos(c) - 3*b*d^2*cos(c) - d^3*cos(c))*sin(-(3*b - d)*
x - 3*a) + 3*(9*b^3*cos(c) - 9*b^2*d*cos(c) - b*d^2*cos(c) + d^3*cos(c))*si
n((b + d)*x + a + 2*c) + 3*(9*b^3*cos(c) - 9*b^2*d*cos(c) - b*d^2*cos(c) +
d^3*cos(c))*sin((b + d)*x + a) + 3*(9*b^3*cos(c) + 9*b^2*d*cos(c) - b*d^2*c
os(c) - d^3*cos(c))*sin(-(b - d)*x - a + 2*c) + 3*(9*b^3*cos(c) + 9*b^2*d*c
os(c) - b*d^2*cos(c) - d^3*cos(c))*sin(-(b - d)*x - a))/(9*b^4*cos(c)^2 + 9
*b^4*sin(c)^2 + (cos(c)^2 + sin(c)^2)*d^4 - 10*(b^2*cos(c)^2 + b^2*sin(c)^2
)*d^2)
```

Fricas [A] time = 0.49894, size = 246, normalized size = 2.54

$$\frac{\left(7\,b^2d-d^3-\left(b^2d-d^3\right)\cos\left(bx+a\right)^2\right)\cos\left(dx+c\right)\sin\left(bx+a\right)+3\left(\left(b^3-bd^2\right)\cos\left(bx+a\right)^3-\left(3\,b^3-bd^2\right)\cos\left(bx+a\right)\right)\mathrm{s}}{9\,b^4-10\,b^2d^2+d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] ((7*b^2*d - d^3 - (b^2*d - d^3)*cos(b*x + a)^2)*cos(d*x + c)*sin(b*x + a) +
 3*((b^3 - b*d^2)*cos(b*x + a)^3 - (3*b^3 - b*d^2)*cos(b*x + a))*sin(d*x +
c))/(9*b^4 - 10*b^2*d^2 + d^4)
```

Sympy [A] time = 88.9814, size = 957, normalized size = 9.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(d*x+c),x)
```

```
[Out] Piecewise((x*sin(a)**3*sin(c), Eq(b, 0) & Eq(d, 0)), (3*x*sin(a - d*x)**3*s
in(c + d*x)/8 - 3*x*sin(a - d*x)**2*cos(a - d*x)*cos(c + d*x)/8 + 3*x*sin(a
 - d*x)*sin(c + d*x)*cos(a - d*x)**2/8 - 3*x*cos(a - d*x)**3*cos(c + d*x)/8
 + 5*sin(a - d*x)**2*sin(c + d*x)*cos(a - d*x)/(8*d) - sin(a - d*x)*cos(a -
 d*x)**2*cos(c + d*x)/(8*d) + sin(c + d*x)*cos(a - d*x)**3/(4*d), Eq(b, -d)
), (x*sin(a - d*x/3)**3*sin(c + d*x)/8 - 3*x*sin(a - d*x/3)**2*cos(a - d*x/
3)*cos(c + d*x)/8 - 3*x*sin(a - d*x/3)*sin(c + d*x)*cos(a - d*x/3)**2/8 + x
*cos(a - d*x/3)**3*cos(c + d*x)/8 + 21*sin(a - d*x/3)**2*sin(c + d*x)*cos(a
 - d*x/3)/(8*d) - 27*sin(a - d*x/3)*cos(a - d*x/3)**2*cos(c + d*x)/(8*d) -
5*sin(c + d*x)*cos(a - d*x/3)**3/(4*d), Eq(b, -d/3)), (x*sin(a + d*x/3)**3*
sin(c + d*x)/8 + 3*x*sin(a + d*x/3)**2*cos(a + d*x/3)*cos(c + d*x)/8 - 3*x*
sin(a + d*x/3)*sin(c + d*x)*cos(a + d*x/3)**2/8 - x*cos(a + d*x/3)**3*cos(c
 + d*x)/8 - 21*sin(a + d*x/3)**2*sin(c + d*x)*cos(a + d*x/3)/(8*d) - 27*sin
(a + d*x/3)*cos(a + d*x/3)**2*cos(c + d*x)/(8*d) + 5*sin(c + d*x)*cos(a + d
*x/3)**3/(4*d), Eq(b, d/3)), (3*x*sin(a + d*x)**3*sin(c + d*x)/8 + 3*x*sin(
a + d*x)**2*cos(a + d*x)*cos(c + d*x)/8 + 3*x*sin(a + d*x)*sin(c + d*x)*cos
(a + d*x)**2/8 + 3*x*cos(a + d*x)**3*cos(c + d*x)/8 + sin(a + d*x)**3*cos(c
 + d*x)/(8*d) - 3*sin(a + d*x)**2*sin(c + d*x)*cos(a + d*x)/(4*d) - 3*sin(c
 + d*x)*cos(a + d*x)**3/(8*d), Eq(b, d)), (-9*b**3*sin(a + b*x)**2*sin(c +
d*x)*cos(a + b*x)/(9*b**4 - 10*b**2*d**2 + d**4) - 6*b**3*sin(c + d*x)*cos(
a + b*x)**3/(9*b**4 - 10*b**2*d**2 + d**4) + 7*b**2*d*sin(a + b*x)**3*cos(c
 + d*x)/(9*b**4 - 10*b**2*d**2 + d**4) + 6*b**2*d*sin(a + b*x)*cos(a + b*x)
**2*cos(c + d*x)/(9*b**4 - 10*b**2*d**2 + d**4) + 3*b*d**2*sin(a + b*x)**2*
sin(c + d*x)*cos(a + b*x)/(9*b**4 - 10*b**2*d**2 + d**4) - d**3*sin(a + b*x
)**3*cos(c + d*x)/(9*b**4 - 10*b**2*d**2 + d**4), True))
```

Giac [A] time = 1.1318, size = 120, normalized size = 1.24

$$\frac{\sin\left(3\,bx + dx + 3\,a + c\right)}{8\,(3\,b + d)} - \frac{\sin\left(3\,bx - dx + 3\,a - c\right)}{8\,(3\,b - d)} - \frac{3\,\sin\left(bx + dx + a + c\right)}{8\,(b + d)} + \frac{3\,\sin\left(bx - dx + a - c\right)}{8\,(b - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c),x, algorithm="giac")
```

```
[Out] 1/8*sin(3*b*x + d*x + 3*a + c)/(3*b + d) - 1/8*sin(3*b*x - d*x + 3*a - c)/(
3*b - d) - 3/8*sin(b*x + d*x + a + c)/(b + d) + 3/8*sin(b*x - d*x + a - c)/
(b - d)
```

3.207 $\int \sin^3(a+bx)\sin^2(c+dx)\,dx$

Optimal. Leaf size=138

$$\frac{3\cos(a+x(b-2d)-2c)}{16(b-2d)}-\frac{\cos(3a+x(3b-2d)-2c)}{16(3b-2d)}+\frac{3\cos(a+x(b+2d)+2c)}{16(b+2d)}-\frac{\cos(3a+x(3b+2d)+2c)}{16(3b+2d)}-\frac{3\cos(\ldots}{8\ldots}$$

```
[Out] (-3*Cos[a + b*x])/(8*b) + Cos[3*a + 3*b*x]/(24*b) + (3*Cos[a - 2*c + (b - 2
*d)*x])/(16*(b - 2*d)) - Cos[3*a - 2*c + (3*b - 2*d)*x]/(16*(3*b - 2*d)) +
(3*Cos[a + 2*c + (b + 2*d)*x])/(16*(b + 2*d)) - Cos[3*a + 2*c + (3*b + 2*d)
*x]/(16*(3*b + 2*d))
```

Rubi [A] time = 0.0984789, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4569, 2638}

$$\frac{3\cos(a+x(b-2d)-2c)}{16(b-2d)}-\frac{\cos(3a+x(3b-2d)-2c)}{16(3b-2d)}+\frac{3\cos(a+x(b+2d)+2c)}{16(b+2d)}-\frac{\cos(3a+x(3b+2d)+2c)}{16(3b+2d)}-\frac{3\cos(\ldots}{8\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[c + d*x]^2,x]
```

```
[Out] (-3*Cos[a + b*x])/(8*b) + Cos[3*a + 3*b*x]/(24*b) + (3*Cos[a - 2*c + (b - 2
*d)*x])/(16*(b - 2*d)) - Cos[3*a - 2*c + (3*b - 2*d)*x]/(16*(3*b - 2*d)) +
(3*Cos[a + 2*c + (b + 2*d)*x])/(16*(b + 2*d)) - Cos[3*a + 2*c + (3*b + 2*d)
*x]/(16*(3*b + 2*d))
```

Rule 4569

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin^3(a+bx)\sin^2(c+dx)\,dx &= \int\left(\frac{3}{8}\sin(a+bx)-\frac{1}{8}\sin(3a+3bx)-\frac{3}{16}\sin(a-2c+(b-2d)x)+\frac{1}{16}\sin(3a-2c+(3\ldots\right.\\ &= \frac{1}{16}\int\sin(3a-2c+(3b-2d)x)\,dx+\frac{1}{16}\int\sin(3a+2c+(3b+2d)x)\,dx-\frac{1}{8}\int\sin(3a\ldots\\ &= -\frac{3\cos(a+bx)}{8b}+\frac{\cos(3a+3bx)}{24b}+\frac{3\cos(a-2c+(b-2d)x)}{16(b-2d)}-\frac{\cos(3a-2c+(3b-2d)\ldots}{16(3b-2d)}\end{aligned}$$

Mathematica [A] time = 1.67213, size = 153, normalized size = 1.11

$$\frac{1}{48}\left(\frac{9\cos(a+bx-2c-2dx)}{b-2d}-\frac{3\cos(3a+3bx-2c-2dx)}{3b-2d}+\frac{9\cos(a+bx+2c+2dx)}{b+2d}-\frac{3\cos(3a+3bx+2c+2dx)}{3b+2d}+\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sin[c + d*x]^2,x]
```

```
[Out] ((-18*Cos[a]*Cos[b*x])/b + (2*Cos[3*a]*Cos[3*b*x])/b + (9*Cos[a - 2*c + b*x
 - 2*d*x])/(b - 2*d) - (3*Cos[3*a - 2*c + 3*b*x - 2*d*x])/(3*b - 2*d) + (9*
Cos[a + 2*c + b*x + 2*d*x])/(b + 2*d) - (3*Cos[3*a + 2*c + 3*b*x + 2*d*x])/
(3*b + 2*d) + (18*Sin[a]*Sin[b*x])/b - (2*Sin[3*a]*Sin[3*b*x])/b)/48
```

Maple [A] time = 0.022, size = 127, normalized size = 0.9

$$-\frac{3\cos(bx+a)}{8b}+\frac{\cos(3bx+3a)}{24b}+\frac{3\cos(a-2c+(b-2d)x)}{16b-32d}-\frac{\cos(3a-2c+(3b-2d)x)}{48b-32d}+\frac{3\cos(a+2c+(b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(d*x+c)^2,x)
```

```
[Out] -3/8*cos(b*x+a)/b+1/24*cos(3*b*x+3*a)/b+3/16*cos(a-2*c+(b-2*d)*x)/(b-2*d)-1
/16*cos(3*a-2*c+(3*b-2*d)*x)/(3*b-2*d)+3/16*cos(a+2*c+(b+2*d)*x)/(b+2*d)-1/
16*cos(3*a+2*c+(3*b+2*d)*x)/(3*b+2*d)
```

Maxima [B] time = 1.55865, size = 1836, normalized size = 13.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/96*(3*(3*b^4*cos(2*c) - 2*b^3*d*cos(2*c) - 12*b^2*d^2*cos(2*c) + 8*b*d^3
*cos(2*c))*cos((3*b + 2*d)*x + 3*a + 4*c) + 3*(3*b^4*cos(2*c) - 2*b^3*d*cos
(2*c) - 12*b^2*d^2*cos(2*c) + 8*b*d^3*cos(2*c))*cos((3*b + 2*d)*x + 3*a) +
3*(3*b^4*cos(2*c) + 2*b^3*d*cos(2*c) - 12*b^2*d^2*cos(2*c) - 8*b*d^3*cos(2*
c))*cos(-(3*b - 2*d)*x - 3*a + 4*c) + 3*(3*b^4*cos(2*c) + 2*b^3*d*cos(2*c)
 - 12*b^2*d^2*cos(2*c) - 8*b*d^3*cos(2*c))*cos(-(3*b - 2*d)*x - 3*a) - 9*(9*
b^4*cos(2*c) - 18*b^3*d*cos(2*c) - 4*b^2*d^2*cos(2*c) + 8*b*d^3*cos(2*c))*c
os((b + 2*d)*x + a + 4*c) - 9*(9*b^4*cos(2*c) - 18*b^3*d*cos(2*c) - 4*b^2*d
^2*cos(2*c) + 8*b*d^3*cos(2*c))*cos((b + 2*d)*x + a) - 9*(9*b^4*cos(2*c) +
18*b^3*d*cos(2*c) - 4*b^2*d^2*cos(2*c) - 8*b*d^3*cos(2*c))*cos(-(b - 2*d)*x
 - a + 4*c) - 9*(9*b^4*cos(2*c) + 18*b^3*d*cos(2*c) - 4*b^2*d^2*cos(2*c) -
8*b*d^3*cos(2*c))*cos(-(b - 2*d)*x - a) - 2*(9*b^4*cos(2*c) - 40*b^2*d^2*co
s(2*c) + 16*d^4*cos(2*c))*cos(3*b*x + 3*a + 2*c) - 2*(9*b^4*cos(2*c) - 40*b
^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(3*b*x + 3*a - 2*c) + 18*(9*b^4*cos(2
*c) - 40*b^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(b*x + a + 2*c) + 18*(9*b^4
*cos(2*c) - 40*b^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(b*x + a - 2*c) + 3*(
3*b^4*sin(2*c) - 2*b^3*d*sin(2*c) - 12*b^2*d^2*sin(2*c) + 8*b*d^3*sin(2*c))
*sin((3*b + 2*d)*x + 3*a + 4*c) - 3*(3*b^4*sin(2*c) - 2*b^3*d*sin(2*c) - 12
*b^2*d^2*sin(2*c) + 8*b*d^3*sin(2*c))*sin((3*b + 2*d)*x + 3*a) + 3*(3*b^4*s
in(2*c) + 2*b^3*d*sin(2*c) - 12*b^2*d^2*sin(2*c) - 8*b*d^3*sin(2*c))*sin(-(
3*b - 2*d)*x - 3*a + 4*c) - 3*(3*b^4*sin(2*c) + 2*b^3*d*sin(2*c) - 12*b^2*d
^2*sin(2*c) - 8*b*d^3*sin(2*c))*sin(-(3*b - 2*d)*x - 3*a) - 9*(9*b^4*sin(2*
c) - 18*b^3*d*sin(2*c) - 4*b^2*d^2*sin(2*c) + 8*b*d^3*sin(2*c))*sin((b + 2*
d)*x + a + 4*c) + 9*(9*b^4*sin(2*c) - 18*b^3*d*sin(2*c) - 4*b^2*d^2*sin(2*c
) + 8*b*d^3*sin(2*c))*sin((b + 2*d)*x + a) - 9*(9*b^4*sin(2*c) + 18*b^3*d*s
in(2*c) - 4*b^2*d^2*sin(2*c) - 8*b*d^3*sin(2*c))*sin(-(b - 2*d)*x - a + 4*c
```

```
) + 9*(9*b^4*sin(2*c) + 18*b^3*d*sin(2*c) - 4*b^2*d^2*sin(2*c) - 8*b*d^3*si
n(2*c))*sin(-(b - 2*d)*x - a) - 2*(9*b^4*sin(2*c) - 40*b^2*d^2*sin(2*c) + 1
6*d^4*sin(2*c))*sin(3*b*x + 3*a + 2*c) + 2*(9*b^4*sin(2*c) - 40*b^2*d^2*sin
(2*c) + 16*d^4*sin(2*c))*sin(3*b*x + 3*a - 2*c) + 18*(9*b^4*sin(2*c) - 40*b
^2*d^2*sin(2*c) + 16*d^4*sin(2*c))*sin(b*x + a + 2*c) - 18*(9*b^4*sin(2*c)
- 40*b^2*d^2*sin(2*c) + 16*d^4*sin(2*c))*sin(b*x + a - 2*c))/(9*b^5*cos(2*c
)^2 + 9*b^5*sin(2*c)^2 + 16*(b*cos(2*c)^2 + b*sin(2*c)^2)*d^4 - 40*(b^3*cos
(2*c)^2 + b^3*sin(2*c)^2)*d^2)
```

Fricas [A] time = 0.533839, size = 427, normalized size = 3.09

$$\frac{\left(9\,b^4 - 38\,b^2d^2 + 8\,d^4\right)\cos\left(bx+a\right)^3 + 6\left(7\,b^3d - 4\,bd^3 - \left(b^3d - 4\,bd^3\right)\cos\left(bx+a\right)^2\right)\cos\left(dx+c\right)\sin\left(bx+a\right)\sin\left(dx+c\right)}{3\left(9\,b^5 - 40\,b^3d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/3*((9*b^4 - 38*b^2*d^2 + 8*d^4)*cos(b*x + a)^3 + 6*(7*b^3*d - 4*b*d^3 - (
b^3*d - 4*b*d^3)*cos(b*x + a)^2)*cos(d*x + c)*sin(b*x + a)*sin(d*x + c) - 9
*((b^4 - 4*b^2*d^2)*cos(b*x + a)^3 - (3*b^4 - 4*b^2*d^2)*cos(b*x + a))*cos(
d*x + c)^2 - 3*(9*b^4 - 26*b^2*d^2 + 8*d^4)*cos(b*x + a))/(9*b^5 - 40*b^3*d
^2 + 16*b*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11666, size = 167, normalized size = 1.21

$$-\frac{\cos\left(3\,bx+2\,dx+3\,a+2\,c\right)}{16\left(3\,b+2\,d\right)} - \frac{\cos\left(3\,bx-2\,dx+3\,a-2\,c\right)}{16\left(3\,b-2\,d\right)} + \frac{\cos\left(3\,bx+3\,a\right)}{24\,b} + \frac{3\,\cos\left(bx+2\,dx+a+2\,c\right)}{16\left(b+2\,d\right)} + \frac{3\,\cos\left(bx\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/16*cos(3*b*x + 2*d*x + 3*a + 2*c)/(3*b + 2*d) - 1/16*cos(3*b*x - 2*d*x +
 3*a - 2*c)/(3*b - 2*d) + 1/24*cos(3*b*x + 3*a)/b + 3/16*cos(b*x + 2*d*x +
a + 2*c)/(b + 2*d) + 3/16*cos(b*x - 2*d*x + a - 2*c)/(b - 2*d) - 3/8*cos(b*
x + a)/b
```

3.208 $\int \sin^3(a+bx)\sin^3(c+dx)\,dx$

Optimal. Leaf size=195

$$-\frac{3\sin(a+x(b-3d)-3c)}{32(b-3d)}+\frac{9\sin(a+x(b-d)-c)}{32(b-d)}+\frac{\sin(3(a-c)+3x(b-d))}{96(b-d)}-\frac{3\sin(3a+x(3b-d)-c)}{32(3b-d)}-\frac{9\sin(a}{3}$$

```
[Out] (-3*Sin[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) + (9*Sin[a - c + (b - d)*x])
/(32*(b - d)) + Sin[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) - (3*Sin[3*a - c
+ (3*b - d)*x])/(32*(3*b - d)) - (9*Sin[a + c + (b + d)*x])/(32*(b + d)) -
Sin[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) + (3*Sin[3*a + c + (3*b + d)*x])/
(32*(3*b + d)) + (3*Sin[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

Rubi [A] time = 0.131891, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4569, 2637}

$$-\frac{3\sin(a+x(b-3d)-3c)}{32(b-3d)}+\frac{9\sin(a+x(b-d)-c)}{32(b-d)}+\frac{\sin(3(a-c)+3x(b-d))}{96(b-d)}-\frac{3\sin(3a+x(3b-d)-c)}{32(3b-d)}-\frac{9\sin(a}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[c + d*x]^3,x]
```


```
[Out] (-3*Sin[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) + (9*Sin[a - c + (b - d)*x])
/(32*(b - d)) + Sin[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) - (3*Sin[3*a - c
+ (3*b - d)*x])/(32*(3*b - d)) - (9*Sin[a + c + (b + d)*x])/(32*(b + d)) -
Sin[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) + (3*Sin[3*a + c + (3*b + d)*x])/
(32*(3*b + d)) + (3*Sin[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```


Rule 4569

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin^3(a+bx)\sin^3(c+dx)\,dx &= \int \left(-\frac{3}{32}\cos(a-3c+(b-3d)x)+\frac{9}{32}\cos(a-c+(b-d)x)+\frac{1}{32}\cos(3(a-c)+3(b\right.\\ &= \frac{1}{32}\int \cos(3(a-c)+3(b-d)x)\,dx-\frac{1}{32}\int \cos(3(a+c)+3(b+d)x)\,dx-\frac{3}{32}\int \cos\\ &= -\frac{3\sin(a-3c+(b-3d)x)}{32(b-3d)}+\frac{9\sin(a-c+(b-d)x)}{32(b-d)}+\frac{\sin(3(a-c)+3(b-d)x)}{96(b-d)}-\frac{3}{}\end{aligned}$$

Mathematica [A] time = 1.65731, size = 177, normalized size = 0.91

$$\frac{1}{96}\left(-\frac{9\sin(a+bx-3c-3dx)}{b-3d}+\frac{27\sin(a+bx-c-dx)}{b-d}+\frac{\sin(3(a+bx-c-dx))}{b-d}-\frac{9\sin(3a+3bx-c-dx)}{3b-d}+\frac{9\sin}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sin[c + d*x]^3,x]
```

```
[Out] ((-9*Sin[a - 3*c + b*x - 3*d*x])/(b - 3*d) + (27*Sin[a - c + b*x - d*x])/(b
 - d) + Sin[3*(a - c + b*x - d*x)]/(b - d) - (9*Sin[3*a - c + 3*b*x - d*x])
/(3*b - d) + (9*Sin[3*a + c + 3*b*x + d*x])/(3*b + d) + (9*Sin[a + 3*c + b*
x + 3*d*x])/(b + 3*d) - (27*Sin[a + c + (b + d)*x])/(b + d) - Sin[3*(a + c
+ (b + d)*x)]/(b + d))/96
```

Maple [A] time = 0.046, size = 184, normalized size = 0.9

$$-\frac{3\sin\left(a-3c+(b-3d)x\right)}{32b-96d}+\frac{9\sin\left(a-c+(b-d)x\right)}{32b-32d}-\frac{9\sin\left(a+c+(b+d)x\right)}{32b+32d}+\frac{3\sin\left(a+3c+(b+3d)x\right)}{32b+96d}+\frac{\sin((}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(d*x+c)^3,x)
```

```
[Out] -3/32*sin(a-3*c+(b-3*d)*x)/(b-3*d)+9/32*sin(a-c+(b-d)*x)/(b-d)-9/32*sin(a+c
+(b+d)*x)/(b+d)+3/32*sin(a+3*c+(b+3*d)*x)/(b+3*d)+1/96/(b-d)*sin((3*b-3*d)*
x-3*c+3*a)-3/32*sin(3*a-c+(3*b-d)*x)/(3*b-d)+3/32*sin(3*a+c+(3*b+d)*x)/(3*b
+d)-1/96/(b+d)*sin((3*b+3*d)*x+3*c+3*a)
```

Maxima [B] time = 2.03564, size = 3526, normalized size = 18.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/192*(9*(3*b^5*sin(3*c) - b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) + 10*b^2*d
^3*sin(3*c) + 27*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos((3*b + d)*x + 3*a + 4
*c) - 9*(3*b^5*sin(3*c) - b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) + 10*b^2*d^3
*sin(3*c) + 27*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos((3*b + d)*x + 3*a - 2*c
) + 9*(3*b^5*sin(3*c) + b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) - 10*b^2*d^3*s
in(3*c) + 27*b*d^4*sin(3*c) + 9*d^5*sin(3*c))*cos(-(3*b - d)*x - 3*a + 4*c)
 - 9*(3*b^5*sin(3*c) + b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) - 10*b^2*d^3*si
n(3*c) + 27*b*d^4*sin(3*c) + 9*d^5*sin(3*c))*cos(-(3*b - d)*x - 3*a - 2*c)
+ 9*(9*b^5*sin(3*c) - 27*b^4*d*sin(3*c) - 10*b^3*d^2*sin(3*c) + 30*b^2*d^3*
sin(3*c) + b*d^4*sin(3*c) - 3*d^5*sin(3*c))*cos((b + 3*d)*x + a + 6*c) - 9*
(9*b^5*sin(3*c) - 27*b^4*d*sin(3*c) - 10*b^3*d^2*sin(3*c) + 30*b^2*d^3*sin(
3*c) + b*d^4*sin(3*c) - 3*d^5*sin(3*c))*cos((b + 3*d)*x + a) - (9*b^5*sin(3
*c) - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^
4*sin(3*c) - 9*d^5*sin(3*c))*cos(3*(b + d)*x + 3*a + 6*c) + (9*b^5*sin(3*c)
 - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^4*s
in(3*c) - 9*d^5*sin(3*c))*cos(3*(b + d)*x + 3*a) - 27*(9*b^5*sin(3*c) - 9*b
^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c
) - 9*d^5*sin(3*c))*cos((b + d)*x + a + 4*c) + 27*(9*b^5*sin(3*c) - 9*b^4*d
*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) -
9*d^5*sin(3*c))*cos((b + d)*x + a - 2*c) - 27*(9*b^5*sin(3*c) + 9*b^4*d*sin
(3*c) - 82*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^
5*sin(3*c))*cos(-(b - d)*x - a + 4*c) + 27*(9*b^5*sin(3*c) + 9*b^4*d*sin(3*
```

```
c) - 82*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^5*s
in(3*c))*cos(-(b - d)*x - a - 2*c) - (9*b^5*sin(3*c) + 9*b^4*d*sin(3*c) - 8
2*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^5*sin(3*c
))*cos(-3*(b - d)*x - 3*a + 6*c) + (9*b^5*sin(3*c) + 9*b^4*d*sin(3*c) - 82*
b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^5*sin(3*c))
*cos(-3*(b - d)*x - 3*a) + 9*(9*b^5*sin(3*c) + 27*b^4*d*sin(3*c) - 10*b^3*d
^2*sin(3*c) - 30*b^2*d^3*sin(3*c) + b*d^4*sin(3*c) + 3*d^5*sin(3*c))*cos(-(
b - 3*d)*x - a + 6*c) - 9*(9*b^5*sin(3*c) + 27*b^4*d*sin(3*c) - 10*b^3*d^2*
sin(3*c) - 30*b^2*d^3*sin(3*c) + b*d^4*sin(3*c) + 3*d^5*sin(3*c))*cos(-(b -
 3*d)*x - a) - 9*(3*b^5*cos(3*c) - b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) + 1
0*b^2*d^3*cos(3*c) + 27*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*sin((3*b + d)*x +
3*a + 4*c) - 9*(3*b^5*cos(3*c) - b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) + 10*
b^2*d^3*cos(3*c) + 27*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*sin((3*b + d)*x + 3*
a - 2*c) - 9*(3*b^5*cos(3*c) + b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) - 10*b^
2*d^3*cos(3*c) + 27*b*d^4*cos(3*c) + 9*d^5*cos(3*c))*sin(-(3*b - d)*x - 3*a
 + 4*c) - 9*(3*b^5*cos(3*c) + b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) - 10*b^2
*d^3*cos(3*c) + 27*b*d^4*cos(3*c) + 9*d^5*cos(3*c))*sin(-(3*b - d)*x - 3*a
- 2*c) - 9*(9*b^5*cos(3*c) - 27*b^4*d*cos(3*c) - 10*b^3*d^2*cos(3*c) + 30*b
^2*d^3*cos(3*c) + b*d^4*cos(3*c) - 3*d^5*cos(3*c))*sin((b + 3*d)*x + a + 6*
c) - 9*(9*b^5*cos(3*c) - 27*b^4*d*cos(3*c) - 10*b^3*d^2*cos(3*c) + 30*b^2*d
^3*cos(3*c) + b*d^4*cos(3*c) - 3*d^5*cos(3*c))*sin((b + 3*d)*x + a) + (9*b^
5*cos(3*c) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) +
 9*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*sin(3*(b + d)*x + 3*a + 6*c) + (9*b^5*c
os(3*c) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*
b*d^4*cos(3*c) - 9*d^5*cos(3*c))*sin(3*(b + d)*x + 3*a) + 27*(9*b^5*cos(3*c
) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*b*d^4*
cos(3*c) - 9*d^5*cos(3*c))*sin((b + d)*x + a + 4*c) + 27*(9*b^5*cos(3*c) -
9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(
3*c) - 9*d^5*cos(3*c))*sin((b + d)*x + a - 2*c) + 27*(9*b^5*cos(3*c) + 9*b^
4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c)
 + 9*d^5*cos(3*c))*sin(-(b - d)*x - a + 4*c) + 27*(9*b^5*cos(3*c) + 9*b^4*d
*cos(3*c) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) +
9*d^5*cos(3*c))*sin(-(b - d)*x - a - 2*c) + (9*b^5*cos(3*c) + 9*b^4*d*cos(3
*c) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) + 9*d^5*
cos(3*c))*sin(-3*(b - d)*x - 3*a + 6*c) + (9*b^5*cos(3*c) + 9*b^4*d*cos(3*c
) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) + 9*d^5*co
s(3*c))*sin(-3*(b - d)*x - 3*a) - 9*(9*b^5*cos(3*c) + 27*b^4*d*cos(3*c) - 1
0*b^3*d^2*cos(3*c) - 30*b^2*d^3*cos(3*c) + b*d^4*cos(3*c) + 3*d^5*cos(3*c))
*sin(-(b - 3*d)*x - a + 6*c) - 9*(9*b^5*cos(3*c) + 27*b^4*d*cos(3*c) - 10*b
^3*d^2*cos(3*c) - 30*b^2*d^3*cos(3*c) + b*d^4*cos(3*c) + 3*d^5*cos(3*c))*si
n(-(b - 3*d)*x - a))/(9*b^6*cos(3*c)^2 + 9*b^6*sin(3*c)^2 - 9*(cos(3*c)^2 +
 sin(3*c)^2)*d^6 + 91*(b^2*cos(3*c)^2 + b^2*sin(3*c)^2)*d^4 - 91*(b^4*cos(3
*c)^2 + b^4*sin(3*c)^2)*d^2)
```

Fricas [A] time = 0.57385, size = 652, normalized size = 3.34

$$-\frac{\left(\left(63\,b^4d-88\,b^2d^3+9\,d^5-\left(9\,b^4d-82\,b^2d^3+9\,d^5\right)\cos\left(bx+a\right)^2\right)\cos\left(dx+c\right)^3-3\left(21\,b^4d-70\,b^2d^3+9\,d^5-\left(3\,b^4d\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(((63*b^4*d - 88*b^2*d^3 + 9*d^5 - (9*b^4*d - 82*b^2*d^3 + 9*d^5)*cos(
b*x + a)^2)*cos(d*x + c)^3 - 3*(21*b^4*d - 70*b^2*d^3 + 9*d^5 - (3*b^4*d -
28*b^2*d^3 + 9*d^5)*cos(b*x + a)^2)*cos(d*x + c))*sin(b*x + a) - ((9*b^5 -
88*b^3*d^2 + 63*b*d^4)*cos(b*x + a)^3 - ((9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cos
```

```
(b*x + a)^3 - 3*(9*b^5 - 28*b^3*d^2 + 3*b*d^4)*cos(b*x + a))*cos(d*x + c)^2
 - 3*(9*b^5 - 70*b^3*d^2 + 21*b*d^4)*cos(b*x + a))*sin(d*x + c))/(9*b^6 - 9
1*b^4*d^2 + 91*b^2*d^4 - 9*d^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12141, size = 244, normalized size = 1.25

$$-\frac{\sin(3\,bx+3\,dx+3\,a+3\,c)}{96\,(b+d)}+\frac{3\,\sin(3\,bx+dx+3\,a+c)}{32\,(3\,b+d)}-\frac{3\,\sin(3\,bx-dx+3\,a-c)}{32\,(3\,b-d)}+\frac{\sin(3\,bx-3\,dx+3\,a-3\,c)}{96\,(b-d)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/96*sin(3*b*x + 3*d*x + 3*a + 3*c)/(b + d) + 3/32*sin(3*b*x + d*x + 3*a +
 c)/(3*b + d) - 3/32*sin(3*b*x - d*x + 3*a - c)/(3*b - d) + 1/96*sin(3*b*x
- 3*d*x + 3*a - 3*c)/(b - d) + 3/32*sin(b*x + 3*d*x + a + 3*c)/(b + 3*d) -
9/32*sin(b*x + d*x + a + c)/(b + d) + 9/32*sin(b*x - d*x + a - c)/(b - d) -
 3/32*sin(b*x - 3*d*x + a - 3*c)/(b - 3*d)
```

3.209 $\int \cos^n(c+dx)\sin(a+bx)\,dx$

Optimal. Leaf size=277

$$-\frac{2^{-n-1}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\left(1+e^{2ic+2idx}\right)^{-n}\text{Hypergeometric2F1}\left(-n,\frac{b-dn}{2d},\frac{1}{2}\left(\frac{b}{d}-n+2\right),-e^{2i(c+dx)}\right)\exp(i(a-cn)\cdot}{b-dn}$$

```
[Out] -((2^(-1 - n)*E^(I*(a - c*n) + I*(b - d*n)*x + I*n*(c + d*x))*(E^((-I)*(c +
 d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, (b - d*n)/(2*d), (2 + b/d
 - n)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x))^n*(b - d*n))
) - (2^(-1 - n)*E^((-I)*(a + c*n) - I*(b + d*n)*x + I*n*(c + d*x))*(E^((-I)
*(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, -(b + d*n)/(2*d), 1
- (b + d*n)/(2*d), -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x))^n*
(b + d*n))
```

Rubi [A] time = 0.588679, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4555, 2285, 2253, 2251}

$$-\frac{2^{-n-1}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\left(1+e^{2ic+2idx}\right)^{-n}\,{}_2F_1\left(-n,\frac{b-dn}{2d};\frac{1}{2}\left(\frac{b}{d}-n+2\right);-e^{2i(c+dx)}\right)\exp(i(a-cn)+ix(b-dn)+in(c}{b-dn}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^n*Sin[a + b*x],x]
```

```
[Out] -((2^(-1 - n)*E^(I*(a - c*n) + I*(b - d*n)*x + I*n*(c + d*x))*(E^((-I)*(c +
 d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, (b - d*n)/(2*d), (2 + b/d
 - n)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x))^n*(b - d*n))
) - (2^(-1 - n)*E^((-I)*(a + c*n) - I*(b + d*n)*x + I*n*(c + d*x))*(E^((-I)
*(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, -(b + d*n)/(2*d), 1
- (b + d*n)/(2*d), -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x))^n*
(b + d*n))
```

Rule 4555

```
Int[Cos[(c_.) + (d_.)*(x_)]^(q_.)*Sin[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol]
:> Dist[1/2^(p + q), Int[ExpandIntegrand[(E^(-(I*(c + d*x))) + E^(I*(c + d*
x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
b, c, d, q}, x] && IGtQ[p, 0] && !IntegerQ[q]
```

Rule 2285

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Dist[(a*F^
v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n), Int[u*F^(n*v)*(a
+ b*F^ExpandToSum[w - v, x])^n, x], x] /; FreeQ[{F, a, b, n}, x] && !Integ
erQ[n] && LinearQ[{v, w}, x]
```

Rule 2253

```
Int[((a_) + (b_.)*(F_)^((e_.)*(v_)))^(p_)*(G_)^((h_.)*(u_)), x_Symbol] :> I
nt[G^(h*ExpandToSum[u, x])*(a + b*F^(e*ExpandToSum[v, x]))^p, x] /; FreeQ[{
F, G, a, b, e, h, p}, x] && LinearQ[{u, v}, x] && !LinearMatchQ[{u, v}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \cos^n(c+dx)\sin(a+bx)\,dx = 2^{-1-n}\int \left(ie^{-ia-ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n - ie^{ia+ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\right)dx$$
$$= \left(i2^{-1-n}\right)\int e^{-ia-ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n dx - \left(i2^{-1-n}\right)\int e^{ia+ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n dx$$
$$= \left(i2^{-1-n}e^{in(c+dx)}\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\right)\int e^{-ia-ibx-in(c+dx)}\left(1+e^{2ic+2idx}\right)^n d$$
$$= -\left(\left(i2^{-1-n}e^{in(c+dx)}\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\right)\int e^{i(a-cn)+i(b-dn)x}\left(1+e^{2ic+2idx}\right)^n$$
$$= -\frac{2^{-1-n}\exp(i(a-cn)+i(b-dn)x+in(c+dx))\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n {}_2F_1}{b-dn}$$

Mathematica [A] time = 0.975377, size = 202, normalized size = 0.73

$$-\frac{2^{-n-1}e^{i(c-bx)}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^{n+1}\left(e^{idx}(\cos(a)-i\sin(a))(b-dn)\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(-\frac{b}{d}+n+2\right),-\frac{b+d(n}{2d}\right.\right.}{(b-dn)(b+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^n*Sin[a + b*x],x]
```

```
[Out] -((2^(-1 - n)*E^(I*(c - b*x))*((1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x)))^(
1 + n)*(E^(I*d*x)*(b - d*n)*Hypergeometric2F1[1, (2 - b/d + n)/2, -(b + d*(
-2 + n))/(2*d), -E^((2*I)*(c + d*x))]*(Cos[a] - I*Sin[a]) + E^(I*(2*b + d)*
x)*(b + d*n)*Hypergeometric2F1[1, (b + d*(2 + n))/(2*d), (2 + b/d - n)/2, -
E^((2*I)*(c + d*x))]*(Cos[a] + I*Sin[a])))/((b - d*n)*(b + d*n)))
```

Maple [F] time = 0.698, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^n \sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^n*sin(b*x+a),x)
```

```
[Out] int(cos(d*x+c)^n*sin(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(dx+c)^n \sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^n*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^n*sin(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cos\left(dx+c\right)^{n}\sin\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^n*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^n*sin(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**n*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(dx+c\right)^{n}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^n*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^n*sin(b*x + a), x)
```

3.210 $\int \cos^3(c+dx)\sin(a+bx)\,dx$

Optimal. Leaf size=91

$$-\frac{\cos(a+x(b-3d)-3c)}{8(b-3d)}-\frac{3\cos(a+x(b-d)-c)}{8(b-d)}-\frac{3\cos(a+x(b+d)+c)}{8(b+d)}-\frac{\cos(a+x(b+3d)+3c)}{8(b+3d)}$$

```
[Out] -Cos[a - 3*c + (b - 3*d)*x]/(8*(b - 3*d)) - (3*Cos[a - c + (b - d)*x])/(8*(
b - d)) - (3*Cos[a + c + (b + d)*x])/(8*(b + d)) - Cos[a + 3*c + (b + 3*d)*
x]/(8*(b + 3*d))
```

Rubi [A] time = 0.0662915, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4574, 2638}

$$-\frac{\cos(a+x(b-3d)-3c)}{8(b-3d)}-\frac{3\cos(a+x(b-d)-c)}{8(b-d)}-\frac{3\cos(a+x(b+d)+c)}{8(b+d)}-\frac{\cos(a+x(b+3d)+3c)}{8(b+3d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sin[a + b*x],x]
```

```
[Out] -Cos[a - 3*c + (b - 3*d)*x]/(8*(b - 3*d)) - (3*Cos[a - c + (b - d)*x])/(8*(
b - d)) - (3*Cos[a + c + (b + d)*x])/(8*(b + d)) - Cos[a + 3*c + (b + 3*d)*
x]/(8*(b + 3*d))
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)\sin(a+bx)\,dx &= \int \left(\frac{1}{8}\sin(a-3c+(b-3d)x)+\frac{3}{8}\sin(a-c+(b-d)x)+\frac{3}{8}\sin(a+c+(b+d)x)+\frac{1}{8}\sin\right.\\ &= \frac{1}{8}\int \sin(a-3c+(b-3d)x)\,dx+\frac{1}{8}\int \sin(a+3c+(b+3d)x)\,dx+\frac{3}{8}\int \sin(a-c+(b-\\ &= -\frac{\cos(a-3c+(b-3d)x)}{8(b-3d)}-\frac{3\cos(a-c+(b-d)x)}{8(b-d)}-\frac{3\cos(a+c+(b+d)x)}{8(b+d)}-\frac{\cos(a+}{8}\end{aligned}$$

Mathematica [A] time = 0.509112, size = 87, normalized size = 0.96

$$\frac{1}{8}\left(-\frac{\cos(a+bx-3c-3dx)}{b-3d}-\frac{3\cos(a+bx-c-dx)}{b-d}-\frac{\cos(a+bx+3c+3dx)}{b+3d}-\frac{3\cos(a+x(b+d)+c)}{b+d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sin[a + b*x],x]
```

```
[Out] (-(Cos[a - 3*c + b*x - 3*d*x]/(b - 3*d)) - (3*Cos[a - c + b*x - d*x])/(b -
d) - Cos[a + 3*c + b*x + 3*d*x]/(b + 3*d) - (3*Cos[a + c + (b + d)*x])/(b +
 d))/8
```

Maple [A] time = 0.019, size = 84, normalized size = 0.9

$$-\frac{\cos\left(a-3c+(b-3d)x\right)}{8b-24d}-\frac{3\cos\left(a-c+(b-d)x\right)}{8b-8d}-\frac{3\cos\left(a+c+(b+d)x\right)}{8b+8d}-\frac{\cos\left(a+3c+(b+3d)x\right)}{8b+24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(b*x+a),x)
```

```
[Out] -1/8*cos(a-3*c+(b-3*d)*x)/(b-3*d)-3/8*cos(a-c+(b-d)*x)/(b-d)-3/8*cos(a+c+(b
+d)*x)/(b+d)-1/8*cos(a+3*c+(b+3*d)*x)/(b+3*d)
```

Maxima [B] time = 1.2841, size = 1231, normalized size = 13.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/16*((b^3*cos(3*c) - 3*b^2*d*cos(3*c) - b*d^2*cos(3*c) + 3*d^3*cos(3*c))*
cos((b + 3*d)*x + a + 6*c) + (b^3*cos(3*c) - 3*b^2*d*cos(3*c) - b*d^2*cos(3
*c) + 3*d^3*cos(3*c))*cos((b + 3*d)*x + a) + 3*(b^3*cos(3*c) - b^2*d*cos(3*
c) - 9*b*d^2*cos(3*c) + 9*d^3*cos(3*c))*cos((b + d)*x + a + 4*c) + 3*(b^3*c
os(3*c) - b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) + 9*d^3*cos(3*c))*cos((b + d)*x
 + a - 2*c) + 3*(b^3*cos(3*c) + b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) - 9*d^3*c
os(3*c))*cos(-(b - d)*x - a + 4*c) + 3*(b^3*cos(3*c) + b^2*d*cos(3*c) - 9*b
*d^2*cos(3*c) - 9*d^3*cos(3*c))*cos(-(b - d)*x - a - 2*c) + (b^3*cos(3*c) +
 3*b^2*d*cos(3*c) - b*d^2*cos(3*c) - 3*d^3*cos(3*c))*cos(-(b - 3*d)*x - a +
 6*c) + (b^3*cos(3*c) + 3*b^2*d*cos(3*c) - b*d^2*cos(3*c) - 3*d^3*cos(3*c))
*cos(-(b - 3*d)*x - a) + (b^3*sin(3*c) - 3*b^2*d*sin(3*c) - b*d^2*sin(3*c)
+ 3*d^3*sin(3*c))*sin((b + 3*d)*x + a + 6*c) - (b^3*sin(3*c) - 3*b^2*d*sin(
3*c) - b*d^2*sin(3*c) + 3*d^3*sin(3*c))*sin((b + 3*d)*x + a) + 3*(b^3*sin(3
*c) - b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) + 9*d^3*sin(3*c))*sin((b + d)*x + a
 + 4*c) - 3*(b^3*sin(3*c) - b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) + 9*d^3*sin(3
*c))*sin((b + d)*x + a - 2*c) + 3*(b^3*sin(3*c) + b^2*d*sin(3*c) - 9*b*d^2*
sin(3*c) - 9*d^3*sin(3*c))*sin(-(b - d)*x - a + 4*c) - 3*(b^3*sin(3*c) + b^
2*d*sin(3*c) - 9*b*d^2*sin(3*c) - 9*d^3*sin(3*c))*sin(-(b - d)*x - a - 2*c)
 + (b^3*sin(3*c) + 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) - 3*d^3*sin(3*c))*sin(
-(b - 3*d)*x - a + 6*c) - (b^3*sin(3*c) + 3*b^2*d*sin(3*c) - b*d^2*sin(3*c)
 - 3*d^3*sin(3*c))*sin(-(b - 3*d)*x - a))/(b^4*cos(3*c)^2 + b^4*sin(3*c)^2
+ 9*(cos(3*c)^2 + sin(3*c)^2)*d^4 - 10*(b^2*cos(3*c)^2 + b^2*sin(3*c)^2)*d^
2)
```

Fricas [A] time = 0.512338, size = 239, normalized size = 2.63

$$\frac{6\,bd^{2}\cos\left(bx+a\right)\cos\left(dx+c\right)-\left(b^{3}-bd^{2}\right)\cos\left(bx+a\right)\cos\left(dx+c\right)^{3}+3\left(2\,d^{3}-\left(b^{2}d-d^{3}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(bx+a\right)}{b^{4}-10\,b^{2}d^{2}+9\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] (6*b*d^2*cos(b*x + a)*cos(d*x + c) - (b^3 - b*d^2)*cos(b*x + a)*cos(d*x + c
)^3 + 3*(2*d^3 - (b^2*d - d^3)*cos(d*x + c)^2)*sin(b*x + a)*sin(d*x + c))/(
b^4 - 10*b^2*d^2 + 9*d^4)
```

Sympy [A] time = 79.4673, size = 923, normalized size = 10.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(b*x+a),x)
```

```
[Out] Piecewise((x*sin(a)*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (-3*x*sin(a - 3*d*x)*s
in(c + d*x)**2*cos(c + d*x)/8 + x*sin(a - 3*d*x)*cos(c + d*x)**3/8 - x*sin(
c + d*x)**3*cos(a - 3*d*x)/8 + 3*x*sin(c + d*x)*cos(a - 3*d*x)*cos(c + d*x)
**2/8 - sin(a - 3*d*x)*sin(c + d*x)**3/(24*d) - sin(a - 3*d*x)*sin(c + d*x)
*cos(c + d*x)**2/(4*d) + 3*cos(a - 3*d*x)*cos(c + d*x)**3/(8*d), Eq(b, -3*d
)), (3*x*sin(a - d*x)*sin(c + d*x)**2*cos(c + d*x)/8 + 3*x*sin(a - d*x)*cos
(c + d*x)**3/8 + 3*x*sin(c + d*x)**3*cos(a - d*x)/8 + 3*x*sin(c + d*x)*cos(
a - d*x)*cos(c + d*x)**2/8 + sin(a - d*x)*sin(c + d*x)**3/(4*d) + 5*sin(a -
 d*x)*sin(c + d*x)*cos(c + d*x)**2/(8*d) + sin(c + d*x)**2*cos(a - d*x)*cos
(c + d*x)/(8*d), Eq(b, -d)), (3*x*sin(a + d*x)*sin(c + d*x)**2*cos(c + d*x)
/8 + 3*x*sin(a + d*x)*cos(c + d*x)**3/8 - 3*x*sin(c + d*x)**3*cos(a + d*x)/
8 - 3*x*sin(c + d*x)*cos(a + d*x)*cos(c + d*x)**2/8 + 3*sin(a + d*x)*sin(c
+ d*x)**3/(8*d) + 3*sin(a + d*x)*sin(c + d*x)*cos(c + d*x)**2/(4*d) + cos(a
 + d*x)*cos(c + d*x)**3/(8*d), Eq(b, d)), (-3*x*sin(a + 3*d*x)*sin(c + d*x)
**2*cos(c + d*x)/8 + x*sin(a + 3*d*x)*cos(c + d*x)**3/8 + x*sin(c + d*x)**3
*cos(a + 3*d*x)/8 - 3*x*sin(c + d*x)*cos(a + 3*d*x)*cos(c + d*x)**2/8 - sin
(a + 3*d*x)*sin(c + d*x)**3/(24*d) - sin(a + 3*d*x)*sin(c + d*x)*cos(c + d*
x)**2/(4*d) - 3*cos(a + 3*d*x)*cos(c + d*x)**3/(8*d), Eq(b, 3*d)), (-b**3*c
os(a + b*x)*cos(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) - 3*b**2*d*sin(a
 + b*x)*sin(c + d*x)*cos(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4) + 6*b*d
**2*sin(c + d*x)**2*cos(a + b*x)*cos(c + d*x)/(b**4 - 10*b**2*d**2 + 9*d**4
) + 7*b*d**2*cos(a + b*x)*cos(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) +
6*d**3*sin(a + b*x)*sin(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) + 9*d**3
*sin(a + b*x)*sin(c + d*x)*cos(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4),
True))
```

Giac [A] time = 1.1603, size = 113, normalized size = 1.24

$$-\frac{\cos\left(bx + 3\,dx + a + 3\,c\right)}{8\,(b + 3\,d)} - \frac{3\,\cos\left(bx + dx + a + c\right)}{8\,(b + d)} - \frac{3\,\cos\left(bx - dx + a - c\right)}{8\,(b - d)} - \frac{\cos\left(bx - 3\,dx + a - 3\,c\right)}{8\,(b - 3\,d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*cos(b*x + 3*d*x + a + 3*c)/(b + 3*d) - 3/8*cos(b*x + d*x + a + c)/(b +
 d) - 3/8*cos(b*x - d*x + a - c)/(b - d) - 1/8*cos(b*x - 3*d*x + a - 3*c)/(
b - 3*d)
```

3.211 $\int \cos^2(c+dx)\sin(a+bx)\,dx$

Optimal. Leaf size=62

$$-\frac{\cos(a+x(b-2d)-2c)}{4(b-2d)}-\frac{\cos(a+x(b+2d)+2c)}{4(b+2d)}-\frac{\cos(a+bx)}{2b}$$

```
[Out] -Cos[a + b*x]/(2*b) - Cos[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) - Cos[a + 2*
c + (b + 2*d)*x]/(4*(b + 2*d))
```

Rubi [A] time = 0.0469182, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4574, 2638}

$$-\frac{\cos(a+x(b-2d)-2c)}{4(b-2d)}-\frac{\cos(a+x(b+2d)+2c)}{4(b+2d)}-\frac{\cos(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[a + b*x],x]
```

```
[Out] -Cos[a + b*x]/(2*b) - Cos[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) - Cos[a + 2*
c + (b + 2*d)*x]/(4*(b + 2*d))
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(c+dx)\sin(a+bx)\,dx &= \int \left(\frac{1}{2}\sin(a+bx)+\frac{1}{4}\sin(a-2c+(b-2d)x)+\frac{1}{4}\sin(a+2c+(b+2d)x)\right)dx\\
&= \frac{1}{4}\int \sin(a-2c+(b-2d)x)\,dx+\frac{1}{4}\int \sin(a+2c+(b+2d)x)\,dx+\frac{1}{2}\int \sin(a+bx)\\
&= -\frac{\cos(a+bx)}{2b}-\frac{\cos(a-2c+(b-2d)x)}{4(b-2d)}-\frac{\cos(a+2c+(b+2d)x)}{4(b+2d)}
\end{aligned}$$

Mathematica [A] time = 0.780621, size = 71, normalized size = 1.15

$$\frac{1}{4}\left(-\frac{\cos(a+bx-2c-2dx)}{b-2d}-\frac{\cos(a+bx+2c+2dx)}{b+2d}+\frac{2\sin(a)\sin(bx)}{b}-\frac{2\cos(a)\cos(bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[a + b*x],x]
```

```
[Out] ((-2*Cos[a]*Cos[b*x])/b - Cos[a - 2*c + b*x - 2*d*x]/(b - 2*d) - Cos[a + 2*
c + b*x + 2*d*x]/(b + 2*d) + (2*Sin[a]*Sin[b*x])/b)/4
```

Maple [A] time = 0.016, size = 57, normalized size = 0.9

$$-\frac{\cos(bx+a)}{2b}-\frac{\cos(a-2c+(b-2d)x)}{4b-8d}-\frac{\cos(a+2c+(b+2d)x)}{4b+8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(b*x+a),x)
```

```
[Out] -1/2*cos(b*x+a)/b-1/4*cos(a-2*c+(b-2*d)*x)/(b-2*d)-1/4*cos(a+2*c+(b+2*d)*x)
/(b+2*d)
```

Maxima [B] time = 1.21244, size = 559, normalized size = 9.02

$$-\frac{\left(b^2\cos(2c)-2bd\cos(2c)\right)\cos((b+2d)x+a+4c)+\left(b^2\cos(2c)-2bd\cos(2c)\right)\cos((b+2d)x+a)+\left(b^2\cos(2c)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*((b^2*cos(2*c) - 2*b*d*cos(2*c))*cos((b + 2*d)*x + a + 4*c) + (b^2*cos
(2*c) - 2*b*d*cos(2*c))*cos((b + 2*d)*x + a) + (b^2*cos(2*c) + 2*b*d*cos(2*
c))*cos(-(b - 2*d)*x - a + 4*c) + (b^2*cos(2*c) + 2*b*d*cos(2*c))*cos(-(b -
 2*d)*x - a) + 2*(b^2*cos(2*c) - 4*d^2*cos(2*c))*cos(b*x + a + 2*c) + 2*(b^
2*cos(2*c) - 4*d^2*cos(2*c))*cos(b*x + a - 2*c) + (b^2*sin(2*c) - 2*b*d*sin
(2*c))*sin((b + 2*d)*x + a + 4*c) - (b^2*sin(2*c) - 2*b*d*sin(2*c))*sin((b
+ 2*d)*x + a) + (b^2*sin(2*c) + 2*b*d*sin(2*c))*sin(-(b - 2*d)*x - a + 4*c)
 - (b^2*sin(2*c) + 2*b*d*sin(2*c))*sin(-(b - 2*d)*x - a) + 2*(b^2*sin(2*c)
- 4*d^2*sin(2*c))*sin(b*x + a + 2*c) - 2*(b^2*sin(2*c) - 4*d^2*sin(2*c))*si
n(b*x + a - 2*c))/(b^3*cos(2*c)^2 + b^3*sin(2*c)^2 - 4*(b*cos(2*c)^2 + b*si
n(2*c)^2)*d^2)
```

Fricas [A] time = 0.490526, size = 162, normalized size = 2.61

$$-\frac{b^2\cos(bx+a)\cos(dx+c)^2+2bd\cos(dx+c)\sin(bx+a)\sin(dx+c)-2d^2\cos(bx+a)}{b^3-4bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -(b^2*cos(b*x + a)*cos(d*x + c)^2 + 2*b*d*cos(d*x + c)*sin(b*x + a)*sin(d*x
 + c) - 2*d^2*cos(b*x + a))/(b^3 - 4*b*d^2)
```

Sympy [A] time = 12.1257, size = 401, normalized size = 6.47

$$\begin{cases} x\sin(a)\cos^2(c) \\ \left(\frac{x\sin^2(c+dx)}{2}+\frac{x\cos^2(c+dx)}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)\sin(a) \\ -\frac{x\sin(a-2dx)\sin^2(c+dx)}{4}+\frac{x\sin(a-2dx)\cos^2(c+dx)}{4}+\frac{x\sin(c+dx)\cos(a-2dx)\cos(c+dx)}{2}+\frac{\sin^2(c+dx)\cos(a-2dx)}{8d}+\frac{3\cos(a-2dx)\cos^2(c+dx)}{8d} \\ -\frac{x\sin(a+2dx)\sin^2(c+dx)}{4}+\frac{x\sin(a+2dx)\cos^2(c+dx)}{4}-\frac{x\sin(c+dx)\cos(a+2dx)\cos(c+dx)}{2}-\frac{\sin(a+2dx)\sin(c+dx)\cos(c+dx)}{4d}-\frac{\cos(a+2dx)\cos}{2d} \\ -\frac{b^2\cos(a+bx)\cos^2(c+dx)}{b^3-4bd^2}-\frac{2bd\sin(a+bx)\sin(c+dx)\cos(c+dx)}{b^3-4bd^2}+\frac{2d^2\sin^2(c+dx)\cos(a+bx)}{b^3-4bd^2}+\frac{2d^2\cos(a+bx)\cos^2(c+dx)}{b^3-4bd^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(b*x+a),x)
```

```
[Out] Piecewise((x*sin(a)*cos(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**2/2
+ x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(2*d))*sin(a), Eq(b, 0)),
 (-x*sin(a - 2*d*x)*sin(c + d*x)**2/4 + x*sin(a - 2*d*x)*cos(c + d*x)**2/4
+ x*sin(c + d*x)*cos(a - 2*d*x)*cos(c + d*x)/2 + sin(c + d*x)**2*cos(a - 2*
d*x)/(8*d) + 3*cos(a - 2*d*x)*cos(c + d*x)**2/(8*d), Eq(b, -2*d)), (-x*sin(
a + 2*d*x)*sin(c + d*x)**2/4 + x*sin(a + 2*d*x)*cos(c + d*x)**2/4 - x*sin(c
 + d*x)*cos(a + 2*d*x)*cos(c + d*x)/2 - sin(a + 2*d*x)*sin(c + d*x)*cos(c +
 d*x)/(4*d) - cos(a + 2*d*x)*cos(c + d*x)**2/(2*d), Eq(b, 2*d)), (-b**2*cos
(a + b*x)*cos(c + d*x)**2/(b**3 - 4*b*d**2) - 2*b*d*sin(a + b*x)*sin(c + d*
x)*cos(c + d*x)/(b**3 - 4*b*d**2) + 2*d**2*sin(c + d*x)**2*cos(a + b*x)/(b*
*3 - 4*b*d**2) + 2*d**2*cos(a + b*x)*cos(c + d*x)**2/(b**3 - 4*b*d**2), Tru
e))
```

Giac [A] time = 1.13856, size = 76, normalized size = 1.23

$$-\frac{\cos(bx+2\,dx+a+2\,c)}{4\,(b+2\,d)}-\frac{\cos(bx-2\,dx+a-2\,c)}{4\,(b-2\,d)}-\frac{\cos(bx+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*cos(b*x + 2*d*x + a + 2*c)/(b + 2*d) - 1/4*cos(b*x - 2*d*x + a - 2*c)/
(b - 2*d) - 1/2*cos(b*x + a)/b
```

3.212 $\int \cos(c+dx)\sin(a+bx)\,dx$

Optimal. Leaf size=43

$$-\frac{\cos(a+x(b-d)-c)}{2(b-d)}-\frac{\cos(a+x(b+d)+c)}{2(b+d)}$$

```
[Out] -Cos[a - c + (b - d)*x]/(2*(b - d)) - Cos[a + c + (b + d)*x]/(2*(b + d))
```

Rubi [A] time = 0.0357136, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4574, 2638}

$$-\frac{\cos(a+x(b-d)-c)}{2(b-d)}-\frac{\cos(a+x(b+d)+c)}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[a + b*x],x]
```

```
[Out] -Cos[a - c + (b - d)*x]/(2*(b - d)) - Cos[a + c + (b + d)*x]/(2*(b + d))
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sin(a+bx)\,dx &= \int \left(\frac{1}{2}\sin(a-c+(b-d)x)+\frac{1}{2}\sin(a+c+(b+d)x)\right)dx\\ &= \frac{1}{2}\int \sin(a-c+(b-d)x)\,dx+\frac{1}{2}\int \sin(a+c+(b+d)x)\,dx\\ &= -\frac{\cos(a-c+(b-d)x)}{2(b-d)}-\frac{\cos(a+c+(b+d)x)}{2(b+d)}\end{aligned}$$

Mathematica [A] time = 0.195882, size = 43, normalized size = 1.

$$-\frac{\cos(a+x(b-d)-c)}{2(b-d)}-\frac{\cos(a+x(b+d)+c)}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[a + b*x],x]
```

```
[Out] -Cos[a - c + (b - d)*x]/(2*(b - d)) - Cos[a + c + (b + d)*x]/(2*(b + d))
```

Maple [A] time = 0.014, size = 40, normalized size = 0.9

$$-\frac{\cos\left(a-c+\left(b-d\right)x\right)}{2b-2d}-\frac{\cos\left(a+c+\left(b+d\right)x\right)}{2b+2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(b*x+a),x)
```

```
[Out] -1/2*cos(a-c+(b-d)*x)/(b-d)-1/2*cos(a+c+(b+d)*x)/(b+d)
```

Maxima [A] time = 1.17808, size = 54, normalized size = 1.26

$$-\frac{\cos\left(bx+dx+a+c\right)}{2\left(b+d\right)}-\frac{\cos\left(-bx+dx-a+c\right)}{2\left(b-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*cos(b*x + d*x + a + c)/(b + d) - 1/2*cos(-b*x + d*x - a + c)/(b - d)
```

Fricas [A] time = 0.473851, size = 100, normalized size = 2.33

$$-\frac{b\cos\left(bx+a\right)\cos\left(dx+c\right)+d\sin\left(bx+a\right)\sin\left(dx+c\right)}{b^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -(b*cos(b*x + a)*cos(d*x + c) + d*sin(b*x + a)*sin(d*x + c))/(b^2 - d^2)
```

Sympy [A] time = 2.87205, size = 155, normalized size = 3.6

$$\begin{cases} x\sin\left(a\right)\cos\left(c\right) & \text{for}\ b=0\wedge d=0\\ \frac{x\sin\left(a-dx\right)\cos\left(c+dx\right)}{2}+\frac{x\sin\left(c+dx\right)\cos\left(a-dx\right)}{2}+\frac{\sin\left(a-dx\right)\sin\left(c+dx\right)}{2d} & \text{for}\ b=-d\\ \frac{x\sin\left(a+dx\right)\cos\left(c+dx\right)}{2}-\frac{x\sin\left(c+dx\right)\cos\left(a+dx\right)}{2}-\frac{\cos\left(a+dx\right)\cos\left(c+dx\right)}{2d} & \text{for}\ b=d\\ -\frac{b\cos\left(a+bx\right)\cos\left(c+dx\right)}{b^2-d^2}-\frac{d\sin\left(a+bx\right)\sin\left(c+dx\right)}{b^2-d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a),x)
```

```
[Out] Piecewise((x*sin(a)*cos(c), Eq(b, 0) & Eq(d, 0)), (x*sin(a - d*x)*cos(c + d
*x)/2 + x*sin(c + d*x)*cos(a - d*x)/2 + sin(a - d*x)*sin(c + d*x)/(2*d), Eq
(b, -d)), (x*sin(a + d*x)*cos(c + d*x)/2 - x*sin(c + d*x)*cos(a + d*x)/2 -
cos(a + d*x)*cos(c + d*x)/(2*d), Eq(b, d)), (-b*cos(a + b*x)*cos(c + d*x)/(
b**2 - d**2) - d*sin(a + b*x)*sin(c + d*x)/(b**2 - d**2), True))
```

Giac [A] time = 1.13548, size = 54, normalized size = 1.26

$$-\frac{\cos(bx + dx + a + c)}{2(b + d)} - \frac{\cos(bx - dx + a - c)}{2(b - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*cos(b*x + d*x + a + c)/(b + d) - 1/2*cos(b*x - d*x + a - c)/(b - d)
```

3.213 $\int \sec(c + bx)\sin(a + bx)\,dx$

Optimal. Leaf size=27

$$x\sin(a-c)-\frac{\cos(a-c)\log(\cos(bx+c))}{b}$$

```
[Out] -((Cos[a - c]*Log[Cos[c + b*x]])/b) + x*Sin[a - c]
```

Rubi [A] time = 0.0175518, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4580, 3475, 8}

$$x\sin(a-c)-\frac{\cos(a-c)\log(\cos(bx+c))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + b*x]*Sin[a + b*x],x]
```

```
[Out] -((Cos[a - c]*Log[Cos[c + b*x]])/b) + x*Sin[a - c]
```

Rule 4580

```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Cos[v - w], Int[Tan[w]*Sec[w]^(n - 1), x], x] + Dist[Sin[v - w], Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w, x] && NeQ[w, v]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sec(c+bx)\sin(a+bx)\,dx &= \cos(a-c)\int \tan(c+bx)\,dx+\sin(a-c)\int 1\,dx\\ &= -\frac{\cos(a-c)\log(\cos(c+bx))}{b}+x\sin(a-c)\end{aligned}$$

Mathematica [A] time = 0.142382, size = 27, normalized size = 1.

$$x\sin(a-c)-\frac{\cos(a-c)\log(\cos(bx+c))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + b*x]*Sin[a + b*x],x]
```

```
[Out] -((Cos[a - c]*Log[Cos[c + b*x]])/b) + x*Sin[a - c]
```

Maple [B] time = 0.201, size = 563, normalized size = 20.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+c)*sin(b*x+a),x)
```

```
[Out] 1/b/(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^
2)/(sin(a)*cos(c)-cos(a)*sin(c))*ln(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*si
n(a)*cos(c)+cos(a)*cos(c)+sin(a)*sin(c))*cos(a)^2*cos(c)*sin(c)-1/b/(cos(a)
^2*cos(c)^2+cos(a)^2*sin(c)^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)/(sin(a)*
cos(c)-cos(a)*sin(c))*ln(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*sin(a)*cos(c)
+cos(a)*cos(c)+sin(a)*sin(c))*cos(a)*cos(c)^2*sin(a)+1/b/(cos(a)^2*cos(c)^2
+cos(a)^2*sin(c)^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)/(sin(a)*cos(c)-cos(
a)*sin(c))*ln(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*sin(a)*cos(c)+cos(a)*cos
(c)+sin(a)*sin(c))*cos(a)*sin(a)*sin(c)^2-1/b/(cos(a)^2*cos(c)^2+cos(a)^2*s
in(c)^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)/(sin(a)*cos(c)-cos(a)*sin(c))*
ln(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*sin(a)*cos(c)+cos(a)*cos(c)+sin(a)*
sin(c))*cos(c)*sin(a)^2*sin(c)+1/2/b/(cos(c)^2+sin(c)^2)/(cos(a)^2+sin(a)^2
)*ln(1+tan(b*x+a)^2)*cos(a)*cos(c)+1/2/b/(cos(c)^2+sin(c)^2)/(cos(a)^2+sin(
a)^2)*ln(1+tan(b*x+a)^2)*sin(a)*sin(c)-1/b/(cos(c)^2+sin(c)^2)/(cos(a)^2+si
n(a)^2)*cos(a)*sin(c)*arctan(tan(b*x+a))+1/b/(cos(c)^2+sin(c)^2)/(cos(a)^2+
sin(a)^2)*cos(c)*sin(a)*arctan(tan(b*x+a))
```

Maxima [B] time = 1.25903, size = 99, normalized size = 3.67

$$-\frac{2\,bx\sin\left(-a+c\right)+\cos\left(-a+c\right)\log\left(\cos\left(2\,bx\right)^{2}+2\,\cos\left(2\,bx\right)\cos\left(2\,c\right)+\cos\left(2\,c\right)^{2}+\sin\left(2\,bx\right)^{2}-2\,\sin\left(2\,bx\right)\sin\left(2\,c\right)+\cdots\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x*sin(-a + c) + cos(-a + c)*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2
*c) + cos(2*c)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*c) + sin(2*c)^2))/b
```

Fricas [A] time = 0.49795, size = 74, normalized size = 2.74

$$-\frac{bx\sin\left(-a+c\right)+\cos\left(-a+c\right)\log\left(-\cos\left(bx+c\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -(b*x*sin(-a + c) + cos(-a + c)*log(-cos(b*x + c)))/b
```

Sympy [B] time = 169.994, size = 435, normalized size = 16.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)*sin(b*x+a),x)
```

```
[Out] Piecewise((0, Eq(b, 0)), (-x, Eq(c, pi/2)), (x, Eq(c, -pi/2)), (-2*b*x*tan(
c/2)/(b*tan(c/2)**2 + b) - log(tan(b*x/2)**2 + 1)*tan(c/2)**2/(b*tan(c/2)**
2 + b) + log(tan(b*x/2)**2 + 1)/(b*tan(c/2)**2 + b) + log(tan(b*x/2) - tan(
c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**2/(b*tan(c/2)**2 + b) - l
og(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))/(b*tan(c/2)**2
+ b) + log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2
)**2/(b*tan(c/2)**2 + b) - log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(ta
n(c/2) + 1))/(b*tan(c/2)**2 + b), True))*cos(a) + Piecewise((x/cos(c), Eq(b
, 0)), (-log(sin(b*x))/b, Eq(c, pi/2)), (log(sin(b*x))/b, Eq(c, -pi/2)), (-
b*x*tan(c/2)**2/(b*tan(c/2)**2 + b) + b*x/(b*tan(c/2)**2 + b) + 2*log(tan(b
*x/2)**2 + 1)*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b*x/2) - tan(c/2)/(t
an(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b
*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)/(b*tan(c/2)**2
 + b), True))*sin(a)
```

Giac [B] time = 1.16072, size = 213, normalized size = 7.89

$$\frac{\frac{4\left(\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^{2}+\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}c\right)\right)(bx+c)}{\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}+\tan\left(\frac{1}{2}a\right)^{2}+\tan\left(\frac{1}{2}c\right)^{2}+1}+\frac{\left(\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}-\tan\left(\frac{1}{2}a\right)^{2}+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}c\right)^{2}+1\right)\log(\mathrm{ta}}{\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}+\tan\left(\frac{1}{2}a\right)^{2}+\tan\left(\frac{1}{2}c\right)^{2}+1}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(4*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - ta
n(1/2*c))*(b*x + c)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^
2 + 1) + (tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c
) - tan(1/2*c)^2 + 1)*log(tan(b*x + c)^2 + 1)/(tan(1/2*a)^2*tan(1/2*c)^2 +
tan(1/2*a)^2 + tan(1/2*c)^2 + 1))/b
```

3.214 $\int \sec^2(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=34

$$\frac{\sin(a-c)\tanh^{-1}(\sin(bx+c))}{b}+\frac{\cos(a-c)\sec(bx+c)}{b}$$

```
[Out] (Cos[a - c]*Sec[c + b*x])/b + (ArcTanh[Sin[c + b*x]]*Sin[a - c])/b
```

Rubi [A] time = 0.0268219, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4580, 2606, 8, 3770}

$$\frac{\sin(a-c)\tanh^{-1}(\sin(bx+c))}{b}+\frac{\cos(a-c)\sec(bx+c)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (Cos[a - c]*Sec[c + b*x])/b + (ArcTanh[Sin[c + b*x]]*Sin[a - c])/b
```

Rule 4580

```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Cos[v - w], Int[Tan[w]*Sec[w]^
(n - 1), x], x] + Dist[Sin[v - w], Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+bx)\sin(a+bx)\,dx &= \cos(a-c)\int \sec(c+bx)\tan(c+bx)\,dx+\sin(a-c)\int \sec(c+bx)\,dx\\ &= \frac{\tanh^{-1}(\sin(c+bx))\sin(a-c)}{b}+\frac{\cos(a-c)\,\mathrm{Subst}(\int 1\,dx,x,\sec(c+bx))}{b}\\ &= \frac{\cos(a-c)\sec(c+bx)}{b}+\frac{\tanh^{-1}(\sin(c+bx))\sin(a-c)}{b}\end{aligned}$$

Mathematica [C] time = 0.0914404, size = 88, normalized size = 2.59

$$\frac{\cos(a-c)\sec(bx+c)}{b} - \frac{2i\sin(a-c)\tan^{-1}\left(\frac{(\sin(c)+i\cos(c))\left(\sin(c)\cos\left(\frac{bx}{2}\right)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\sin(c)\cos\left(\frac{bx}{2}\right)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (Cos[a - c]*Sec[c + b*x])/b - ((2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x)
/2]*Sin[c] + Cos[c]*Sin[(b*x)/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Si
n[c])]*Sin[a - c])/b
```

Maple [B] time = 0.424, size = 888, normalized size = 26.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+c)^2*sin(b*x+a),x)
```

```
[Out] -8/b/(-4*cos(a)^2*cos(c)^2-4*cos(a)^2*sin(c)^2-4*cos(c)^2*sin(a)^2-4*sin(a)
^2*sin(c)^2)/(cos(a)*cos(c)*tan(1/2*b*x+1/2*a)^2+sin(a)*sin(c)*tan(1/2*b*x+
1/2*a)^2+2*tan(1/2*b*x+1/2*a)*cos(a)*sin(c)-2*tan(1/2*b*x+1/2*a)*sin(a)*cos
(c)-cos(a)*cos(c)-sin(a)*sin(c))*tan(1/2*b*x+1/2*a)*cos(a)*sin(c)+8/b/(-4*c
os(a)^2*cos(c)^2-4*cos(a)^2*sin(c)^2-4*cos(c)^2*sin(a)^2-4*sin(a)^2*sin(c)^
2)/(cos(a)*cos(c)*tan(1/2*b*x+1/2*a)^2+sin(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+2
*tan(1/2*b*x+1/2*a)*cos(a)*sin(c)-2*tan(1/2*b*x+1/2*a)*sin(a)*cos(c)-cos(a)
*cos(c)-sin(a)*sin(c))*tan(1/2*b*x+1/2*a)*sin(a)*cos(c)+8/b/(-4*cos(a)^2*co
s(c)^2-4*cos(a)^2*sin(c)^2-4*cos(c)^2*sin(a)^2-4*sin(a)^2*sin(c)^2)/(cos(a)
*cos(c)*tan(1/2*b*x+1/2*a)^2+sin(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+2*tan(1/2*b
*x+1/2*a)*cos(a)*sin(c)-2*tan(1/2*b*x+1/2*a)*sin(a)*cos(c)-cos(a)*cos(c)-si
n(a)*sin(c))*cos(a)*cos(c)+8/b/(-4*cos(a)^2*cos(c)^2-4*cos(a)^2*sin(c)^2-4*
cos(c)^2*sin(a)^2-4*sin(a)^2*sin(c)^2)/(cos(a)*cos(c)*tan(1/2*b*x+1/2*a)^2+
sin(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+2*tan(1/2*b*x+1/2*a)*cos(a)*sin(c)-2*tan
(1/2*b*x+1/2*a)*sin(a)*cos(c)-cos(a)*cos(c)-sin(a)*sin(c))*sin(a)*sin(c)-8/
b/(-4*cos(a)^2*cos(c)^2-4*cos(a)^2*sin(c)^2-4*cos(c)^2*sin(a)^2-4*sin(a)^2*
sin(c)^2)/(-cos(a)^2*cos(c)^2-cos(a)^2*sin(c)^2-cos(c)^2*sin(a)^2-sin(a)^2*
sin(c)^2)^(1/2)*arctan(1/2*(2*(cos(a)*cos(c)+sin(a)*sin(c))*tan(1/2*b*x+1/2
*a)+2*cos(a)*sin(c)-2*sin(a)*cos(c))/(-cos(a)^2*cos(c)^2-cos(a)^2*sin(c)^2-
cos(c)^2*sin(a)^2-sin(a)^2*sin(c)^2)^(1/2))*cos(a)*sin(c)+8/b/(-4*cos(a)^2*
cos(c)^2-4*cos(a)^2*sin(c)^2-4*cos(c)^2*sin(a)^2-4*sin(a)^2*sin(c)^2)/(-cos
(a)^2*cos(c)^2-cos(a)^2*sin(c)^2-cos(c)^2*sin(a)^2-sin(a)^2*sin(c)^2)^(1/2)
*arctan(1/2*(2*(cos(a)*cos(c)+sin(a)*sin(c))*tan(1/2*b*x+1/2*a)+2*cos(a)*si
n(c)-2*sin(a)*cos(c))/(-cos(a)^2*cos(c)^2-cos(a)^2*sin(c)^2-cos(c)^2*sin(a)
^2-sin(a)^2*sin(c)^2)^(1/2))*sin(a)*cos(c)
```

Maxima [B] time = 2.04218, size = 522, normalized size = 15.35

$$\frac{2\left(\cos\left(bx+2a\right)+\cos\left(bx+2c\right)\right)\cos\left(2bx+a+2c\right)+2\cos\left(bx+2a\right)\cos\left(a\right)+2\cos\left(bx+2c\right)\cos\left(a\right)+\left(\cos\left(2bx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(cos(b*x + 2*a) + cos(b*x + 2*c))*cos(2*b*x + a + 2*c) + 2*cos(b*x +
 2*a)*cos(a) + 2*cos(b*x + 2*c)*cos(a) + (cos(2*b*x + a + 2*c)^2*sin(-a + c
) + 2*cos(2*b*x + a + 2*c)*cos(a)*sin(-a + c) + sin(2*b*x + a + 2*c)^2*sin(
-a + c) + 2*sin(2*b*x + a + 2*c)*sin(a)*sin(-a + c) + (cos(a)^2 + sin(a)^2)
*sin(-a + c))*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) +
sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 +
cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*si
n(c) + sin(c)^2)) + 2*(sin(b*x + 2*a) + sin(b*x + 2*c))*sin(2*b*x + a + 2*c
) + 2*sin(b*x + 2*a)*sin(a) + 2*sin(b*x + 2*c)*sin(a))/(b*cos(2*b*x + a + 2
*c)^2 + 2*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(2*b*x + a + 2*c)^2 + 2*b*si
n(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b)
```

Fricas [B] time = 0.503744, size = 186, normalized size = 5.47

$$-\frac{\cos\left(bx+c\right)\log\left(\sin\left(bx+c\right)+1\right)\sin\left(-a+c\right)-\cos\left(bx+c\right)\log\left(-\sin\left(bx+c\right)+1\right)\sin\left(-a+c\right)-2\,\cos\left(-a+c\right)}{2\,b\cos\left(bx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(cos(b*x + c)*log(sin(b*x + c) + 1)*sin(-a + c) - cos(b*x + c)*log(-si
n(b*x + c) + 1)*sin(-a + c) - 2*cos(-a + c))/(b*cos(b*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)**2*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17763, size = 335, normalized size = 9.85

$$\frac{2\left(\frac{\left(\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)-\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,a\right)-\tan\left(\frac{1}{2}\,c\right)\right)\log\left(\left|\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,c\right)+1\right|\right)}{\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,a\right)^{2}+\tan\left(\frac{1}{2}\,c\right)^{2}+1}-\frac{\left(\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)-\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,a\right)-\tan\left(\frac{1}{2}\,c\right.}{\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,a\right)^{2}+\tan\left(\frac{1}{2}\,c\right.}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 2*((tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/
2*c))*log(abs(tan(1/2*b*x + 1/2*c) + 1))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1
/2*a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2
*c)^2 + tan(1/2*a) - tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2*c) - 1))/(tan(1/
```

```
2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*a)^2*tan(
1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1)/((tan
(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*(tan(1/2*b*x + 1/
2*c)^2 - 1)))/b
```

3.215 $\int \sec^3(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\sin(a-c)\tan(bx+c)}{b}+\frac{\cos(a-c)\sec^2(bx+c)}{2b}$$

```
[Out] (Cos[a - c]*Sec[c + b*x]^2)/(2*b) + (Sin[a - c]*Tan[c + b*x])/b
```

Rubi [A] time = 0.0367189, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4580, 2606, 30, 3767, 8}

$$\frac{\sin(a-c)\tan(bx+c)}{b}+\frac{\cos(a-c)\sec^2(bx+c)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (Cos[a - c]*Sec[c + b*x]^2)/(2*b) + (Sin[a - c]*Tan[c + b*x])/b
```

Rule 4580

```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Cos[v - w], Int[Tan[w]*Sec[w]^
(n - 1), x], x] + Dist[Sin[v - w], Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec^3(c+bx)\sin(a+bx)\,dx = \cos(a-c)\int \sec^2(c+bx)\tan(c+bx)\,dx + \sin(a-c)\int \sec^2(c+bx)\,dx$$
$$= \frac{\cos(a-c)\,\mathrm{Subst}(\int x\,dx, x, \sec(c+bx))}{b} - \frac{\sin(a-c)\,\mathrm{Subst}(\int 1\,dx, x, -\tan(c+bx))}{b}$$
$$= \frac{\cos(a-c)\sec^2(c+bx)}{2b} + \frac{\sin(a-c)\tan(c+bx)}{b}$$

Mathematica [A] time = 0.173792, size = 34, normalized size = 0.89

$$\frac{\sec(c)\sec^2(bx+c)(\sin(a-c)\sin(2bx+c)+\cos(a))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (Sec[c]*Sec[c + b*x]^2*(Cos[a] + Sin[a - c]*Sin[c + 2*b*x]))/(2*b)
```

Maple [B] time = 0.595, size = 150, normalized size = 4.

$$\frac{1}{b}\left(\frac{-\cos(a)\cos(c)-\sin(a)\sin(c)}{(2\cos(a)\sin(c)-2\sin(a)\cos(c))(\sin(a)\cos(c)-\cos(a)\sin(c))(-\tan(bx+a)\cos(a)\sin(c)+\tan(bx+a)\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+c)^3*sin(b*x+a),x)
```

```
[Out] 1/b*(1/2*(-cos(a)*cos(c)-sin(a)*sin(c))/(cos(a)*sin(c)-sin(a)*cos(c))/(sin(
a)*cos(c)-cos(a)*sin(c))/(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*sin(a)*cos(c
)+cos(a)*cos(c)+sin(a)*sin(c))^2+1/(cos(a)*sin(c)-sin(a)*cos(c))/(sin(a)*co
s(c)-cos(a)*sin(c))/(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*sin(a)*cos(c)+cos
(a)*cos(c)+sin(a)*sin(c)))
```

Maxima [B] time = 1.18113, size = 528, normalized size = 13.89

$$\frac{(2\cos(2bx+2a+2c)+\cos(2a)-\cos(2c))\cos(4bx+a+5c)+2(2\cos(2bx+2a+2c)+\cos(2a)-\cos(2c))}{b\cos(4bx+a+5c)^2+4b\cos(2bx+a+3c)^2+4b\cos(2bx+a+3c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] ((2*cos(2*b*x + 2*a + 2*c) + cos(2*a) - cos(2*c))*cos(4*b*x + a + 5*c) + 2*
(2*cos(2*b*x + 2*a + 2*c) + cos(2*a) - cos(2*c))*cos(2*b*x + a + 3*c) + (co
s(2*a) - cos(2*c))*cos(a + c) + 2*cos(2*b*x + 2*a + 2*c)*cos(a + c) + (2*si
n(2*b*x + 2*a + 2*c) + sin(2*a) - sin(2*c))*sin(4*b*x + a + 5*c) + 2*(2*sin
(2*b*x + 2*a + 2*c) + sin(2*a) - sin(2*c))*sin(2*b*x + a + 3*c) + (sin(2*a)
 - sin(2*c))*sin(a + c) + 2*sin(2*b*x + 2*a + 2*c)*sin(a + c))/(b*cos(4*b*x
 + a + 5*c)^2 + 4*b*cos(2*b*x + a + 3*c)^2 + 4*b*cos(2*b*x + a + 3*c)*cos(a
 + c) + b*cos(a + c)^2 + b*sin(4*b*x + a + 5*c)^2 + 4*b*sin(2*b*x + a + 3*c
)^2 + 4*b*sin(2*b*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 + 2*(2*b*cos(2*b
```

```
*x + a + 3*c) + b*cos(a + c))*cos(4*b*x + a + 5*c) + 2*(2*b*sin(2*b*x + a +
 3*c) + b*sin(a + c))*sin(4*b*x + a + 5*c))
```

Fricas [A] time = 0.490323, size = 109, normalized size = 2.87

$$-\frac{2\,\cos\left(bx+c\right)\sin\left(bx+c\right)\sin\left(-a+c\right)-\cos\left(-a+c\right)}{2\,b\cos\left(bx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*cos(b*x + c)*sin(b*x + c)*sin(-a + c) - cos(-a + c))/(b*cos(b*x + c
)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)**3*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19662, size = 235, normalized size = 6.18

$$\frac{\tan\left(bx+c\right)^{2}\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}-\tan\left(bx+c\right)^{2}\tan\left(\frac{1}{2}\,a\right)^{2}+4\,\tan\left(bx+c\right)^{2}\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)+4\,\tan\left(bx+c\right)\tan\left(\right.}{2\left(\tan\left(\frac{1}{2}\,a\right)^{2}\,\mathrm{t}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(tan(b*x + c)^2*tan(1/2*a)^2*tan(1/2*c)^2 - tan(b*x + c)^2*tan(1/2*a)^2
 + 4*tan(b*x + c)^2*tan(1/2*a)*tan(1/2*c) + 4*tan(b*x + c)*tan(1/2*a)^2*tan
(1/2*c) - tan(b*x + c)^2*tan(1/2*c)^2 - 4*tan(b*x + c)*tan(1/2*a)*tan(1/2*c
)^2 + tan(b*x + c)^2 + 4*tan(b*x + c)*tan(1/2*a) - 4*tan(b*x + c)*tan(1/2*c
))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*b)
```

3.216 $\int \sec^4(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=67

$$\frac{\sin(a-c)\tanh^{-1}(\sin(bx+c))}{2b}+\frac{\cos(a-c)\sec^3(bx+c)}{3b}+\frac{\sin(a-c)\tan(bx+c)\sec(bx+c)}{2b}$$

```
[Out] (Cos[a - c]*Sec[c + b*x]^3)/(3*b) + (ArcTanh[Sin[c + b*x]]*Sin[a - c])/(2*b
) + (Sec[c + b*x]*Sin[a - c]*Tan[c + b*x])/(2*b)
```

Rubi [A] time = 0.0417556, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4580, 2606, 30, 3768, 3770}

$$\frac{\sin(a-c)\tanh^{-1}(\sin(bx+c))}{2b}+\frac{\cos(a-c)\sec^3(bx+c)}{3b}+\frac{\sin(a-c)\tan(bx+c)\sec(bx+c)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + b*x]^4*Sin[a + b*x],x]
```

```
[Out] (Cos[a - c]*Sec[c + b*x]^3)/(3*b) + (ArcTanh[Sin[c + b*x]]*Sin[a - c])/(2*b
) + (Sec[c + b*x]*Sin[a - c]*Tan[c + b*x])/(2*b)
```

Rule 4580

```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Cos[v - w], Int[Tan[w]*Sec[w]^
(n - 1), x], x] + Dist[Sin[v - w], Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^4(c+bx)\sin(a+bx)\,dx = \cos(a-c)\int \sec^3(c+bx)\tan(c+bx)\,dx + \sin(a-c)\int \sec^3(c+bx)\,dx$$
$$= \frac{\sec(c+bx)\sin(a-c)\tan(c+bx)}{2b} + \frac{\cos(a-c)\,\mathrm{Subst}\left(\int x^2\,dx, x, \sec(c+bx)\right)}{b} + \frac{1}{2}\sin(a$$
$$= \frac{\cos(a-c)\sec^3(c+bx)}{3b} + \frac{\tanh^{-1}(\sin(c+bx))\sin(a-c)}{2b} + \frac{\sec(c+bx)\sin(a-c)\tan(c}{2b}$$

Mathematica [A] time = 0.430755, size = 64, normalized size = 0.96

$$\frac{\sec^3(bx+c)(3\sin(a-c)\sin(2(bx+c))+4\cos(a-c))+12\sin(a-c)\tanh^{-1}\left(\cos(c)\tan\left(\frac{bx}{2}\right)+\sin(c)\right)}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + b*x]^4*Sin[a + b*x],x]
```

```
[Out] (12*ArcTanh[Sin[c] + Cos[c]*Tan[(b*x)/2]]*Sin[a - c] + Sec[c + b*x]^3*(4*Co
s[a - c] + 3*Sin[a - c]*Sin[2*(c + b*x)]))/(12*b)
```

Maple [B] time = 2.318, size = 14825, normalized size = 221.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+c)^4*sin(b*x+a),x)
```

```
[Out] result too large to display
```

Maxima [B] time = 2.16094, size = 1922, normalized size = 28.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^4*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/12*(2*(3*cos(5*b*x + 2*a + 4*c) - 3*cos(5*b*x + 6*c) - 8*cos(3*b*x + 2*a
 + 2*c) - 8*cos(3*b*x + 4*c) - 3*cos(b*x + 2*a) + 3*cos(b*x + 2*c))*cos(6*b
*x + a + 6*c) + 6*(3*cos(4*b*x + a + 4*c) + 3*cos(2*b*x + a + 2*c) + cos(a)
)*cos(5*b*x + 2*a + 4*c) - 6*(3*cos(4*b*x + a + 4*c) + 3*cos(2*b*x + a + 2*
c) + cos(a))*cos(5*b*x + 6*c) - 6*(8*cos(3*b*x + 2*a + 2*c) + 8*cos(3*b*x +
 4*c) + 3*cos(b*x + 2*a) - 3*cos(b*x + 2*c))*cos(4*b*x + a + 4*c) - 16*(3*c
os(2*b*x + a + 2*c) + cos(a))*cos(3*b*x + 2*a + 2*c) - 16*(3*cos(2*b*x + a
+ 2*c) + cos(a))*cos(3*b*x + 4*c) - 18*(cos(b*x + 2*a) - cos(b*x + 2*c))*co
s(2*b*x + a + 2*c) - 6*cos(b*x + 2*a)*cos(a) + 6*cos(b*x + 2*c)*cos(a) - 3*
(cos(6*b*x + a + 6*c)^2*sin(-a + c) + 9*cos(4*b*x + a + 4*c)^2*sin(-a + c)
+ 9*cos(2*b*x + a + 2*c)^2*sin(-a + c) + 6*cos(2*b*x + a + 2*c)*cos(a)*sin(
-a + c) + sin(6*b*x + a + 6*c)^2*sin(-a + c) + 9*sin(4*b*x + a + 4*c)^2*sin
(-a + c) + 9*sin(2*b*x + a + 2*c)^2*sin(-a + c) + 6*sin(2*b*x + a + 2*c)*si
```

```
n(a)*sin(-a + c) + 2*(3*cos(4*b*x + a + 4*c)*sin(-a + c) + 3*cos(2*b*x + a
+ 2*c)*sin(-a + c) + cos(a)*sin(-a + c))*cos(6*b*x + a + 6*c) + 6*(3*cos(2*
b*x + a + 2*c)*sin(-a + c) + cos(a)*sin(-a + c))*cos(4*b*x + a + 4*c) + 2*(
3*sin(4*b*x + a + 4*c)*sin(-a + c) + 3*sin(2*b*x + a + 2*c)*sin(-a + c) + s
in(a)*sin(-a + c))*sin(6*b*x + a + 6*c) + 6*(3*sin(2*b*x + a + 2*c)*sin(-a
+ c) + sin(a)*sin(-a + c))*sin(4*b*x + a + 4*c) + (cos(a)^2 + sin(a)^2)*sin
(-a + c))*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(
b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(
c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c)
 + sin(c)^2)) + 2*(3*sin(5*b*x + 2*a + 4*c) - 3*sin(5*b*x + 6*c) - 8*sin(3*
b*x + 2*a + 2*c) - 8*sin(3*b*x + 4*c) - 3*sin(b*x + 2*a) + 3*sin(b*x + 2*c)
)*sin(6*b*x + a + 6*c) + 6*(3*sin(4*b*x + a + 4*c) + 3*sin(2*b*x + a + 2*c)
 + sin(a))*sin(5*b*x + 2*a + 4*c) - 6*(3*sin(4*b*x + a + 4*c) + 3*sin(2*b*x
 + a + 2*c) + sin(a))*sin(5*b*x + 6*c) - 6*(8*sin(3*b*x + 2*a + 2*c) + 8*si
n(3*b*x + 4*c) + 3*sin(b*x + 2*a) - 3*sin(b*x + 2*c))*sin(4*b*x + a + 4*c)
- 16*(3*sin(2*b*x + a + 2*c) + sin(a))*sin(3*b*x + 2*a + 2*c) - 16*(3*sin(2
*b*x + a + 2*c) + sin(a))*sin(3*b*x + 4*c) - 18*(sin(b*x + 2*a) - sin(b*x +
 2*c))*sin(2*b*x + a + 2*c) - 6*sin(b*x + 2*a)*sin(a) + 6*sin(b*x + 2*c)*si
n(a))/(b*cos(6*b*x + a + 6*c)^2 + 9*b*cos(4*b*x + a + 4*c)^2 + 9*b*cos(2*b*
x + a + 2*c)^2 + 6*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(6*b*x + a + 6*c)^2
 + 9*b*sin(4*b*x + a + 4*c)^2 + 9*b*sin(2*b*x + a + 2*c)^2 + 6*b*sin(2*b*x
+ a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b + 2*(3*b*cos(4*b*x + a + 4*c) +
 3*b*cos(2*b*x + a + 2*c) + b*cos(a))*cos(6*b*x + a + 6*c) + 6*(3*b*cos(2*b
*x + a + 2*c) + b*cos(a))*cos(4*b*x + a + 4*c) + 2*(3*b*sin(4*b*x + a + 4*c
) + 3*b*sin(2*b*x + a + 2*c) + b*sin(a))*sin(6*b*x + a + 6*c) + 6*(3*b*sin(
2*b*x + a + 2*c) + b*sin(a))*sin(4*b*x + a + 4*c))
```

Fricas [A] time = 0.5273, size = 258, normalized size = 3.85

$$-\frac{3\,\cos\left(bx+c\right)^{3}\log\left(\sin\left(bx+c\right)+1\right)\sin\left(-a+c\right)-3\,\cos\left(bx+c\right)^{3}\log\left(-\sin\left(bx+c\right)+1\right)\sin\left(-a+c\right)+6\,\cos\left(bx+\right.}{12\,b\cos\left(bx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^4*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/12*(3*cos(b*x + c)^3*log(sin(b*x + c) + 1)*sin(-a + c) - 3*cos(b*x + c)^
3*log(-sin(b*x + c) + 1)*sin(-a + c) + 6*cos(b*x + c)*sin(b*x + c)*sin(-a +
 c) - 4*cos(-a + c))/(b*cos(b*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)**4*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20114, size = 668, normalized size = 9.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^4*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/3*(3*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - ta
n(1/2*c))*log(abs(tan(1/2*b*x + 1/2*c) + 1))/(tan(1/2*a)^2*tan(1/2*c)^2 + t
an(1/2*a)^2 + tan(1/2*c)^2 + 1) - 3*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*t
an(1/2*c)^2 + tan(1/2*a) - tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2*c) - 1))/(
tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + 2*(3*tan(1/2
*b*x + 1/2*c)^5*tan(1/2*a)^2*tan(1/2*c) - 3*tan(1/2*b*x + 1/2*c)^5*tan(1/2*
a)*tan(1/2*c)^2 - 3*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^2*tan(1/2*c)^2 + 3*ta
n(1/2*b*x + 1/2*c)^5*tan(1/2*a) + 3*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^2 - 3
*tan(1/2*b*x + 1/2*c)^5*tan(1/2*c) - 12*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)*t
an(1/2*c) + 3*tan(1/2*b*x + 1/2*c)^4*tan(1/2*c)^2 - 3*tan(1/2*b*x + 1/2*c)^
4 - 3*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2*tan(1/2*c) + 3*tan(1/2*b*x + 1/2*c)
*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^2 - 3*tan(1/2*b*x + 1/2*
c)*tan(1/2*a) + tan(1/2*a)^2 + 3*tan(1/2*b*x + 1/2*c)*tan(1/2*c) - 4*tan(1/
2*a)*tan(1/2*c) + tan(1/2*c)^2 - 1)/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a
)^2 + tan(1/2*c)^2 + 1)*(tan(1/2*b*x + 1/2*c)^2 - 1)^3))/b
```

3.217 $\int \sec^5(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=59

$$\frac{\sin(a-c)\tan^3(bx+c)}{3b}+\frac{\sin(a-c)\tan(bx+c)}{b}+\frac{\cos(a-c)\sec^4(bx+c)}{4b}$$

```
[Out] (Cos[a - c]*Sec[c + b*x]^4)/(4*b) + (Sin[a - c]*Tan[c + b*x])/b + (Sin[a - c]*Tan[c + b*x]^3)/(3*b)
```

Rubi [A] time = 0.0513425, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4580, 2606, 30, 3767}

$$\frac{\sin(a-c)\tan^3(bx+c)}{3b}+\frac{\sin(a-c)\tan(bx+c)}{b}+\frac{\cos(a-c)\sec^4(bx+c)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + b*x]^5*Sin[a + b*x],x]
```

```
[Out] (Cos[a - c]*Sec[c + b*x]^4)/(4*b) + (Sin[a - c]*Tan[c + b*x])/b + (Sin[a - c]*Tan[c + b*x]^3)/(3*b)
```

Rule 4580

```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Cos[v - w], Int[Tan[w]*Sec[w]^(n - 1), x], x] + Dist[Sin[v - w], Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w, x] && NeQ[w, v]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^5(c+bx)\sin(a+bx)\,dx &= \cos(a-c)\int \sec^4(c+bx)\tan(c+bx)\,dx+\sin(a-c)\int \sec^4(c+bx)\,dx\\ &= \frac{\cos(a-c)\,\text{Subst}\left(\int x^3\,dx,x,\sec(c+bx)\right)}{b}-\frac{\sin(a-c)\,\text{Subst}\left(\int \left(1+x^2\right)dx,x,-\tan\right.}{b}\\ &= \frac{\cos(a-c)\sec^4(c+bx)}{4b}+\frac{\sin(a-c)\tan(c+bx)}{b}+\frac{\sin(a-c)\tan^3(c+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.338964, size = 48, normalized size = 0.81

$$\frac{\sec(c)\sec^4(bx+c)(\sin(a-c)(4\sin(2bx+c)+\sin(4bx+3c))+3\cos(a))}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + b*x]^5*Sin[a + b*x],x]
```

```
[Out] (Sec[c]*Sec[c + b*x]^4*(3*Cos[a] + Sin[a - c]*(4*Sin[c + 2*b*x] + Sin[3*c +
 4*b*x])))/(12*b)
```

Maple [B] time = 1.819, size = 381, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+c)^5*sin(b*x+a),x)
```

```
[Out] 1/b*(1/2*(-3*cos(a)*cos(c)-3*sin(a)*sin(c))/(cos(a)*sin(c)-sin(a)*cos(c))^3
/(sin(a)*cos(c)-cos(a)*sin(c))/(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*sin(a)
*cos(c)+cos(a)*cos(c)+sin(a)*sin(c))^2-1/3*(-3*cos(a)^2*cos(c)^2-cos(a)^2*s
in(c)^2-4*cos(a)*cos(c)*sin(a)*sin(c)-cos(c)^2*sin(a)^2-3*sin(a)^2*sin(c)^2
)/(cos(a)*sin(c)-sin(a)*cos(c))^3/(sin(a)*cos(c)-cos(a)*sin(c))/(-tan(b*x+a
)*cos(a)*sin(c)+tan(b*x+a)*sin(a)*cos(c)+cos(a)*cos(c)+sin(a)*sin(c))^3+1/(
cos(a)*sin(c)-sin(a)*cos(c))^3/(sin(a)*cos(c)-cos(a)*sin(c))/(-tan(b*x+a)*c
os(a)*sin(c)+tan(b*x+a)*sin(a)*cos(c)+cos(a)*cos(c)+sin(a)*sin(c))-1/4*(cos
(a)*cos(c)+sin(a)*sin(c))*(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^2+cos(c)^2*sin
(a)^2+sin(a)^2*sin(c)^2)/(cos(a)*sin(c)-sin(a)*cos(c))^3/(sin(a)*cos(c)-cos
(a)*sin(c))/(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*sin(a)*cos(c)+cos(a)*cos(
c)+sin(a)*sin(c))^4)
```

Maxima [B] time = 1.19081, size = 1450, normalized size = 24.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^5*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 2/3*((6*cos(4*b*x + 2*a + 4*c) + 4*cos(2*b*x + 2*a + 2*c) - 4*cos(2*b*x + 4
*c) + cos(2*a) - cos(2*c))*cos(8*b*x + a + 9*c) + 4*(6*cos(4*b*x + 2*a + 4*
c) + 4*cos(2*b*x + 2*a + 2*c) - 4*cos(2*b*x + 4*c) + cos(2*a) - cos(2*c))*c
os(6*b*x + a + 7*c) + 6*(4*cos(2*b*x + a + 3*c) + cos(a + c))*cos(4*b*x + 2
*a + 4*c) + 6*(6*cos(4*b*x + 2*a + 4*c) + 4*cos(2*b*x + 2*a + 2*c) - 4*cos(
2*b*x + 4*c) + cos(2*a) - cos(2*c))*cos(4*b*x + a + 5*c) + 4*(4*cos(2*b*x +
 2*a + 2*c) + cos(2*a) - cos(2*c))*cos(2*b*x + a + 3*c) - 4*(4*cos(2*b*x +
a + 3*c) + cos(a + c))*cos(2*b*x + 4*c) + (cos(2*a) - cos(2*c))*cos(a + c)
+ 4*cos(2*b*x + 2*a + 2*c)*cos(a + c) + (6*sin(4*b*x + 2*a + 4*c) + 4*sin(2
*b*x + 2*a + 2*c) - 4*sin(2*b*x + 4*c) + sin(2*a) - sin(2*c))*sin(8*b*x + a
 + 9*c) + 4*(6*sin(4*b*x + 2*a + 4*c) + 4*sin(2*b*x + 2*a + 2*c) - 4*sin(2*
b*x + 4*c) + sin(2*a) - sin(2*c))*sin(6*b*x + a + 7*c) + 6*(4*sin(2*b*x + a
 + 3*c) + sin(a + c))*sin(4*b*x + 2*a + 4*c) + 6*(6*sin(4*b*x + 2*a + 4*c)
```

```
+ 4*sin(2*b*x + 2*a + 2*c) - 4*sin(2*b*x + 4*c) + sin(2*a) - sin(2*c))*sin(
4*b*x + a + 5*c) + 4*(4*sin(2*b*x + 2*a + 2*c) + sin(2*a) - sin(2*c))*sin(2
*b*x + a + 3*c) - 4*(4*sin(2*b*x + a + 3*c) + sin(a + c))*sin(2*b*x + 4*c)
+ (sin(2*a) - sin(2*c))*sin(a + c) + 4*sin(2*b*x + 2*a + 2*c)*sin(a + c))/(
b*cos(8*b*x + a + 9*c)^2 + 16*b*cos(6*b*x + a + 7*c)^2 + 36*b*cos(4*b*x + a
 + 5*c)^2 + 16*b*cos(2*b*x + a + 3*c)^2 + 8*b*cos(2*b*x + a + 3*c)*cos(a +
c) + b*cos(a + c)^2 + b*sin(8*b*x + a + 9*c)^2 + 16*b*sin(6*b*x + a + 7*c)^
2 + 36*b*sin(4*b*x + a + 5*c)^2 + 16*b*sin(2*b*x + a + 3*c)^2 + 8*b*sin(2*b
*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 + 2*(4*b*cos(6*b*x + a + 7*c) + 6
*b*cos(4*b*x + a + 5*c) + 4*b*cos(2*b*x + a + 3*c) + b*cos(a + c))*cos(8*b*
x + a + 9*c) + 8*(6*b*cos(4*b*x + a + 5*c) + 4*b*cos(2*b*x + a + 3*c) + b*c
os(a + c))*cos(6*b*x + a + 7*c) + 12*(4*b*cos(2*b*x + a + 3*c) + b*cos(a +
c))*cos(4*b*x + a + 5*c) + 2*(4*b*sin(6*b*x + a + 7*c) + 6*b*sin(4*b*x + a
+ 5*c) + 4*b*sin(2*b*x + a + 3*c) + b*sin(a + c))*sin(8*b*x + a + 9*c) + 8*
(6*b*sin(4*b*x + a + 5*c) + 4*b*sin(2*b*x + a + 3*c) + b*sin(a + c))*sin(6*
b*x + a + 7*c) + 12*(4*b*sin(2*b*x + a + 3*c) + b*sin(a + c))*sin(4*b*x + a
 + 5*c))
```

Fricas [A] time = 0.495206, size = 142, normalized size = 2.41

$$-\frac{4\left(2\cos\left(bx+c\right)^{3}+\cos\left(bx+c\right)\right)\sin\left(bx+c\right)\sin\left(-a+c\right)-3\cos\left(-a+c\right)}{12\,b\cos\left(bx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^5*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/12*(4*(2*cos(b*x + c)^3 + cos(b*x + c))*sin(b*x + c)*sin(-a + c) - 3*cos
(-a + c))/(b*cos(b*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)**5*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24585, size = 441, normalized size = 7.47

$$\frac{3\tan\left(bx+c\right)^{4}\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}-3\tan\left(bx+c\right)^{4}\tan\left(\frac{1}{2}a\right)^{2}+12\tan\left(bx+c\right)^{4}\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)+8\tan(bx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^5*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/12*(3*tan(b*x + c)^4*tan(1/2*a)^2*tan(1/2*c)^2 - 3*tan(b*x + c)^4*tan(1/2
*a)^2 + 12*tan(b*x + c)^4*tan(1/2*a)*tan(1/2*c) + 8*tan(b*x + c)^3*tan(1/2*
a)^2*tan(1/2*c) - 3*tan(b*x + c)^4*tan(1/2*c)^2 - 8*tan(b*x + c)^3*tan(1/2*
a)*tan(1/2*c)^2 + 6*tan(b*x + c)^2*tan(1/2*a)^2*tan(1/2*c)^2 + 3*tan(b*x +
c)^4 + 8*tan(b*x + c)^3*tan(1/2*a) - 6*tan(b*x + c)^2*tan(1/2*a)^2 - 8*tan(
b*x + c)^3*tan(1/2*c) + 24*tan(b*x + c)^2*tan(1/2*a)*tan(1/2*c) + 24*tan(b*
x + c)*tan(1/2*a)^2*tan(1/2*c) - 6*tan(b*x + c)^2*tan(1/2*c)^2 - 24*tan(b*x
 + c)*tan(1/2*a)*tan(1/2*c)^2 + 6*tan(b*x + c)^2 + 24*tan(b*x + c)*tan(1/2*
a) - 24*tan(b*x + c)*tan(1/2*c))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2
 + tan(1/2*c)^2 + 1)*b)
```

3.218 $\int \sec^6(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=94

$$\frac{3\sin(a-c)\tanh^{-1}(\sin(bx+c))}{8b}+\frac{\cos(a-c)\sec^5(bx+c)}{5b}+\frac{\sin(a-c)\tan(bx+c)\sec^3(bx+c)}{4b}+\frac{3\sin(a-c)\tan(bx}{8b}$$

```
[Out] (Cos[a - c]*Sec[c + b*x]^5)/(5*b) + (3*ArcTanh[Sin[c + b*x]]*Sin[a - c])/(8
*b) + (3*Sec[c + b*x]*Sin[a - c]*Tan[c + b*x])/(8*b) + (Sec[c + b*x]^3*Sin[
a - c]*Tan[c + b*x])/(4*b)
```

Rubi [A] time = 0.0703124, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4580, 2606, 30, 3768, 3770}

$$\frac{3\sin(a-c)\tanh^{-1}(\sin(bx+c))}{8b}+\frac{\cos(a-c)\sec^5(bx+c)}{5b}+\frac{\sin(a-c)\tan(bx+c)\sec^3(bx+c)}{4b}+\frac{3\sin(a-c)\tan(bx}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + b*x]^6*Sin[a + b*x],x]
```

```
[Out] (Cos[a - c]*Sec[c + b*x]^5)/(5*b) + (3*ArcTanh[Sin[c + b*x]]*Sin[a - c])/(8
*b) + (3*Sec[c + b*x]*Sin[a - c]*Tan[c + b*x])/(8*b) + (Sec[c + b*x]^3*Sin[
a - c]*Tan[c + b*x])/(4*b)
```

Rule 4580

```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Cos[v - w], Int[Tan[w]*Sec[w]^
(n - 1), x], x] + Dist[Sin[v - w], Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^6(c+bx)\sin(a+bx)\,dx = \cos(a-c)\int \sec^5(c+bx)\tan(c+bx)\,dx + \sin(a-c)\int \sec^5(c+bx)\,dx$$
$$= \frac{\sec^3(c+bx)\sin(a-c)\tan(c+bx)}{4b} + \frac{\cos(a-c)\,\mathrm{Subst}\left(\int x^4\,dx, x, \sec(c+bx)\right)}{b} + \frac{1}{4}(3\mathrm{si}$$
$$= \frac{\cos(a-c)\sec^5(c+bx)}{5b} + \frac{3\sec(c+bx)\sin(a-c)\tan(c+bx)}{8b} + \frac{\sec^3(c+bx)\sin(a-c)\,\mathrm{t}}{4b}$$
$$= \frac{\cos(a-c)\sec^5(c+bx)}{5b} + \frac{3\tanh^{-1}(\sin(c+bx))\sin(a-c)}{8b} + \frac{3\sec(c+bx)\sin(a-c)\,\mathrm{tan}}{8b}$$

Mathematica [A] time = 1.00265, size = 78, normalized size = 0.83

$$\frac{2\sec^5(bx+c)(5\sin(a-c)(14\sin(2(bx+c))+3\sin(4(bx+c)))+64\cos(a-c))+480\sin(a-c)\tanh^{-1}\left(\cos(c)\tan\left(\frac{bx}{2}\right)\right.}{640b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + b*x]^6*Sin[a + b*x],x]
```

```
[Out] (480*ArcTanh[Sin[c] + Cos[c]*Tan[(b*x)/2]]*Sin[a - c] + 2*Sec[c + b*x]^5*(6
4*Cos[a - c] + 5*Sin[a - c]*(14*Sin[2*(c + b*x)] + 3*Sin[4*(c + b*x)])))/(6
40*b)
```

Maple [B] time = 9.186, size = 97703, normalized size = 1039.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+c)^6*sin(b*x+a),x)
```

```
[Out] result too large to display
```

Maxima [B] time = 2.6606, size = 4180, normalized size = 44.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^6*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/80*(2*(15*cos(9*b*x + 2*a + 8*c) - 15*cos(9*b*x + 10*c) + 70*cos(7*b*x +
 2*a + 6*c) - 70*cos(7*b*x + 8*c) - 128*cos(5*b*x + 2*a + 4*c) - 128*cos(5*
b*x + 6*c) - 70*cos(3*b*x + 2*a + 2*c) + 70*cos(3*b*x + 4*c) - 15*cos(b*x +
 2*a) + 15*cos(b*x + 2*c))*cos(10*b*x + a + 10*c) + 30*(5*cos(8*b*x + a + 8
*c) + 10*cos(6*b*x + a + 6*c) + 10*cos(4*b*x + a + 4*c) + 5*cos(2*b*x + a +
 2*c) + cos(a))*cos(9*b*x + 2*a + 8*c) - 30*(5*cos(8*b*x + a + 8*c) + 10*co
s(6*b*x + a + 6*c) + 10*cos(4*b*x + a + 4*c) + 5*cos(2*b*x + a + 2*c) + cos
(a))*cos(9*b*x + 10*c) + 10*(70*cos(7*b*x + 2*a + 6*c) - 70*cos(7*b*x + 8*c
) - 128*cos(5*b*x + 2*a + 4*c) - 128*cos(5*b*x + 6*c) - 70*cos(3*b*x + 2*a
+ 2*c) + 70*cos(3*b*x + 4*c) - 15*cos(b*x + 2*a) + 15*cos(b*x + 2*c))*cos(8
```

*b*x + a + 8*c) + 140*(10*cos(6*b*x + a + 6*c) + 10*cos(4*b*x + a + 4*c) + 5*cos(2*b*x + a + 2*c) + cos(a))*cos(7*b*x + 2*a + 6*c) - 140*(10*cos(6*b*x + a + 6*c) + 10*cos(4*b*x + a + 4*c) + 5*cos(2*b*x + a + 2*c) + cos(a))*cos(7*b*x + 8*c) - 20*(128*cos(5*b*x + 2*a + 4*c) + 128*cos(5*b*x + 6*c) + 70*cos(3*b*x + 2*a + 2*c) - 70*cos(3*b*x + 4*c) + 15*cos(b*x + 2*a) - 15*cos(b*x + 2*c))*cos(6*b*x + a + 6*c) - 256*(10*cos(4*b*x + a + 4*c) + 5*cos(2*b*x + a + 2*c) + cos(a))*cos(5*b*x + 2*a + 4*c) - 256*(10*cos(4*b*x + a + 4*c) + 5*cos(2*b*x + a + 2*c) + cos(a))*cos(5*b*x + 6*c) - 100*(14*cos(3*b*x + 2*a + 2*c) - 14*cos(3*b*x + 4*c) + 3*cos(b*x + 2*a) - 3*cos(b*x + 2*c))*cos(4*b*x + a + 4*c) - 140*(5*cos(2*b*x + a + 2*c) + cos(a))*cos(3*b*x + 2*a + 2*c) + 140*(5*cos(2*b*x + a + 2*c) + cos(a))*cos(3*b*x + 4*c) - 150*(cos(b*x + 2*a) - cos(b*x + 2*c))*cos(2*b*x + a + 2*c) - 30*cos(b*x + 2*a)*cos(a) + 30*cos(b*x + 2*c)*cos(a) - 15*(cos(10*b*x + a + 10*c)^2*sin(-a + c) + 25*cos(8*b*x + a + 8*c)^2*sin(-a + c) + 100*cos(6*b*x + a + 6*c)^2*sin(-a + c) + 100*cos(4*b*x + a + 4*c)^2*sin(-a + c) + 25*cos(2*b*x + a + 2*c)^2*sin(-a + c) + 10*cos(2*b*x + a + 2*c)*cos(a)*sin(-a + c) + sin(10*b*x + a + 10*c)^2*sin(-a + c) + 25*sin(8*b*x + a + 8*c)^2*sin(-a + c) + 100*sin(6*b*x + a + 6*c)^2*sin(-a + c) + 100*sin(4*b*x + a + 4*c)^2*sin(-a + c) + 25*sin(2*b*x + a + 2*c)^2*sin(-a + c) + 10*sin(2*b*x + a + 2*c)*sin(a)*sin(-a + c) + 2*(5*cos(8*b*x + a + 8*c)*sin(-a + c) + 10*cos(6*b*x + a + 6*c)*sin(-a + c) + 10*cos(4*b*x + a + 4*c)*sin(-a + c) + 5*cos(2*b*x + a + 2*c)*sin(-a + c) + cos(a)*sin(-a + c))*cos(10*b*x + a + 10*c) + 10*(10*cos(6*b*x + a + 6*c)*sin(-a + c) + 10*cos(4*b*x + a + 4*c)*sin(-a + c) + 5*cos(2*b*x + a + 2*c)*sin(-a + c) + cos(a)*sin(-a + c))*cos(8*b*x + a + 8*c) + 20*(10*cos(4*b*x + a + 4*c)*sin(-a + c) + 5*cos(2*b*x + a + 2*c)*sin(-a + c) + cos(a)*sin(-a + c))*cos(6*b*x + a + 6*c) + 20*(5*cos(2*b*x + a + 2*c)*sin(-a + c) + cos(a)*sin(-a + c))*cos(4*b*x + a + 4*c) + 2*(5*sin(8*b*x + a + 8*c)*sin(-a + c) + 10*sin(6*b*x + a + 6*c)*sin(-a + c) + 10*sin(4*b*x + a + 4*c)*sin(-a + c) + 5*sin(2*b*x + a + 2*c)*sin(-a + c) + sin(a)*sin(-a + c))*sin(10*b*x + a + 10*c) + 10*(10*sin(6*b*x + a + 6*c)*sin(-a + c) + 10*sin(4*b*x + a + 4*c)*sin(-a + c) + 5*sin(2*b*x + a + 2*c)*sin(-a + c) + sin(a)*sin(-a + c))*sin(8*b*x + a + 8*c) + 20*(10*sin(4*b*x + a + 4*c)*sin(-a + c) + 5*sin(2*b*x + a + 2*c)*sin(-a + c) + sin(a)*sin(-a + c))*sin(6*b*x + a + 6*c) + 20*(5*sin(2*b*x + a + 2*c)*sin(-a + c) + sin(a)*sin(-a + c))*sin(4*b*x + a + 4*c) + (cos(a)^2 + sin(a)^2)*sin(-a + c))*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)) + 2*(15*sin(9*b*x + 2*a + 8*c) - 15*sin(9*b*x + 10*c) + 70*sin(7*b*x + 2*a + 6*c) - 70*sin(7*b*x + 8*c) - 128*sin(5*b*x + 2*a + 4*c) - 128*sin(5*b*x + 6*c) - 70*sin(3*b*x + 2*a + 2*c) + 70*sin(3*b*x + 4*c) - 15*sin(b*x + 2*a) + 15*sin(b*x + 2*c))*sin(10*b*x + a + 10*c) + 30*(5*sin(8*b*x + a + 8*c) + 10*sin(6*b*x + a + 6*c) + 10*sin(4*b*x + a + 4*c) + 5*sin(2*b*x + a + 2*c) + sin(a))*sin(9*b*x + 2*a + 8*c) - 30*(5*sin(8*b*x + a + 8*c) + 10*sin(6*b*x + a + 6*c) + 10*sin(4*b*x + a + 4*c) + 5*sin(2*b*x + a + 2*c) + sin(a))*sin(9*b*x + 10*c) + 10*(70*sin(7*b*x + 2*a + 6*c) - 70*sin(7*b*x + 8*c) - 128*sin(5*b*x + 2*a + 4*c) - 128*sin(5*b*x + 6*c) - 70*sin(3*b*x + 2*a + 2*c) + 70*sin(3*b*x + 4*c) - 15*sin(b*x + 2*a) + 15*sin(b*x + 2*c))*sin(8*b*x + a + 8*c) + 140*(10*sin(6*b*x + a + 6*c) + 10*sin(4*b*x + a + 4*c) + 5*sin(2*b*x + a + 2*c) + sin(a))*sin(7*b*x + 2*a + 6*c) - 140*(10*sin(6*b*x + a + 6*c) + 10*sin(4*b*x + a + 4*c) + 5*sin(2*b*x + a + 2*c) + sin(a))*sin(7*b*x + 8*c) - 20*(128*sin(5*b*x + 2*a + 4*c) + 128*sin(5*b*x + 6*c) + 70*sin(3*b*x + 2*a + 2*c) - 70*sin(3*b*x + 4*c) + 15*sin(b*x + 2*a) - 15*sin(b*x + 2*c))*sin(6*b*x + a + 6*c) - 256*(10*sin(4*b*x + a + 4*c) + 5*sin(2*b*x + a + 2*c) + sin(a))*sin(5*b*x + 2*a + 4*c) - 256*(10*sin(4*b*x + a + 4*c) + 5*sin(2*b*x + a + 2*c) + sin(a))*sin(5*b*x + 6*c) - 100*(14*sin(3*b*x + 2*a + 2*c) - 14*sin(3*b*x + 4*c) + 3*sin(b*x + 2*a) - 3*sin(b*x + 2*c))*sin(4*b*x + a + 4*c) - 140*(5*sin(2*b*x + a + 2*c) + sin(a))*sin(3*b*x + 2*a + 2*c) + 140*(5*sin(2*b*x + a + 2*c) + sin(a))*sin(3*b*x + 4*c) - 150*(sin(b*x + 2*a) - sin(b*x + 2*c))*sin(2

```
*b*x + a + 2*c) - 30*sin(b*x + 2*a)*sin(a) + 30*sin(b*x + 2*c)*sin(a))/(b*c
os(10*b*x + a + 10*c)^2 + 25*b*cos(8*b*x + a + 8*c)^2 + 100*b*cos(6*b*x + a
 + 6*c)^2 + 100*b*cos(4*b*x + a + 4*c)^2 + 25*b*cos(2*b*x + a + 2*c)^2 + 10
*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(10*b*x + a + 10*c)^2 + 25*b*sin(8*b*
x + a + 8*c)^2 + 100*b*sin(6*b*x + a + 6*c)^2 + 100*b*sin(4*b*x + a + 4*c)^
2 + 25*b*sin(2*b*x + a + 2*c)^2 + 10*b*sin(2*b*x + a + 2*c)*sin(a) + (cos(a
)^2 + sin(a)^2)*b + 2*(5*b*cos(8*b*x + a + 8*c) + 10*b*cos(6*b*x + a + 6*c)
 + 10*b*cos(4*b*x + a + 4*c) + 5*b*cos(2*b*x + a + 2*c) + b*cos(a))*cos(10*
b*x + a + 10*c) + 10*(10*b*cos(6*b*x + a + 6*c) + 10*b*cos(4*b*x + a + 4*c)
 + 5*b*cos(2*b*x + a + 2*c) + b*cos(a))*cos(8*b*x + a + 8*c) + 20*(10*b*cos
(4*b*x + a + 4*c) + 5*b*cos(2*b*x + a + 2*c) + b*cos(a))*cos(6*b*x + a + 6*
c) + 20*(5*b*cos(2*b*x + a + 2*c) + b*cos(a))*cos(4*b*x + a + 4*c) + 2*(5*b
*sin(8*b*x + a + 8*c) + 10*b*sin(6*b*x + a + 6*c) + 10*b*sin(4*b*x + a + 4*
c) + 5*b*sin(2*b*x + a + 2*c) + b*sin(a))*sin(10*b*x + a + 10*c) + 10*(10*b
*sin(6*b*x + a + 6*c) + 10*b*sin(4*b*x + a + 4*c) + 5*b*sin(2*b*x + a + 2*c
) + b*sin(a))*sin(8*b*x + a + 8*c) + 20*(10*b*sin(4*b*x + a + 4*c) + 5*b*si
n(2*b*x + a + 2*c) + b*sin(a))*sin(6*b*x + a + 6*c) + 20*(5*b*sin(2*b*x + a
 + 2*c) + b*sin(a))*sin(4*b*x + a + 4*c))
```

Fricas [A] time = 0.547882, size = 294, normalized size = 3.13

$$-\frac{15\cos(bx+c)^5\log(\sin(bx+c)+1)\sin(-a+c) - 15\cos(bx+c)^5\log(-\sin(bx+c)+1)\sin(-a+c) + 10\left(3\cos(bx\right.}{80\,b\cos(bx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^6*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/80*(15*cos(b*x + c)^5*log(sin(b*x + c) + 1)*sin(-a + c) - 15*cos(b*x + c
)^5*log(-sin(b*x + c) + 1)*sin(-a + c) + 10*(3*cos(b*x + c)^3 + 2*cos(b*x +
 c))*sin(b*x + c)*sin(-a + c) - 16*cos(-a + c))/(b*cos(b*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)**6*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21352, size = 1021, normalized size = 10.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^6*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/20*(15*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) -
tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2*c) + 1))/(tan(1/2*a)^2*tan(1/2*c)^2 +
```

```
 tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - 15*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a
)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2*c) - 1)
)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + 2*(25*tan
(1/2*b*x + 1/2*c)^9*tan(1/2*a)^2*tan(1/2*c) - 25*tan(1/2*b*x + 1/2*c)^9*tan
(1/2*a)*tan(1/2*c)^2 - 20*tan(1/2*b*x + 1/2*c)^8*tan(1/2*a)^2*tan(1/2*c)^2
+ 25*tan(1/2*b*x + 1/2*c)^9*tan(1/2*a) + 20*tan(1/2*b*x + 1/2*c)^8*tan(1/2*
a)^2 - 25*tan(1/2*b*x + 1/2*c)^9*tan(1/2*c) - 80*tan(1/2*b*x + 1/2*c)^8*tan
(1/2*a)*tan(1/2*c) - 10*tan(1/2*b*x + 1/2*c)^7*tan(1/2*a)^2*tan(1/2*c) + 20
*tan(1/2*b*x + 1/2*c)^8*tan(1/2*c)^2 + 10*tan(1/2*b*x + 1/2*c)^7*tan(1/2*a)
*tan(1/2*c)^2 - 20*tan(1/2*b*x + 1/2*c)^8 - 10*tan(1/2*b*x + 1/2*c)^7*tan(1
/2*a) + 10*tan(1/2*b*x + 1/2*c)^7*tan(1/2*c) - 40*tan(1/2*b*x + 1/2*c)^4*ta
n(1/2*a)^2*tan(1/2*c)^2 + 40*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^2 - 160*tan(
1/2*b*x + 1/2*c)^4*tan(1/2*a)*tan(1/2*c) + 10*tan(1/2*b*x + 1/2*c)^3*tan(1/
2*a)^2*tan(1/2*c) + 40*tan(1/2*b*x + 1/2*c)^4*tan(1/2*c)^2 - 10*tan(1/2*b*x
 + 1/2*c)^3*tan(1/2*a)*tan(1/2*c)^2 - 40*tan(1/2*b*x + 1/2*c)^4 + 10*tan(1/
2*b*x + 1/2*c)^3*tan(1/2*a) - 10*tan(1/2*b*x + 1/2*c)^3*tan(1/2*c) - 25*tan
(1/2*b*x + 1/2*c)*tan(1/2*a)^2*tan(1/2*c) + 25*tan(1/2*b*x + 1/2*c)*tan(1/2
*a)*tan(1/2*c)^2 - 4*tan(1/2*a)^2*tan(1/2*c)^2 - 25*tan(1/2*b*x + 1/2*c)*ta
n(1/2*a) + 4*tan(1/2*a)^2 + 25*tan(1/2*b*x + 1/2*c)*tan(1/2*c) - 16*tan(1/2
*a)*tan(1/2*c) + 4*tan(1/2*c)^2 - 4)/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*
a)^2 + tan(1/2*c)^2 + 1)*(tan(1/2*b*x + 1/2*c)^2 - 1)^5))/b
```

3.219 $\int \cos^n(c+dx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=386

$$-\frac{i2^{-n-2}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\left(1+e^{2ic+2idx}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(-\frac{2b}{d}-n\right),-n,\frac{1}{2}\left(-\frac{2b}{d}-n+2\right),-e^{2i(c+dx)}\right)\exp(-i}{2b+dn}$$

```
[Out] ((-I)*2^(-2 - n)*E^((-I)*(2*a + c*n) - I*(2*b + d*n)*x + I*n*(c + d*x))*(E^
((-I)*(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[((-2*b)/d - n)/2, -
n, (2 - (2*b)/d - n)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x
))^n*(2*b + d*n)) + (I*2^(-2 - n)*E^(I*(2*a - c*n) + I*(2*b - d*n)*x + I*n*
(c + d*x))*(E^((-I)*(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[((2*b
)/d - n)/2, -n, (2 + (2*b)/d - n)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*
c + (2*I)*d*x))^n*(2*b - d*n)) + (I*2^(-1 - n)*(E^((-I)*(c + d*x)) + E^(I*(
c + d*x)))^n*Hypergeometric2F1[-n, -n/2, 1 - n/2, -E^((2*I)*(c + d*x))])/(d
*(1 + E^((2*I)*(c + d*x)))^n*n)
```

Rubi [A] time = 0.700175, antiderivative size = 386, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4555, 2282, 2032, 364, 2285, 2253, 2251}

$$-\frac{i2^{-n-2}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\left(1+e^{2ic+2idx}\right)^{-n}{}_2F_1\left(\frac{1}{2}\left(-\frac{2b}{d}-n\right),-n;\frac{1}{2}\left(-\frac{2b}{d}-n+2\right);-e^{2i(c+dx)}\right)\exp(-i(2a+cn)-ix(2b+}{2b+dn}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^n*Sin[a + b*x]^2,x]
```

```
[Out] ((-I)*2^(-2 - n)*E^((-I)*(2*a + c*n) - I*(2*b + d*n)*x + I*n*(c + d*x))*(E^
((-I)*(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[((-2*b)/d - n)/2, -
n, (2 - (2*b)/d - n)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x
))^n*(2*b + d*n)) + (I*2^(-2 - n)*E^(I*(2*a - c*n) + I*(2*b - d*n)*x + I*n*
(c + d*x))*(E^((-I)*(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[((2*b
)/d - n)/2, -n, (2 + (2*b)/d - n)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*
c + (2*I)*d*x))^n*(2*b - d*n)) + (I*2^(-1 - n)*(E^((-I)*(c + d*x)) + E^(I*(
c + d*x)))^n*Hypergeometric2F1[-n, -n/2, 1 - n/2, -E^((2*I)*(c + d*x))])/(d
*(1 + E^((2*I)*(c + d*x)))^n*n)
```

Rule 4555

```
Int[Cos[(c_.) + (d_.)*(x_)]^(q_.)*Sin[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol]
:> Dist[1/2^(p + q), Int[ExpandIntegrand[(E^(-(I*(c + d*x))) + E^(I*(c + d*
x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2285

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Dist[(a*F^
v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n), Int[u*F^(n*v)*(a
+ b*F^ExpandToSum[w - v, x])^n, x], x] /; FreeQ[{F, a, b, n}, x] &&  !Integ
erQ[n] && LinearQ[{v, w}, x]
```

Rule 2253

```
Int[((a_) + (b_.)*(F_)^((e_.)*(v_)))^(p_)*(G_)^((h_.)*(u_)), x_Symbol] :> I
nt[G^(h*ExpandToSum[u, x])*(a + b*F^(e*ExpandToSum[v, x]))^p, x] /; FreeQ[{
F, G, a, b, e, h, p}, x] && LinearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \cos^n(c+dx)\sin^2(a+bx)\,dx = 2^{-2-n}\int \Big(2\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n - e^{-2ia-2ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n - e^{2ia+2ibx}\Big(e^{-i(c+dx)}$$

$$= -\left(2^{-2-n}\int e^{-2ia-2ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\,dx\right) - 2^{-2-n}\int e^{2ia+2ibx}\Big(e^{-i(c+dx)}+e^{i(c+dx)}\Big)$$

$$= -\frac{\left(i2^{-1-n}\right)\text{Subst}\left(\int \frac{\left(\frac{1}{x}+x\right)^n}{x}\,dx, x, e^{i(c+dx)}\right)}{d} - \Big(2^{-2-n}e^{in(c+dx)}\left(1+e^{2ic+2idx}\right)^{-n}\Big(e^{-i(c+dx)}$$

$$= -\Big(\Big(2^{-2-n}e^{in(c+dx)}\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\Big)\int e^{i(2a-cn)+i(2b-dn)x}\Big(1+e^{2ic+}$$

$$= -\frac{i2^{-2-n}\exp(-i(2a+cn)-i(2b+dn)x+in(c+dx))\left(1+e^{2ic+2idx}\right)^{-n}\Big(e^{-i(c+dx)}+e^{i(c-}}{2b+dn}$$

Mathematica [A] time = 1.78569, size = 242, normalized size = 0.63

$$-\frac{i2^{-n-2}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^{n+1}e^{i(c+dx)-2i(a+bx)}\Big(e^{2i(a+bx)}(2b+dn)\Big(dne^{2i(a+bx)}\text{Hypergeometric2F1}\Big(1,\frac{b}{d}+\frac{n}{2}+1,\frac{b}{d}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^n*Sin[a + b*x]^2,x]
```

```
[Out] ((-I)*2^(-2 - n)*E^((-2*I)*(a + b*x) + I*(c + d*x))*((1 + E^((2*I)*(c + d*x
)))/E^(I*(c + d*x)))^(1 + n)*(d*n*(-2*b + d*n)*Hypergeometric2F1[1, 1 - b/d
 + n/2, 1 - b/d - n/2, -E^((2*I)*(c + d*x))] + E^((2*I)*(a + b*x))*(2*b + d
*n)*(d*E^((2*I)*(a + b*x))*n*Hypergeometric2F1[1, 1 + b/d + n/2, 1 + b/d -
n/2, -E^((2*I)*(c + d*x))] + 2*(2*b - d*n)*Hypergeometric2F1[1, (2 + n)/2,
1 - n/2, -E^((2*I)*(c + d*x))])))/(-4*b^2*d*n + d^3*n^3)
```

Maple [F] time = 0.751, size = 0, normalized size = 0.

$$\int (\cos (dx+c))^{n} (\sin (bx+a))^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^n*sin(b*x+a)^2,x)
```

```
[Out] int(cos(d*x+c)^n*sin(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos (dx+c)^{n} \sin (bx+a)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^n*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^n*sin(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(\cos (bx+a)^{2}-1\right)\cos (dx+c)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^n*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*cos(d*x + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**n*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(dx+c)^n \sin(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^n*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^n*sin(b*x + a)^2, x)
```

3.220 $\int \cos(c+dx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=68

$$-\frac{\sin(2a+x(2b-d)-c)}{4(2b-d)}-\frac{\sin(2a+x(2b+d)+c)}{4(2b+d)}+\frac{\sin(c+dx)}{2d}$$

```
[Out] -Sin[2*a - c + (2*b - d)*x]/(4*(2*b - d)) + Sin[c + d*x]/(2*d) - Sin[2*a +
c + (2*b + d)*x]/(4*(2*b + d))
```

Rubi [A] time = 0.0510584, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4574, 2637}

$$-\frac{\sin(2a+x(2b-d)-c)}{4(2b-d)}-\frac{\sin(2a+x(2b+d)+c)}{4(2b+d)}+\frac{\sin(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[a + b*x]^2,x]
```

```
[Out] -Sin[2*a - c + (2*b - d)*x]/(4*(2*b - d)) + Sin[c + d*x]/(2*d) - Sin[2*a +
c + (2*b + d)*x]/(4*(2*b + d))
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sin^2(a+bx)\,dx &= \int \left(-\frac{1}{4}\cos(2a-c+(2b-d)x)+\frac{1}{2}\cos(c+dx)-\frac{1}{4}\cos(2a+c+(2b+d)x)\right)dx\\ &= -\left(\frac{1}{4}\int \cos(2a-c+(2b-d)x)\,dx\right)-\frac{1}{4}\int \cos(2a+c+(2b+d)x)\,dx+\frac{1}{2}\int \cos(c+dx\\ &= -\frac{\sin(2a-c+(2b-d)x)}{4(2b-d)}+\frac{\sin(c+dx)}{2d}-\frac{\sin(2a+c+(2b+d)x)}{4(2b+d)}\end{aligned}$$

Mathematica [A] time = 0.784375, size = 76, normalized size = 1.12

$$\frac{1}{4}\left(-\frac{\sin(2a+2bx-c-dx)}{2b-d}-\frac{\sin(2a+2bx+c+dx)}{2b+d}+\frac{2\sin(c)\cos(dx)}{d}+\frac{2\cos(c)\sin(dx)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((2*Cos[d*x]*Sin[c])/d + (2*Cos[c]*Sin[d*x])/d - Sin[2*a - c + 2*b*x - d*x]
/(2*b - d) - Sin[2*a + c + 2*b*x + d*x]/(2*b + d))/4
```

Maple [A] time = 0.023, size = 63, normalized size = 0.9

$$-\frac{\sin(2a-c+(2b-d)x)}{8b-4d}+\frac{\sin(dx+c)}{2d}-\frac{\sin(2a+c+(2b+d)x)}{8b+4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(b*x+a)^2,x)
```

```
[Out] -1/4*sin(2*a-c+(2*b-d)*x)/(2*b-d)+1/2*sin(d*x+c)/d-1/4*sin(2*a+c+(2*b+d)*x)
/(2*b+d)
```

Maxima [B] time = 1.2133, size = 501, normalized size = 7.37

$$-\frac{\left(2\,bd\sin(c)-d^2\sin(c)\right)\cos((2\,b+d)x+2\,a+2\,c)-\left(2\,bd\sin(c)-d^2\sin(c)\right)\cos((2\,b+d)x+2\,a)-\left(2\,bd\sin(c)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*((2*b*d*sin(c) - d^2*sin(c))*cos((2*b + d)*x + 2*a + 2*c) - (2*b*d*sin
(c) - d^2*sin(c))*cos((2*b + d)*x + 2*a) - (2*b*d*sin(c) + d^2*sin(c))*cos(
-(2*b - d)*x - 2*a + 2*c) + (2*b*d*sin(c) + d^2*sin(c))*cos(-(2*b - d)*x -
2*a) - 2*(4*b^2*sin(c) - d^2*sin(c))*cos(d*x + 2*c) + 2*(4*b^2*sin(c) - d^2
*sin(c))*cos(d*x) - (2*b*d*cos(c) - d^2*cos(c))*sin((2*b + d)*x + 2*a + 2*c
) - (2*b*d*cos(c) - d^2*cos(c))*sin((2*b + d)*x + 2*a) + (2*b*d*cos(c) + d^
2*cos(c))*sin(-(2*b - d)*x - 2*a + 2*c) + (2*b*d*cos(c) + d^2*cos(c))*sin(-
(2*b - d)*x - 2*a) + 2*(4*b^2*cos(c) - d^2*cos(c))*sin(d*x + 2*c) + 2*(4*b^
2*cos(c) - d^2*cos(c))*sin(d*x))/((cos(c)^2 + sin(c)^2)*d^3 - 4*(b^2*cos(c)
^2 + b^2*sin(c)^2)*d)
```

Fricas [A] time = 0.488515, size = 155, normalized size = 2.28

$$-\frac{2\,bd\cos(bx+a)\cos(dx+c)\sin(bx+a)-\left(d^2\cos(bx+a)^2+2\,b^2-d^2\right)\sin(dx+c)}{4\,b^2d-d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*b*d*cos(b*x + a)*cos(d*x + c)*sin(b*x + a) - (d^2*cos(b*x + a)^2 + 2*b^
2 - d^2)*sin(d*x + c))/(4*b^2*d - d^3)
```

Sympy [A] time = 9.75465, size = 410, normalized size = 6.03

$$\begin{cases} x\sin^2(a)\cos(c) \\ \frac{x\sin^2\left(a-\frac{dx}{2}\right)\cos(c+dx)}{4} + \frac{x\sin\left(a-\frac{dx}{2}\right)\sin(c+dx)\cos\left(a-\frac{dx}{2}\right)}{2} - \frac{x\cos^2\left(a-\frac{dx}{2}\right)\cos(c+dx)}{4} + \frac{\sin^2\left(a-\frac{dx}{2}\right)\sin(c+dx)}{d} - \frac{\sin\left(a-\frac{dx}{2}\right)\cos\left(a-\frac{dx}{2}\right)\cos(c+dx)}{2d} \\ \frac{x\sin^2\left(a+\frac{dx}{2}\right)\cos(c+dx)}{4} - \frac{x\sin\left(a+\frac{dx}{2}\right)\sin(c+dx)\cos\left(a+\frac{dx}{2}\right)}{2} - \frac{x\cos^2\left(a+\frac{dx}{2}\right)\cos(c+dx)}{4} - \frac{3\sin\left(a+\frac{dx}{2}\right)\cos\left(a+\frac{dx}{2}\right)\cos(c+dx)}{2d} + \frac{\sin(c+dx)\cos^2\left(a+\right.}{d} \\ \left(\frac{x\sin^2(a+bx)}{2} + \frac{x\cos^2(a+bx)}{2} - \frac{\sin(a+bx)\cos(a+bx)}{2b}\right)\cos(c) \\ \frac{2b^2\sin^2(a+bx)\sin(c+dx)}{4b^2d-d^3} + \frac{2b^2\sin(c+dx)\cos^2(a+bx)}{4b^2d-d^3} - \frac{2bd\sin(a+bx)\cos(a+bx)\cos(c+dx)}{4b^2d-d^3} - \frac{d^2\sin^2(a+bx)\sin(c+dx)}{4b^2d-d^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sin(a)**2*cos(c), Eq(b, 0) & Eq(d, 0)), (x*sin(a - d*x/2)**2*c
os(c + d*x)/4 + x*sin(a - d*x/2)*sin(c + d*x)*cos(a - d*x/2)/2 - x*cos(a -
d*x/2)**2*cos(c + d*x)/4 + sin(a - d*x/2)**2*sin(c + d*x)/d - sin(a - d*x/2
)*cos(a - d*x/2)*cos(c + d*x)/(2*d), Eq(b, -d/2)), (x*sin(a + d*x/2)**2*cos
(c + d*x)/4 - x*sin(a + d*x/2)*sin(c + d*x)*cos(a + d*x/2)/2 - x*cos(a + d*
x/2)**2*cos(c + d*x)/4 - 3*sin(a + d*x/2)*cos(a + d*x/2)*cos(c + d*x)/(2*d)
 + sin(c + d*x)*cos(a + d*x/2)**2/d, Eq(b, d/2)), ((x*sin(a + b*x)**2/2 + x
*cos(a + b*x)**2/2 - sin(a + b*x)*cos(a + b*x)/(2*b))*cos(c), Eq(d, 0)), (2
*b**2*sin(a + b*x)**2*sin(c + d*x)/(4*b**2*d - d**3) + 2*b**2*sin(c + d*x)*
cos(a + b*x)**2/(4*b**2*d - d**3) - 2*b*d*sin(a + b*x)*cos(a + b*x)*cos(c +
 d*x)/(4*b**2*d - d**3) - d**2*sin(a + b*x)**2*sin(c + d*x)/(4*b**2*d - d**
3), True))
```

Giac [A] time = 1.15072, size = 82, normalized size = 1.21

$$-\frac{\sin(2\,bx + dx + 2\,a + c)}{4\,(2\,b + d)} - \frac{\sin(2\,bx - dx + 2\,a - c)}{4\,(2\,b - d)} + \frac{\sin(dx + c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*sin(2*b*x + d*x + 2*a + c)/(2*b + d) - 1/4*sin(2*b*x - d*x + 2*a - c)/
(2*b - d) + 1/2*sin(d*x + c)/d
```

3.221 $\int \cos^2(c+dx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=88

$$-\frac{\sin(2(a-c)+2x(b-d))}{16(b-d)}-\frac{\sin(2(a+c)+2x(b+d))}{16(b+d)}-\frac{\sin(2a+2bx)}{8b}+\frac{\sin(2c+2dx)}{8d}+\frac{x}{4}$$

```
[Out] x/4 - Sin[2*a + 2*b*x]/(8*b) - Sin[2*(a - c) + 2*(b - d)*x]/(16*(b - d)) +
Sin[2*c + 2*d*x]/(8*d) - Sin[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

Rubi [A] time = 0.067894, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4574, 2637}

$$-\frac{\sin(2(a-c)+2x(b-d))}{16(b-d)}-\frac{\sin(2(a+c)+2x(b+d))}{16(b+d)}-\frac{\sin(2a+2bx)}{8b}+\frac{\sin(2c+2dx)}{8d}+\frac{x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] x/4 - Sin[2*a + 2*b*x]/(8*b) - Sin[2*(a - c) + 2*(b - d)*x]/(16*(b - d)) +
Sin[2*c + 2*d*x]/(8*d) - Sin[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)\sin^2(a+bx)\,dx &= \int \left(\frac{1}{4}-\frac{1}{4}\cos(2a+2bx)-\frac{1}{8}\cos(2(a-c)+2(b-d)x)+\frac{1}{4}\cos(2c+2dx)-\frac{1}{8}\cos(2\right.\\ &= \frac{x}{4}-\frac{1}{8}\int \cos(2(a-c)+2(b-d)x)\,dx-\frac{1}{8}\int \cos(2(a+c)+2(b+d)x)\,dx-\frac{1}{4}\int \cos\\ &= \frac{x}{4}-\frac{\sin(2a+2bx)}{8b}-\frac{\sin(2(a-c)+2(b-d)x)}{16(b-d)}+\frac{\sin(2c+2dx)}{8d}-\frac{\sin(2(a+c)+2(b}{16(b+d)}\end{aligned}$$

Mathematica [A] time = 0.781524, size = 108, normalized size = 1.23

$$\frac{\left(2d^3-2b^2d\right)\sin(2(a+bx))-bd(b+d)\sin(2(a+x(b-d)-c))+b(b-d)(-d\sin(2(a+x(b+d)+c))+2(b+d)\sin(2}{16bd(b-d)(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] ((-2*b^2*d + 2*d^3)*Sin[2*(a + b*x)] - b*d*(b + d)*Sin[2*(a - c + (b - d)*x
)] + b*(b - d)*(4*d*(b + d)*x + 2*(b + d)*Sin[2*(c + d*x)] - d*Sin[2*(a + c
 + (b + d)*x)]))/(16*b*(b - d)*d*(b + d))
```

Maple [A] time = 0.028, size = 83, normalized size = 0.9

$$\frac{x}{4}-\frac{\sin(2bx+2a)}{8b}+\frac{\sin(2dx+2c)}{8d}-\frac{\sin((2b-2d)x-2c+2a)}{16b-16d}-\frac{\sin((2b+2d)x+2a+2c)}{16b+16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(b*x+a)^2,x)
```

```
[Out] 1/4*x-1/8*sin(2*b*x+2*a)/b+1/8*sin(2*d*x+2*c)/d-1/16/(b-d)*sin((2*b-2*d)*x-
2*c+2*a)-1/16/(b+d)*sin((2*b+2*d)*x+2*a+2*c)
```

Maxima [B] time = 1.21011, size = 837, normalized size = 9.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(8*((b*cos(2*c)^2 + b*sin(2*c)^2)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c)
^2)*d)*x - (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a + 4*c) +
 (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a) + (b^2*d*sin(2*c)
 + b*d^2*sin(2*c))*cos(-2*(b - d)*x - 2*a + 4*c) - (b^2*d*sin(2*c) + b*d^2*
sin(2*c))*cos(-2*(b - d)*x - 2*a) - 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*cos(2
*b*x + 2*a + 2*c) + 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*cos(2*b*x + 2*a - 2*c
) - 2*(b^3*sin(2*c) - b*d^2*sin(2*c))*cos(2*d*x) + 2*(b^3*sin(2*c) - b*d^2*
sin(2*c))*cos(2*d*x + 4*c) + (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(2*(b + d
)*x + 2*a + 4*c) + (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(2*(b + d)*x + 2*a)
 - (b^2*d*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a + 4*c) - (b^2*d
*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a) + 2*(b^2*d*cos(2*c) - d
^3*cos(2*c))*sin(2*b*x + 2*a + 2*c) + 2*(b^2*d*cos(2*c) - d^3*cos(2*c))*sin
(2*b*x + 2*a - 2*c) - 2*(b^3*cos(2*c) - b*d^2*cos(2*c))*sin(2*d*x) - 2*(b^3
*cos(2*c) - b*d^2*cos(2*c))*sin(2*d*x + 4*c))/((b*cos(2*c)^2 + b*sin(2*c)^2
)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c)^2)*d)
```

Fricas [A] time = 0.508512, size = 246, normalized size = 2.8

$$\frac{\left(2\,bd^2\cos(bx+a)^2+b^3-2\,bd^2\right)\cos(dx+c)\sin(dx+c)+\left(b^3d-bd^3\right)x-\left(2\,b^2d\cos(bx+a)\cos(dx+c)^2-d^3\cos(bx\right.}{4\left(b^3d-bd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*b*d^2*cos(b*x + a)^2 + b^3 - 2*b*d^2)*cos(d*x + c)*sin(d*x + c) + (
b^3*d - b*d^3)*x - (2*b^2*d*cos(b*x + a)*cos(d*x + c)^2 - d^3*cos(b*x + a))
```

```
*sin(b*x + a))/(b^3*d - b*d^3)
```

Sympy [A] time = 151.378, size = 1027, normalized size = 11.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sin(a)**2*cos(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**2
/2 + x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(2*d))*sin(a)**2, Eq(b
, 0)), (x*sin(a - d*x)**2*sin(c + d*x)**2/8 + 3*x*sin(a - d*x)**2*cos(c + d
*x)**2/8 + x*sin(a - d*x)*sin(c + d*x)*cos(a - d*x)*cos(c + d*x)/2 + 3*x*si
n(c + d*x)**2*cos(a - d*x)**2/8 + x*cos(a - d*x)**2*cos(c + d*x)**2/8 + sin
(a - d*x)**2*sin(c + d*x)*cos(c + d*x)/(8*d) + sin(a - d*x)*cos(a - d*x)*co
s(c + d*x)**2/(2*d) + 3*sin(c + d*x)*cos(a - d*x)**2*cos(c + d*x)/(8*d), Eq
(b, -d)), (x*sin(a + d*x)**2*sin(c + d*x)**2/8 + 3*x*sin(a + d*x)**2*cos(c
+ d*x)**2/8 - x*sin(a + d*x)*sin(c + d*x)*cos(a + d*x)*cos(c + d*x)/2 + 3*x
*sin(c + d*x)**2*cos(a + d*x)**2/8 + x*cos(a + d*x)**2*cos(c + d*x)**2/8 +
sin(a + d*x)**2*sin(c + d*x)*cos(c + d*x)/(8*d) - sin(a + d*x)*cos(a + d*x)
*cos(c + d*x)**2/(2*d) + 3*sin(c + d*x)*cos(a + d*x)**2*cos(c + d*x)/(8*d),
 Eq(b, d)), ((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 - sin(a + b*x)*cos(
a + b*x)/(2*b))*cos(c)**2, Eq(d, 0)), (b**3*d*x*sin(a + b*x)**2*sin(c + d*x
)**2/(4*b**3*d - 4*b*d**3) + b**3*d*x*sin(a + b*x)**2*cos(c + d*x)**2/(4*b*
*3*d - 4*b*d**3) + b**3*d*x*sin(c + d*x)**2*cos(a + b*x)**2/(4*b**3*d - 4*b
*d**3) + b**3*d*x*cos(a + b*x)**2*cos(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b
**3*sin(a + b*x)**2*sin(c + d*x)*cos(c + d*x)/(4*b**3*d - 4*b*d**3) + b**3*
sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x)/(4*b**3*d - 4*b*d**3) - 2*b**2*d*
sin(a + b*x)*cos(a + b*x)*cos(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*
sin(a + b*x)**2*sin(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*sin(a + b*
x)**2*cos(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*sin(c + d*x)**2*cos(
a + b*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*cos(a + b*x)**2*cos(c + d*x)**
2/(4*b**3*d - 4*b*d**3) - 2*b*d**2*sin(a + b*x)**2*sin(c + d*x)*cos(c + d*x
)/(4*b**3*d - 4*b*d**3) + d**3*sin(a + b*x)*sin(c + d*x)**2*cos(a + b*x)/(4
*b**3*d - 4*b*d**3) + d**3*sin(a + b*x)*cos(a + b*x)*cos(c + d*x)**2/(4*b**
3*d - 4*b*d**3), True))
```

Giac [A] time = 1.12695, size = 108, normalized size = 1.23

$$\frac{1}{4}x - \frac{\sin\left(2\,bx + 2\,dx + 2\,a + 2\,c\right)}{16\,(b+d)} - \frac{\sin\left(2\,bx - 2\,dx + 2\,a - 2\,c\right)}{16\,(b-d)} - \frac{\sin\left(2\,bx + 2\,a\right)}{8\,b} + \frac{\sin\left(2\,dx + 2\,c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*x - 1/16*sin(2*b*x + 2*d*x + 2*a + 2*c)/(b + d) - 1/16*sin(2*b*x - 2*d*
x + 2*a - 2*c)/(b - d) - 1/8*sin(2*b*x + 2*a)/b + 1/8*sin(2*d*x + 2*c)/d
```

3.222 $\int \cos^3(c+dx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=144

$$-\frac{\sin(2a+x(2b-3d)-3c)}{16(2b-3d)}-\frac{3\sin(2a+x(2b-d)-c)}{16(2b-d)}-\frac{3\sin(2a+x(2b+d)+c)}{16(2b+d)}-\frac{\sin(2a+x(2b+3d)+3c)}{16(2b+3d)}+\frac{3\sin(\ldots}{8\ldots}$$

```
[Out] -Sin[2*a - 3*c + (2*b - 3*d)*x]/(16*(2*b - 3*d)) - (3*Sin[2*a - c + (2*b -
d)*x])/(16*(2*b - d)) + (3*Sin[c + d*x])/(8*d) + Sin[3*c + 3*d*x]/(24*d) -
(3*Sin[2*a + c + (2*b + d)*x])/(16*(2*b + d)) - Sin[2*a + 3*c + (2*b + 3*d)
*x]/(16*(2*b + 3*d))
```

Rubi [A] time = 0.0957972, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4574, 2637}

$$-\frac{\sin(2a+x(2b-3d)-3c)}{16(2b-3d)}-\frac{3\sin(2a+x(2b-d)-c)}{16(2b-d)}-\frac{3\sin(2a+x(2b+d)+c)}{16(2b+d)}-\frac{\sin(2a+x(2b+3d)+3c)}{16(2b+3d)}+\frac{3\sin(\ldots}{8\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] -Sin[2*a - 3*c + (2*b - 3*d)*x]/(16*(2*b - 3*d)) - (3*Sin[2*a - c + (2*b -
d)*x])/(16*(2*b - d)) + (3*Sin[c + d*x])/(8*d) + Sin[3*c + 3*d*x]/(24*d) -
(3*Sin[2*a + c + (2*b + d)*x])/(16*(2*b + d)) - Sin[2*a + 3*c + (2*b + 3*d)
*x]/(16*(2*b + 3*d))
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)\sin^2(a+bx)\,dx &= \int \left(-\frac{1}{16}\cos(2a-3c+(2b-3d)x)-\frac{3}{16}\cos(2a-c+(2b-d)x)+\frac{3}{8}\cos(c+dx)+\frac{1}{8}\cos\ldots\right.\\ &= -\left(\frac{1}{16}\int \cos(2a-3c+(2b-3d)x)\,dx\right)-\frac{1}{16}\int \cos(2a+3c+(2b+3d)x)\,dx+\frac{1}{8}\int \cos\ldots\\ &= -\frac{\sin(2a-3c+(2b-3d)x)}{16(2b-3d)}-\frac{3\sin(2a-c+(2b-d)x)}{16(2b-d)}+\frac{3\sin(c+dx)}{8d}+\frac{\sin(3c+3dx\ldots}{24d}\end{aligned}$$

Mathematica [A] time = 1.82646, size = 158, normalized size = 1.1

$$\frac{1}{48}\left(-\frac{3\sin(2a+2bx-3c-3dx)}{2b-3d}-\frac{9\sin(2a+2bx-c-dx)}{2b-d}-\frac{9\sin(2a+2bx+c+dx)}{2b+d}-\frac{3\sin(2a+2bx+3c+3dx)}{2b+3d}+\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((18*Cos[d*x]*Sin[c])/d + (2*Cos[3*d*x]*Sin[3*c])/d + (18*Cos[c]*Sin[d*x])/
d + (2*Cos[3*c]*Sin[3*d*x])/d - (3*Sin[2*a - 3*c + 2*b*x - 3*d*x])/(2*b - 3
*d) - (9*Sin[2*a - c + 2*b*x - d*x])/(2*b - d) - (9*Sin[2*a + c + 2*b*x + d
*x])/(2*b + d) - (3*Sin[2*a + 3*c + 2*b*x + 3*d*x])/(2*b + 3*d))/48
```

Maple [A] time = 0.036, size = 133, normalized size = 0.9

$$-\frac{\sin\left(2a-3c+(2b-3d)x\right)}{32b-48d}-\frac{3\sin\left(2a-c+(2b-d)x\right)}{32b-16d}+\frac{3\sin\left(dx+c\right)}{8d}+\frac{\sin\left(3dx+3c\right)}{24d}-\frac{3\sin\left(2a+c+(2b\right.}{32b+16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(b*x+a)^2,x)
```

```
[Out] -1/16*sin(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)-3/16*sin(2*a-c+(2*b-d)*x)/(2*b-d)+
3/8*sin(d*x+c)/d+1/24*sin(3*d*x+3*c)/d-3/16*sin(2*a+c+(2*b+d)*x)/(2*b+d)-1/
16*sin(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

Maxima [B] time = 1.41638, size = 1839, normalized size = 12.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/96*(3*(8*b^3*d*sin(3*c) - 12*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c) + 3*d^4*
sin(3*c))*cos((2*b + 3*d)*x + 2*a + 6*c) - 3*(8*b^3*d*sin(3*c) - 12*b^2*d^2
*sin(3*c) - 2*b*d^3*sin(3*c) + 3*d^4*sin(3*c))*cos((2*b + 3*d)*x + 2*a) + 9
*(8*b^3*d*sin(3*c) - 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c) + 9*d^4*sin(3*c
))*cos((2*b + d)*x + 2*a + 4*c) - 9*(8*b^3*d*sin(3*c) - 4*b^2*d^2*sin(3*c)
- 18*b*d^3*sin(3*c) + 9*d^4*sin(3*c))*cos((2*b + d)*x + 2*a - 2*c) - 9*(8*b
^3*d*sin(3*c) + 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c) - 9*d^4*sin(3*c))*co
s(-(2*b - d)*x - 2*a + 4*c) + 9*(8*b^3*d*sin(3*c) + 4*b^2*d^2*sin(3*c) - 18
*b*d^3*sin(3*c) - 9*d^4*sin(3*c))*cos(-(2*b - d)*x - 2*a - 2*c) - 3*(8*b^3*
d*sin(3*c) + 12*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c) - 3*d^4*sin(3*c))*cos(-
(2*b - 3*d)*x - 2*a + 6*c) + 3*(8*b^3*d*sin(3*c) + 12*b^2*d^2*sin(3*c) - 2*
b*d^3*sin(3*c) - 3*d^4*sin(3*c))*cos(-(2*b - 3*d)*x - 2*a) + 2*(16*b^4*sin(
3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(3*d*x) - 2*(16*b^4*sin(3*c
) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(3*d*x + 6*c) - 18*(16*b^4*sin
(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(d*x + 4*c) + 18*(16*b^4*s
in(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(d*x - 2*c) - 3*(8*b^3*d
*cos(3*c) - 12*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) + 3*d^4*cos(3*c))*sin((2
*b + 3*d)*x + 2*a + 6*c) - 3*(8*b^3*d*cos(3*c) - 12*b^2*d^2*cos(3*c) - 2*b*
d^3*cos(3*c) + 3*d^4*cos(3*c))*sin((2*b + 3*d)*x + 2*a) - 9*(8*b^3*d*cos(3*
c) - 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c) + 9*d^4*cos(3*c))*sin((2*b + d)
*x + 2*a + 4*c) - 9*(8*b^3*d*cos(3*c) - 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3
*c) + 9*d^4*cos(3*c))*sin((2*b + d)*x + 2*a - 2*c) + 9*(8*b^3*d*cos(3*c) +
4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c) - 9*d^4*cos(3*c))*sin(-(2*b - d)*x -
 2*a + 4*c) + 9*(8*b^3*d*cos(3*c) + 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c)
- 9*d^4*cos(3*c))*sin(-(2*b - d)*x - 2*a - 2*c) + 3*(8*b^3*d*cos(3*c) + 12*
b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) - 3*d^4*cos(3*c))*sin(-(2*b - 3*d)*x -
```

```
2*a + 6*c) + 3*(8*b^3*d*cos(3*c) + 12*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) -
 3*d^4*cos(3*c))*sin(-(2*b - 3*d)*x - 2*a) + 2*(16*b^4*cos(3*c) - 40*b^2*d^
2*cos(3*c) + 9*d^4*cos(3*c))*sin(3*d*x) + 2*(16*b^4*cos(3*c) - 40*b^2*d^2*c
os(3*c) + 9*d^4*cos(3*c))*sin(3*d*x + 6*c) + 18*(16*b^4*cos(3*c) - 40*b^2*d
^2*cos(3*c) + 9*d^4*cos(3*c))*sin(d*x + 4*c) + 18*(16*b^4*cos(3*c) - 40*b^2
*d^2*cos(3*c) + 9*d^4*cos(3*c))*sin(d*x - 2*c))/(9*(cos(3*c)^2 + sin(3*c)^2
)*d^5 - 40*(b^2*cos(3*c)^2 + b^2*sin(3*c)^2)*d^3 + 16*(b^4*cos(3*c)^2 + b^4
*sin(3*c)^2)*d)
```

Fricas [A] time = 0.526663, size = 393, normalized size = 2.73

$$\frac{6\left(6\,bd^3\cos\left(bx+a\right)\cos\left(dx+c\right)-\left(4\,b^3d-bd^3\right)\cos\left(bx+a\right)\cos\left(dx+c\right)^3\right)\sin\left(bx+a\right)-\left(18\,d^4\cos\left(bx+a\right)^2-16\,b^4+4\right.}{3\left(16\,b^4d-40\,b^2d^3+9\,d^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(6*(6*b*d^3*cos(b*x + a)*cos(d*x + c) - (4*b^3*d - b*d^3)*cos(b*x + a)*
cos(d*x + c)^3)*sin(b*x + a) - (18*d^4*cos(b*x + a)^2 - 16*b^4 + 40*b^2*d^2
 - 18*d^4 - (8*b^4 - 38*b^2*d^2 + 9*d^4 + 9*(4*b^2*d^2 - d^4)*cos(b*x + a)^
2)*cos(d*x + c)^2)*sin(d*x + c))/(16*b^4*d - 40*b^2*d^3 + 9*d^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14089, size = 174, normalized size = 1.21

$$-\frac{\sin\left(2\,bx+3\,dx+2\,a+3\,c\right)}{16\left(2\,b+3\,d\right)}-\frac{3\,\sin\left(2\,bx+dx+2\,a+c\right)}{16\left(2\,b+d\right)}-\frac{3\,\sin\left(2\,bx-dx+2\,a-c\right)}{16\left(2\,b-d\right)}-\frac{\sin\left(2\,bx-3\,dx+2\,a-3\,c\right)}{16\left(2\,b-3\,d\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*sin(2*b*x + 3*d*x + 2*a + 3*c)/(2*b + 3*d) - 3/16*sin(2*b*x + d*x + 2
*a + c)/(2*b + d) - 3/16*sin(2*b*x - d*x + 2*a - c)/(2*b - d) - 1/16*sin(2*
b*x - 3*d*x + 2*a - 3*c)/(2*b - 3*d) + 1/24*sin(3*d*x + 3*c)/d + 3/8*sin(d*
x + c)/d
```

3.223 $\int \cos^n(c+dx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=568

$$\frac{2^{-n-3}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\left(1+e^{2ic+2idx}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(\frac{3b}{d}-n\right),-n,\frac{1}{2}\left(\frac{3b}{d}-n+2\right),-e^{2i(c+dx)}\right)\exp(i(3a}{3b-dn}$$

```
[Out] (2^(-3 - n)*E^(I*(3*a - c*n) + I*(3*b - d*n)*x + I*n*(c + d*x))*(E^((-I)*(c
 + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[((3*b)/d - n)/2, -n, (2 + (
3*b)/d - n)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x))^n*(3*b
 - d*n)) - (3*2^(-3 - n)*E^(I*(a - c*n) + I*(b - d*n)*x + I*n*(c + d*x))*(E
^((-I)*(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, (b - d*n)/(2*d
), (2 + b/d - n)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x))^n
*(b - d*n)) - (3*2^(-3 - n)*E^((-I)*(a + c*n) - I*(b + d*n)*x + I*n*(c + d*
x))*(E^((-I)*(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, -(b + d*
n)/(2*d), 1 - (b + d*n)/(2*d), -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2
*I)*d*x))^n*(b + d*n)) + (2^(-3 - n)*E^((-I)*(3*a + c*n) - I*(3*b + d*n)*x
+ I*n*(c + d*x))*(E^((-I)*(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1
[-n, -(3*b + d*n)/(2*d), (2 - (3*b)/d - n)/2, -E^((2*I)*(c + d*x))])/((1 +
E^((2*I)*c + (2*I)*d*x))^n*(3*b + d*n))
```

Rubi [A] time = 1.18036, antiderivative size = 568, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4555, 2285, 2253, 2251}

$$\frac{2^{-n-3}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\left(1+e^{2ic+2idx}\right)^{-n}{}_2F_1\left(\frac{1}{2}\left(\frac{3b}{d}-n\right),-n;\frac{1}{2}\left(\frac{3b}{d}-n+2\right);-e^{2i(c+dx)}\right)\exp(i(3a-cn)+ix(3b-dn}{3b-dn}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^n*Sin[a + b*x]^3,x]
```


```
[Out] (2^(-3 - n)*E^(I*(3*a - c*n) + I*(3*b - d*n)*x + I*n*(c + d*x))*(E^((-I)*(c
 + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[((3*b)/d - n)/2, -n, (2 + (
3*b)/d - n)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x))^n*(3*b
 - d*n)) - (3*2^(-3 - n)*E^(I*(a - c*n) + I*(b - d*n)*x + I*n*(c + d*x))*(E
^((-I)*(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, (b - d*n)/(2*d
), (2 + b/d - n)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x))^n
*(b - d*n)) - (3*2^(-3 - n)*E^((-I)*(a + c*n) - I*(b + d*n)*x + I*n*(c + d*
x))*(E^((-I)*(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, -(b + d*
n)/(2*d), 1 - (b + d*n)/(2*d), -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2
*I)*d*x))^n*(b + d*n)) + (2^(-3 - n)*E^((-I)*(3*a + c*n) - I*(3*b + d*n)*x
+ I*n*(c + d*x))*(E^((-I)*(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1
[-n, -(3*b + d*n)/(2*d), (2 - (3*b)/d - n)/2, -E^((2*I)*(c + d*x))])/((1 +
E^((2*I)*c + (2*I)*d*x))^n*(3*b + d*n))
```


Rule 4555

```
Int[Cos[(c_.) + (d_.)*(x_)]^(q_.)*Sin[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol]
:> Dist[1/2^(p + q), Int[ExpandIntegrand[(E^(-(I*(c + d*x))) + E^(I*(c + d*
x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Rule 2285

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Dist[(a*F^
v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n), Int[u*F^(n*v)*(a
```

```
+ b*F^ExpandToSum[w - v, x])^n, x], x] /; FreeQ[{F, a, b, n}, x] &&  !Integ
erQ[n] && LinearQ[{v, w}, x]
```

Rule 2253

```
Int[((a_) + (b_.)*(F_)^((e_.)*(v_)))^(p_)*(G_)^((h_.)*(u_)), x_Symbol] :> I
nt[G^(h*ExpandToSum[u, x])*(a + b*F^(e*ExpandToSum[v, x]))^p, x] /; FreeQ[{
F, G, a, b, e, h, p}, x] && LinearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \cos^n(c+dx)\sin^3(a+bx)\,dx = 2^{-3-n}\int \left(3ie^{-ia-ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n - 3ie^{ia+ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n - ie^{-3ia-3ibx}\left(e^{-i(\right.\right.$$
$$= -\left(\left(i2^{-3-n}\right)\int e^{-3ia-3ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n dx\right) + \left(i2^{-3-n}\right)\int e^{3ia+3ibx}\left(e^{-i(c+dx)}+e^{i(c+d\right.$$
$$= -\left(\left(i2^{-3-n}e^{in(c+dx)}\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\right)\int e^{-3ia-3ibx-in(c+dx)}\left(1+e^{2ic+2i\right.$$
$$= \left(i2^{-3-n}e^{in(c+dx)}\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\right)\int e^{i(3a-cn)+i(3b-dn)x}\left(1+e^{2ic+2idx}\right)^n$$
$$= \frac{2^{-3-n}\exp(i(3a-cn)+i(3b-dn)x+in(c+dx))\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\ 2}{3b-dn}$$

Mathematica [A] time = 24.6439, size = 329, normalized size = 0.58

$$2^{-n-3}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^{n+1}e^{i(-3a+c+d(n+1)x)}\left(-\frac{3e^{2ia-ix(b+dn)}\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(-\frac{b}{d}+n+2\right),-\frac{b+d(n-2)}{2d},-e^{2i(}\right.}{b+dn}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^n*Sin[a + b*x]^3,x]
```

```
[Out] 2^(-3 - n)*E^(I*(-3*a + c + d*(1 + n)*x))*((1 + E^((2*I)*(c + d*x)))/E^(I*(
c + d*x)))^(1 + n)*(Hypergeometric2F1[1, (2 - (3*b)/d + n)/2, 1 - (3*b)/(2*
d) - n/2, -E^((2*I)*(c + d*x))]/(E^(I*(3*b + d*n)*x)*(3*b + d*n)) - (3*E^((
2*I)*a - I*(b + d*n)*x)*Hypergeometric2F1[1, (2 - b/d + n)/2, -(b + d*(-2 +
 n))/(2*d), -E^((2*I)*(c + d*x))])/(b + d*n) + E^(I*(4*a + b*x - d*n*x))*((
E^((2*I)*(a + b*x))*Hypergeometric2F1[1, (2 + (3*b)/d + n)/2, 1 + (3*b)/(2*
d) - n/2, -E^((2*I)*(c + d*x))])/(3*b - d*n) - (3*Hypergeometric2F1[1, (b +
 d*(2 + n))/(2*d), (2 + b/d - n)/2, -E^((2*I)*(c + d*x))])/(b - d*n)))
```

Maple [F] time = 0.762, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^n(\sin(bx+a))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^n*sin(b*x+a)^3,x)

[Out] int(cos(d*x+c)^n*sin(b*x+a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(dx+c)^n \sin(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^n*sin(b*x+a)^3,x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^n*sin(b*x + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos(bx+a)^2-1\right)\cos(dx+c)^n\sin(bx+a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^n*sin(b*x+a)^3,x, algorithm="fricas")

[Out] integral(-(cos(b*x + a)^2 - 1)*cos(d*x + c)^n*sin(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**n*sin(b*x+a)**3,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(dx+c)^n \sin(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^n*sin(b*x+a)^3,x, algorithm="giac")

[Out] integrate(cos(d*x + c)^n*sin(b*x + a)^3, x)
```

3.224 $\int \cos(c+dx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=97

$$-\frac{3\cos(a+x(b-d)-c)}{8(b-d)}+\frac{\cos(3a+x(3b-d)-c)}{8(3b-d)}-\frac{3\cos(a+x(b+d)+c)}{8(b+d)}+\frac{\cos(3a+x(3b+d)+c)}{8(3b+d)}$$

```
[Out] (-3*Cos[a - c + (b - d)*x])/(8*(b - d)) + Cos[3*a - c + (3*b - d)*x]/(8*(3*
b - d)) - (3*Cos[a + c + (b + d)*x])/(8*(b + d)) + Cos[3*a + c + (3*b + d)*
x]/(8*(3*b + d))
```

Rubi [A] time = 0.0682744, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4574, 2638}

$$-\frac{3\cos(a+x(b-d)-c)}{8(b-d)}+\frac{\cos(3a+x(3b-d)-c)}{8(3b-d)}-\frac{3\cos(a+x(b+d)+c)}{8(b+d)}+\frac{\cos(3a+x(3b+d)+c)}{8(3b+d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-3*Cos[a - c + (b - d)*x])/(8*(b - d)) + Cos[3*a - c + (3*b - d)*x]/(8*(3*
b - d)) - (3*Cos[a + c + (b + d)*x])/(8*(b + d)) + Cos[3*a + c + (3*b + d)*
x]/(8*(3*b + d))
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{8}\sin(a-c+(b-d)x)-\frac{1}{8}\sin(3a-c+(3b-d)x)+\frac{3}{8}\sin(a+c+(b+d)x)-\frac{1}{8}\sin$$

$$= -\left(\frac{1}{8}\int \sin(3a-c+(3b-d)x)\,dx\right)-\frac{1}{8}\int \sin(3a+c+(3b+d)x)\,dx+\frac{3}{8}\int \sin(a-c+$$

$$= -\frac{3\cos(a-c+(b-d)x)}{8(b-d)}+\frac{\cos(3a-c+(3b-d)x)}{8(3b-d)}-\frac{3\cos(a+c+(b+d)x)}{8(b+d)}+\frac{\cos(3a+}{8}$$

Mathematica [A] time = 0.523228, size = 90, normalized size = 0.93

$$\frac{1}{8}\left(-\frac{3\cos(a+bx-c-dx)}{b-d}+\frac{\cos(3a+3bx-c-dx)}{3b-d}+\frac{\cos(3a+3bx+c+dx)}{3b+d}-\frac{3\cos(a+x(b+d)+c)}{b+d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[a + b*x]^3,x]
```

```
[Out] ((-3*Cos[a - c + b*x - d*x])/(b - d) + Cos[3*a - c + 3*b*x - d*x]/(3*b - d)
 + Cos[3*a + c + 3*b*x + d*x]/(3*b + d) - (3*Cos[a + c + (b + d)*x])/(b + d
))/8
```

Maple [A] time = 0.02, size = 90, normalized size = 0.9

$$-\frac{3\cos(a-c+(b-d)x)}{8b-8d}+\frac{\cos(3a-c+(3b-d)x)}{24b-8d}-\frac{3\cos(a+c+(b+d)x)}{8b+8d}+\frac{\cos(3a+c+(3b+d)x)}{24b+8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(b*x+a)^3,x)
```

```
[Out] -3/8*cos(a-c+(b-d)*x)/(b-d)+1/8*cos(3*a-c+(3*b-d)*x)/(3*b-d)-3/8*cos(a+c+(b
+d)*x)/(b+d)+1/8*cos(3*a+c+(3*b+d)*x)/(3*b+d)
```

Maxima [B] time = 1.26686, size = 1060, normalized size = 10.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((3*b^3*cos(c) - b^2*d*cos(c) - 3*b*d^2*cos(c) + d^3*cos(c))*cos((3*b
+ d)*x + 3*a + 2*c) + (3*b^3*cos(c) - b^2*d*cos(c) - 3*b*d^2*cos(c) + d^3*c
os(c))*cos((3*b + d)*x + 3*a) + (3*b^3*cos(c) + b^2*d*cos(c) - 3*b*d^2*cos(
c) - d^3*cos(c))*cos(-(3*b - d)*x - 3*a + 2*c) + (3*b^3*cos(c) + b^2*d*cos(
c) - 3*b*d^2*cos(c) - d^3*cos(c))*cos(-(3*b - d)*x - 3*a) - 3*(9*b^3*cos(c)
 - 9*b^2*d*cos(c) - b*d^2*cos(c) + d^3*cos(c))*cos((b + d)*x + a + 2*c) - 3
*(9*b^3*cos(c) - 9*b^2*d*cos(c) - b*d^2*cos(c) + d^3*cos(c))*cos((b + d)*x
+ a) - 3*(9*b^3*cos(c) + 9*b^2*d*cos(c) - b*d^2*cos(c) - d^3*cos(c))*cos(-(
b - d)*x - a + 2*c) - 3*(9*b^3*cos(c) + 9*b^2*d*cos(c) - b*d^2*cos(c) - d^3
*cos(c))*cos(-(b - d)*x - a) + (3*b^3*sin(c) - b^2*d*sin(c) - 3*b*d^2*sin(c
) + d^3*sin(c))*sin((3*b + d)*x + 3*a + 2*c) - (3*b^3*sin(c) - b^2*d*sin(c)
 - 3*b*d^2*sin(c) + d^3*sin(c))*sin((3*b + d)*x + 3*a) + (3*b^3*sin(c) + b^
2*d*sin(c) - 3*b*d^2*sin(c) - d^3*sin(c))*sin(-(3*b - d)*x - 3*a + 2*c) - (
3*b^3*sin(c) + b^2*d*sin(c) - 3*b*d^2*sin(c) - d^3*sin(c))*sin(-(3*b - d)*x
 - 3*a) - 3*(9*b^3*sin(c) - 9*b^2*d*sin(c) - b*d^2*sin(c) + d^3*sin(c))*sin
((b + d)*x + a + 2*c) + 3*(9*b^3*sin(c) - 9*b^2*d*sin(c) - b*d^2*sin(c) + d
^3*sin(c))*sin((b + d)*x + a) - 3*(9*b^3*sin(c) + 9*b^2*d*sin(c) - b*d^2*si
n(c) - d^3*sin(c))*sin(-(b - d)*x - a + 2*c) + 3*(9*b^3*sin(c) + 9*b^2*d*si
n(c) - b*d^2*sin(c) - d^3*sin(c))*sin(-(b - d)*x - a))/(9*b^4*cos(c)^2 + 9*
b^4*sin(c)^2 + (cos(c)^2 + sin(c)^2)*d^4 - 10*(b^2*cos(c)^2 + b^2*sin(c)^2)
*d^2)
```

Fricas [A] time = 0.504789, size = 247, normalized size = 2.55

$$-\frac{\left(7\,b^2d-d^3-\left(b^2d-d^3\right)\cos\left(bx+a\right)^2\right)\sin\left(bx+a\right)\sin\left(dx+c\right)-3\left(\left(b^3-bd^2\right)\cos\left(bx+a\right)^3-\left(3\,b^3-bd^2\right)\cos\left(bx+\right.\right.}{9\,b^4-10\,b^2d^2+d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -((7*b^2*d - d^3 - (b^2*d - d^3)*cos(b*x + a)^2)*sin(b*x + a)*sin(d*x + c)
- 3*((b^3 - b*d^2)*cos(b*x + a)^3 - (3*b^3 - b*d^2)*cos(b*x + a))*cos(d*x +
 c))/(9*b^4 - 10*b^2*d^2 + d^4)
```

Sympy [A] time = 88.4002, size = 964, normalized size = 9.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((x*sin(a)**3*cos(c), Eq(b, 0) & Eq(d, 0)), (3*x*sin(a - d*x)**3*c
os(c + d*x)/8 + 3*x*sin(a - d*x)**2*sin(c + d*x)*cos(a - d*x)/8 + 3*x*sin(a
 - d*x)*cos(a - d*x)**2*cos(c + d*x)/8 + 3*x*sin(c + d*x)*cos(a - d*x)**3/8
 + 5*sin(a - d*x)**2*cos(a - d*x)*cos(c + d*x)/(8*d) + sin(a - d*x)*sin(c +
 d*x)*cos(a - d*x)**2/(8*d) + cos(a - d*x)**3*cos(c + d*x)/(4*d), Eq(b, -d)
), (x*sin(a - d*x/3)**3*cos(c + d*x)/8 + 3*x*sin(a - d*x/3)**2*sin(c + d*x)
*cos(a - d*x/3)/8 - 3*x*sin(a - d*x/3)*cos(a - d*x/3)**2*cos(c + d*x)/8 - x
*sin(c + d*x)*cos(a - d*x/3)**3/8 + 21*sin(a - d*x/3)**2*cos(a - d*x/3)*cos
(c + d*x)/(8*d) + 27*sin(a - d*x/3)*sin(c + d*x)*cos(a - d*x/3)**2/(8*d) -
5*cos(a - d*x/3)**3*cos(c + d*x)/(4*d), Eq(b, -d/3)), (x*sin(a + d*x/3)**3*
cos(c + d*x)/8 - 3*x*sin(a + d*x/3)**2*sin(c + d*x)*cos(a + d*x/3)/8 - 3*x*
sin(a + d*x/3)*cos(a + d*x/3)**2*cos(c + d*x)/8 + x*sin(c + d*x)*cos(a + d*
x/3)**3/8 - 21*sin(a + d*x/3)**2*cos(a + d*x/3)*cos(c + d*x)/(8*d) + 27*sin
(a + d*x/3)*sin(c + d*x)*cos(a + d*x/3)**2/(8*d) + 5*cos(a + d*x/3)**3*cos(
c + d*x)/(4*d), Eq(b, d/3)), (3*x*sin(a + d*x)**3*cos(c + d*x)/8 - 3*x*sin(
a + d*x)**2*sin(c + d*x)*cos(a + d*x)/8 + 3*x*sin(a + d*x)*cos(a + d*x)**2*
cos(c + d*x)/8 - 3*x*sin(c + d*x)*cos(a + d*x)**3/8 + sin(a + d*x)**3*sin(c
 + d*x)/(4*d) - 3*sin(a + d*x)**2*cos(a + d*x)*cos(c + d*x)/(8*d) + 3*sin(a
 + d*x)*sin(c + d*x)*cos(a + d*x)**2/(8*d), Eq(b, d)), (-9*b**3*sin(a + b*x
)**2*cos(a + b*x)*cos(c + d*x)/(9*b**4 - 10*b**2*d**2 + d**4) - 6*b**3*cos(
a + b*x)**3*cos(c + d*x)/(9*b**4 - 10*b**2*d**2 + d**4) - 7*b**2*d*sin(a +
b*x)**3*sin(c + d*x)/(9*b**4 - 10*b**2*d**2 + d**4) - 6*b**2*d*sin(a + b*x)
*sin(c + d*x)*cos(a + b*x)**2/(9*b**4 - 10*b**2*d**2 + d**4) + 3*b*d**2*sin
(a + b*x)**2*cos(a + b*x)*cos(c + d*x)/(9*b**4 - 10*b**2*d**2 + d**4) + d**
3*sin(a + b*x)**3*sin(c + d*x)/(9*b**4 - 10*b**2*d**2 + d**4), True))
```

Giac [A] time = 1.127, size = 120, normalized size = 1.24

$$\frac{\cos\left(3\,bx + dx + 3\,a + c\right)}{8\,(3\,b + d)} + \frac{\cos\left(3\,bx - dx + 3\,a - c\right)}{8\,(3\,b - d)} - \frac{3\,\cos\left(bx + dx + a + c\right)}{8\,(b + d)} - \frac{3\,\cos\left(bx - dx + a - c\right)}{8\,(b - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*cos(3*b*x + d*x + 3*a + c)/(3*b + d) + 1/8*cos(3*b*x - d*x + 3*a - c)/(
3*b - d) - 3/8*cos(b*x + d*x + a + c)/(b + d) - 3/8*cos(b*x - d*x + a - c)/
(b - d)
```

3.225 $\int \cos^2(c+dx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=138

$$-\frac{3\cos(a+x(b-2d)-2c)}{16(b-2d)}+\frac{\cos(3a+x(3b-2d)-2c)}{16(3b-2d)}-\frac{3\cos(a+x(b+2d)+2c)}{16(b+2d)}+\frac{\cos(3a+x(3b+2d)+2c)}{16(3b+2d)}-\frac{3}{}$$

```
[Out] (-3*Cos[a + b*x])/(8*b) + Cos[3*a + 3*b*x]/(24*b) - (3*Cos[a - 2*c + (b - 2
*d)*x])/(16*(b - 2*d)) + Cos[3*a - 2*c + (3*b - 2*d)*x]/(16*(3*b - 2*d)) -
(3*Cos[a + 2*c + (b + 2*d)*x])/(16*(b + 2*d)) + Cos[3*a + 2*c + (3*b + 2*d)
*x]/(16*(3*b + 2*d))
```

Rubi [A] time = 0.095632, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4574, 2638}

$$-\frac{3\cos(a+x(b-2d)-2c)}{16(b-2d)}+\frac{\cos(3a+x(3b-2d)-2c)}{16(3b-2d)}-\frac{3\cos(a+x(b+2d)+2c)}{16(b+2d)}+\frac{\cos(3a+x(3b+2d)+2c)}{16(3b+2d)}-\frac{3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[a + b*x]^3,x]
```


```
[Out] (-3*Cos[a + b*x])/(8*b) + Cos[3*a + 3*b*x]/(24*b) - (3*Cos[a - 2*c + (b - 2
*d)*x])/(16*(b - 2*d)) + Cos[3*a - 2*c + (3*b - 2*d)*x]/(16*(3*b - 2*d)) -
(3*Cos[a + 2*c + (b + 2*d)*x])/(16*(b + 2*d)) + Cos[3*a + 2*c + (3*b + 2*d)
*x]/(16*(3*b + 2*d))
```


Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)\sin^3(a+bx)\,dx &= \int \left(\frac{3}{8}\sin(a+bx)-\frac{1}{8}\sin(3a+3bx)+\frac{3}{16}\sin(a-2c+(b-2d)x)-\frac{1}{16}\sin(3a-2c+\right.\\ &= -\left(\frac{1}{16}\int \sin(3a-2c+(3b-2d)x)\,dx\right)-\frac{1}{16}\int \sin(3a+2c+(3b+2d)x)\,dx-\frac{1}{8}\int\\ &= -\frac{3\cos(a+bx)}{8b}+\frac{\cos(3a+3bx)}{24b}-\frac{3\cos(a-2c+(b-2d)x)}{16(b-2d)}+\frac{\cos(3a-2c+(3b-}{16(3b-2d)}\end{aligned}$$

Mathematica [A] time = 1.64136, size = 153, normalized size = 1.11

$$\frac{1}{48}\left(-\frac{9\cos(a+bx-2c-2dx)}{b-2d}+\frac{3\cos(3a+3bx-2c-2dx)}{3b-2d}-\frac{9\cos(a+bx+2c+2dx)}{b+2d}+\frac{3\cos(3a+3bx+2c+2d}{3b+2d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] ((-18*Cos[a]*Cos[b*x])/b + (2*Cos[3*a]*Cos[3*b*x])/b - (9*Cos[a - 2*c + b*x
 - 2*d*x])/(b - 2*d) + (3*Cos[3*a - 2*c + 3*b*x - 2*d*x])/(3*b - 2*d) - (9*
Cos[a + 2*c + b*x + 2*d*x])/(b + 2*d) + (3*Cos[3*a + 2*c + 3*b*x + 2*d*x])/
(3*b + 2*d) + (18*Sin[a]*Sin[b*x])/b - (2*Sin[3*a]*Sin[3*b*x])/b)/48
```

Maple [A] time = 0.023, size = 127, normalized size = 0.9

$$-\frac{3\cos(bx+a)}{8b}+\frac{\cos(3bx+3a)}{24b}-\frac{3\cos(a-2c+(b-2d)x)}{16b-32d}+\frac{\cos(3a-2c+(3b-2d)x)}{48b-32d}-\frac{3\cos(a+2c+(b+2}{16b+32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(b*x+a)^3,x)
```

```
[Out] -3/8*cos(b*x+a)/b+1/24*cos(3*b*x+3*a)/b-3/16*cos(a-2*c+(b-2*d)*x)/(b-2*d)+1
/16*cos(3*a-2*c+(3*b-2*d)*x)/(3*b-2*d)-3/16*cos(a+2*c+(b+2*d)*x)/(b+2*d)+1/
16*cos(3*a+2*c+(3*b+2*d)*x)/(3*b+2*d)
```

Maxima [B] time = 1.42997, size = 1836, normalized size = 13.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/96*(3*(3*b^4*cos(2*c) - 2*b^3*d*cos(2*c) - 12*b^2*d^2*cos(2*c) + 8*b*d^3*
cos(2*c))*cos((3*b + 2*d)*x + 3*a + 4*c) + 3*(3*b^4*cos(2*c) - 2*b^3*d*cos(
2*c) - 12*b^2*d^2*cos(2*c) + 8*b*d^3*cos(2*c))*cos((3*b + 2*d)*x + 3*a) + 3
*(3*b^4*cos(2*c) + 2*b^3*d*cos(2*c) - 12*b^2*d^2*cos(2*c) - 8*b*d^3*cos(2*c
))*cos(-(3*b - 2*d)*x - 3*a + 4*c) + 3*(3*b^4*cos(2*c) + 2*b^3*d*cos(2*c) -
 12*b^2*d^2*cos(2*c) - 8*b*d^3*cos(2*c))*cos(-(3*b - 2*d)*x - 3*a) - 9*(9*b
^4*cos(2*c) - 18*b^3*d*cos(2*c) - 4*b^2*d^2*cos(2*c) + 8*b*d^3*cos(2*c))*co
s((b + 2*d)*x + a + 4*c) - 9*(9*b^4*cos(2*c) - 18*b^3*d*cos(2*c) - 4*b^2*d^
2*cos(2*c) + 8*b*d^3*cos(2*c))*cos((b + 2*d)*x + a) - 9*(9*b^4*cos(2*c) + 1
8*b^3*d*cos(2*c) - 4*b^2*d^2*cos(2*c) - 8*b*d^3*cos(2*c))*cos(-(b - 2*d)*x
- a + 4*c) - 9*(9*b^4*cos(2*c) + 18*b^3*d*cos(2*c) - 4*b^2*d^2*cos(2*c) - 8
*b*d^3*cos(2*c))*cos(-(b - 2*d)*x - a) + 2*(9*b^4*cos(2*c) - 40*b^2*d^2*cos
(2*c) + 16*d^4*cos(2*c))*cos(3*b*x + 3*a + 2*c) + 2*(9*b^4*cos(2*c) - 40*b^
2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(3*b*x + 3*a - 2*c) - 18*(9*b^4*cos(2*
c) - 40*b^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(b*x + a + 2*c) - 18*(9*b^4*
cos(2*c) - 40*b^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(b*x + a - 2*c) + 3*(3
*b^4*sin(2*c) - 2*b^3*d*sin(2*c) - 12*b^2*d^2*sin(2*c) + 8*b*d^3*sin(2*c))*
sin((3*b + 2*d)*x + 3*a + 4*c) - 3*(3*b^4*sin(2*c) - 2*b^3*d*sin(2*c) - 12*
b^2*d^2*sin(2*c) + 8*b*d^3*sin(2*c))*sin((3*b + 2*d)*x + 3*a) + 3*(3*b^4*si
n(2*c) + 2*b^3*d*sin(2*c) - 12*b^2*d^2*sin(2*c) - 8*b*d^3*sin(2*c))*sin(-(3
*b - 2*d)*x - 3*a + 4*c) - 3*(3*b^4*sin(2*c) + 2*b^3*d*sin(2*c) - 12*b^2*d^
2*sin(2*c) - 8*b*d^3*sin(2*c))*sin(-(3*b - 2*d)*x - 3*a) - 9*(9*b^4*sin(2*c
) - 18*b^3*d*sin(2*c) - 4*b^2*d^2*sin(2*c) + 8*b*d^3*sin(2*c))*sin((b + 2*d
)*x + a + 4*c) + 9*(9*b^4*sin(2*c) - 18*b^3*d*sin(2*c) - 4*b^2*d^2*sin(2*c)
 + 8*b*d^3*sin(2*c))*sin((b + 2*d)*x + a) - 9*(9*b^4*sin(2*c) + 18*b^3*d*si
n(2*c) - 4*b^2*d^2*sin(2*c) - 8*b*d^3*sin(2*c))*sin(-(b - 2*d)*x - a + 4*c)
```

```
 + 9*(9*b^4*sin(2*c) + 18*b^3*d*sin(2*c) - 4*b^2*d^2*sin(2*c) - 8*b*d^3*sin
(2*c))*sin(-(b - 2*d)*x - a) + 2*(9*b^4*sin(2*c) - 40*b^2*d^2*sin(2*c) + 16
*d^4*sin(2*c))*sin(3*b*x + 3*a + 2*c) - 2*(9*b^4*sin(2*c) - 40*b^2*d^2*sin(
2*c) + 16*d^4*sin(2*c))*sin(3*b*x + 3*a - 2*c) - 18*(9*b^4*sin(2*c) - 40*b^
2*d^2*sin(2*c) + 16*d^4*sin(2*c))*sin(b*x + a + 2*c) + 18*(9*b^4*sin(2*c) -
 40*b^2*d^2*sin(2*c) + 16*d^4*sin(2*c))*sin(b*x + a - 2*c))/(9*b^5*cos(2*c)
^2 + 9*b^5*sin(2*c)^2 + 16*(b*cos(2*c)^2 + b*sin(2*c)^2)*d^4 - 40*(b^3*cos(
2*c)^2 + b^3*sin(2*c)^2)*d^2)
```

Fricas [A] time = 0.539745, size = 404, normalized size = 2.93

$$-\frac{2\left(b^2d^2-4\,d^4\right)\cos\left(bx+a\right)^3+6\left(7\,b^3d-4\,bd^3-\left(b^3d-4\,bd^3\right)\cos\left(bx+a\right)^2\right)\cos\left(dx+c\right)\sin\left(bx+a\right)\sin\left(dx+c\right)-9}{3\left(9\,b^5-40\,b^3d^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(b^2*d^2 - 4*d^4)*cos(b*x + a)^3 + 6*(7*b^3*d - 4*b*d^3 - (b^3*d -
4*b*d^3)*cos(b*x + a)^2)*cos(d*x + c)*sin(b*x + a)*sin(d*x + c) - 9*((b^4 -
 4*b^2*d^2)*cos(b*x + a)^3 - (3*b^4 - 4*b^2*d^2)*cos(b*x + a))*cos(d*x + c)
^2 - 6*(7*b^2*d^2 - 4*d^4)*cos(b*x + a))/(9*b^5 - 40*b^3*d^2 + 16*b*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14146, size = 167, normalized size = 1.21

$$\frac{\cos\left(3\,bx+2\,dx+3\,a+2\,c\right)}{16\,(3\,b+2\,d)}+\frac{\cos\left(3\,bx-2\,dx+3\,a-2\,c\right)}{16\,(3\,b-2\,d)}+\frac{\cos\left(3\,bx+3\,a\right)}{24\,b}-\frac{3\,\cos\left(bx+2\,dx+a+2\,c\right)}{16\,(b+2\,d)}-\frac{3\,\cos\left(b\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*cos(3*b*x + 2*d*x + 3*a + 2*c)/(3*b + 2*d) + 1/16*cos(3*b*x - 2*d*x +
3*a - 2*c)/(3*b - 2*d) + 1/24*cos(3*b*x + 3*a)/b - 3/16*cos(b*x + 2*d*x + a
 + 2*c)/(b + 2*d) - 3/16*cos(b*x - 2*d*x + a - 2*c)/(b - 2*d) - 3/8*cos(b*x
 + a)/b
```

3.226 $\int \cos^3(c+dx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=195

$$-\frac{3\cos(a+x(b-3d)-3c)}{32(b-3d)}-\frac{9\cos(a+x(b-d)-c)}{32(b-d)}+\frac{\cos(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\cos(3a+x(3b-d)-c)}{32(3b-d)}-\frac{9\cos(a+\ldots}{32\ldots}$$

```
[Out] (-3*Cos[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) - (9*Cos[a - c + (b - d)*x])
/(32*(b - d)) + Cos[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) + (3*Cos[3*a - c
+ (3*b - d)*x])/(32*(3*b - d)) - (9*Cos[a + c + (b + d)*x])/(32*(b + d)) +
Cos[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) + (3*Cos[3*a + c + (3*b + d)*x])/
(32*(3*b + d)) - (3*Cos[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

Rubi [A] time = 0.124218, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4574, 2638}

$$-\frac{3\cos(a+x(b-3d)-3c)}{32(b-3d)}-\frac{9\cos(a+x(b-d)-c)}{32(b-d)}+\frac{\cos(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\cos(3a+x(3b-d)-c)}{32(3b-d)}-\frac{9\cos(a+\ldots}{32\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-3*Cos[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) - (9*Cos[a - c + (b - d)*x])
/(32*(b - d)) + Cos[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) + (3*Cos[3*a - c
+ (3*b - d)*x])/(32*(3*b - d)) - (9*Cos[a + c + (b + d)*x])/(32*(b + d)) +
Cos[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) + (3*Cos[3*a + c + (3*b + d)*x])/
(32*(3*b + d)) - (3*Cos[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)\sin^3(a+bx)\,dx &= \int \left(\frac{3}{32}\sin(a-3c+(b-3d)x)+\frac{9}{32}\sin(a-c+(b-d)x)-\frac{1}{32}\sin(3(a-c)+3(b-d)x\ldots\right.\\ &= -\left(\frac{1}{32}\int \sin(3(a-c)+3(b-d)x)\,dx\right)-\frac{1}{32}\int \sin(3(a+c)+3(b+d)x)\,dx+\frac{3}{32}\int \sin\ldots\\ &= -\frac{3\cos(a-3c+(b-3d)x)}{32(b-3d)}-\frac{9\cos(a-c+(b-d)x)}{32(b-d)}+\frac{\cos(3(a-c)+3(b-d)x)}{96(b-d)}+\frac{3\cos\ldots}{\ldots}\end{aligned}$$

Mathematica [A] time = 1.60172, size = 176, normalized size = 0.9

$$\frac{1}{96}\left(-\frac{9\cos(a+bx-3c-3dx)}{b-3d}-\frac{27\cos(a+bx-c-dx)}{b-d}+\frac{\cos(3(a+bx-c-dx))}{b-d}+\frac{9\cos(3a+3bx-c-dx)}{3b-d}+\frac{9\cos(\ldots}{\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] ((-9*Cos[a - 3*c + b*x - 3*d*x])/(b - 3*d) - (27*Cos[a - c + b*x - d*x])/(b
 - d) + Cos[3*(a - c + b*x - d*x)]/(b - d) + (9*Cos[3*a - c + 3*b*x - d*x])
/(3*b - d) + (9*Cos[3*a + c + 3*b*x + d*x])/(3*b + d) - (9*Cos[a + 3*c + b*
x + 3*d*x])/(b + 3*d) - (27*Cos[a + c + (b + d)*x])/(b + d) + Cos[3*(a + c
+ (b + d)*x)]/(b + d))/96
```

Maple [A] time = 0.029, size = 184, normalized size = 0.9

$$-\frac{3\cos(a-3c+(b-3d)x)}{32b-96d}-\frac{9\cos(a-c+(b-d)x)}{32b-32d}-\frac{9\cos(a+c+(b+d)x)}{32b+32d}-\frac{3\cos(a+3c+(b+3d)x)}{32b+96d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(b*x+a)^3,x)
```

```
[Out] -3/32*cos(a-3*c+(b-3*d)*x)/(b-3*d)-9/32*cos(a-c+(b-d)*x)/(b-d)-9/32*cos(a+c
+(b+d)*x)/(b+d)-3/32*cos(a+3*c+(b+3*d)*x)/(b+3*d)+1/96/(b-d)*cos((3*b-3*d)*
x-3*c+3*a)+3/32*cos(3*a-c+(3*b-d)*x)/(3*b-d)+3/32*cos(3*a+c+(3*b+d)*x)/(3*b
+d)+1/96/(b+d)*cos((3*b+3*d)*x+3*c+3*a)
```

Maxima [B] time = 1.8888, size = 3526, normalized size = 18.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/192*(9*(3*b^5*cos(3*c) - b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) + 10*b^2*d^
3*cos(3*c) + 27*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*cos((3*b + d)*x + 3*a + 4*
c) + 9*(3*b^5*cos(3*c) - b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) + 10*b^2*d^3*
cos(3*c) + 27*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*cos((3*b + d)*x + 3*a - 2*c)
 + 9*(3*b^5*cos(3*c) + b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) - 10*b^2*d^3*co
s(3*c) + 27*b*d^4*cos(3*c) + 9*d^5*cos(3*c))*cos(-(3*b - d)*x - 3*a + 4*c)
+ 9*(3*b^5*cos(3*c) + b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) - 10*b^2*d^3*cos
(3*c) + 27*b*d^4*cos(3*c) + 9*d^5*cos(3*c))*cos(-(3*b - d)*x - 3*a - 2*c) -
 9*(9*b^5*cos(3*c) - 27*b^4*d*cos(3*c) - 10*b^3*d^2*cos(3*c) + 30*b^2*d^3*c
os(3*c) + b*d^4*cos(3*c) - 3*d^5*cos(3*c))*cos((b + 3*d)*x + a + 6*c) - 9*(
9*b^5*cos(3*c) - 27*b^4*d*cos(3*c) - 10*b^3*d^2*cos(3*c) + 30*b^2*d^3*cos(3
*c) + b*d^4*cos(3*c) - 3*d^5*cos(3*c))*cos((b + 3*d)*x + a) + (9*b^5*cos(3*
c) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*b*d^4
*cos(3*c) - 9*d^5*cos(3*c))*cos(3*(b + d)*x + 3*a + 6*c) + (9*b^5*cos(3*c)
- 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*b*d^4*co
s(3*c) - 9*d^5*cos(3*c))*cos(3*(b + d)*x + 3*a) - 27*(9*b^5*cos(3*c) - 9*b^
4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c)
 - 9*d^5*cos(3*c))*cos((b + d)*x + a + 4*c) - 27*(9*b^5*cos(3*c) - 9*b^4*d*
cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) - 9
*d^5*cos(3*c))*cos((b + d)*x + a - 2*c) - 27*(9*b^5*cos(3*c) + 9*b^4*d*cos(
3*c) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) + 9*d^5
*cos(3*c))*cos(-(b - d)*x - a + 4*c) - 27*(9*b^5*cos(3*c) + 9*b^4*d*cos(3*c
```

```
) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) + 9*d^5*co
s(3*c))*cos(-(b - d)*x - a - 2*c) + (9*b^5*cos(3*c) + 9*b^4*d*cos(3*c) - 82
*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) + 9*d^5*cos(3*c)
)*cos(-3*(b - d)*x - 3*a + 6*c) + (9*b^5*cos(3*c) + 9*b^4*d*cos(3*c) - 82*b
^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) + 9*d^5*cos(3*c))*
cos(-3*(b - d)*x - 3*a) - 9*(9*b^5*cos(3*c) + 27*b^4*d*cos(3*c) - 10*b^3*d^
2*cos(3*c) - 30*b^2*d^3*cos(3*c) + b*d^4*cos(3*c) + 3*d^5*cos(3*c))*cos(-(b
 - 3*d)*x - a + 6*c) - 9*(9*b^5*cos(3*c) + 27*b^4*d*cos(3*c) - 10*b^3*d^2*c
os(3*c) - 30*b^2*d^3*cos(3*c) + b*d^4*cos(3*c) + 3*d^5*cos(3*c))*cos(-(b -
3*d)*x - a) + 9*(3*b^5*sin(3*c) - b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) + 10
*b^2*d^3*sin(3*c) + 27*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*sin((3*b + d)*x + 3
*a + 4*c) - 9*(3*b^5*sin(3*c) - b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) + 10*b
^2*d^3*sin(3*c) + 27*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*sin((3*b + d)*x + 3*a
 - 2*c) + 9*(3*b^5*sin(3*c) + b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) - 10*b^2
*d^3*sin(3*c) + 27*b*d^4*sin(3*c) + 9*d^5*sin(3*c))*sin(-(3*b - d)*x - 3*a
+ 4*c) - 9*(3*b^5*sin(3*c) + b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) - 10*b^2*
d^3*sin(3*c) + 27*b*d^4*sin(3*c) + 9*d^5*sin(3*c))*sin(-(3*b - d)*x - 3*a -
 2*c) - 9*(9*b^5*sin(3*c) - 27*b^4*d*sin(3*c) - 10*b^3*d^2*sin(3*c) + 30*b^
2*d^3*sin(3*c) + b*d^4*sin(3*c) - 3*d^5*sin(3*c))*sin((b + 3*d)*x + a + 6*c
) + 9*(9*b^5*sin(3*c) - 27*b^4*d*sin(3*c) - 10*b^3*d^2*sin(3*c) + 30*b^2*d^
3*sin(3*c) + b*d^4*sin(3*c) - 3*d^5*sin(3*c))*sin((b + 3*d)*x + a) + (9*b^5
*sin(3*c) - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) +
9*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*sin(3*(b + d)*x + 3*a + 6*c) - (9*b^5*si
n(3*c) - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b
*d^4*sin(3*c) - 9*d^5*sin(3*c))*sin(3*(b + d)*x + 3*a) - 27*(9*b^5*sin(3*c)
 - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^4*s
in(3*c) - 9*d^5*sin(3*c))*sin((b + d)*x + a + 4*c) + 27*(9*b^5*sin(3*c) - 9
*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3
*c) - 9*d^5*sin(3*c))*sin((b + d)*x + a - 2*c) - 27*(9*b^5*sin(3*c) + 9*b^4
*d*sin(3*c) - 82*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c)
+ 9*d^5*sin(3*c))*sin(-(b - d)*x - a + 4*c) + 27*(9*b^5*sin(3*c) + 9*b^4*d*
sin(3*c) - 82*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9
*d^5*sin(3*c))*sin(-(b - d)*x - a - 2*c) + (9*b^5*sin(3*c) + 9*b^4*d*sin(3*
c) - 82*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^5*s
in(3*c))*sin(-3*(b - d)*x - 3*a + 6*c) - (9*b^5*sin(3*c) + 9*b^4*d*sin(3*c)
 - 82*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^5*sin
(3*c))*sin(-3*(b - d)*x - 3*a) - 9*(9*b^5*sin(3*c) + 27*b^4*d*sin(3*c) - 10
*b^3*d^2*sin(3*c) - 30*b^2*d^3*sin(3*c) + b*d^4*sin(3*c) + 3*d^5*sin(3*c))*
sin(-(b - 3*d)*x - a + 6*c) + 9*(9*b^5*sin(3*c) + 27*b^4*d*sin(3*c) - 10*b^
3*d^2*sin(3*c) - 30*b^2*d^3*sin(3*c) + b*d^4*sin(3*c) + 3*d^5*sin(3*c))*sin
(-(b - 3*d)*x - a))/(9*b^6*cos(3*c)^2 + 9*b^6*sin(3*c)^2 - 9*(cos(3*c)^2 +
sin(3*c)^2)*d^6 + 91*(b^2*cos(3*c)^2 + b^2*sin(3*c)^2)*d^4 - 91*(b^4*cos(3*
c)^2 + b^4*sin(3*c)^2)*d^2)
```

Fricas [A] time = 0.588335, size = 591, normalized size = 3.03

$$\left(\left(9\,b^5 - 82\,b^3d^2 + 9\,bd^4\right)\cos\left(bx+a\right)^3 - 3\left(9\,b^5 - 28\,b^3d^2 + 3\,bd^4\right)\cos\left(bx+a\right)\right)\cos\left(dx+c\right)^3 + \left(122\,b^2d^3 - 18\,d^5 - 2\left(b^2d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(((9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cos(b*x + a)^3 - 3*(9*b^5 - 28*b^3*d^2
 + 3*b*d^4)*cos(b*x + a))*cos(d*x + c)^3 + (122*b^2*d^3 - 18*d^5 - 2*(b^2*d
^3 - 9*d^5)*cos(b*x + a)^2 - (63*b^4*d - 88*b^2*d^3 + 9*d^5 - (9*b^4*d - 82
*b^2*d^3 + 9*d^5)*cos(b*x + a)^2)*cos(d*x + c)^2)*sin(b*x + a)*sin(d*x + c)
```

```
 - 6*((b^3*d^2 - 9*b*d^4)*cos(b*x + a)^3 - 3*(7*b^3*d^2 - 3*b*d^4)*cos(b*x
+ a))*cos(d*x + c))/(9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13263, size = 244, normalized size = 1.25

$$\frac{\cos\left(3\,bx+3\,dx+3\,a+3\,c\right)}{96\,(b+d)}+\frac{3\,\cos\left(3\,bx+dx+3\,a+c\right)}{32\,(3\,b+d)}+\frac{3\,\cos\left(3\,bx-dx+3\,a-c\right)}{32\,(3\,b-d)}+\frac{\cos\left(3\,bx-3\,dx+3\,a-3\,c\right)}{96\,(b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/96*cos(3*b*x + 3*d*x + 3*a + 3*c)/(b + d) + 3/32*cos(3*b*x + d*x + 3*a +
c)/(3*b + d) + 3/32*cos(3*b*x - d*x + 3*a - c)/(3*b - d) + 1/96*cos(3*b*x -
 3*d*x + 3*a - 3*c)/(b - d) - 3/32*cos(b*x + 3*d*x + a + 3*c)/(b + 3*d) - 9
/32*cos(b*x + d*x + a + c)/(b + d) - 9/32*cos(b*x - d*x + a - c)/(b - d) -
3/32*cos(b*x - 3*d*x + a - 3*c)/(b - 3*d)
```

3.227 $\int \cos(a+bx)\csc(c+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\cos(a-c)\log(\sin(bx+c))}{b}-x\sin(a-c)$$

```
[Out] (Cos[a - c]*Log[Sin[c + b*x]])/b - x*Sin[a - c]
```

Rubi [A] time = 0.0168558, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4581, 3475, 8}

$$\frac{\cos(a-c)\log(\sin(bx+c))}{b}-x\sin(a-c)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Csc[c + b*x],x]
```

```
[Out] (Cos[a - c]*Log[Sin[c + b*x]])/b - x*Sin[a - c]
```

Rule 4581

```
Int[Cos[v_]*Csc[w_]^(n_.), x_Symbol] :> Dist[Cos[v - w], Int[Cot[w]*Csc[w]^
(n - 1), x], x] - Dist[Sin[v - w], Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\csc(c+bx)\,dx &= \cos(a-c)\int \cot(c+bx)\,dx-\sin(a-c)\int 1\,dx\\ &= \frac{\cos(a-c)\log(\sin(c+bx))}{b}-x\sin(a-c)\end{aligned}$$

Mathematica [C] time = 0.173198, size = 58, normalized size = 2.15

$$\frac{-2bx\sin(a-c)-2i\cos(a-c)\tan^{-1}(\tan(bx+c))+\cos(a-c)\left(\log\left(\sin^2(bx+c)\right)+2ibx\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Csc[c + b*x],x]
```

```
[Out] ((-2*I)*ArcTan[Tan[c + b*x]]*Cos[a - c] + Cos[a - c]*((2*I)*b*x + Log[Sin[c
 + b*x]^2]) - 2*b*x*Sin[a - c])/(2*b)
```

Maple [B] time = 0.184, size = 325, normalized size = 12.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(b*x+c),x)
```

```
[Out] -1/2/b/(cos(c)^2+sin(c)^2)/(cos(a)^2+sin(a)^2)*ln(1+tan(b*x+a)^2)*cos(a)*co
s(c)-1/2/b/(cos(c)^2+sin(c)^2)/(cos(a)^2+sin(a)^2)*ln(1+tan(b*x+a)^2)*sin(a
)*sin(c)+1/b/(cos(c)^2+sin(c)^2)/(cos(a)^2+sin(a)^2)*cos(a)*sin(c)*arctan(t
an(b*x+a))-1/b/(cos(c)^2+sin(c)^2)/(cos(a)^2+sin(a)^2)*cos(c)*sin(a)*arctan
(tan(b*x+a))+1/b/(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^2+cos(c)^2*sin(a)^2+sin
(a)^2*sin(c)^2)*ln(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*sin(a)*sin(c)+cos(a)
*sin(c)-sin(a)*cos(c))*cos(a)*cos(c)+1/b/(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)
^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)*ln(tan(b*x+a)*cos(a)*cos(c)+tan(b*x
+a)*sin(a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c))*sin(a)*sin(c)
```

Maxima [B] time = 1.113, size = 143, normalized size = 5.3

$$\frac{2\,bx\sin\left(-a+c\right)+\cos\left(-a+c\right)\log\left(\cos\left(bx\right)^{2}+2\,\cos\left(bx\right)\cos\left(c\right)+\cos\left(c\right)^{2}+\sin\left(bx\right)^{2}-2\,\sin\left(bx\right)\sin\left(c\right)+\sin\left(c\right)^{2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x*sin(-a + c) + cos(-a + c)*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + c
os(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) + cos(-a + c)*log(cos(
b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + si
n(c)^2))/b
```

Fricas [A] time = 0.50052, size = 77, normalized size = 2.85

$$\frac{bx\sin\left(-a+c\right)+\cos\left(-a+c\right)\log\left(\frac{1}{2}\,\sin\left(bx+c\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c),x, algorithm="fricas")
```

```
[Out] (b*x*sin(-a + c) + cos(-a + c)*log(1/2*sin(b*x + c)))/b
```

Sympy [B] time = 10.3423, size = 335, normalized size = 12.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c),x)
```

```
[Out] -Piecewise((0, Eq(b, 0) & (Eq(b, 0) | Eq(c, 0))), (x, Eq(c, 0)), (-b*x*tan(
c/2)**2/(b*tan(c/2)**2 + b) + b*x/(b*tan(c/2)**2 + b) - 2*log(tan(c/2) + ta
n(b*x/2))*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b*x/2) - 1/tan(c/2))*tan
(c/2)/(b*tan(c/2)**2 + b) + 2*log(tan(b*x/2)**2 + 1)*tan(c/2)/(b*tan(c/2)**
2 + b), True))*sin(a) + Piecewise((zoo*x, Eq(b, 0) & Eq(c, 0)), (x/sin(c),
Eq(b, 0)), (log(sin(b*x))/b, Eq(c, 0)), (2*b*x*tan(c/2)/(b*tan(c/2)**2 + b)
 - log(tan(c/2) + tan(b*x/2))*tan(c/2)**2/(b*tan(c/2)**2 + b) + log(tan(c/2
) + tan(b*x/2))/(b*tan(c/2)**2 + b) - log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)
**2/(b*tan(c/2)**2 + b) + log(tan(b*x/2) - 1/tan(c/2))/(b*tan(c/2)**2 + b)
+ log(tan(b*x/2)**2 + 1)*tan(c/2)**2/(b*tan(c/2)**2 + b) - log(tan(b*x/2)**
2 + 1)/(b*tan(c/2)**2 + b), True))*cos(a)
```

Giac [B] time = 1.228, size = 651, normalized size = 24.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(4*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - t
an(1/2*c))*(b*x + a)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)
^2 + 1) + (tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*
c) - tan(1/2*c)^2 + 1)*log(tan(b*x + a)^2 + 1)/(tan(1/2*a)^2*tan(1/2*c)^2 +
 tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - 2*(tan(1/2*a)^4*tan(1/2*c)^4 - 2*tan(1/
2*a)^4*tan(1/2*c)^2 + 8*tan(1/2*a)^3*tan(1/2*c)^3 - 2*tan(1/2*a)^2*tan(1/2*
c)^4 + tan(1/2*a)^4 - 8*tan(1/2*a)^3*tan(1/2*c) + 20*tan(1/2*a)^2*tan(1/2*c
)^2 - 8*tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*c)^4 - 2*tan(1/2*a)^2 + 8*tan(1/2
*a)*tan(1/2*c) - 2*tan(1/2*c)^2 + 1)*log(abs(tan(b*x + a)*tan(1/2*a)^2*tan(
1/2*c)^2 - tan(b*x + a)*tan(1/2*a)^2 + 4*tan(b*x + a)*tan(1/2*a)*tan(1/2*c)
 - 2*tan(1/2*a)^2*tan(1/2*c) - tan(b*x + a)*tan(1/2*c)^2 + 2*tan(1/2*a)*tan
(1/2*c)^2 + tan(b*x + a) - 2*tan(1/2*a) + 2*tan(1/2*c)))/(tan(1/2*a)^4*tan(
1/2*c)^4 + 4*tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^4 + 4*tan(1/2*a)^3*tan(
1/2*c) + 4*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 + 4*tan(1/2*a)*tan(1/2*c)
 + 1))/b
```

3.228 $\int \cos(a+bx)\csc^2(c+bx)\,dx$

Optimal. Leaf size=35

$$\frac{\sin(a-c)\tanh^{-1}(\cos(bx+c))}{b}-\frac{\cos(a-c)\csc(bx+c)}{b}$$

```
[Out] -((Cos[a - c]*Csc[c + b*x])/b) + (ArcTanh[Cos[c + b*x]]*Sin[a - c])/b
```

Rubi [A] time = 0.028113, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4581, 2606, 8, 3770}

$$\frac{\sin(a-c)\tanh^{-1}(\cos(bx+c))}{b}-\frac{\cos(a-c)\csc(bx+c)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Csc[c + b*x]^2,x]
```

```
[Out] -((Cos[a - c]*Csc[c + b*x])/b) + (ArcTanh[Cos[c + b*x]]*Sin[a - c])/b
```

Rule 4581

```
Int[Cos[v_]*Csc[w_]^(n_.), x_Symbol] :> Dist[Cos[v - w], Int[Cot[w]*Csc[w]^
(n - 1), x], x] - Dist[Sin[v - w], Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\csc^2(c+bx)\,dx &= \cos(a-c)\int \cot(c+bx)\csc(c+bx)\,dx-\sin(a-c)\int \csc(c+bx)\,dx\\ &= \frac{\tanh^{-1}(\cos(c+bx))\sin(a-c)}{b}-\frac{\cos(a-c)\,\mathrm{Subst}\left(\int 1\,dx,x,\csc(c+bx)\right)}{b}\\ &= -\frac{\cos(a-c)\csc(c+bx)}{b}+\frac{\tanh^{-1}(\cos(c+bx))\sin(a-c)}{b}\end{aligned}$$

Mathematica [C] time = 0.0955334, size = 90, normalized size = 2.57

$$-\frac{\cos(a-c)\csc(bx+c)}{b}+\frac{2i\sin(a-c)\tan^{-1}\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{\sin(c)\cos\left(\frac{bx}{2}\right)+i\cos(c)\cos\left(\frac{bx}{2}\right)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Csc[c + b*x]^2,x]
```

```
[Out] -((Cos[a - c]*Csc[c + b*x])/b) + ((2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c]
*Cos[(b*x)/2] - Sin[c]*Sin[(b*x)/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/2]
*Sin[c])]*Sin[a - c])/b
```

Maple [B] time = 0.452, size = 1062, normalized size = 30.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(b*x+c)^2,x)
```

```
[Out] -1/b/(-1/2*cos(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+1/2*cos(c)*sin(a)*tan(1/2*b*x
+1/2*a)^2+tan(1/2*b*x+1/2*a)*cos(a)*cos(c)+tan(1/2*b*x+1/2*a)*sin(a)*sin(c)
+1/2*cos(a)*sin(c)-1/2*sin(a)*cos(c))/(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^2+
cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)/(cos(a)*sin(c)-sin(a)*cos(c))*tan(1/2*
b*x+1/2*a)*cos(a)^2*cos(c)^2-2/b/(-1/2*cos(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+1
/2*cos(c)*sin(a)*tan(1/2*b*x+1/2*a)^2+tan(1/2*b*x+1/2*a)*cos(a)*cos(c)+tan(
1/2*b*x+1/2*a)*sin(a)*sin(c)+1/2*cos(a)*sin(c)-1/2*sin(a)*cos(c))/(cos(a)^2
*cos(c)^2+cos(a)^2*sin(c)^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)/(cos(a)*si
n(c)-sin(a)*cos(c))*tan(1/2*b*x+1/2*a)*cos(a)*cos(c)*sin(a)*sin(c)-1/b/(-1/
2*cos(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+1/2*cos(c)*sin(a)*tan(1/2*b*x+1/2*a)^2
+tan(1/2*b*x+1/2*a)*cos(a)*cos(c)+tan(1/2*b*x+1/2*a)*sin(a)*sin(c)+1/2*cos(
a)*sin(c)-1/2*sin(a)*cos(c))/(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^2+cos(c)^2*
sin(a)^2+sin(a)^2*sin(c)^2)/(cos(a)*sin(c)-sin(a)*cos(c))*tan(1/2*b*x+1/2*a
)*sin(a)^2*sin(c)^2-1/b/(-1/2*cos(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+1/2*cos(c)
*sin(a)*tan(1/2*b*x+1/2*a)^2+tan(1/2*b*x+1/2*a)*cos(a)*cos(c)+tan(1/2*b*x+1
/2*a)*sin(a)*sin(c)+1/2*cos(a)*sin(c)-1/2*sin(a)*cos(c))/(cos(a)^2*cos(c)^2
+cos(a)^2*sin(c)^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)*cos(a)*cos(c)-1/b/(
-1/2*cos(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+1/2*cos(c)*sin(a)*tan(1/2*b*x+1/2*a
)^2+tan(1/2*b*x+1/2*a)*cos(a)*cos(c)+tan(1/2*b*x+1/2*a)*sin(a)*sin(c)+1/2*c
os(a)*sin(c)-1/2*sin(a)*cos(c))/(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^2+cos(c)
^2*sin(a)^2+sin(a)^2*sin(c)^2)*sin(a)*sin(c)+4/b/(2*cos(a)^2*cos(c)^2+2*cos
(a)^2*sin(c)^2+2*cos(c)^2*sin(a)^2+2*sin(a)^2*sin(c)^2)/(-cos(a)^2*cos(c)^2
-cos(a)^2*sin(c)^2-cos(c)^2*sin(a)^2-sin(a)^2*sin(c)^2)^(1/2)*arctan(1/2*(2
*(sin(a)*cos(c)-cos(a)*sin(c))*tan(1/2*b*x+1/2*a)+2*cos(a)*cos(c)+2*sin(a)*
sin(c))/(-cos(a)^2*cos(c)^2-cos(a)^2*sin(c)^2-cos(c)^2*sin(a)^2-sin(a)^2*si
n(c)^2)^(1/2))*cos(a)*sin(c)-4/b/(2*cos(a)^2*cos(c)^2+2*cos(a)^2*sin(c)^2+2
*cos(c)^2*sin(a)^2+2*sin(a)^2*sin(c)^2)/(-cos(a)^2*cos(c)^2-cos(a)^2*sin(c)
^2-cos(c)^2*sin(a)^2-sin(a)^2*sin(c)^2)^(1/2)*arctan(1/2*(2*(sin(a)*cos(c)-
cos(a)*sin(c))*tan(1/2*b*x+1/2*a)+2*cos(a)*cos(c)+2*sin(a)*sin(c))/(-cos(a)
^2*cos(c)^2-cos(a)^2*sin(c)^2-cos(c)^2*sin(a)^2-sin(a)^2*sin(c)^2)^(1/2))*s
in(a)*cos(c)
```

Maxima [B] time = 1.3102, size = 608, normalized size = 17.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(sin(b*x + 2*a) + sin(b*x + 2*c))*cos(2*b*x + a + 2*c) - (cos(2*b*x
+ a + 2*c)^2*sin(-a + c) - 2*cos(2*b*x + a + 2*c)*cos(a)*sin(-a + c) + sin(
2*b*x + a + 2*c)^2*sin(-a + c) - 2*sin(2*b*x + a + 2*c)*sin(a)*sin(-a + c)
+ (cos(a)^2 + sin(a)^2)*sin(-a + c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + c
os(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) + (cos(2*b*x + a + 2*c
)^2*sin(-a + c) - 2*cos(2*b*x + a + 2*c)*cos(a)*sin(-a + c) + sin(2*b*x + a
 + 2*c)^2*sin(-a + c) - 2*sin(2*b*x + a + 2*c)*sin(a)*sin(-a + c) + (cos(a)
^2 + sin(a)^2)*sin(-a + c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 +
 sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) - 2*(cos(b*x + 2*a) + cos(b*x +
 2*c))*sin(2*b*x + a + 2*c) - 2*cos(a)*sin(b*x + 2*a) - 2*cos(a)*sin(b*x +
2*c) + 2*cos(b*x + 2*a)*sin(a) + 2*cos(b*x + 2*c)*sin(a))/(b*cos(2*b*x + a
+ 2*c)^2 - 2*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(2*b*x + a + 2*c)^2 - 2*b
*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b)
```

Fricas [B] time = 0.517224, size = 203, normalized size = 5.8

$$-\frac{\log\left(\frac{1}{2}\cos(bx+c)+\frac{1}{2}\right)\sin(bx+c)\sin(-a+c)-\log\left(-\frac{1}{2}\cos(bx+c)+\frac{1}{2}\right)\sin(bx+c)\sin(-a+c)+2\cos(-a+}{2b\sin(bx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c)*sin(-a + c) - log(-1/2*cos(b
*x + c) + 1/2)*sin(b*x + c)*sin(-a + c) + 2*cos(-a + c))/(b*sin(b*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.3323, size = 1326, normalized size = 37.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*(tan(1/2*a)^3*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^3 + 2*tan(1/2*
a)^2*tan(1/2*c) - 2*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c))*log(
abs(tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c) + tan(1/2*b*x + 1/2*a) - tan
(1/2*a) + tan(1/2*c)))/(tan(1/2*a)^3*tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c)
 + tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)^2 + tan
(1/2*a)*tan(1/2*c) + tan(1/2*c)^2 + 1) - 4*(tan(1/2*a)^3*tan(1/2*c) - 2*tan
(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)^2 - 2*tan(1/2
*a)*tan(1/2*c) + tan(1/2*c)^2)*log(abs(tan(1/2*b*x + 1/2*a)*tan(1/2*a) - ta
n(1/2*b*x + 1/2*a)*tan(1/2*c) + tan(1/2*a)*tan(1/2*c) + 1))/(tan(1/2*a)^3*t
an(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^3 - tan(1/2*a)^2*tan(1
/2*c) + tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*c)^3 + tan(1/2*a) - tan(1/2*c)) -
 (tan(1/2*b*x + 1/2*a)*tan(1/2*a)^4*tan(1/2*c)^4 - 2*tan(1/2*b*x + 1/2*a)*t
an(1/2*a)^4*tan(1/2*c)^2 + 8*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^3*tan(1/2*c)^3
 - 2*tan(1/2*a)^4*tan(1/2*c)^3 - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/
2*c)^4 + 2*tan(1/2*a)^3*tan(1/2*c)^4 + tan(1/2*b*x + 1/2*a)*tan(1/2*a)^4 -
8*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^3*tan(1/2*c) + 2*tan(1/2*a)^4*tan(1/2*c)
+ 20*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c)^2 - 12*tan(1/2*a)^3*tan(1
/2*c)^2 - 8*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^3 + 12*tan(1/2*a)^2*
tan(1/2*c)^3 + tan(1/2*b*x + 1/2*a)*tan(1/2*c)^4 - 2*tan(1/2*a)*tan(1/2*c)^
4 - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2 + 2*tan(1/2*a)^3 + 8*tan(1/2*b*x +
1/2*a)*tan(1/2*a)*tan(1/2*c) - 12*tan(1/2*a)^2*tan(1/2*c) - 2*tan(1/2*b*x +
 1/2*a)*tan(1/2*c)^2 + 12*tan(1/2*a)*tan(1/2*c)^2 - 2*tan(1/2*c)^3 + tan(1/
2*b*x + 1/2*a) - 2*tan(1/2*a) + 2*tan(1/2*c))/((tan(1/2*b*x + 1/2*a)^2*tan(
1/2*a)^2*tan(1/2*c) - tan(1/2*b*x + 1/2*a)^2*tan(1/2*a)*tan(1/2*c)^2 + tan(
1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*b*x + 1/2*a)^2*tan(1/2
*a) - tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2 - tan(1/2*b*x + 1/2*a)^2*tan(1/2*c)
 + 4*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c) - tan(1/2*a)^2*tan(1/2*c) -
 tan(1/2*b*x + 1/2*a)*tan(1/2*c)^2 + tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*b*x
+ 1/2*a) - tan(1/2*a) + tan(1/2*c))*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*t
an(1/2*c)^2 + tan(1/2*a) - tan(1/2*c))))/b
```

3.229 $\int \cos(a+bx)\csc^3(c+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\sin(a-c)\cot(bx+c)}{b}-\frac{\cos(a-c)\csc^2(bx+c)}{2b}$$

```
[Out] -(Cos[a - c]*Csc[c + b*x]^2)/(2*b) + (Cot[c + b*x]*Sin[a - c])/b
```

Rubi [A] time = 0.0425368, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4581, 2606, 30, 3767, 8}

$$\frac{\sin(a-c)\cot(bx+c)}{b}-\frac{\cos(a-c)\csc^2(bx+c)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Csc[c + b*x]^3,x]
```

```
[Out] -(Cos[a - c]*Csc[c + b*x]^2)/(2*b) + (Cot[c + b*x]*Sin[a - c])/b
```

Rule 4581

```
Int[Cos[v_]*Csc[w_]^(n_.), x_Symbol] :> Dist[Cos[v - w], Int[Cot[w]*Csc[w]^
(n - 1), x], x] - Dist[Sin[v - w], Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos(a+bx)\csc^3(c+bx)\,dx = \cos(a-c)\int \cot(c+bx)\csc^2(c+bx)\,dx - \sin(a-c)\int \csc^2(c+bx)\,dx$$
$$= -\frac{\cos(a-c)\,\text{Subst}\left(\int x\,dx, x, \csc(c+bx)\right)}{b} + \frac{\sin(a-c)\,\text{Subst}\left(\int 1\,dx, x, \cot(c+bx)\right)}{b}$$
$$= -\frac{\cos(a-c)\csc^2(c+bx)}{2b} + \frac{\cot(c+bx)\sin(a-c)}{b}$$

Mathematica [A] time = 0.196821, size = 35, normalized size = 0.92

$$-\frac{\csc(c)\csc^2(bx+c)(\sin(a)-\sin(a-c)\cos(2bx+c))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Csc[c + b*x]^3,x]
```

```
[Out] -(Csc[c]*Csc[c + b*x]^2*(Sin[a] - Cos[c + 2*b*x]*Sin[a - c]))/(2*b)
```

Maple [A] time = 0.544, size = 55, normalized size = 1.5

$$-\frac{1}{2\,b\,(\cos(a)\cos(c)+\sin(a)\sin(c))\,(\tan(bx+a)\cos(a)\cos(c)+\tan(bx+a)\sin(a)\sin(c)+\cos(a)\sin(c)-\sin(a)\cos(c)) ...}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(b*x+c)^3,x)
```

```
[Out] -1/2/b/(cos(a)*cos(c)+sin(a)*sin(c))/(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*s
in(a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c))^2
```

Maxima [B] time = 1.12199, size = 533, normalized size = 14.03

$$\frac{(2\cos(2bx+2a+2c)-\cos(2a)+\cos(2c))\cos(4bx+a+5c)-2(2\cos(2bx+2a+2c)-\cos(2a)+\cos(2c))\cos ...}{b\cos(4bx+a+5c)^2+4b\cos(2bx+a+3c)^2-4b\cos(2bx+a+3c)\cos ...}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] ((2*cos(2*b*x + 2*a + 2*c) - cos(2*a) + cos(2*c))*cos(4*b*x + a + 5*c) - 2*
(2*cos(2*b*x + 2*a + 2*c) - cos(2*a) + cos(2*c))*cos(2*b*x + a + 3*c) - (co
s(2*a) - cos(2*c))*cos(a + c) + 2*cos(2*b*x + 2*a + 2*c)*cos(a + c) + (2*si
n(2*b*x + 2*a + 2*c) - sin(2*a) + sin(2*c))*sin(4*b*x + a + 5*c) - 2*(2*sin
(2*b*x + 2*a + 2*c) - sin(2*a) + sin(2*c))*sin(2*b*x + a + 3*c) - (sin(2*a)
 - sin(2*c))*sin(a + c) + 2*sin(2*b*x + 2*a + 2*c)*sin(a + c))/(b*cos(4*b*x
 + a + 5*c)^2 + 4*b*cos(2*b*x + a + 3*c)^2 - 4*b*cos(2*b*x + a + 3*c)*cos(a
 + c) + b*cos(a + c)^2 + b*sin(4*b*x + a + 5*c)^2 + 4*b*sin(2*b*x + a + 3*c
)^2 - 4*b*sin(2*b*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 - 2*(2*b*cos(2*b
*x + a + 3*c) - b*cos(a + c))*cos(4*b*x + a + 5*c) - 2*(2*b*sin(2*b*x + a +
 3*c) - b*sin(a + c))*sin(4*b*x + a + 5*c))
```

Fricas [A] time = 0.469258, size = 113, normalized size = 2.97

$$\frac{2\cos(bx+c)\sin(bx+c)\sin(-a+c)+\cos(-a+c)}{2\left(b\cos(bx+c)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*cos(b*x + c)*sin(b*x + c)*sin(-a + c) + cos(-a + c))/(b*cos(b*x + c)
^2 - b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26156, size = 441, normalized size = 11.61

$$-\frac{\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^6+3\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^4+3\tan\left(\frac{1}{2}a\right)^4\tan\left(\frac{1}{2}c\right)^6+3\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^2+9\tan\left(\frac{1}{2}a\right)^4\cdots}{2\left(\tan(bx+a)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-\tan(bx+a)\tan\left(\frac{1}{2}a\right)^2+4\tan(bx+a)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*(tan(1/2*a)^6*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 3*tan(1/2*a
)^4*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^4*tan(1/2*c)^
4 + 3*tan(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*a)^6 + 9*tan(1/2*a)^4*tan(1/2*c)^
2 + 9*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*c)^6 + 3*tan(1/2*a)^4 + 9*tan(1/2
*a)^2*tan(1/2*c)^2 + 3*tan(1/2*c)^4 + 3*tan(1/2*a)^2 + 3*tan(1/2*c)^2 + 1)/
((tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c)^2 - tan(b*x + a)*tan(1/2*a)^2 + 4*ta
n(b*x + a)*tan(1/2*a)*tan(1/2*c) - 2*tan(1/2*a)^2*tan(1/2*c) - tan(b*x + a)
*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/2*c)^2 + tan(b*x + a) - 2*tan(1/2*a) + 2
*tan(1/2*c))^2*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan
(1/2*c) - tan(1/2*c)^2 + 1)*b)
```

3.230 $\int \sin(a+bx)\tan^3(c+bx)\,dx$

Optimal. Leaf size=72

$$\frac{\sin(a-c)\sec(bx+c)}{b}-\frac{3\cos(a-c)\tanh^{-1}(\sin(bx+c))}{2b}+\frac{\cos(a-c)\tan(bx+c)\sec(bx+c)}{2b}+\frac{\sin(a+bx)}{b}$$

```
[Out] (-3*ArcTanh[Sin[c + b*x]]*Cos[a - c])/(2*b) + (Sec[c + b*x]*Sin[a - c])/b +
 Sin[a + b*x]/b + (Cos[a - c]*Sec[c + b*x]*Tan[c + b*x])/(2*b)
```

Rubi [A] time = 0.0707118, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {4576, 4579, 2637, 3770, 2606, 8, 2611}

$$\frac{\sin(a-c)\sec(bx+c)}{b}-\frac{3\cos(a-c)\tanh^{-1}(\sin(bx+c))}{2b}+\frac{\cos(a-c)\tan(bx+c)\sec(bx+c)}{2b}+\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Tan[c + b*x]^3,x]
```

```
[Out] (-3*ArcTanh[Sin[c + b*x]]*Cos[a - c])/(2*b) + (Sec[c + b*x]*Sin[a - c])/b +
 Sin[a + b*x]/b + (Cos[a - c]*Sec[c + b*x]*Tan[c + b*x])/(2*b)
```

Rule 4576

```
Int[Sin[v_]*Tan[w_]^(n_.), x_Symbol] :> -Int[Cos[v]*Tan[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Rule 4579

```
Int[Cos[v_]*Tan[w_]^(n_.), x_Symbol] :> Int[Sin[v]*Tan[w]^(n - 1), x] - Dis
t[Sin[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \sin(a+bx)\tan^3(c+bx)\,dx = \cos(a-c)\int \sec(c+bx)\tan^2(c+bx)\,dx - \int \cos(a+bx)\tan^2(c+bx)\,dx$$
$$= \frac{\cos(a-c)\sec(c+bx)\tan(c+bx)}{2b} - \frac{1}{2}\cos(a-c)\int \sec(c+bx)\,dx + \sin(a-c)\int \mathrm{se}$$
$$= -\frac{\tanh^{-1}(\sin(c+bx))\cos(a-c)}{2b} + \frac{\cos(a-c)\sec(c+bx)\tan(c+bx)}{2b} - \cos(a-c)\int$$
$$= -\frac{3\tanh^{-1}(\sin(c+bx))\cos(a-c)}{2b} + \frac{\sec(c+bx)\sin(a-c)}{b} + \frac{\sin(a+bx)}{b} + \frac{\cos(a-}{}$$

Mathematica [A] time = 0.363034, size = 70, normalized size = 0.97

$$\frac{\sec^2(bx+c)(2\sin(a-bx-2c)+\sin(a+3bx+2c)+5\sin(a+bx)) - 12\cos(a-c)\tanh^{-1}\left(\cos(c)\tan\left(\frac{bx}{2}\right)+\sin(c)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Tan[c + b*x]^3,x]
```

```
[Out] (-12*ArcTanh[Sin[c] + Cos[c]*Tan[(b*x)/2]]*Cos[a - c] + Sec[c + b*x]^2*(2*S
in[a - 2*c - b*x] + 5*Sin[a + b*x] + Sin[a + 2*c + 3*b*x]))/(4*b)
```

Maple [C] time = 0.115, size = 186, normalized size = 2.6

$$\frac{-\frac{i}{2}e^{i(bx+a)}}{b} + \frac{\frac{i}{2}e^{-i(bx+a)}}{b} - \frac{\frac{i}{2}\left(3\,e^{i(3\,bx+5\,a+2\,c)} - e^{i(3\,bx+3\,a+4\,c)} + e^{i(bx+5\,a)} - 3\,e^{i(bx+3\,a+2\,c)}\right)}{b\left(e^{2\,i(bx+a+c)} + e^{2\,ia}\right)^2} + \frac{3\,\ln\left(e^{i(bx+a)} - ie^{i(a-c)}\right)\cos(a}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*tan(b*x+c)^3,x)
```

```
[Out] -1/2*I*exp(I*(b*x+a))/b+1/2*I/b*exp(-I*(b*x+a))-1/2*I/b/(exp(2*I*(b*x+a+c))
+exp(2*I*a))^2*(3*exp(I*(3*b*x+5*a+2*c))-exp(I*(3*b*x+3*a+4*c))+exp(I*(b*x+
5*a))-3*exp(I*(b*x+3*a+2*c)))+3/2/b*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))*cos(a
-c)-3/2/b*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))*cos(a-c)
```

Maxima [B] time = 1.97578, size = 1386, normalized size = 19.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(sin(5*b*x + a + 4*c) + 2*sin(3*b*x + a + 2*c) + sin(b*x + a))*cos(
6*b*x + 2*a + 4*c) - 2*(5*sin(4*b*x + 2*a + 2*c) - 2*sin(4*b*x + 4*c) + 2*s
in(2*b*x + 2*a) - 5*sin(2*b*x + 2*c))*cos(5*b*x + a + 4*c) + 10*(2*sin(3*b*
x + a + 2*c) + sin(b*x + a))*cos(4*b*x + 2*a + 2*c) - 4*(2*sin(3*b*x + a +
2*c) + sin(b*x + a))*cos(4*b*x + 4*c) - 4*(2*sin(2*b*x + 2*a) - 5*sin(2*b*x
 + 2*c))*cos(3*b*x + a + 2*c) - 3*(cos(5*b*x + a + 4*c)^2*cos(-a + c) + 4*c
os(3*b*x + a + 2*c)^2*cos(-a + c) + 4*cos(3*b*x + a + 2*c)*cos(b*x + a)*cos
(-a + c) + cos(b*x + a)^2*cos(-a + c) + cos(-a + c)*sin(5*b*x + a + 4*c)^2
+ 4*cos(-a + c)*sin(3*b*x + a + 2*c)^2 + 4*cos(-a + c)*sin(3*b*x + a + 2*c)
*sin(b*x + a) + cos(-a + c)*sin(b*x + a)^2 + 2*(2*cos(3*b*x + a + 2*c)*cos(
-a + c) + cos(b*x + a)*cos(-a + c))*cos(5*b*x + a + 4*c) + 2*(2*cos(-a + c)
*sin(3*b*x + a + 2*c) + cos(-a + c)*sin(b*x + a))*sin(5*b*x + a + 4*c))*log
((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2
+ 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos(
c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2))
 - 2*(cos(5*b*x + a + 4*c) + 2*cos(3*b*x + a + 2*c) + cos(b*x + a))*sin(6*b
*x + 2*a + 4*c) + 2*(5*cos(4*b*x + 2*a + 2*c) - 2*cos(4*b*x + 4*c) + 2*cos(
2*b*x + 2*a) - 5*cos(2*b*x + 2*c) - 1)*sin(5*b*x + a + 4*c) - 10*(2*cos(3*b
*x + a + 2*c) + cos(b*x + a))*sin(4*b*x + 2*a + 2*c) + 4*(2*cos(3*b*x + a +
 2*c) + cos(b*x + a))*sin(4*b*x + 4*c) + 4*(2*cos(2*b*x + 2*a) - 5*cos(2*b*
x + 2*c) - 1)*sin(3*b*x + a + 2*c) - 4*cos(b*x + a)*sin(2*b*x + 2*a) + 10*c
os(b*x + a)*sin(2*b*x + 2*c) + 4*cos(2*b*x + 2*a)*sin(b*x + a) - 10*cos(2*b
*x + 2*c)*sin(b*x + a) - 2*sin(b*x + a))/(b*cos(5*b*x + a + 4*c)^2 + 4*b*co
s(3*b*x + a + 2*c)^2 + 4*b*cos(3*b*x + a + 2*c)*cos(b*x + a) + b*cos(b*x +
a)^2 + b*sin(5*b*x + a + 4*c)^2 + 4*b*sin(3*b*x + a + 2*c)^2 + 4*b*sin(3*b*
x + a + 2*c)*sin(b*x + a) + b*sin(b*x + a)^2 + 2*(2*b*cos(3*b*x + a + 2*c)
+ b*cos(b*x + a))*cos(5*b*x + a + 4*c) + 2*(2*b*sin(3*b*x + a + 2*c) + b*si
n(b*x + a))*sin(5*b*x + a + 4*c))
```

Fricas [B] time = 0.564859, size = 1015, normalized size = 14.1

$$-\frac{\frac{3\sqrt{2}\left(2(\cos(-2a+2c)+1)\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-2\left(\cos(-2a+2c)^2+\cos(-2a+2c)\right)\cos(bx+a)^2+\cos(-2a+2c)^2-1\right)\log\left(-\frac{2\cos(bx+a)^2\cos(-2a+2c)-\cdots}{\cdots}\right.}{\sqrt{\cos(-2a+2c)+1}}}{8\left(2b\cos(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(3*sqrt(2)*(2*(cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(b*x + a)*sin(-2*a
 + 2*c) - 2*(cos(-2*a + 2*c)^2 + cos(-2*a + 2*c))*cos(b*x + a)^2 + cos(-2*a
 + 2*c)^2 - 1)*log(-(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(
b*x + a)*sin(-2*a + 2*c) + 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*sin(b*x + a) +
cos(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2*a + 2*c) -
 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*
a + 2*c) - cos(-2*a + 2*c) + 1))/sqrt(cos(-2*a + 2*c) + 1) - 4*(4*cos(b*x +
 a)^2*cos(-2*a + 2*c) - 3*cos(-2*a + 2*c) + 5)*sin(b*x + a) - 4*(4*cos(b*x
+ a)^3 - 5*cos(b*x + a))*sin(-2*a + 2*c))/(2*b*cos(b*x + a)^2*cos(-2*a + 2*
c) - 2*b*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - b*cos(-2*a + 2*c) + b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(bx+a)\tan(bx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)*tan(b*x + c)^3, x)
```

3.231 $\int \sin(a + bx) \tan^2(c + bx) \, dx$

Optimal. Leaf size=44

$$\frac{\sin(a-c)\tanh^{-1}(\sin(bx+c))}{b}+\frac{\cos(a-c)\sec(bx+c)}{b}+\frac{\cos(a+bx)}{b}$$

```
[Out] Cos[a + b*x]/b + (Cos[a - c]*Sec[c + b*x])/b + (ArcTanh[Sin[c + b*x]]*Sin[a
 - c])/b
```

Rubi [A] time = 0.035963, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4576, 4579, 2638, 3770, 2606, 8}

$$\frac{\sin(a-c)\tanh^{-1}(\sin(bx+c))}{b}+\frac{\cos(a-c)\sec(bx+c)}{b}+\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Tan[c + b*x]^2,x]
```

```
[Out] Cos[a + b*x]/b + (Cos[a - c]*Sec[c + b*x])/b + (ArcTanh[Sin[c + b*x]]*Sin[a
 - c])/b
```

Rule 4576

```
Int[Sin[v_]*Tan[w_]^(n_.), x_Symbol] :> -Int[Cos[v]*Tan[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Rule 4579

```
Int[Cos[v_]*Tan[w_]^(n_.), x_Symbol] :> Int[Sin[v]*Tan[w]^(n - 1), x] - Dis
t[Sin[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sin(a+bx)\tan^2(c+bx)\,dx = \cos(a-c)\int \sec(c+bx)\tan(c+bx)\,dx - \int \cos(a+bx)\tan(c+bx)\,dx$$
$$= \frac{\cos(a-c)\,\mathrm{Subst}(\int 1\,dx, x, \sec(c+bx))}{b} + \sin(a-c)\int \sec(c+bx)\,dx - \int \sin(a+b$$
$$= \frac{\cos(a+bx)}{b} + \frac{\cos(a-c)\sec(c+bx)}{b} + \frac{\tanh^{-1}(\sin(c+bx))\sin(a-c)}{b}$$

Mathematica [C] time = 0.10031, size = 109, normalized size = 2.48

$$\frac{\cos(a-c)\sec(bx+c)}{b} - \frac{2i\sin(a-c)\tan^{-1}\left(\frac{(\sin(c)+i\cos(c))\left(\sin(c)\cos\left(\frac{bx}{2}\right)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\sin(c)\cos\left(\frac{bx}{2}\right)}\right)}{b} - \frac{\sin(a)\sin(bx)}{b} + \frac{\cos(a)\cos(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Tan[c + b*x]^2,x]
```

```
[Out] (Cos[a]*Cos[b*x])/b + (Cos[a - c]*Sec[c + b*x])/b - ((2*I)*ArcTan[((I*Cos[c
] + Sin[c])*(Cos[(b*x)/2]*Sin[c] + Cos[c]*Sin[(b*x)/2]))/(Cos[c]*Cos[(b*x)/
2] - I*Cos[(b*x)/2]*Sin[c])]*Sin[a - c])/b - (Sin[a]*Sin[b*x])/b
```

Maple [C] time = 0.083, size = 143, normalized size = 3.3

$$\frac{e^{i(bx+a)}}{2b} + \frac{e^{-i(bx+a)}}{2b} + \frac{e^{i(bx+3a)} + e^{i(bx+a+2c)}}{b\left(e^{2i(bx+a+c)} + e^{2ia}\right)} - \frac{\ln\left(e^{i(bx+a)} - ie^{i(a-c)}\right)\sin(a-c)}{b} + \frac{\ln\left(e^{i(bx+a)} + ie^{i(a-c)}\right)\sin(a-c)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*tan(b*x+c)^2,x)
```

```
[Out] 1/2*exp(I*(b*x+a))/b+1/2/b*exp(-I*(b*x+a))+1/b/(exp(2*I*(b*x+a+c))+exp(2*I*
a))*(exp(I*(b*x+3*a))+exp(I*(b*x+a+2*c)))-1/b*ln(exp(I*(b*x+a))-I*exp(I*(a-
c)))*sin(a-c)+1/b*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))*sin(a-c)
```

Maxima [B] time = 1.99207, size = 702, normalized size = 15.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((cos(3*b*x + a + 2*c) + cos(b*x + a))*cos(4*b*x + 2*a + 2*c) + (3*cos(
2*b*x + 2*a) + 3*cos(2*b*x + 2*c) + 1)*cos(3*b*x + a + 2*c) + 3*cos(2*b*x +
 2*a)*cos(b*x + a) + 3*cos(2*b*x + 2*c)*cos(b*x + a) + (cos(3*b*x + a + 2*c
)^2*sin(-a + c) + 2*cos(3*b*x + a + 2*c)*cos(b*x + a)*sin(-a + c) + cos(b*x
 + a)^2*sin(-a + c) + sin(3*b*x + a + 2*c)^2*sin(-a + c) + 2*sin(3*b*x + a
+ 2*c)*sin(b*x + a)*sin(-a + c) + sin(b*x + a)^2*sin(-a + c))*log((cos(b*x
+ 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 + 2*cos(b*
```

```
x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)*sin(b*x
 + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)) + (sin(3*
b*x + a + 2*c) + sin(b*x + a))*sin(4*b*x + 2*a + 2*c) + 3*(sin(2*b*x + 2*a)
 + sin(2*b*x + 2*c))*sin(3*b*x + a + 2*c) + 3*sin(2*b*x + 2*a)*sin(b*x + a)
 + 3*sin(2*b*x + 2*c)*sin(b*x + a) + cos(b*x + a))/(b*cos(3*b*x + a + 2*c)^
2 + 2*b*cos(3*b*x + a + 2*c)*cos(b*x + a) + b*cos(b*x + a)^2 + b*sin(3*b*x
+ a + 2*c)^2 + 2*b*sin(3*b*x + a + 2*c)*sin(b*x + a) + b*sin(b*x + a)^2)
```

Fricas [B] time = 0.538988, size = 857, normalized size = 19.48

$$-\frac{4\left(\cos\left(-2\,a+2\,c\right)+1\right)\cos\left(bx+a\right)^{2}-4\,\cos\left(bx+a\right)\sin\left(bx+a\right)\sin\left(-2\,a+2\,c\right)+\frac{\sqrt{2}\left(\left(\cos(-2\,a+2\,c)+1\right)\cos(bx+a)\sin(-2\,a+2\,c)+\cdots\right.}{\cdots}}{4\left(b\sin\left(bx+a\right)\sin\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*(cos(-2*a + 2*c) + 1)*cos(b*x + a)^2 - 4*cos(b*x + a)*sin(b*x + a)*
sin(-2*a + 2*c) + sqrt(2)*((cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(-2*a + 2*
c) + (cos(-2*a + 2*c)^2 - 1)*sin(b*x + a))*log(-(2*cos(b*x + a)^2*cos(-2*a
+ 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) + 2*sqrt(2)*((cos(-2*a
 + 2*c) + 1)*sin(b*x + a) + cos(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2
*c) + 1) - cos(-2*a + 2*c) - 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b
*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) + 1))/sqrt(cos(-2*a
+ 2*c) + 1) + 4*cos(-2*a + 2*c) + 4)/(b*sin(b*x + a)*sin(-2*a + 2*c) - (b*c
os(-2*a + 2*c) + b)*cos(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(a+bx\right)\tan^{2}\left(bx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)*tan(b*x + c)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(bx+a\right)\tan\left(bx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)*tan(b*x + c)^2, x)
```

3.232 $\int \sin(a+bx)\tan(c+bx)\,dx$

Optimal. Leaf size=29

$$\frac{\cos(a-c)\tanh^{-1}(\sin(bx+c))}{b}-\frac{\sin(a+bx)}{b}$$

```
[Out] (ArcTanh[Sin[c + b*x]]*Cos[a - c])/b - Sin[a + b*x]/b
```

Rubi [A] time = 0.015958, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4576, 2637, 3770}

$$\frac{\cos(a-c)\tanh^{-1}(\sin(bx+c))}{b}-\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Tan[c + b*x],x]

[Out] (ArcTanh[Sin[c + b*x]]*Cos[a - c])/b - Sin[a + b*x]/b
```

Rule 4576

```
Int[Sin[v_]*Tan[w_]^(n_.), x_Symbol] :> -Int[Cos[v]*Tan[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\tan(c+bx)\,dx &= \cos(a-c)\int \sec(c+bx)\,dx-\int \cos(a+bx)\,dx\\ &= \frac{\tanh^{-1}(\sin(c+bx))\cos(a-c)}{b}-\frac{\sin(a+bx)}{b}\end{aligned}$$

Mathematica [C] time = 0.0486601, size = 94, normalized size = 3.24

$$-\frac{2i\cos(a-c)\tan^{-1}\left(\frac{(\sin(c)+i\cos(c))\left(\sin(c)\cos\left(\frac{bx}{2}\right)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\sin(c)\cos\left(\frac{bx}{2}\right)}\right)}{b}-\frac{\sin(a)\cos(bx)}{b}-\frac{\cos(a)\sin(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Tan[c + b*x],x]
```

```
[Out] ((-2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x)/2]*Sin[c] + Cos[c]*Sin[(b*x)
/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b - (Cos[b
*x]*Sin[a])/b - (Cos[a]*Sin[b*x])/b
```

Maple [C] time = 0.066, size = 99, normalized size = 3.4

$$\frac{\frac{i}{2}e^{i(bx+a)}}{b} - \frac{\frac{i}{2}e^{-i(bx+a)}}{b} - \frac{\ln\left(e^{i(bx+a)} - ie^{i(a-c)}\right)\cos(a-c)}{b} + \frac{\ln\left(e^{i(bx+a)} + ie^{i(a-c)}\right)\cos(a-c)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*tan(b*x+c),x)
```

```
[Out] 1/2*I*exp(I*(b*x+a))/b-1/2*I/b*exp(-I*(b*x+a))-1/b*ln(exp(I*(b*x+a))-I*exp(
I*(a-c)))*cos(a-c)+1/b*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))*cos(a-c)
```

Maxima [B] time = 1.91388, size = 177, normalized size = 6.1

$$-\frac{\cos(-a+c)\log\left(\frac{\cos(bx+2c)^2+\cos(c)^2-2\cos(c)\sin(bx+2c)+\sin(bx+2c)^2+2\cos(bx+2c)\sin(c)+\sin(c)^2}{\cos(bx+2c)^2+\cos(c)^2+2\cos(c)\sin(bx+2c)+\sin(bx+2c)^2-2\cos(bx+2c)\sin(c)+\sin(c)^2}\right)+2\sin(bx+a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(cos(-a + c)*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c
) + sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^
2 + cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c
)*sin(c) + sin(c)^2)) + 2*sin(b*x + a))/b
```

Fricas [B] time = 0.516937, size = 509, normalized size = 17.55

$$\frac{\sqrt{2}\sqrt{\cos(-2a+2c)+1}\log\left(\frac{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\frac{2\sqrt{2}((\cos(-2a+2c)+1)\sin(bx+a)+\cos(bx+a)\sin(-2a+2c))}{\sqrt{\cos(-2a+2c)+1}}-c}{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\cos(-2a+2c)+1}\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*sqrt(cos(-2*a + 2*c) + 1)*log((2*cos(b*x + a)^2*cos(-2*a + 2*c
) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a + 2*
c) + 1)*sin(b*x + a) + cos(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) +
 1) - cos(-2*a + 2*c) - 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x +
a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) + 1)) - 4*sin(b*x + a))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(a+bx)\tan(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c),x)
```

```
[Out] Integral(sin(a + b*x)*tan(b*x + c), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(bx+a)\tan(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)*tan(b*x + c), x)
```

3.233 $\int \cot(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=29

$$\frac{\sin(a+bx)}{b}-\frac{\sin(a-c)\tanh^{-1}(\cos(bx+c))}{b}$$

```
[Out] -((ArcTanh[Cos[c + b*x]]*Sin[a - c])/b) + Sin[a + b*x]/b
```

Rubi [A] time = 0.0155587, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4578, 2637, 3770}

$$\frac{\sin(a+bx)}{b}-\frac{\sin(a-c)\tanh^{-1}(\cos(bx+c))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + b*x]*Sin[a + b*x],x]
```

```
[Out] -((ArcTanh[Cos[c + b*x]]*Sin[a - c])/b) + Sin[a + b*x]/b
```

Rule 4578

```
Int[Cot[w_]^(n_.)*Sin[v_], x_Symbol] :> Int[Cos[v]*Cot[w]^(n - 1), x] + Dis
t[Sin[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot(c+bx)\sin(a+bx)\,dx &= \sin(a-c)\int \csc(c+bx)\,dx+\int \cos(a+bx)\,dx\\ &= -\frac{\tanh^{-1}(\cos(c+bx))\sin(a-c)}{b}+\frac{\sin(a+bx)}{b}\end{aligned}$$

Mathematica [C] time = 0.0525759, size = 93, normalized size = 3.21

$$-\frac{2i\sin(a-c)\tan^{-1}\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{\sin(c)\cos\left(\frac{bx}{2}\right)+i\cos(c)\cos\left(\frac{bx}{2}\right)}\right)}{b}+\frac{\sin(a)\cos(bx)}{b}+\frac{\cos(a)\sin(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + b*x]*Sin[a + b*x],x]
```

```
[Out] (Cos[b*x]*Sin[a])/b - ((2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c]*Cos[(b*x)/
2] - Sin[c]*Sin[(b*x)/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/2]*Sin[c])]*S
in[a - c])/b + (Cos[a]*Sin[b*x])/b
```

Maple [C] time = 0.085, size = 95, normalized size = 3.3

$$\frac{-\frac{i}{2}e^{i(bx+a)}}{b}+\frac{\frac{i}{2}e^{-i(bx+a)}}{b}+\frac{\ln\left(e^{i(bx+a)}-e^{i(a-c)}\right)\sin(a-c)}{b}-\frac{\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)\sin(a-c)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+c)*sin(b*x+a),x)
```

```
[Out] -1/2*I*exp(I*(b*x+a))/b+1/2*I/b*exp(-I*(b*x+a))+1/b*ln(exp(I*(b*x+a))-exp(I
*(a-c)))*sin(a-c)-1/b*ln(exp(I*(b*x+a))+exp(I*(a-c)))*sin(a-c)
```

Maxima [B] time = 1.16384, size = 142, normalized size = 4.9

$$\frac{\log\left(\cos(bx)^2+2\cos(bx)\cos(c)+\cos(c)^2+\sin(bx)^2-2\sin(bx)\sin(c)+\sin(c)^2\right)\sin(-a+c)-\log\left(\cos(bx)^2-2\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x
)*sin(c) + sin(c)^2)*sin(-a + c) - log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos
(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2)*sin(-a + c) + 2*sin(b*x
+ a))/b
```

Fricas [B] time = 0.53109, size = 531, normalized size = 18.31

$$\frac{\frac{\sqrt{2}\log\left(\frac{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)+\frac{2\sqrt{2}((\cos(-2a+2c)+1)\cos(bx+a)-\sin(bx+a)\sin(-2a+2c))}{\sqrt{\cos(-2a+2c)+1}}-\cos(-2a+2c)+3}{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\cos(-2a+2c)-1}\right)\sin(-2a+2c)}{\sqrt{\cos(-2a+2c)+1}}+4\,\mathrm{si}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*log((2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x
 + a)*sin(-2*a + 2*c) + 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*cos(b*x + a) - sin
(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2*a + 2*c) + 3)
/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a +
 2*c) - cos(-2*a + 2*c) - 1))*sin(-2*a + 2*c)/sqrt(cos(-2*a + 2*c) + 1) + 4
*sin(b*x + a))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(a+bx)\cot(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)*sin(b*x+a),x)
```

```
[Out] Integral(sin(a + b*x)*cot(b*x + c), x)
```

Giac [B] time = 1.19333, size = 305, normalized size = 10.52

$$-\frac{2\left(\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}c\right)^2\right)\log\left(\left|\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}c\right)-1\right|\right)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)+\tan\left(\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}c\right)}-\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2+t\cdots\right.}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\cdots}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -2*((tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)*tan(1
/2*c) - tan(1/2*c)^2)*log(abs(tan(1/2*b*x)*tan(1/2*c) - 1))/(tan(1/2*a)^2*t
an(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c) + tan(1/2*c)^3 + tan(1/2*c)) - (tan(1
/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c))*log
(abs(tan(1/2*b*x) + tan(1/2*c)))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2
+ tan(1/2*c)^2 + 1) + (tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*b*x) - 2*tan(1/2
*a))/((tan(1/2*b*x)^2 + 1)*(tan(1/2*a)^2 + 1)))/b
```

3.234 $\int \cot^2(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\cos(a-c)\tanh^{-1}(\cos(bx+c))}{b}-\frac{\sin(a-c)\csc(bx+c)}{b}+\frac{\cos(a+bx)}{b}$$

```
[Out] -((ArcTanh[Cos[c + b*x]]*Cos[a - c])/b) + Cos[a + b*x]/b - (Csc[c + b*x]*Si
n[a - c])/b
```

Rubi [A] time = 0.0404453, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4578, 4577, 2638, 3770, 2606, 8}

$$-\frac{\cos(a-c)\tanh^{-1}(\cos(bx+c))}{b}-\frac{\sin(a-c)\csc(bx+c)}{b}+\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -((ArcTanh[Cos[c + b*x]]*Cos[a - c])/b) + Cos[a + b*x]/b - (Csc[c + b*x]*Si
n[a - c])/b
```

Rule 4578

```
Int[Cot[w_]^(n_.)*Sin[v_], x_Symbol] :> Int[Cos[v]*Cot[w]^(n - 1), x] + Dis
t[Sin[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rule 4577

```
Int[Cos[v_]*Cot[w_]^(n_.), x_Symbol] :> -Int[Sin[v]*Cot[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c+bx)\sin(a+bx)\,dx &= \sin(a-c)\int \cot(c+bx)\csc(c+bx)\,dx + \int \cos(a+bx)\cot(c+bx)\,dx\\ &= \cos(a-c)\int \csc(c+bx)\,dx - \frac{\sin(a-c)\,\mathrm{Subst}\left(\int 1\,dx, x, \csc(c+bx)\right)}{b} - \int \sin(a+bx)\,d\\ &= -\frac{\tanh^{-1}(\cos(c+bx))\cos(a-c)}{b} + \frac{\cos(a+bx)}{b} - \frac{\csc(c+bx)\sin(a-c)}{b}\end{aligned}$$

Mathematica [C] time = 0.0972564, size = 111, normalized size = 2.41

$$-\frac{\sin(a-c)\csc(bx+c)}{b} - \frac{2i\cos(a-c)\tan^{-1}\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{\sin(c)\cos\left(\frac{bx}{2}\right)+i\cos(c)\cos\left(\frac{bx}{2}\right)}\right)}{b} - \frac{\sin(a)\sin(bx)}{b} + \frac{\cos(a)\cos(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + b*x]^2*Sin[a + b*x],x]
```

```
[Out] ((-2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c]*Cos[(b*x)/2] - Sin[c]*Sin[(b*x)
/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b + (Cos[a
]*Cos[b*x])/b - (Csc[c + b*x]*Sin[a - c])/b - (Sin[a]*Sin[b*x])/b
```

Maple [C] time = 0.089, size = 143, normalized size = 3.1

$$\frac{e^{i(bx+a)}}{2b} + \frac{e^{-i(bx+a)}}{2b} + \frac{e^{i(bx+3a)} - e^{i(bx+a+2c)}}{b\left(-e^{2i(bx+a+c)} + e^{2ia}\right)} - \frac{\ln\left(e^{i(bx+a)} + e^{i(a-c)}\right)\cos(a-c)}{b} + \frac{\ln\left(e^{i(bx+a)} - e^{i(a-c)}\right)\cos(a-c)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+c)^2*sin(b*x+a),x)
```

```
[Out] 1/2*exp(I*(b*x+a))/b+1/2/b*exp(-I*(b*x+a))+1/b/(-exp(2*I*(b*x+a+c))+exp(2*I
*a))*(exp(I*(b*x+3*a))-exp(I*(b*x+a+2*c)))-1/b*ln(exp(I*(b*x+a))+exp(I*(a-c
)))*cos(a-c)+1/b*ln(exp(I*(b*x+a))-exp(I*(a-c)))*cos(a-c)
```

Maxima [B] time = 1.33538, size = 826, normalized size = 17.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*((cos(3*b*x + a + 2*c) - cos(b*x + a))*cos(4*b*x + 2*a + 2*c) - (3*cos(
2*b*x + 2*a) - 3*cos(2*b*x + 2*c) + 1)*cos(3*b*x + a + 2*c) + 3*cos(2*b*x +
 2*a)*cos(b*x + a) - 3*cos(2*b*x + 2*c)*cos(b*x + a) - (cos(3*b*x + a + 2*c
)^2*cos(-a + c) - 2*cos(3*b*x + a + 2*c)*cos(b*x + a)*cos(-a + c) + cos(b*x
 + a)^2*cos(-a + c) + cos(-a + c)*sin(3*b*x + a + 2*c)^2 - 2*cos(-a + c)*si
n(3*b*x + a + 2*c)*sin(b*x + a) + cos(-a + c)*sin(b*x + a)^2)*log(cos(b*x)^
2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^
```

```
2) + (cos(3*b*x + a + 2*c)^2*cos(-a + c) - 2*cos(3*b*x + a + 2*c)*cos(b*x +
 a)*cos(-a + c) + cos(b*x + a)^2*cos(-a + c) + cos(-a + c)*sin(3*b*x + a +
2*c)^2 - 2*cos(-a + c)*sin(3*b*x + a + 2*c)*sin(b*x + a) + cos(-a + c)*sin(
b*x + a)^2)*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*
sin(b*x)*sin(c) + sin(c)^2) + (sin(3*b*x + a + 2*c) - sin(b*x + a))*sin(4*b
*x + 2*a + 2*c) - 3*(sin(2*b*x + 2*a) - sin(2*b*x + 2*c))*sin(3*b*x + a + 2
*c) + 3*sin(2*b*x + 2*a)*sin(b*x + a) - 3*sin(2*b*x + 2*c)*sin(b*x + a) + c
os(b*x + a))/(b*cos(3*b*x + a + 2*c)^2 - 2*b*cos(3*b*x + a + 2*c)*cos(b*x +
 a) + b*cos(b*x + a)^2 + b*sin(3*b*x + a + 2*c)^2 - 2*b*sin(3*b*x + a + 2*c
)*sin(b*x + a) + b*sin(b*x + a)^2)
```

Fricas [B] time = 0.553371, size = 859, normalized size = 18.67

$$\frac{4\left(\cos\left(-2\,a+2\,c\right)+1\right)\cos\left(bx+a\right)\sin\left(bx+a\right)+\frac{\sqrt{2}\left(\left(\cos(-2\,a+2\,c)+1\right)\cos(bx+a)\sin(-2\,a+2\,c)+\left(\cos(-2\,a+2\,c)^2+2\,\cos(-2\,a+2\,c)+1\right)\sin(\cdots}{\cdots}}{4\left(b\cos\left(bx+a\right)\sin\left(-\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(4*(cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*((cos(-2*a
 + 2*c) + 1)*cos(b*x + a)*sin(-2*a + 2*c) + (cos(-2*a + 2*c)^2 + 2*cos(-2*a
 + 2*c) + 1)*sin(b*x + a))*log(-(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b
*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*cos
(b*x + a) - sin(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-
2*a + 2*c) + 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x
+ a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) - 1))/sqrt(cos(-2*a + 2*c) + 1) + 4*
(cos(b*x + a)^2 + 1)*sin(-2*a + 2*c))/(b*cos(b*x + a)*sin(-2*a + 2*c) + (b*
cos(-2*a + 2*c) + b)*sin(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(a+bx\right)\cot^{2}\left(bx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)**2*sin(b*x+a),x)
```

```
[Out] Integral(sin(a + b*x)*cot(b*x + c)**2, x)
```

Giac [B] time = 1.20986, size = 779, normalized size = 16.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -((tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)^2*tan(1/2*c) + 4*tan(1/2*a)*tan(1
/2*c)^2 - tan(1/2*c)^3 + tan(1/2*c))*log(abs(tan(1/2*b*x)*tan(1/2*c) - 1))/
```

```
(tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c) + tan(1/2*c)^3 + tan(1
/2*c)) - (tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c
) - tan(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x) + tan(1/2*c)))/(tan(1/2*a)^2*tan
(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + (tan(1/2*b*x)^3*tan(1/2*a)^2
*tan(1/2*c)^3 - tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*c)^4 - tan(1/2*b*x)^3*tan
(1/2*a)^2*tan(1/2*c) + 6*tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*b
*x)^3*tan(1/2*c)^3 + 6*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*c)^3 + 3*tan(1/2*b
*x)*tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*b*x)*tan(1/2*a)*tan(1/2*c)^4 - tan(
1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^3*tan(1/2*c) - 6*tan(1/2*b*x)^2*tan(1/
2*a)*tan(1/2*c) - 3*tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*c) - 2*tan(1/2*b*x)*t
an(1/2*a)*tan(1/2*c)^2 - 4*tan(1/2*a)^2*tan(1/2*c)^2 - 3*tan(1/2*b*x)*tan(1
/2*c)^3 + 2*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*b*x)*tan(1/2*a) + 3*tan(1/2*b
*x)*tan(1/2*c) - 2*tan(1/2*a)*tan(1/2*c) + 4*tan(1/2*c)^2)/((tan(1/2*b*x)^4
*tan(1/2*c) + tan(1/2*b*x)^3*tan(1/2*c)^2 - tan(1/2*b*x)^3 + tan(1/2*b*x)*t
an(1/2*c)^2 - tan(1/2*b*x) - tan(1/2*c))*(tan(1/2*a)^2*tan(1/2*c) + tan(1/2
*c))))/b
```

3.235 $\int \cot^3(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=74

$$-\frac{\cos(a-c)\csc(bx+c)}{b}+\frac{3\sin(a-c)\tanh^{-1}(\cos(bx+c))}{2b}-\frac{\sin(a-c)\cot(bx+c)\csc(bx+c)}{2b}-\frac{\sin(a+bx)}{b}$$

```
[Out] -((Cos[a - c]*Csc[c + b*x])/b) + (3*ArcTanh[Cos[c + b*x]]*Sin[a - c])/(2*b)
 - (Cot[c + b*x]*Csc[c + b*x]*Sin[a - c])/(2*b) - Sin[a + b*x]/b
```

Rubi [A] time = 0.0717082, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {4578, 4577, 2637, 3770, 2606, 8, 2611}

$$-\frac{\cos(a-c)\csc(bx+c)}{b}+\frac{3\sin(a-c)\tanh^{-1}(\cos(bx+c))}{2b}-\frac{\sin(a-c)\cot(bx+c)\csc(bx+c)}{2b}-\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -((Cos[a - c]*Csc[c + b*x])/b) + (3*ArcTanh[Cos[c + b*x]]*Sin[a - c])/(2*b)
 - (Cot[c + b*x]*Csc[c + b*x]*Sin[a - c])/(2*b) - Sin[a + b*x]/b
```

Rule 4578

```
Int[Cot[w_]^(n_.)*Sin[v_], x_Symbol] :> Int[Cos[v]*Cot[w]^(n - 1), x] + Dis
t[Sin[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rule 4577

```
Int[Cos[v_]*Cot[w_]^(n_.), x_Symbol] :> -Int[Sin[v]*Cot[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \cot^3(c+bx)\sin(a+bx)\,dx = \sin(a-c)\int \cot^2(c+bx)\csc(c+bx)\,dx + \int \cos(a+bx)\cot^2(c+bx)\,dx$$
$$= -\frac{\cot(c+bx)\csc(c+bx)\sin(a-c)}{2b} + \cos(a-c)\int \cot(c+bx)\csc(c+bx)\,dx - \frac{1}{2}\sin(a$$
$$= \frac{\tanh^{-1}(\cos(c+bx))\sin(a-c)}{2b} - \frac{\cot(c+bx)\csc(c+bx)\sin(a-c)}{2b} - \frac{\cos(a-c)\,\mathrm{Subst}(}{}$$
$$= -\frac{\cos(a-c)\csc(c+bx)}{b} + \frac{3\tanh^{-1}(\cos(c+bx))\sin(a-c)}{2b} - \frac{\cot(c+bx)\csc(c+bx)\sin}{2b}$$

Mathematica [A] time = 0.350771, size = 71, normalized size = 0.96

$$\frac{\csc^2(bx+c)(2\sin(a-bx-2c)+\sin(a+3bx+2c)-5\sin(a+bx))+12\sin(a-c)\tanh^{-1}\left(\cos(c)-\sin(c)\tan\left(\frac{bx}{2}\right)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (12*ArcTanh[Cos[c] - Sin[c]*Tan[(b*x)/2]]*Sin[a - c] + Csc[c + b*x]^2*(2*Si
n[a - 2*c - b*x] - 5*Sin[a + b*x] + Sin[a + 2*c + 3*b*x]))/(4*b)
```

Maple [C] time = 0.104, size = 184, normalized size = 2.5

$$\frac{\frac{i}{2}e^{i(bx+a)}}{b} - \frac{\frac{i}{2}e^{-i(bx+a)}}{b} + \frac{\frac{i}{2}\left(-3\,e^{i(3\,bx+5\,a+2\,c)} - e^{i(3\,bx+3\,a+4\,c)} + e^{i(bx+5\,a)} + 3\,e^{i(bx+3\,a+2\,c)}\right)}{b\left(-e^{2\,i(bx+a+c)} + e^{2\,ia}\right)^2} - \frac{3\ln\left(e^{i(bx+a)} - e^{i(a-c)}\right)\sin(a-c)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+c)^3*sin(b*x+a),x)
```

```
[Out] 1/2*I*exp(I*(b*x+a))/b-1/2*I/b*exp(-I*(b*x+a))+1/2*I/b/(-exp(2*I*(b*x+a+c))
+exp(2*I*a))^2*(-3*exp(I*(3*b*x+5*a+2*c))-exp(I*(3*b*x+3*a+4*c))+exp(I*(b*x
+5*a))+3*exp(I*(b*x+3*a+2*c)))-3/2/b*ln(exp(I*(b*x+a))-exp(I*(a-c)))*sin(a-
c)+3/2/b*ln(exp(I*(b*x+a))+exp(I*(a-c)))*sin(a-c)
```

Maxima [B] time = 1.24852, size = 1693, normalized size = 22.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(sin(5*b*x + a + 4*c) - 2*sin(3*b*x + a + 2*c) + sin(b*x + a))*cos(6
*b*x + 2*a + 4*c) + 2*(5*sin(4*b*x + 2*a + 2*c) + 2*sin(4*b*x + 4*c) - 2*si
n(2*b*x + 2*a) - 5*sin(2*b*x + 2*c))*cos(5*b*x + a + 4*c) + 10*(2*sin(3*b*x
 + a + 2*c) - sin(b*x + a))*cos(4*b*x + 2*a + 2*c) + 4*(2*sin(3*b*x + a + 2
*c) - sin(b*x + a))*cos(4*b*x + 4*c) + 4*(2*sin(2*b*x + 2*a) + 5*sin(2*b*x
+ 2*c))*cos(3*b*x + a + 2*c) - 3*(cos(5*b*x + a + 4*c)^2*sin(-a + c) + 4*co
s(3*b*x + a + 2*c)^2*sin(-a + c) - 4*cos(3*b*x + a + 2*c)*cos(b*x + a)*sin(
-a + c) + cos(b*x + a)^2*sin(-a + c) + sin(5*b*x + a + 4*c)^2*sin(-a + c) +
 4*sin(3*b*x + a + 2*c)^2*sin(-a + c) - 4*sin(3*b*x + a + 2*c)*sin(b*x + a)
*sin(-a + c) + sin(b*x + a)^2*sin(-a + c) - 2*(2*cos(3*b*x + a + 2*c)*sin(-
a + c) - cos(b*x + a)*sin(-a + c))*cos(5*b*x + a + 4*c) - 2*(2*sin(3*b*x +
a + 2*c)*sin(-a + c) - sin(b*x + a)*sin(-a + c))*sin(5*b*x + a + 4*c))*log(
cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c)
+ sin(c)^2) + 3*(cos(5*b*x + a + 4*c)^2*sin(-a + c) + 4*cos(3*b*x + a + 2*c
)^2*sin(-a + c) - 4*cos(3*b*x + a + 2*c)*cos(b*x + a)*sin(-a + c) + cos(b*x
 + a)^2*sin(-a + c) + sin(5*b*x + a + 4*c)^2*sin(-a + c) + 4*sin(3*b*x + a
+ 2*c)^2*sin(-a + c) - 4*sin(3*b*x + a + 2*c)*sin(b*x + a)*sin(-a + c) + si
n(b*x + a)^2*sin(-a + c) - 2*(2*cos(3*b*x + a + 2*c)*sin(-a + c) - cos(b*x
+ a)*sin(-a + c))*cos(5*b*x + a + 4*c) - 2*(2*sin(3*b*x + a + 2*c)*sin(-a +
 c) - sin(b*x + a)*sin(-a + c))*sin(5*b*x + a + 4*c))*log(cos(b*x)^2 - 2*co
s(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) - 2*(
cos(5*b*x + a + 4*c) - 2*cos(3*b*x + a + 2*c) + cos(b*x + a))*sin(6*b*x + 2
*a + 4*c) - 2*(5*cos(4*b*x + 2*a + 2*c) + 2*cos(4*b*x + 4*c) - 2*cos(2*b*x
+ 2*a) - 5*cos(2*b*x + 2*c) + 1)*sin(5*b*x + a + 4*c) - 10*(2*cos(3*b*x + a
 + 2*c) - cos(b*x + a))*sin(4*b*x + 2*a + 2*c) - 4*(2*cos(3*b*x + a + 2*c)
- cos(b*x + a))*sin(4*b*x + 4*c) - 4*(2*cos(2*b*x + 2*a) + 5*cos(2*b*x + 2*
c) - 1)*sin(3*b*x + a + 2*c) - 4*cos(b*x + a)*sin(2*b*x + 2*a) - 10*cos(b*x
 + a)*sin(2*b*x + 2*c) + 4*cos(2*b*x + 2*a)*sin(b*x + a) + 10*cos(2*b*x + 2
*c)*sin(b*x + a) - 2*sin(b*x + a))/(b*cos(5*b*x + a + 4*c)^2 + 4*b*cos(3*b*
x + a + 2*c)^2 - 4*b*cos(3*b*x + a + 2*c)*cos(b*x + a) + b*cos(b*x + a)^2 +
 b*sin(5*b*x + a + 4*c)^2 + 4*b*sin(3*b*x + a + 2*c)^2 - 4*b*sin(3*b*x + a
+ 2*c)*sin(b*x + a) + b*sin(b*x + a)^2 - 2*(2*b*cos(3*b*x + a + 2*c) - b*co
s(b*x + a))*cos(5*b*x + a + 4*c) - 2*(2*b*sin(3*b*x + a + 2*c) - b*sin(b*x
+ a))*sin(5*b*x + a + 4*c))
```

Fricas [B] time = 0.56521, size = 987, normalized size = 13.34

$$\frac{\frac{3\sqrt{2}\left(2\left(\cos(-2a+2c)^2-1\right)\cos(bx+a)\sin(bx+a)+\left(2\cos(bx+a)^2\cos(-2a+2c)-\cos(-2a+2c)-1\right)\sin(-2a+2c)\right)\log\left(-\frac{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin\cdots}{2\cos(bx+a)^2\cdots}\right.}{\sqrt{\cos(-2a+2c)+1}}\cdots}{8\left(2b\cos(bx+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(3*sqrt(2)*(2*(cos(-2*a + 2*c)^2 - 1)*cos(b*x + a)*sin(b*x + a) + (2*co
s(b*x + a)^2*cos(-2*a + 2*c) - cos(-2*a + 2*c) - 1)*sin(-2*a + 2*c))*log(-(
2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2
*c) - 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*cos(b*x + a) - sin(b*x + a)*sin(-2*a
 + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2*a + 2*c) + 3)/(2*cos(b*x + a)^2
*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a +
 2*c) - 1))/sqrt(cos(-2*a + 2*c) + 1) - 4*(4*cos(b*x + a)^2*cos(-2*a + 2*c)
 - 3*cos(-2*a + 2*c) - 5)*sin(b*x + a) - 4*(4*cos(b*x + a)^3 - 5*cos(b*x +
```

```
a))*sin(-2*a + 2*c))/(2*b*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*b*cos(b*x + a)
*sin(b*x + a)*sin(-2*a + 2*c) - b*cos(-2*a + 2*c) - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(a+bx)\cot^3(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)**3*sin(b*x+a),x)
```

```
[Out] Integral(sin(a + b*x)*cot(b*x + c)**3, x)
```

Giac [B] time = 1.33205, size = 1175, normalized size = 15.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(12*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)*t
an(1/2*c) - tan(1/2*c)^2)*log(abs(tan(1/2*b*x)*tan(1/2*c) - 1))/(tan(1/2*a)
^2*tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c) + tan(1/2*c)^3 + tan(1/2*c)) - 12
*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*
c))*log(abs(tan(1/2*b*x) + tan(1/2*c)))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/
2*a)^2 + tan(1/2*c)^2 + 1) + 8*(tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*b*x) -
2*tan(1/2*a))/((tan(1/2*b*x)^2 + 1)*(tan(1/2*a)^2 + 1)) - (2*tan(1/2*b*x)^3
*tan(1/2*a)*tan(1/2*c)^7 + tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*c)^7 + tan(1
/2*b*x)^2*tan(1/2*a)*tan(1/2*c)^8 - 4*tan(1/2*b*x)^3*tan(1/2*a)^2*tan(1/2*c
)^4 + 6*tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*c)^5 - 5*tan(1/2*b*x)^2*tan(1/2*a
)^2*tan(1/2*c)^5 + 2*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*c)^6 - 4*tan(1/2*b*x
)*tan(1/2*a)^2*tan(1/2*c)^6 - tan(1/2*b*x)^2*tan(1/2*c)^7 - 2*tan(1/2*b*x)*
tan(1/2*a)*tan(1/2*c)^7 - 6*tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*c)^3 + 5*tan(
1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*c)^3 + 4*tan(1/2*b*x)^3*tan(1/2*c)^4 - 22*t
an(1/2*b*x)^2*tan(1/2*a)*tan(1/2*c)^4 + 4*tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2
*c)^4 + 5*tan(1/2*b*x)^2*tan(1/2*c)^5 - 14*tan(1/2*b*x)*tan(1/2*a)*tan(1/2*
c)^5 + 2*tan(1/2*a)^2*tan(1/2*c)^5 + 4*tan(1/2*b*x)*tan(1/2*c)^6 + 2*tan(1/
2*a)*tan(1/2*c)^6 - 2*tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*c) - tan(1/2*b*x)^2
*tan(1/2*a)^2*tan(1/2*c) + 2*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*c)^2 - 4*tan
(1/2*b*x)*tan(1/2*a)^2*tan(1/2*c)^2 - 5*tan(1/2*b*x)^2*tan(1/2*c)^3 + 14*ta
n(1/2*b*x)*tan(1/2*a)*tan(1/2*c)^3 - 2*tan(1/2*a)^2*tan(1/2*c)^3 - 4*tan(1/
2*b*x)*tan(1/2*c)^4 + 12*tan(1/2*a)*tan(1/2*c)^4 - 2*tan(1/2*c)^5 + tan(1/2
*b*x)^2*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*c) + 2*tan(1/2*b*x)*tan(1/2*a)*
tan(1/2*c) + 4*tan(1/2*b*x)*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/2*c)^2 + 2*ta
n(1/2*c)^3)/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^2)*(tan(1/2*b*x)^2*tan
(1/2*c) + tan(1/2*b*x)*tan(1/2*c)^2 - tan(1/2*b*x) - tan(1/2*c))^2))/b
```

3.236 $\int \sin(a+bx)\tan(c+dx)\,dx$

Optimal. Leaf size=143

$$-\frac{ie^{-i(a+bx)}\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2i(c+dx)}\right)}{b}-\frac{ie^{i(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{b}{2d},\frac{b}{2d}+1,-e^{2i(c+dx)}\right)}{b}+$$

```
[Out] (I/2)/(b*E^(I*(a + b*x))) + ((I/2)*E^(I*(a + b*x)))/b - (I*Hypergeometric2F
1[1, -b/(2*d), 1 - b/(2*d), -E^((2*I)*(c + d*x))])/(b*E^(I*(a + b*x))) - (I
*E^(I*(a + b*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), -E^((2*I)*(c +
d*x))])/b
```

Rubi [A] time = 0.112093, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4557, 2194, 2251}

$$-\frac{ie^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2i(c+dx)}\right)}{b}-\frac{ie^{i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;-e^{2i(c+dx)}\right)}{b}+\frac{ie^{-i(a+bx)}}{2b}+\frac{ie^{i(a+bx)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Tan[c + d*x],x]
```

```
[Out] (I/2)/(b*E^(I*(a + b*x))) + ((I/2)*E^(I*(a + b*x)))/b - (I*Hypergeometric2F
1[1, -b/(2*d), 1 - b/(2*d), -E^((2*I)*(c + d*x))])/(b*E^(I*(a + b*x))) - (I
*E^(I*(a + b*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), -E^((2*I)*(c +
d*x))])/b
```

Rule 4557

```
Int[Sin[(a_.) + (b_.)*(x_)]*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[1/(E^
(I*(a + b*x))*2) - E^(I*(a + b*x))/2 - 1/(E^(I*(a + b*x))*(1 + E^(2*I*(c +
d*x)))) + E^(I*(a + b*x))/(1 + E^(2*I*(c + d*x))), x] /; FreeQ[{a, b, c, d}
, x] && NeQ[b^2 - d^2, 0]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \sin(a+bx)\tan(c+dx)\,dx = \int \left(\frac{1}{2}e^{-i(a+bx)} - \frac{1}{2}e^{i(a+bx)} - \frac{e^{-i(a+bx)}}{1+e^{2i(c+dx)}} + \frac{e^{i(a+bx)}}{1+e^{2i(c+dx)}}\right)dx$$
$$= \frac{1}{2}\int e^{-i(a+bx)}\,dx - \frac{1}{2}\int e^{i(a+bx)}\,dx - \int \frac{e^{-i(a+bx)}}{1+e^{2i(c+dx)}}\,dx + \int \frac{e^{i(a+bx)}}{1+e^{2i(c+dx)}}\,dx$$
$$= \frac{ie^{-i(a+bx)}}{2b} + \frac{ie^{i(a+bx)}}{2b} - \frac{ie^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2i(c+dx)}\right)}{b} - \frac{ie^{i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};1+\frac{b}{2d};\right.}{b}$$

Mathematica [A] time = 1.78747, size = 116, normalized size = 0.81

$$-\frac{ie^{-i(a+bx)}\left(2e^{2i(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{b}{2d},\frac{b}{2d}+1,-e^{2i(c+dx)}\right)+2\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2i(c+dx)}\right)\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Tan[c + d*x],x]
```

```
[Out] ((-I/2)*(-1 - E^((2*I)*(a + b*x)) + 2*Hypergeometric2F1[1, -b/(2*d), 1 - b/
(2*d), -E^((2*I)*(c + d*x))] + 2*E^((2*I)*(a + b*x))*Hypergeometric2F1[1, b
/(2*d), 1 + b/(2*d), -E^((2*I)*(c + d*x))]))/(b*E^(I*(a + b*x)))
```

Maple [F] time = 0.181, size = 0, normalized size = 0.

$$\int \sin(bx+a)\tan(dx+c)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*tan(d*x+c),x)
```

```
[Out] int(sin(b*x+a)*tan(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(bx+a)\tan(dx+c)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)*tan(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sin(bx+a)\tan(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sin(b*x + a)*tan(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(a + bx)\tan(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)*tan(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(bx + a)\tan(dx + c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)*tan(d*x + c), x)
```

3.237 $\int \cot(c+dx)\sin(a+bx)\,dx$

Optimal. Leaf size=139

$$\frac{ie^{-i(a+bx)}\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},e^{2i(c+dx)}\right)}{b}+\frac{ie^{i(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{b}{2d},\frac{b}{2d}+1,e^{2i(c+dx)}\right)}{b}-\frac{ie^{-i(a+}}{2b}$$

```
[Out] (-I/2)/(b*E^(I*(a + b*x))) - ((I/2)*E^(I*(a + b*x)))/b + (I*Hypergeometric2
F1[1, -b/(2*d), 1 - b/(2*d), E^((2*I)*(c + d*x))])/(b*E^(I*(a + b*x))) + (I
*E^(I*(a + b*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), E^((2*I)*(c + d
*x))])/b
```

Rubi [A] time = 0.110764, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4559, 2194, 2251}

$$\frac{ie^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2i(c+dx)}\right)}{b}+\frac{ie^{i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;e^{2i(c+dx)}\right)}{b}-\frac{ie^{-i(a+bx)}}{2b}-\frac{ie^{i(a+bx)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Sin[a + b*x],x]
```


```
[Out] (-I/2)/(b*E^(I*(a + b*x))) - ((I/2)*E^(I*(a + b*x)))/b + (I*Hypergeometric2
F1[1, -b/(2*d), 1 - b/(2*d), E^((2*I)*(c + d*x))])/(b*E^(I*(a + b*x))) + (I
*E^(I*(a + b*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), E^((2*I)*(c + d
*x))])/b
```


Rule 4559

```
Int[Cot[(c_.) + (d_.)*(x_)]*Sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Int[-(1/(
E^(I*(a + b*x))*2)) + E^(I*(a + b*x))/2 + 1/(E^(I*(a + b*x))*(1 - E^(2*I*(c
 + d*x)))) - E^(I*(a + b*x))/(1 - E^(2*I*(c + d*x))), x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b^2 - d^2, 0]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \cot(c+dx)\sin(a+bx)\,dx = \int \left(-\frac{1}{2}e^{-i(a+bx)} + \frac{1}{2}e^{i(a+bx)} + \frac{e^{-i(a+bx)}}{1-e^{2i(c+dx)}} - \frac{e^{i(a+bx)}}{1-e^{2i(c+dx)}}\right)dx$$
$$= -\left(\frac{1}{2}\int e^{-i(a+bx)}\,dx\right) + \frac{1}{2}\int e^{i(a+bx)}\,dx + \int \frac{e^{-i(a+bx)}}{1-e^{2i(c+dx)}}\,dx - \int \frac{e^{i(a+bx)}}{1-e^{2i(c+dx)}}\,dx$$
$$= -\frac{ie^{-i(a+bx)}}{2b} - \frac{ie^{i(a+bx)}}{2b} + \frac{ie^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2i(c+dx)}\right)}{b} + \frac{ie^{i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};1+\right.}{b}$$

Mathematica [A] time = 3.59495, size = 260, normalized size = 1.87

$$\frac{-\frac{ie^{-i(a+bx-2c)}\left(be^{2idx}\text{Hypergeometric2F1}\left(1,1-\frac{b}{2d},2-\frac{b}{2d},e^{2i(c+dx)}\right)-(b-2d)\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},e^{2i(c+dx)}\right)\right)}{\left(-1+e^{2ic}\right)(b-2d)} - \frac{ie^{i(a+bx+2c)}\left(be^{2idx}\text{Hypergeome}\right.}{}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Sin[a + b*x],x]
```

```
[Out] (-(Cos[a]*Cos[b*x]*Cot[c]) - (I*(b*E^((2*I)*d*x)*Hypergeometric2F1[1, 1 - b
/(2*d), 2 - b/(2*d), E^((2*I)*(c + d*x))] - (b - 2*d)*Hypergeometric2F1[1,
-b/(2*d), 1 - b/(2*d), E^((2*I)*(c + d*x))]))/((b - 2*d)*E^(I*(a - 2*c + b*
x))*(-1 + E^((2*I)*c))) - (I*E^(I*(a + 2*c + b*x))*(b*E^((2*I)*d*x)*Hyperge
ometric2F1[1, 1 + b/(2*d), 2 + b/(2*d), E^((2*I)*(c + d*x))] - (b + 2*d)*Hy
pergeometric2F1[1, b/(2*d), 1 + b/(2*d), E^((2*I)*(c + d*x))]))/((b + 2*d)*
(-1 + E^((2*I)*c))) + Cot[c]*Sin[a]*Sin[b*x])/b
```

Maple [F] time = 0.208, size = 0, normalized size = 0.

$$\int \cot(dx+c)\sin(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*sin(b*x+a),x)
```

```
[Out] int(cot(d*x+c)*sin(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cot(dx+c)\sin(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)*sin(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cot(dx+c)\sin(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(cot(d*x + c)*sin(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(a+bx)\cot(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*sin(b*x+a),x)
```

```
[Out] Integral(sin(a + b*x)*cot(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cot(dx+c)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)*sin(b*x + a), x)
```

3.238 $\int \cos(a+bx)\cos^3(c+dx)\,dx$

Optimal. Leaf size=91

$$\frac{\sin(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\sin(a+x(b-d)-c)}{8(b-d)}+\frac{3\sin(a+x(b+d)+c)}{8(b+d)}+\frac{\sin(a+x(b+3d)+3c)}{8(b+3d)}$$

```
[Out] Sin[a - 3*c + (b - 3*d)*x]/(8*(b - 3*d)) + (3*Sin[a - c + (b - d)*x])/(8*(b
 - d)) + (3*Sin[a + c + (b + d)*x])/(8*(b + d)) + Sin[a + 3*c + (b + 3*d)*x
]/(8*(b + 3*d))
```

Rubi [A] time = 0.0654962, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4570, 2637}

$$\frac{\sin(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\sin(a+x(b-d)-c)}{8(b-d)}+\frac{3\sin(a+x(b+d)+c)}{8(b+d)}+\frac{\sin(a+x(b+3d)+3c)}{8(b+3d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cos[c + d*x]^3,x]
```

```
[Out] Sin[a - 3*c + (b - 3*d)*x]/(8*(b - 3*d)) + (3*Sin[a - c + (b - d)*x])/(8*(b
 - d)) + (3*Sin[a + c + (b + d)*x])/(8*(b + d)) + Sin[a + 3*c + (b + 3*d)*x
]/(8*(b + 3*d))
```

Rule 4570

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\cos^3(c+dx)\,dx &= \int \left(\frac{1}{8}\cos(a-3c+(b-3d)x)+\frac{3}{8}\cos(a-c+(b-d)x)+\frac{3}{8}\cos(a+c+(b+d)x)+\right.\\ &= \frac{1}{8}\int \cos(a-3c+(b-3d)x)\,dx+\frac{1}{8}\int \cos(a+3c+(b+3d)x)\,dx+\frac{3}{8}\int \cos(a-c+\\ &= \frac{\sin(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\sin(a-c+(b-d)x)}{8(b-d)}+\frac{3\sin(a+c+(b+d)x)}{8(b+d)}+\frac{\sin(a+}{}\end{aligned}$$

Mathematica [A] time = 0.48636, size = 85, normalized size = 0.93

$$\frac{1}{8}\left(\frac{\sin(a+bx-3c-3dx)}{b-3d}+\frac{3\sin(a+bx-c-dx)}{b-d}+\frac{\sin(a+bx+3c+3dx)}{b+3d}+\frac{3\sin(a+x(b+d)+c)}{b+d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cos[c + d*x]^3,x]
```

```
[Out] (Sin[a - 3*c + b*x - 3*d*x]/(b - 3*d) + (3*Sin[a - c + b*x - d*x])/(b - d)
+ Sin[a + 3*c + b*x + 3*d*x]/(b + 3*d) + (3*Sin[a + c + (b + d)*x])/(b + d)
)/8
```

Maple [A] time = 0.027, size = 84, normalized size = 0.9

$$\frac{\sin\left(a-3c+\left(b-3d\right)x\right)}{8b-24d}+\frac{3\sin\left(a-c+\left(b-d\right)x\right)}{8b-8d}+\frac{3\sin\left(a+c+\left(b+d\right)x\right)}{8b+8d}+\frac{\sin\left(a+3c+\left(b+3d\right)x\right)}{8b+24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cos(d*x+c)^3,x)
```

```
[Out] 1/8*sin(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sin(a-c+(b-d)*x)/(b-d)+3/8*sin(a+c+(b+
d)*x)/(b+d)+1/8*sin(a+3*c+(b+3*d)*x)/(b+3*d)
```

Maxima [B] time = 1.45457, size = 1234, normalized size = 13.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((b^3*sin(3*c) - 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) + 3*d^3*sin(3*c))*
cos((b + 3*d)*x + a + 6*c) - (b^3*sin(3*c) - 3*b^2*d*sin(3*c) - b*d^2*sin(3
*c) + 3*d^3*sin(3*c))*cos((b + 3*d)*x + a) + 3*(b^3*sin(3*c) - b^2*d*sin(3*
c) - 9*b*d^2*sin(3*c) + 9*d^3*sin(3*c))*cos((b + d)*x + a + 4*c) - 3*(b^3*s
in(3*c) - b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) + 9*d^3*sin(3*c))*cos((b + d)*x
 + a - 2*c) - 3*(b^3*sin(3*c) + b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) - 9*d^3*s
in(3*c))*cos(-(b - d)*x - a + 4*c) + 3*(b^3*sin(3*c) + b^2*d*sin(3*c) - 9*b
*d^2*sin(3*c) - 9*d^3*sin(3*c))*cos(-(b - d)*x - a - 2*c) - (b^3*sin(3*c) +
 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) - 3*d^3*sin(3*c))*cos(-(b - 3*d)*x - a +
 6*c) + (b^3*sin(3*c) + 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) - 3*d^3*sin(3*c))
*cos(-(b - 3*d)*x - a) - (b^3*cos(3*c) - 3*b^2*d*cos(3*c) - b*d^2*cos(3*c)
+ 3*d^3*cos(3*c))*sin((b + 3*d)*x + a + 6*c) - (b^3*cos(3*c) - 3*b^2*d*cos(
3*c) - b*d^2*cos(3*c) + 3*d^3*cos(3*c))*sin((b + 3*d)*x + a) - 3*(b^3*cos(3
*c) - b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) + 9*d^3*cos(3*c))*sin((b + d)*x + a
 + 4*c) - 3*(b^3*cos(3*c) - b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) + 9*d^3*cos(3
*c))*sin((b + d)*x + a - 2*c) + 3*(b^3*cos(3*c) + b^2*d*cos(3*c) - 9*b*d^2*
cos(3*c) - 9*d^3*cos(3*c))*sin(-(b - d)*x - a + 4*c) + 3*(b^3*cos(3*c) + b^
2*d*cos(3*c) - 9*b*d^2*cos(3*c) - 9*d^3*cos(3*c))*sin(-(b - d)*x - a - 2*c)
 + (b^3*cos(3*c) + 3*b^2*d*cos(3*c) - b*d^2*cos(3*c) - 3*d^3*cos(3*c))*sin(
-(b - 3*d)*x - a + 6*c) + (b^3*cos(3*c) + 3*b^2*d*cos(3*c) - b*d^2*cos(3*c)
 - 3*d^3*cos(3*c))*sin(-(b - 3*d)*x - a))/(b^4*cos(3*c)^2 + b^4*sin(3*c)^2
+ 9*(cos(3*c)^2 + sin(3*c)^2)*d^4 - 10*(b^2*cos(3*c)^2 + b^2*sin(3*c)^2)*d^
2)
```

Fricas [A] time = 0.511305, size = 243, normalized size = 2.67

$$-\frac{\left(6\,bd^2\cos\left(dx+c\right)-\left(b^3-bd^2\right)\cos\left(dx+c\right)^3\right)\sin\left(bx+a\right)-3\left(2\,d^3\cos\left(bx+a\right)-\left(b^2d-d^3\right)\cos\left(bx+a\right)\cos\left(dx+c\right)^2\right)}{b^4-10\,b^2d^2+9\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -((6*b*d^2*cos(d*x + c) - (b^3 - b*d^2)*cos(d*x + c)^3)*sin(b*x + a) - 3*(2
*d^3*cos(b*x + a) - (b^2*d - d^3)*cos(b*x + a)*cos(d*x + c)^2)*sin(d*x + c)
)/(b^4 - 10*b^2*d^2 + 9*d^4)
```

Sympy [A] time = 136.5, size = 933, normalized size = 10.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c)**3,x)
```

```
[Out] Piecewise((x*cos(a)*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (x*sin(a - 3*d*x)*sin(
c + d*x)**3/8 - 3*x*sin(a - 3*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 - 3*x*sin
(c + d*x)**2*cos(a - 3*d*x)*cos(c + d*x)/8 + x*cos(a - 3*d*x)*cos(c + d*x)*
*3/8 - 3*sin(a - 3*d*x)*cos(c + d*x)**3/(8*d) - sin(c + d*x)**3*cos(a - 3*d
*x)/(24*d) - sin(c + d*x)*cos(a - 3*d*x)*cos(c + d*x)**2/(4*d), Eq(b, -3*d)
), (-3*x*sin(a - d*x)*sin(c + d*x)**3/8 - 3*x*sin(a - d*x)*sin(c + d*x)*cos
(c + d*x)**2/8 + 3*x*sin(c + d*x)**2*cos(a - d*x)*cos(c + d*x)/8 + 3*x*cos(
a - d*x)*cos(c + d*x)**3/8 + sin(a - d*x)*cos(c + d*x)**3/(8*d) + 3*sin(c +
 d*x)**3*cos(a - d*x)/(8*d) + 3*sin(c + d*x)*cos(a - d*x)*cos(c + d*x)**2/(
4*d), Eq(b, -d)), (3*x*sin(a + d*x)*sin(c + d*x)**3/8 + 3*x*sin(a + d*x)*si
n(c + d*x)*cos(c + d*x)**2/8 + 3*x*sin(c + d*x)**2*cos(a + d*x)*cos(c + d*x
)/8 + 3*x*cos(a + d*x)*cos(c + d*x)**3/8 + sin(a + d*x)*sin(c + d*x)**2*cos
(c + d*x)/(8*d) + sin(c + d*x)**3*cos(a + d*x)/(4*d) + 5*sin(c + d*x)*cos(a
 + d*x)*cos(c + d*x)**2/(8*d), Eq(b, d)), (-x*sin(a + 3*d*x)*sin(c + d*x)**
3/8 + 3*x*sin(a + 3*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 - 3*x*sin(c + d*x)*
*2*cos(a + 3*d*x)*cos(c + d*x)/8 + x*cos(a + 3*d*x)*cos(c + d*x)**3/8 + 9*s
in(a + 3*d*x)*sin(c + d*x)**2*cos(c + d*x)/(8*d) - 5*sin(c + d*x)**3*cos(a
+ 3*d*x)/(12*d) + 7*sin(c + d*x)*cos(a + 3*d*x)*cos(c + d*x)**2/(8*d), Eq(b
, 3*d)), (b**3*sin(a + b*x)*cos(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4)
- 3*b**2*d*sin(c + d*x)*cos(a + b*x)*cos(c + d*x)**2/(b**4 - 10*b**2*d**2 +
 9*d**4) - 6*b*d**2*sin(a + b*x)*sin(c + d*x)**2*cos(c + d*x)/(b**4 - 10*b*
*2*d**2 + 9*d**4) - 7*b*d**2*sin(a + b*x)*cos(c + d*x)**3/(b**4 - 10*b**2*d
**2 + 9*d**4) + 6*d**3*sin(c + d*x)**3*cos(a + b*x)/(b**4 - 10*b**2*d**2 +
9*d**4) + 9*d**3*sin(c + d*x)*cos(a + b*x)*cos(c + d*x)**2/(b**4 - 10*b**2*
d**2 + 9*d**4), True))
```

Giac [A] time = 1.12383, size = 113, normalized size = 1.24

$$\frac{\sin\left(bx + 3\,dx + a + 3\,c\right)}{8\,(b + 3\,d)} + \frac{3\,\sin\left(bx + dx + a + c\right)}{8\,(b + d)} + \frac{3\,\sin\left(bx - dx + a - c\right)}{8\,(b - d)} + \frac{\sin\left(bx - 3\,dx + a - 3\,c\right)}{8\,(b - 3\,d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*sin(b*x + 3*d*x + a + 3*c)/(b + 3*d) + 3/8*sin(b*x + d*x + a + c)/(b +
d) + 3/8*sin(b*x - d*x + a - c)/(b - d) + 1/8*sin(b*x - 3*d*x + a - 3*c)/(b
 - 3*d)
```

3.239 $\int \cos(a+bx)\cos^2(c+dx)\,dx$

Optimal. Leaf size=62

$$\frac{\sin(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\sin(a+x(b+2d)+2c)}{4(b+2d)}+\frac{\sin(a+bx)}{2b}$$

```
[Out] Sin[a + b*x]/(2*b) + Sin[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) + Sin[a + 2*c
 + (b + 2*d)*x]/(4*(b + 2*d))
```

Rubi [A] time = 0.0454893, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4570, 2637}

$$\frac{\sin(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\sin(a+x(b+2d)+2c)}{4(b+2d)}+\frac{\sin(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cos[c + d*x]^2,x]
```

```
[Out] Sin[a + b*x]/(2*b) + Sin[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) + Sin[a + 2*c
 + (b + 2*d)*x]/(4*(b + 2*d))
```

Rule 4570

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos(a+bx)\cos^2(c+dx)\,dx &= \int \left(\frac{1}{2}\cos(a+bx)+\frac{1}{4}\cos(a-2c+(b-2d)x)+\frac{1}{4}\cos(a+2c+(b+2d)x)\right)dx\\
&= \frac{1}{4}\int \cos(a-2c+(b-2d)x)\,dx+\frac{1}{4}\int \cos(a+2c+(b+2d)x)\,dx+\frac{1}{2}\int \cos(a+bx)\,dx\\
&= \frac{\sin(a+bx)}{2b}+\frac{\sin(a-2c+(b-2d)x)}{4(b-2d)}+\frac{\sin(a+2c+(b+2d)x)}{4(b+2d)}
\end{aligned}$$

Mathematica [A] time = 0.713151, size = 69, normalized size = 1.11

$$\frac{1}{4}\left(\frac{\sin(a+bx-2c-2dx)}{b-2d}+\frac{\sin(a+bx+2c+2dx)}{b+2d}+\frac{2\sin(a)\cos(bx)}{b}+\frac{2\cos(a)\sin(bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cos[c + d*x]^2,x]
```

```
[Out] ((2*Cos[b*x]*Sin[a])/b + (2*Cos[a]*Sin[b*x])/b + Sin[a - 2*c + b*x - 2*d*x]
/(b - 2*d) + Sin[a + 2*c + b*x + 2*d*x]/(b + 2*d))/4
```

Maple [A] time = 0.023, size = 57, normalized size = 0.9

$$\frac{\sin(bx+a)}{2b}+\frac{\sin(a-2c+(b-2d)x)}{4b-8d}+\frac{\sin(a+2c+(b+2d)x)}{4b+8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cos(d*x+c)^2,x)
```

```
[Out] 1/2*sin(b*x+a)/b+1/4*sin(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*sin(a+2*c+(b+2*d)*x)/
(b+2*d)
```

Maxima [B] time = 1.24765, size = 562, normalized size = 9.06

$$-\frac{\left(b^2\sin(2c)-2bd\sin(2c)\right)\cos((b+2d)x+a+4c)-\left(b^2\sin(2c)-2bd\sin(2c)\right)\cos((b+2d)x+a)-\left(b^2\sin(2c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/8*((b^2*sin(2*c) - 2*b*d*sin(2*c))*cos((b + 2*d)*x + a + 4*c) - (b^2*sin
(2*c) - 2*b*d*sin(2*c))*cos((b + 2*d)*x + a) - (b^2*sin(2*c) + 2*b*d*sin(2*
c))*cos(-(b - 2*d)*x - a + 4*c) + (b^2*sin(2*c) + 2*b*d*sin(2*c))*cos(-(b -
 2*d)*x - a) + 2*(b^2*sin(2*c) - 4*d^2*sin(2*c))*cos(b*x + a + 2*c) - 2*(b^
2*sin(2*c) - 4*d^2*sin(2*c))*cos(b*x + a - 2*c) - (b^2*cos(2*c) - 2*b*d*cos
(2*c))*sin((b + 2*d)*x + a + 4*c) - (b^2*cos(2*c) - 2*b*d*cos(2*c))*sin((b
+ 2*d)*x + a) + (b^2*cos(2*c) + 2*b*d*cos(2*c))*sin(-(b - 2*d)*x - a + 4*c)
 + (b^2*cos(2*c) + 2*b*d*cos(2*c))*sin(-(b - 2*d)*x - a) - 2*(b^2*cos(2*c)
- 4*d^2*cos(2*c))*sin(b*x + a + 2*c) - 2*(b^2*cos(2*c) - 4*d^2*cos(2*c))*si
n(b*x + a - 2*c))/(b^3*cos(2*c)^2 + b^3*sin(2*c)^2 - 4*(b*cos(2*c)^2 + b*si
n(2*c)^2)*d^2)
```

Fricas [A] time = 0.504163, size = 147, normalized size = 2.37

$$-\frac{2\,bd\cos(bx+a)\cos(dx+c)\sin(dx+c)-\left(b^2\cos(dx+c)^2-2\,d^2\right)\sin(bx+a)}{b^3-4\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(2*b*d*cos(b*x + a)*cos(d*x + c)*sin(d*x + c) - (b^2*cos(d*x + c)^2 - 2*d^
2)*sin(b*x + a))/(b^3 - 4*b*d^2)
```

Sympy [A] time = 12.6248, size = 401, normalized size = 6.47

$$\begin{cases} x\cos(a)\cos^2(c) \\ \left(\frac{x\sin^2(c+dx)}{2}+\frac{x\cos^2(c+dx)}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)\cos(a) \\ -\frac{x\sin(a-2dx)\sin(c+dx)\cos(c+dx)}{2}-\frac{x\sin^2(c+dx)\cos(a-2dx)}{4}+\frac{x\cos(a-2dx)\cos^2(c+dx)}{4}-\frac{\sin(a-2dx)\cos^2(c+dx)}{2d}-\frac{\sin(c+dx)\cos(a-2dx)\cos(c+}{4d} \\ \frac{x\sin(a+2dx)\sin(c+dx)\cos(c+dx)}{2}-\frac{x\sin^2(c+dx)\cos(a+2dx)}{4}+\frac{x\cos(a+2dx)\cos^2(c+dx)}{4}+\frac{\sin(a+2dx)\sin^2(c+dx)}{8d}+\frac{3\sin(a+2dx)\cos^2(c+dx)}{8d} \\ \frac{b^2\sin(a+bx)\cos^2(c+dx)}{b^3-4bd^2}-\frac{2bd\sin(c+dx)\cos(a+bx)\cos(c+dx)}{b^3-4bd^2}-\frac{2d^2\sin(a+bx)\sin^2(c+dx)}{b^3-4bd^2}-\frac{2d^2\sin(a+bx)\cos^2(c+dx)}{b^3-4bd^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c)**2,x)
```

```
[Out] Piecewise((x*cos(a)*cos(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**2/2
+ x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(2*d))*cos(a), Eq(b, 0)),
 (-x*sin(a - 2*d*x)*sin(c + d*x)*cos(c + d*x)/2 - x*sin(c + d*x)**2*cos(a -
 2*d*x)/4 + x*cos(a - 2*d*x)*cos(c + d*x)**2/4 - sin(a - 2*d*x)*cos(c + d*x
)**2/(2*d) - sin(c + d*x)*cos(a - 2*d*x)*cos(c + d*x)/(4*d), Eq(b, -2*d)),
(x*sin(a + 2*d*x)*sin(c + d*x)*cos(c + d*x)/2 - x*sin(c + d*x)**2*cos(a + 2
*d*x)/4 + x*cos(a + 2*d*x)*cos(c + d*x)**2/4 + sin(a + 2*d*x)*sin(c + d*x)*
*2/(8*d) + 3*sin(a + 2*d*x)*cos(c + d*x)**2/(8*d), Eq(b, 2*d)), (b**2*sin(a
 + b*x)*cos(c + d*x)**2/(b**3 - 4*b*d**2) - 2*b*d*sin(c + d*x)*cos(a + b*x)
*cos(c + d*x)/(b**3 - 4*b*d**2) - 2*d**2*sin(a + b*x)*sin(c + d*x)**2/(b**3
 - 4*b*d**2) - 2*d**2*sin(a + b*x)*cos(c + d*x)**2/(b**3 - 4*b*d**2), True)
)
```

Giac [A] time = 1.11932, size = 76, normalized size = 1.23

$$\frac{\sin(bx+2\,dx+a+2\,c)}{4\,(b+2\,d)}+\frac{\sin(bx-2\,dx+a-2\,c)}{4\,(b-2\,d)}+\frac{\sin(bx+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*sin(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/4*sin(b*x - 2*d*x + a - 2*c)/(
b - 2*d) + 1/2*sin(b*x + a)/b
```

3.240 $\int \cos(a+bx)\cos(c+dx)\,dx$

Optimal. Leaf size=43

$$\frac{\sin(a+x(b-d)-c)}{2(b-d)}+\frac{\sin(a+x(b+d)+c)}{2(b+d)}$$

```
[Out] Sin[a - c + (b - d)*x]/(2*(b - d)) + Sin[a + c + (b + d)*x]/(2*(b + d))
```

Rubi [A] time = 0.0334101, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4570, 2637}

$$\frac{\sin(a+x(b-d)-c)}{2(b-d)}+\frac{\sin(a+x(b+d)+c)}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cos[c + d*x],x]
```

```
[Out] Sin[a - c + (b - d)*x]/(2*(b - d)) + Sin[a + c + (b + d)*x]/(2*(b + d))
```

Rule 4570

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\cos(c+dx)\,dx &= \int \left(\frac{1}{2}\cos(a-c+(b-d)x)+\frac{1}{2}\cos(a+c+(b+d)x)\right)dx\\ &= \frac{1}{2}\int \cos(a-c+(b-d)x)\,dx+\frac{1}{2}\int \cos(a+c+(b+d)x)\,dx\\ &= \frac{\sin(a-c+(b-d)x)}{2(b-d)}+\frac{\sin(a+c+(b+d)x)}{2(b+d)}\end{aligned}$$

Mathematica [A] time = 0.180851, size = 43, normalized size = 1.

$$\frac{\sin(a+x(b-d)-c)}{2(b-d)}+\frac{\sin(a+x(b+d)+c)}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cos[c + d*x],x]
```

```
[Out] Sin[a - c + (b - d)*x]/(2*(b - d)) + Sin[a + c + (b + d)*x]/(2*(b + d))
```

Maple [A] time = 0.019, size = 40, normalized size = 0.9

$$\frac{\sin\left(a-c+\left(b-d\right)x\right)}{2b-2d}+\frac{\sin\left(a+c+\left(b+d\right)x\right)}{2b+2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cos(d*x+c),x)
```

```
[Out] 1/2*sin(a-c+(b-d)*x)/(b-d)+1/2*sin(a+c+(b+d)*x)/(b+d)
```

Maxima [A] time = 1.07282, size = 54, normalized size = 1.26

$$\frac{\sin\left(bx+dx+a+c\right)}{2\left(b+d\right)}-\frac{\sin\left(-bx+dx-a+c\right)}{2\left(b-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*sin(b*x + d*x + a + c)/(b + d) - 1/2*sin(-b*x + d*x - a + c)/(b - d)
```

Fricas [A] time = 0.473431, size = 99, normalized size = 2.3

$$\frac{b\cos\left(dx+c\right)\sin\left(bx+a\right)-d\cos\left(bx+a\right)\sin\left(dx+c\right)}{b^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c),x, algorithm="fricas")
```

```
[Out] (b*cos(d*x + c)*sin(b*x + a) - d*cos(b*x + a)*sin(d*x + c))/(b^2 - d^2)
```

Sympy [A] time = 2.41227, size = 153, normalized size = 3.56

$$\begin{cases} x\cos\left(a\right)\cos\left(c\right) & \text{for}\: b=0\wedge d=0\\ -\frac{x\sin\left(a-dx\right)\sin\left(c+dx\right)}{2}+\frac{x\cos\left(a-dx\right)\cos\left(c+dx\right)}{2}-\frac{\sin\left(a-dx\right)\cos\left(c+dx\right)}{2d} & \text{for}\: b=-d\\ \frac{x\sin\left(a+dx\right)\sin\left(c+dx\right)}{2}+\frac{x\cos\left(a+dx\right)\cos\left(c+dx\right)}{2}+\frac{\sin\left(c+dx\right)\cos\left(a+dx\right)}{2d} & \text{for}\: b=d\\ \frac{b\sin\left(a+bx\right)\cos\left(c+dx\right)}{b^2-d^2}-\frac{d\sin\left(c+dx\right)\cos\left(a+bx\right)}{b^2-d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c),x)
```

```
[Out] Piecewise((x*cos(a)*cos(c), Eq(b, 0) & Eq(d, 0)), (-x*sin(a - d*x)*sin(c +
d*x)/2 + x*cos(a - d*x)*cos(c + d*x)/2 - sin(a - d*x)*cos(c + d*x)/(2*d), E
q(b, -d)), (x*sin(a + d*x)*sin(c + d*x)/2 + x*cos(a + d*x)*cos(c + d*x)/2 +
 sin(c + d*x)*cos(a + d*x)/(2*d), Eq(b, d)), (b*sin(a + b*x)*cos(c + d*x)/(
b**2 - d**2) - d*sin(c + d*x)*cos(a + b*x)/(b**2 - d**2), True))
```

Giac [A] time = 1.09072, size = 54, normalized size = 1.26

$$\frac{\sin\left(bx+dx+a+c\right)}{2\left(b+d\right)}+\frac{\sin\left(bx-dx+a-c\right)}{2\left(b-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*sin(b*x + d*x + a + c)/(b + d) + 1/2*sin(b*x - d*x + a - c)/(b - d)
```

3.241 $\int \cos(a+bx)\sec(c+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\sin(a-c)\log(\cos(bx+c))}{b}+x\cos(a-c)$$

```
[Out] x*Cos[a - c] + (Log[Cos[c + b*x]]*Sin[a - c])/b
```

Rubi [A] time = 0.0132681, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4583, 3475, 8}

$$\frac{\sin(a-c)\log(\cos(bx+c))}{b}+x\cos(a-c)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sec[c + b*x],x]

[Out] x*Cos[a - c] + (Log[Cos[c + b*x]]*Sin[a - c])/b
```

Rule 4583

```
Int[Cos[v_]*Sec[w_]^(n_.), x_Symbol] :> -Dist[Sin[v - w], Int[Tan[w]*Sec[w]
^(n - 1), x], x] + Dist[Cos[v - w], Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0]
 && FreeQ[v - w, x] && NeQ[w, v]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sec(c+bx)\,dx &= \cos(a-c)\int 1\,dx-\sin(a-c)\int \tan(c+bx)\,dx\\ &= x\cos(a-c)+\frac{\log(\cos(c+bx))\sin(a-c)}{b}\end{aligned}$$

Mathematica [A] time = 0.121403, size = 26, normalized size = 1.

$$\frac{\sin(a-c)\log(\cos(bx+c))}{b}+x\cos(a-c)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sec[c + b*x],x]

[Out] x*Cos[a - c] + (Log[Cos[c + b*x]]*Sin[a - c])/b
```

Maple [B] time = 0.199, size = 461, normalized size = 17.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sec(b*x+c),x)
```

```
[Out] 1/b/(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^
2)/(sin(a)*cos(c)-cos(a)*sin(c))*ln(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*si
n(a)*cos(c)+cos(a)*cos(c)+sin(a)*sin(c))*cos(a)^2*sin(c)^2-2/b/(cos(a)^2*co
s(c)^2+cos(a)^2*sin(c)^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)/(sin(a)*cos(c
)-cos(a)*sin(c))*ln(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*sin(a)*cos(c)+cos(
a)*cos(c)+sin(a)*sin(c))*cos(a)*cos(c)*sin(a)*sin(c)+1/b/(cos(a)^2*cos(c)^2
+cos(a)^2*sin(c)^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)/(sin(a)*cos(c)-cos(
a)*sin(c))*ln(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*sin(a)*cos(c)+cos(a)*cos
(c)+sin(a)*sin(c))*cos(c)^2*sin(a)^2+1/2/b/(cos(c)^2+sin(c)^2)/(cos(a)^2+si
n(a)^2)*ln(1+tan(b*x+a)^2)*cos(a)*sin(c)-1/2/b/(cos(c)^2+sin(c)^2)/(cos(a)^
2+sin(a)^2)*ln(1+tan(b*x+a)^2)*sin(a)*cos(c)+1/b/(cos(c)^2+sin(c)^2)/(cos(a
)^2+sin(a)^2)*cos(a)*cos(c)*arctan(tan(b*x+a))+1/b/(cos(c)^2+sin(c)^2)/(cos
(a)^2+sin(a)^2)*sin(a)*sin(c)*arctan(tan(b*x+a))
```

Maxima [B] time = 1.18199, size = 100, normalized size = 3.85

$$\frac{2\,bx\cos\left(-a+c\right) - \log\left(\cos\left(2\,bx\right)^{2} + 2\,\cos\left(2\,bx\right)\cos\left(2\,c\right) + \cos\left(2\,c\right)^{2} + \sin\left(2\,bx\right)^{2} - 2\,\sin\left(2\,bx\right)\sin\left(2\,c\right) + \sin\left(2\,c\right)^{2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x*cos(-a + c) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*c
)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*c) + sin(2*c)^2)*sin(-a + c))/b
```

Fricas [A] time = 0.533184, size = 73, normalized size = 2.81

$$\frac{bx\cos\left(-a+c\right) - \log\left(-\cos\left(bx+c\right)\right)\sin\left(-a+c\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c),x, algorithm="fricas")
```

```
[Out] (b*x*cos(-a + c) - log(-cos(b*x + c))*sin(-a + c))/b
```

Sympy [B] time = 109.34, size = 435, normalized size = 16.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c),x)
```

```
[Out] -Piecewise((0, Eq(b, 0)), (-x, Eq(c, pi/2)), (x, Eq(c, -pi/2)), (-2*b*x*tan
(c/2)/(b*tan(c/2)**2 + b) - log(tan(b*x/2)**2 + 1)*tan(c/2)**2/(b*tan(c/2)*
*2 + b) + log(tan(b*x/2)**2 + 1)/(b*tan(c/2)**2 + b) + log(tan(b*x/2) - tan
(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**2/(b*tan(c/2)**2 + b) -
log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))/(b*tan(c/2)**2
 + b) + log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/
2)**2/(b*tan(c/2)**2 + b) - log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(t
an(c/2) + 1))/(b*tan(c/2)**2 + b), True))*sin(a) + Piecewise((x/cos(c), Eq(
b, 0)), (-log(sin(b*x))/b, Eq(c, pi/2)), (log(sin(b*x))/b, Eq(c, -pi/2)), (
-b*x*tan(c/2)**2/(b*tan(c/2)**2 + b) + b*x/(b*tan(c/2)**2 + b) + 2*log(tan(
b*x/2)**2 + 1)*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b*x/2) - tan(c/2)/(
tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(
b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)/(b*tan(c/2)**
2 + b), True))*cos(a)
```

Giac [B] time = 1.21496, size = 594, normalized size = 22.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c),x, algorithm="giac")
```

```
[Out] ((tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c) - tan(
1/2*c)^2 + 1)*(b*x + a)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2
*c)^2 + 1) - (tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a
) - tan(1/2*c))*log(tan(b*x + a)^2 + 1)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/
2*a)^2 + tan(1/2*c)^2 + 1) + 2*(tan(1/2*a)^4*tan(1/2*c)^2 - 2*tan(1/2*a)^3*
tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c)^4 + 2*tan(1/2*a)^3*tan(1/2*c) - 4*ta
n(1/2*a)^2*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)^2 - 2*tan(
1/2*a)*tan(1/2*c) + tan(1/2*c)^2)*log(abs(2*tan(b*x + a)*tan(1/2*a)^2*tan(1
/2*c) - 2*tan(b*x + a)*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a)^2*tan(1/2*c)^2
+ 2*tan(b*x + a)*tan(1/2*a) - tan(1/2*a)^2 - 2*tan(b*x + a)*tan(1/2*c) + 4*
tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1))/(tan(1/2*a)^4*tan(1/2*c)^3 - tan
(1/2*a)^3*tan(1/2*c)^4 + tan(1/2*a)^4*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^4
+ tan(1/2*a)^3 - tan(1/2*c)^3 + tan(1/2*a) - tan(1/2*c)))/b
```

3.242 $\int \cos(a+bx)\sec^2(c+bx)\,dx$

Optimal. Leaf size=35

$$\frac{\cos(a-c)\tanh^{-1}(\sin(bx+c))}{b}-\frac{\sin(a-c)\sec(bx+c)}{b}$$

```
[Out] (ArcTanh[Sin[c + b*x]]*Cos[a - c])/b - (Sec[c + b*x]*Sin[a - c])/b
```

Rubi [A] time = 0.0287126, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4583, 2606, 8, 3770}

$$\frac{\cos(a-c)\tanh^{-1}(\sin(bx+c))}{b}-\frac{\sin(a-c)\sec(bx+c)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sec[c + b*x]^2,x]
```

```
[Out] (ArcTanh[Sin[c + b*x]]*Cos[a - c])/b - (Sec[c + b*x]*Sin[a - c])/b
```

Rule 4583

```
Int[Cos[v_]*Sec[w_]^(n_.), x_Symbol] :> -Dist[Sin[v - w], Int[Tan[w]*Sec[w]
^(n - 1), x], x] + Dist[Cos[v - w], Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0]
 && FreeQ[v - w, x] && NeQ[w, v]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sec^2(c+bx)\,dx &= \cos(a-c)\int \sec(c+bx)\,dx-\sin(a-c)\int \sec(c+bx)\tan(c+bx)\,dx\\ &= \frac{\tanh^{-1}(\sin(c+bx))\cos(a-c)}{b}-\frac{\sin(a-c)\,\mathrm{Subst}(\int 1\,dx,x,\sec(c+bx))}{b}\\ &= \frac{\tanh^{-1}(\sin(c+bx))\cos(a-c)}{b}-\frac{\sec(c+bx)\sin(a-c)}{b}\end{aligned}$$

Mathematica [C] time = 0.089295, size = 89, normalized size = 2.54

$$-\frac{\sin(a-c)\sec(bx+c)}{b}-\frac{2i\cos(a-c)\tan^{-1}\left(\frac{(\sin(c)+i\cos(c))\left(\sin(c)\cos\left(\frac{bx}{2}\right)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\sin(c)\cos\left(\frac{bx}{2}\right)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sec[c + b*x]^2,x]
```

```
[Out] ((-2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x)/2]*Sin[c] + Cos[c]*Sin[(b*x)
/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b - (Sec[c
 + b*x]*Sin[a - c])/b
```

Maple [B] time = 0.451, size = 1049, normalized size = 30.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sec(b*x+c)^2,x)
```

```
[Out] 2/b/(cos(a)*cos(c)*tan(1/2*b*x+1/2*a)^2+sin(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+
2*tan(1/2*b*x+1/2*a)*cos(a)*sin(c)-2*tan(1/2*b*x+1/2*a)*sin(a)*cos(c)-cos(a
)*cos(c)-sin(a)*sin(c))/(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^2+cos(c)^2*sin(a
)^2+sin(a)^2*sin(c)^2)/(cos(a)*cos(c)+sin(a)*sin(c))*tan(1/2*b*x+1/2*a)*cos
(a)^2*sin(c)^2-4/b/(cos(a)*cos(c)*tan(1/2*b*x+1/2*a)^2+sin(a)*sin(c)*tan(1/
2*b*x+1/2*a)^2+2*tan(1/2*b*x+1/2*a)*cos(a)*sin(c)-2*tan(1/2*b*x+1/2*a)*sin(
a)*cos(c)-cos(a)*cos(c)-sin(a)*sin(c))/(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^2
+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)/(cos(a)*cos(c)+sin(a)*sin(c))*tan(1/2
*b*x+1/2*a)*cos(a)*cos(c)*sin(a)*sin(c)+2/b/(cos(a)*cos(c)*tan(1/2*b*x+1/2*
a)^2+sin(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+2*tan(1/2*b*x+1/2*a)*cos(a)*sin(c)-
2*tan(1/2*b*x+1/2*a)*sin(a)*cos(c)-cos(a)*cos(c)-sin(a)*sin(c))/(cos(a)^2*c
os(c)^2+cos(a)^2*sin(c)^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)/(cos(a)*cos(
c)+sin(a)*sin(c))*tan(1/2*b*x+1/2*a)*cos(c)^2*sin(a)^2-2/b/(cos(a)*cos(c)*t
an(1/2*b*x+1/2*a)^2+sin(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+2*tan(1/2*b*x+1/2*a)
*cos(a)*sin(c)-2*tan(1/2*b*x+1/2*a)*sin(a)*cos(c)-cos(a)*cos(c)-sin(a)*sin(
c))/(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^
2)*cos(a)*sin(c)+2/b/(cos(a)*cos(c)*tan(1/2*b*x+1/2*a)^2+sin(a)*sin(c)*tan(
1/2*b*x+1/2*a)^2+2*tan(1/2*b*x+1/2*a)*cos(a)*sin(c)-2*tan(1/2*b*x+1/2*a)*si
n(a)*cos(c)-cos(a)*cos(c)-sin(a)*sin(c))/(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)
^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)*sin(a)*cos(c)-2/b/(cos(a)^2*cos(c)^
2+cos(a)^2*sin(c)^2+cos(c)^2*sin(a)^2+sin(a)^2*sin(c)^2)/(-cos(a)^2*cos(c)^
2-cos(a)^2*sin(c)^2-cos(c)^2*sin(a)^2-sin(a)^2*sin(c)^2)^(1/2)*arctan(1/2*(
2*(cos(a)*cos(c)+sin(a)*sin(c))*tan(1/2*b*x+1/2*a)+2*cos(a)*sin(c)-2*sin(a)
*cos(c))/(-cos(a)^2*cos(c)^2-cos(a)^2*sin(c)^2-cos(c)^2*sin(a)^2-sin(a)^2*s
in(c)^2)^(1/2))*cos(a)*cos(c)-2/b/(cos(a)^2*cos(c)^2+cos(a)^2*sin(c)^2+cos(
c)^2*sin(a)^2+sin(a)^2*sin(c)^2)/(-cos(a)^2*cos(c)^2-cos(a)^2*sin(c)^2-cos(
c)^2*sin(a)^2-sin(a)^2*sin(c)^2)^(1/2)*arctan(1/2*(2*(cos(a)*cos(c)+sin(a)*
sin(c))*tan(1/2*b*x+1/2*a)+2*cos(a)*sin(c)-2*sin(a)*cos(c))/(-cos(a)^2*cos(
c)^2-cos(a)^2*sin(c)^2-cos(c)^2*sin(a)^2-sin(a)^2*sin(c)^2)^(1/2))*sin(a)*s
in(c)
```

Maxima [B] time = 1.86843, size = 528, normalized size = 15.09

$$-\frac{2\left(\sin\left(bx+2\,a\right)-\sin\left(bx+2\,c\right)\right)\cos\left(2\,bx+a+2\,c\right)+\left(\cos\left(2\,bx+a+2\,c\right)^{2}\cos\left(-a+c\right)+2\,\cos\left(2\,bx+a+2\,c\right)\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(sin(b*x + 2*a) - sin(b*x + 2*c))*cos(2*b*x + a + 2*c) + (cos(2*b*x
 + a + 2*c)^2*cos(-a + c) + 2*cos(2*b*x + a + 2*c)*cos(a)*cos(-a + c) + cos
(-a + c)*sin(2*b*x + a + 2*c)^2 + 2*cos(-a + c)*sin(2*b*x + a + 2*c)*sin(a)
 + (cos(a)^2 + sin(a)^2)*cos(-a + c))*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*
cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)
^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)
^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)) - 2*(cos(b*x + 2*a) - cos(b*x + 2
*c))*sin(2*b*x + a + 2*c) + 2*cos(a)*sin(b*x + 2*a) - 2*cos(a)*sin(b*x + 2*
c) - 2*cos(b*x + 2*a)*sin(a) + 2*cos(b*x + 2*c)*sin(a))/(b*cos(2*b*x + a +
2*c)^2 + 2*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(2*b*x + a + 2*c)^2 + 2*b*s
in(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b)
```

Fricas [A] time = 0.535566, size = 185, normalized size = 5.29

$$\frac{\cos\left(bx+c\right)\cos\left(-a+c\right)\log\left(\sin\left(bx+c\right)+1\right)-\cos\left(bx+c\right)\cos\left(-a+c\right)\log\left(-\sin\left(bx+c\right)+1\right)+2\,\sin\left(-a+c\right)}{2\,b\cos\left(bx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(cos(b*x + c)*cos(-a + c)*log(sin(b*x + c) + 1) - cos(b*x + c)*cos(-a +
 c)*log(-sin(b*x + c) + 1) + 2*sin(-a + c))/(b*cos(b*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.37998, size = 1810, normalized size = 51.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c)^2,x, algorithm="giac")
```

```
[Out] -((tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^2 + tan(1/2*a)^2*tan
(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c) + 5*tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2
*a)*tan(1/2*c)^3 + tan(1/2*a)^3 - 5*tan(1/2*a)^2*tan(1/2*c) + 5*tan(1/2*a)*
tan(1/2*c)^2 - tan(1/2*c)^3 - tan(1/2*a)^2 + 5*tan(1/2*a)*tan(1/2*c) - tan(
1/2*c)^2 - tan(1/2*a) + tan(1/2*c) + 1)*log(abs(-tan(1/2*b*x + 1/2*a)*tan(1
/2*a)*tan(1/2*c) + tan(1/2*b*x + 1/2*a)*tan(1/2*a) - tan(1/2*b*x + 1/2*a)*t
an(1/2*c) + tan(1/2*a)*tan(1/2*c) - tan(1/2*b*x + 1/2*a) + tan(1/2*a) - tan
(1/2*c) + 1))/(tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^2 + tan(
1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c) + tan(1/2*a)^2*tan(1/2*c)^2
 + tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*a)^3 + tan(1/2*a)^2*tan(1/2*c) - tan(1
/2*a)*tan(1/2*c)^2 + tan(1/2*c)^3 + tan(1/2*a)^2 + tan(1/2*a)*tan(1/2*c) +
tan(1/2*c)^2 - tan(1/2*a) + tan(1/2*c) + 1) - (tan(1/2*a)^3*tan(1/2*c)^3 +
tan(1/2*a)^3*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/
2*c) + 5*tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*a)^3
 + 5*tan(1/2*a)^2*tan(1/2*c) - 5*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^3 - t
an(1/2*a)^2 + 5*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + tan(1/2*a) - tan(1/2
*c) + 1)*log(abs(-tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c) - tan(1/2*b*x
+ 1/2*a)*tan(1/2*a) + tan(1/2*b*x + 1/2*a)*tan(1/2*c) - tan(1/2*a)*tan(1/2*
c) - tan(1/2*b*x + 1/2*a) + tan(1/2*a) - tan(1/2*c) - 1))/(tan(1/2*a)^3*tan
(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2
*a)^3*tan(1/2*c) + tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)*tan(1/2*c)^3 + ta
n(1/2*a)^3 - tan(1/2*a)^2*tan(1/2*c) + tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*c)
^3 + tan(1/2*a)^2 + tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2 + tan(1/2*a) - tan
(1/2*c) + 1) - 4*(2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^4*tan(1/2*c)^2 - 4*tan(
1/2*b*x + 1/2*a)*tan(1/2*a)^3*tan(1/2*c)^3 + tan(1/2*a)^4*tan(1/2*c)^3 + 2*
tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c)^4 - tan(1/2*a)^3*tan(1/2*c)^4
+ 4*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^3*tan(1/2*c) - tan(1/2*a)^4*tan(1/2*c)
- 8*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c)^2 + 6*tan(1/2*a)^3*tan(1/2
*c)^2 + 4*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^3 - 6*tan(1/2*a)^2*tan
(1/2*c)^3 + tan(1/2*a)*tan(1/2*c)^4 + 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2 -
 tan(1/2*a)^3 - 4*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c) + 6*tan(1/2*a)
^2*tan(1/2*c) + 2*tan(1/2*b*x + 1/2*a)*tan(1/2*c)^2 - 6*tan(1/2*a)*tan(1/2*
c)^2 + tan(1/2*c)^3 + tan(1/2*a) - tan(1/2*c))/((tan(1/2*b*x + 1/2*a)^2*tan
(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*b*x + 1/2*a)^2*tan(1/2*a)^2 + 4*tan(1/2*b*
x + 1/2*a)^2*tan(1/2*a)*tan(1/2*c) - 4*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*ta
n(1/2*c) - tan(1/2*b*x + 1/2*a)^2*tan(1/2*c)^2 + 4*tan(1/2*b*x + 1/2*a)*tan
(1/2*a)*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*b*x + 1/2*a)^2 -
 4*tan(1/2*b*x + 1/2*a)*tan(1/2*a) + tan(1/2*a)^2 + 4*tan(1/2*b*x + 1/2*a)*
tan(1/2*c) - 4*tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2 - 1)*(tan(1/2*a)^2*tan(
1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1)))/b
```

3.243 $\int \cos(a+bx)\sec^3(c+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\cos(a-c)\tan(bx+c)}{b}-\frac{\sin(a-c)\sec^2(bx+c)}{2b}$$

```
[Out] -(Sec[c + b*x]^2*Sin[a - c])/(2*b) + (Cos[a - c]*Tan[c + b*x])/b
```

Rubi [A] time = 0.0390639, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4583, 2606, 30, 3767, 8}

$$\frac{\cos(a-c)\tan(bx+c)}{b}-\frac{\sin(a-c)\sec^2(bx+c)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sec[c + b*x]^3,x]
```

```
[Out] -(Sec[c + b*x]^2*Sin[a - c])/(2*b) + (Cos[a - c]*Tan[c + b*x])/b
```

Rule 4583

```
Int[Cos[v_]*Sec[w_]^(n_.), x_Symbol] :> -Dist[Sin[v - w], Int[Tan[w]*Sec[w]
^(n - 1), x], x] + Dist[Cos[v - w], Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0]
 && FreeQ[v - w, x] && NeQ[w, v]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos(a+bx)\sec^3(c+bx)\,dx = \cos(a-c)\int \sec^2(c+bx)\,dx - \sin(a-c)\int \sec^2(c+bx)\tan(c+bx)\,dx$$
$$= -\frac{\cos(a-c)\,\mathrm{Subst}(\int 1\,dx, x, -\tan(c+bx))}{b} - \frac{\sin(a-c)\,\mathrm{Subst}(\int x\,dx, x, \sec(c+bx))}{b}$$
$$= -\frac{\sec^2(c+bx)\sin(a-c)}{2b} + \frac{\cos(a-c)\tan(c+bx)}{b}$$

Mathematica [A] time = 0.191481, size = 35, normalized size = 0.92

$$-\frac{\sec(c)\sec^2(bx+c)(\sin(a)-\cos(a-c)\sin(2bx+c))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sec[c + b*x]^3,x]
```

```
[Out] -(Sec[c]*Sec[c + b*x]^2*(Sin[a] - Cos[a - c]*Sin[c + 2*b*x]))/(2*b)
```

Maple [A] time = 0.556, size = 56, normalized size = 1.5

$$\frac{1}{2\,b\,(\cos(a)\sin(c)-\sin(a)\cos(c))\,(-\tan(bx+a)\cos(a)\sin(c)+\tan(bx+a)\sin(a)\cos(c)+\cos(a)\cos(c)+\sin(a)\,\mathrm{si}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sec(b*x+c)^3,x)
```

```
[Out] 1/2/b/(cos(a)*sin(c)-sin(a)*cos(c))/(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*s
in(a)*cos(c)+cos(a)*cos(c)+sin(a)*sin(c))^2
```

Maxima [B] time = 1.12904, size = 516, normalized size = 13.58

$$-\frac{(2\sin(2bx+2a+2c)+\sin(2a)+\sin(2c))\cos(4bx+a+5c)+2(2\sin(2bx+2a+2c)+\sin(2a)+\sin(2c))\,\mathrm{cos}}{b\cos(4bx+a+5c)^2+4b\cos(2bx+a+3c)^2+4b\cos(2bx+a+3c)\,\mathrm{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] -((2*sin(2*b*x + 2*a + 2*c) + sin(2*a) + sin(2*c))*cos(4*b*x + a + 5*c) + 2
*(2*sin(2*b*x + 2*a + 2*c) + sin(2*a) + sin(2*c))*cos(2*b*x + a + 3*c) + (s
in(2*a) + sin(2*c))*cos(a + c) - (2*cos(2*b*x + 2*a + 2*c) + cos(2*a) + cos
(2*c))*sin(4*b*x + a + 5*c) + 2*cos(a + c)*sin(2*b*x + 2*a + 2*c) - 2*(2*co
s(2*b*x + 2*a + 2*c) + cos(2*a) + cos(2*c))*sin(2*b*x + a + 3*c) - (cos(2*a
) + cos(2*c))*sin(a + c) - 2*cos(2*b*x + 2*a + 2*c)*sin(a + c))/(b*cos(4*b*
x + a + 5*c)^2 + 4*b*cos(2*b*x + a + 3*c)^2 + 4*b*cos(2*b*x + a + 3*c)*cos(
a + c) + b*cos(a + c)^2 + b*sin(4*b*x + a + 5*c)^2 + 4*b*sin(2*b*x + a + 3*
c)^2 + 4*b*sin(2*b*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 + 2*(2*b*cos(2*
b*x + a + 3*c) + b*cos(a + c))*cos(4*b*x + a + 5*c) + 2*(2*b*sin(2*b*x + a
+ 3*c) + b*sin(a + c))*sin(4*b*x + a + 5*c))
```

Fricas [A] time = 0.485064, size = 108, normalized size = 2.84

$$\frac{2 \cos(bx+c)\cos(-a+c)\sin(bx+c)+\sin(-a+c)}{2\,b\cos(bx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c) + sin(-a + c))/(b*cos(b*x + c)
^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17443, size = 425, normalized size = 11.18

$$-\frac{\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^6+3\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^4+3\tan\left(\frac{1}{2}a\right)^4\tan\left(\frac{1}{2}c\right)^6+3\tan\left(\frac{1}{2}a\right)^6\tan\left(\frac{1}{2}c\right)^2+9\tan\left(\frac{1}{2}a\right)^4 \ta}{4\left(2\tan(bx+a)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-2\tan(bx+a)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*(tan(1/2*a)^6*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 3*tan(1/2*a
)^4*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^4*tan(1/2*c)^
4 + 3*tan(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*a)^6 + 9*tan(1/2*a)^4*tan(1/2*c)^
2 + 9*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*c)^6 + 3*tan(1/2*a)^4 + 9*tan(1/2
*a)^2*tan(1/2*c)^2 + 3*tan(1/2*c)^4 + 3*tan(1/2*a)^2 + 3*tan(1/2*c)^2 + 1)/
((2*tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c) - 2*tan(b*x + a)*tan(1/2*a)*tan(1/
2*c)^2 + tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(b*x + a)*tan(1/2*a) - tan(1/2*a)
^2 - 2*tan(b*x + a)*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1
)^2*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1
/2*c))*b)
```

3.244 $\int \cos^2(a+bx)\cos^3(c+dx)\,dx$

Optimal. Leaf size=144

$$\frac{\sin(2a+x(2b-3d)-3c)}{16(2b-3d)}+\frac{3\sin(2a+x(2b-d)-c)}{16(2b-d)}+\frac{3\sin(2a+x(2b+d)+c)}{16(2b+d)}+\frac{\sin(2a+x(2b+3d)+3c)}{16(2b+3d)}+\frac{3\sin(c}{8d}$$

```
[Out] Sin[2*a - 3*c + (2*b - 3*d)*x]/(16*(2*b - 3*d)) + (3*Sin[2*a - c + (2*b - d
)*x])/(16*(2*b - d)) + (3*Sin[c + d*x])/(8*d) + Sin[3*c + 3*d*x]/(24*d) + (
3*Sin[2*a + c + (2*b + d)*x])/(16*(2*b + d)) + Sin[2*a + 3*c + (2*b + 3*d)*
x]/(16*(2*b + 3*d))
```

Rubi [A] time = 0.0887541, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4570, 2637}

$$\frac{\sin(2a+x(2b-3d)-3c)}{16(2b-3d)}+\frac{3\sin(2a+x(2b-d)-c)}{16(2b-d)}+\frac{3\sin(2a+x(2b+d)+c)}{16(2b+d)}+\frac{\sin(2a+x(2b+3d)+3c)}{16(2b+3d)}+\frac{3\sin(c}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Cos[c + d*x]^3,x]
```


```
[Out] Sin[2*a - 3*c + (2*b - 3*d)*x]/(16*(2*b - 3*d)) + (3*Sin[2*a - c + (2*b - d
)*x])/(16*(2*b - d)) + (3*Sin[c + d*x])/(8*d) + Sin[3*c + 3*d*x]/(24*d) + (
3*Sin[2*a + c + (2*b + d)*x])/(16*(2*b + d)) + Sin[2*a + 3*c + (2*b + 3*d)*
x]/(16*(2*b + 3*d))
```


Rule 4570

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(a+bx)\cos^3(c+dx)\,dx &= \int \left(\frac{1}{16}\cos(2a-3c+(2b-3d)x)+\frac{3}{16}\cos(2a-c+(2b-d)x)+\frac{3}{8}\cos(c+dx)+\frac{1}{8}\cos\right.\\
&= \frac{1}{16}\int \cos(2a-3c+(2b-3d)x)\,dx+\frac{1}{16}\int \cos(2a+3c+(2b+3d)x)\,dx+\frac{1}{8}\int \cos(3c\\
&= \frac{\sin(2a-3c+(2b-3d)x)}{16(2b-3d)}+\frac{3\sin(2a-c+(2b-d)x)}{16(2b-d)}+\frac{3\sin(c+dx)}{8d}+\frac{\sin(3c+3dx)}{24d}
\end{aligned}$$

Mathematica [A] time = 1.63928, size = 158, normalized size = 1.1

$$\frac{1}{48}\left(\frac{3\sin(2a+2bx-3c-3dx)}{2b-3d}+\frac{9\sin(2a+2bx-c-dx)}{2b-d}+\frac{9\sin(2a+2bx+c+dx)}{2b+d}+\frac{3\sin(2a+2bx+3c+3dx)}{2b+3d}+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Cos[c + d*x]^3,x]
```

```
[Out] ((18*Cos[d*x]*Sin[c])/d + (2*Cos[3*d*x]*Sin[3*c])/d + (18*Cos[c]*Sin[d*x])/
d + (2*Cos[3*c]*Sin[3*d*x])/d + (3*Sin[2*a - 3*c + 2*b*x - 3*d*x])/(2*b - 3
*d) + (9*Sin[2*a - c + 2*b*x - d*x])/(2*b - d) + (9*Sin[2*a + c + 2*b*x + d
*x])/(2*b + d) + (3*Sin[2*a + 3*c + 2*b*x + 3*d*x])/(2*b + 3*d))/48
```

Maple [A] time = 0.036, size = 133, normalized size = 0.9

$$\frac{\sin(2a-3c+(2b-3d)x)}{32b-48d}+\frac{3\sin(2a-c+(2b-d)x)}{32b-16d}+\frac{3\sin(dx+c)}{8d}+\frac{\sin(3dx+3c)}{24d}+\frac{3\sin(2a+c+(2b+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*cos(d*x+c)^3,x)
```

```
[Out] 1/16*sin(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)+3/16*sin(2*a-c+(2*b-d)*x)/(2*b-d)+3
/8*sin(d*x+c)/d+1/24*sin(3*d*x+3*c)/d+3/16*sin(2*a+c+(2*b+d)*x)/(2*b+d)+1/1
6*sin(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

Maxima [B] time = 1.57257, size = 1839, normalized size = 12.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cos(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/96*(3*(8*b^3*d*sin(3*c) - 12*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c) + 3*d^4
*sin(3*c))*cos((2*b + 3*d)*x + 2*a + 6*c) - 3*(8*b^3*d*sin(3*c) - 12*b^2*d^
2*sin(3*c) - 2*b*d^3*sin(3*c) + 3*d^4*sin(3*c))*cos((2*b + 3*d)*x + 2*a) +
9*(8*b^3*d*sin(3*c) - 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c) + 9*d^4*sin(3*
c))*cos((2*b + d)*x + 2*a + 4*c) - 9*(8*b^3*d*sin(3*c) - 4*b^2*d^2*sin(3*c)
 - 18*b*d^3*sin(3*c) + 9*d^4*sin(3*c))*cos((2*b + d)*x + 2*a - 2*c) - 9*(8*
b^3*d*sin(3*c) + 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c) - 9*d^4*sin(3*c))*c
os(-(2*b - d)*x - 2*a + 4*c) + 9*(8*b^3*d*sin(3*c) + 4*b^2*d^2*sin(3*c) - 1
8*b*d^3*sin(3*c) - 9*d^4*sin(3*c))*cos(-(2*b - d)*x - 2*a - 2*c) - 3*(8*b^3
*d*sin(3*c) + 12*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c) - 3*d^4*sin(3*c))*cos(
-(2*b - 3*d)*x - 2*a + 6*c) + 3*(8*b^3*d*sin(3*c) + 12*b^2*d^2*sin(3*c) - 2
*b*d^3*sin(3*c) - 3*d^4*sin(3*c))*cos(-(2*b - 3*d)*x - 2*a) - 2*(16*b^4*sin
(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(3*d*x) + 2*(16*b^4*sin(3*
c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(3*d*x + 6*c) + 18*(16*b^4*si
n(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(d*x + 4*c) - 18*(16*b^4*
sin(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(d*x - 2*c) - 3*(8*b^3*
d*cos(3*c) - 12*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) + 3*d^4*cos(3*c))*sin((
2*b + 3*d)*x + 2*a + 6*c) - 3*(8*b^3*d*cos(3*c) - 12*b^2*d^2*cos(3*c) - 2*b
*d^3*cos(3*c) + 3*d^4*cos(3*c))*sin((2*b + 3*d)*x + 2*a) - 9*(8*b^3*d*cos(3
*c) - 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c) + 9*d^4*cos(3*c))*sin((2*b + d
)*x + 2*a + 4*c) - 9*(8*b^3*d*cos(3*c) - 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(
3*c) + 9*d^4*cos(3*c))*sin((2*b + d)*x + 2*a - 2*c) + 9*(8*b^3*d*cos(3*c) +
 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c) - 9*d^4*cos(3*c))*sin(-(2*b - d)*x
- 2*a + 4*c) + 9*(8*b^3*d*cos(3*c) + 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c)
 - 9*d^4*cos(3*c))*sin(-(2*b - d)*x - 2*a - 2*c) + 3*(8*b^3*d*cos(3*c) + 12
*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) - 3*d^4*cos(3*c))*sin(-(2*b - 3*d)*x -
```

```
 2*a + 6*c) + 3*(8*b^3*d*cos(3*c) + 12*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c)
- 3*d^4*cos(3*c))*sin(-(2*b - 3*d)*x - 2*a) - 2*(16*b^4*cos(3*c) - 40*b^2*d
^2*cos(3*c) + 9*d^4*cos(3*c))*sin(3*d*x) - 2*(16*b^4*cos(3*c) - 40*b^2*d^2*
cos(3*c) + 9*d^4*cos(3*c))*sin(3*d*x + 6*c) - 18*(16*b^4*cos(3*c) - 40*b^2*
d^2*cos(3*c) + 9*d^4*cos(3*c))*sin(d*x + 4*c) - 18*(16*b^4*cos(3*c) - 40*b^
2*d^2*cos(3*c) + 9*d^4*cos(3*c))*sin(d*x - 2*c))/(9*(cos(3*c)^2 + sin(3*c)^
2)*d^5 - 40*(b^2*cos(3*c)^2 + b^2*sin(3*c)^2)*d^3 + 16*(b^4*cos(3*c)^2 + b^
4*sin(3*c)^2)*d)
```

Fricas [A] time = 0.533443, size = 370, normalized size = 2.57

$$-\frac{6\left(6\,bd^3\cos\left(bx+a\right)\cos\left(dx+c\right)-\left(4\,b^3d-bd^3\right)\cos\left(bx+a\right)\cos\left(dx+c\right)^3\right)\sin\left(bx+a\right)-\left(18\,d^4\cos\left(bx+a\right)^2+16\,b^4-\right.}{3\left(16\,b^4d-40\,b^2d^3+9\,d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cos(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(6*(6*b*d^3*cos(b*x + a)*cos(d*x + c) - (4*b^3*d - b*d^3)*cos(b*x + a)
*cos(d*x + c)^3)*sin(b*x + a) - (18*d^4*cos(b*x + a)^2 + 16*b^4 - 40*b^2*d^
2 + (8*b^4 - 2*b^2*d^2 - 9*(4*b^2*d^2 - d^4)*cos(b*x + a)^2)*cos(d*x + c)^2
)*sin(d*x + c))/(16*b^4*d - 40*b^2*d^3 + 9*d^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*cos(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12546, size = 174, normalized size = 1.21

$$\frac{\sin\left(2\,bx+3\,dx+2\,a+3\,c\right)}{16\left(2\,b+3\,d\right)}+\frac{3\,\sin\left(2\,bx+dx+2\,a+c\right)}{16\left(2\,b+d\right)}+\frac{3\,\sin\left(2\,bx-dx+2\,a-c\right)}{16\left(2\,b-d\right)}+\frac{\sin\left(2\,bx-3\,dx+2\,a-3\,c\right)}{16\left(2\,b-3\,d\right)}+\frac{s}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cos(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*sin(2*b*x + 3*d*x + 2*a + 3*c)/(2*b + 3*d) + 3/16*sin(2*b*x + d*x + 2*
a + c)/(2*b + d) + 3/16*sin(2*b*x - d*x + 2*a - c)/(2*b - d) + 1/16*sin(2*b
*x - 3*d*x + 2*a - 3*c)/(2*b - 3*d) + 1/24*sin(3*d*x + 3*c)/d + 3/8*sin(d*x
 + c)/d
```

3.245 $\int \cos^2(a+bx)\cos^2(c+dx)\,dx$

Optimal. Leaf size=88

$$\frac{\sin(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sin(2(a+c)+2x(b+d))}{16(b+d)}+\frac{\sin(2a+2bx)}{8b}+\frac{\sin(2c+2dx)}{8d}+\frac{x}{4}$$

```
[Out] x/4 + Sin[2*a + 2*b*x]/(8*b) + Sin[2*(a - c) + 2*(b - d)*x]/(16*(b - d)) +
Sin[2*c + 2*d*x]/(8*d) + Sin[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

Rubi [A] time = 0.0686459, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4570, 2637}

$$\frac{\sin(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sin(2(a+c)+2x(b+d))}{16(b+d)}+\frac{\sin(2a+2bx)}{8b}+\frac{\sin(2c+2dx)}{8d}+\frac{x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Cos[c + d*x]^2,x]
```

```
[Out] x/4 + Sin[2*a + 2*b*x]/(8*b) + Sin[2*(a - c) + 2*(b - d)*x]/(16*(b - d)) +
Sin[2*c + 2*d*x]/(8*d) + Sin[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

Rule 4570

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\cos^2(c+dx)\,dx &= \int \left(\frac{1}{4}+\frac{1}{4}\cos(2a+2bx)+\frac{1}{8}\cos(2(a-c)+2(b-d)x)+\frac{1}{4}\cos(2c+2dx)+\frac{1}{8}\cos(2\right.\\ &= \frac{x}{4}+\frac{1}{8}\int \cos(2(a-c)+2(b-d)x)\,dx+\frac{1}{8}\int \cos(2(a+c)+2(b+d)x)\,dx+\frac{1}{4}\int \mathrm{co}\\ &= \frac{x}{4}+\frac{\sin(2a+2bx)}{8b}+\frac{\sin(2(a-c)+2(b-d)x)}{16(b-d)}+\frac{\sin(2c+2dx)}{8d}+\frac{\sin(2(a+c)+2(}{16(b+d}\end{aligned}$$

Mathematica [A] time = 0.732747, size = 105, normalized size = 1.19

$$\frac{2d\left(b^2-d^2\right)\sin(2(a+bx))+bd(b+d)\sin(2(a+x(b-d)-c))+b(b-d)(d(\sin(2(a+x(b+d)+c))+4x(b+d))+2(}{16bd(b-d)(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Cos[c + d*x]^2,x]
```

```
[Out] (2*d*(b^2 - d^2)*Sin[2*(a + b*x)] + b*d*(b + d)*Sin[2*(a - c + (b - d)*x)]
+ b*(b - d)*(2*(b + d)*Sin[2*(c + d*x)] + d*(4*(b + d)*x + Sin[2*(a + c + (
b + d)*x)])))/(16*b*(b - d)*d*(b + d))
```

Maple [A] time = 0.031, size = 83, normalized size = 0.9

$$\frac{x}{4}+\frac{\sin(2\,bx+2\,a)}{8\,b}+\frac{\sin(2\,dx+2\,c)}{8\,d}+\frac{\sin((2\,b-2\,d)\,x-2\,c+2\,a)}{16\,b-16\,d}+\frac{\sin((2\,b+2\,d)\,x+2\,a+2\,c)}{16\,b+16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*cos(d*x+c)^2,x)
```

```
[Out] 1/4*x+1/8*sin(2*b*x+2*a)/b+1/8*sin(2*d*x+2*c)/d+1/16/(b-d)*sin((2*b-2*d)*x-
2*c+2*a)+1/16/(b+d)*sin((2*b+2*d)*x+2*a+2*c)
```

Maxima [B] time = 1.27885, size = 837, normalized size = 9.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cos(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(8*((b*cos(2*c)^2 + b*sin(2*c)^2)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c)
^2)*d)*x + (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a + 4*c) -
 (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a) - (b^2*d*sin(2*c)
 + b*d^2*sin(2*c))*cos(-2*(b - d)*x - 2*a + 4*c) + (b^2*d*sin(2*c) + b*d^2*
sin(2*c))*cos(-2*(b - d)*x - 2*a) + 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*cos(2
*b*x + 2*a + 2*c) - 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*cos(2*b*x + 2*a - 2*c
) - 2*(b^3*sin(2*c) - b*d^2*sin(2*c))*cos(2*d*x) + 2*(b^3*sin(2*c) - b*d^2*
sin(2*c))*cos(2*d*x + 4*c) - (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(2*(b + d
)*x + 2*a + 4*c) - (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(2*(b + d)*x + 2*a)
 + (b^2*d*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a + 4*c) + (b^2*d
*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a) - 2*(b^2*d*cos(2*c) - d
^3*cos(2*c))*sin(2*b*x + 2*a + 2*c) - 2*(b^2*d*cos(2*c) - d^3*cos(2*c))*sin
(2*b*x + 2*a - 2*c) - 2*(b^3*cos(2*c) - b*d^2*cos(2*c))*sin(2*d*x) - 2*(b^3
*cos(2*c) - b*d^2*cos(2*c))*sin(2*d*x + 4*c))/((b*cos(2*c)^2 + b*sin(2*c)^2
)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c)^2)*d)
```

Fricas [A] time = 0.511237, size = 234, normalized size = 2.66

$$-\frac{\left(2\,bd^2\cos(bx+a)^2-b^3\right)\cos(dx+c)\sin(dx+c)-\left(b^3d-bd^3\right)x-\left(2\,b^2d\cos(bx+a)\cos(dx+c)^2-d^3\cos(bx+a)\right)\sin}{4\left(b^3d-bd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cos(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*((2*b*d^2*cos(b*x + a)^2 - b^3)*cos(d*x + c)*sin(d*x + c) - (b^3*d - b
*d^3)*x - (2*b^2*d*cos(b*x + a)*cos(d*x + c)^2 - d^3*cos(b*x + a))*sin(b*x
```

```
+ a))/(b^3*d - b*d^3)
```

Sympy [A] time = 35.0847, size = 1027, normalized size = 11.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*cos(d*x+c)**2,x)
```

```
[Out] Piecewise((x*cos(a)**2*cos(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**2
/2 + x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(2*d))*cos(a)**2, Eq(b
, 0)), (3*x*sin(a - d*x)**2*sin(c + d*x)**2/8 + x*sin(a - d*x)**2*cos(c + d
*x)**2/8 - x*sin(a - d*x)*sin(c + d*x)*cos(a - d*x)*cos(c + d*x)/2 + x*sin(
c + d*x)**2*cos(a - d*x)**2/8 + 3*x*cos(a - d*x)**2*cos(c + d*x)**2/8 + 3*s
in(a - d*x)**2*sin(c + d*x)*cos(c + d*x)/(8*d) - sin(a - d*x)*cos(a - d*x)*
cos(c + d*x)**2/(2*d) + sin(c + d*x)*cos(a - d*x)**2*cos(c + d*x)/(8*d), Eq
(b, -d)), (3*x*sin(a + d*x)**2*sin(c + d*x)**2/8 + x*sin(a + d*x)**2*cos(c
+ d*x)**2/8 + x*sin(a + d*x)*sin(c + d*x)*cos(a + d*x)*cos(c + d*x)/2 + x*s
in(c + d*x)**2*cos(a + d*x)**2/8 + 3*x*cos(a + d*x)**2*cos(c + d*x)**2/8 +
3*sin(a + d*x)**2*sin(c + d*x)*cos(c + d*x)/(8*d) + sin(a + d*x)*cos(a + d*
x)*cos(c + d*x)**2/(2*d) + sin(c + d*x)*cos(a + d*x)**2*cos(c + d*x)/(8*d),
 Eq(b, d)), ((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 + sin(a + b*x)*cos(
a + b*x)/(2*b))*cos(c)**2, Eq(d, 0)), (b**3*d*x*sin(a + b*x)**2*sin(c + d*x
)**2/(4*b**3*d - 4*b*d**3) + b**3*d*x*sin(a + b*x)**2*cos(c + d*x)**2/(4*b*
*3*d - 4*b*d**3) + b**3*d*x*sin(c + d*x)**2*cos(a + b*x)**2/(4*b**3*d - 4*b
*d**3) + b**3*d*x*cos(a + b*x)**2*cos(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b
**3*sin(a + b*x)**2*sin(c + d*x)*cos(c + d*x)/(4*b**3*d - 4*b*d**3) + b**3*
sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x)/(4*b**3*d - 4*b*d**3) + 2*b**2*d*
sin(a + b*x)*cos(a + b*x)*cos(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*
sin(a + b*x)**2*sin(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*sin(a + b*
x)**2*cos(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*sin(c + d*x)**2*cos(
a + b*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*cos(a + b*x)**2*cos(c + d*x)**
2/(4*b**3*d - 4*b*d**3) - 2*b*d**2*sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x
)/(4*b**3*d - 4*b*d**3) - d**3*sin(a + b*x)*sin(c + d*x)**2*cos(a + b*x)/(4
*b**3*d - 4*b*d**3) - d**3*sin(a + b*x)*cos(a + b*x)*cos(c + d*x)**2/(4*b**
3*d - 4*b*d**3), True))
```

Giac [A] time = 1.12814, size = 108, normalized size = 1.23

$$\frac{1}{4}x + \frac{\sin\left(2\,bx + 2\,dx + 2\,a + 2\,c\right)}{16\,(b + d)} + \frac{\sin\left(2\,bx - 2\,dx + 2\,a - 2\,c\right)}{16\,(b - d)} + \frac{\sin\left(2\,bx + 2\,a\right)}{8\,b} + \frac{\sin\left(2\,dx + 2\,c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cos(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*x + 1/16*sin(2*b*x + 2*d*x + 2*a + 2*c)/(b + d) + 1/16*sin(2*b*x - 2*d*
x + 2*a - 2*c)/(b - d) + 1/8*sin(2*b*x + 2*a)/b + 1/8*sin(2*d*x + 2*c)/d
```

3.246 $\int \cos^3(a+bx)\cos^3(c+dx)\,dx$

Optimal. Leaf size=195

$$\frac{3\sin(a+x(b-3d)-3c)}{32(b-3d)}+\frac{9\sin(a+x(b-d)-c)}{32(b-d)}+\frac{\sin(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\sin(3a+x(3b-d)-c)}{32(3b-d)}+\frac{9\sin(a+x(}{32(b}$$

```
[Out] (3*Sin[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) + (9*Sin[a - c + (b - d)*x])/
(32*(b - d)) + Sin[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) + (3*Sin[3*a - c +
 (3*b - d)*x])/(32*(3*b - d)) + (9*Sin[a + c + (b + d)*x])/(32*(b + d)) + S
in[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) + (3*Sin[3*a + c + (3*b + d)*x])/(
32*(3*b + d)) + (3*Sin[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

Rubi [A] time = 0.134626, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4570, 2637}

$$\frac{3\sin(a+x(b-3d)-3c)}{32(b-3d)}+\frac{9\sin(a+x(b-d)-c)}{32(b-d)}+\frac{\sin(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\sin(3a+x(3b-d)-c)}{32(3b-d)}+\frac{9\sin(a+x(}{32(b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Cos[c + d*x]^3,x]
```

```
[Out] (3*Sin[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) + (9*Sin[a - c + (b - d)*x])/
(32*(b - d)) + Sin[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) + (3*Sin[3*a - c +
 (3*b - d)*x])/(32*(3*b - d)) + (9*Sin[a + c + (b + d)*x])/(32*(b + d)) + S
in[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) + (3*Sin[3*a + c + (3*b + d)*x])/(
32*(3*b + d)) + (3*Sin[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

Rule 4570

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\cos^3(c+dx)\,dx &= \int \left(\frac{3}{32}\cos(a-3c+(b-3d)x)+\frac{9}{32}\cos(a-c+(b-d)x)+\frac{1}{32}\cos(3(a-c)+3(b-d)\right.\\ &= \frac{1}{32}\int \cos(3(a-c)+3(b-d)x)\,dx+\frac{1}{32}\int \cos(3(a+c)+3(b+d)x)\,dx+\frac{3}{32}\int \cos(a\\ &= \frac{3\sin(a-3c+(b-3d)x)}{32(b-3d)}+\frac{9\sin(a-c+(b-d)x)}{32(b-d)}+\frac{\sin(3(a-c)+3(b-d)x)}{96(b-d)}+\frac{3\sin(}{}\end{aligned}$$

Mathematica [A] time = 1.62955, size = 176, normalized size = 0.9

$$\frac{1}{96}\left(\frac{9\sin(a+bx-3c-3dx)}{b-3d}+\frac{27\sin(a+bx-c-dx)}{b-d}+\frac{\sin(3(a+bx-c-dx))}{b-d}+\frac{9\sin(3a+3bx-c-dx)}{3b-d}+\frac{9\sin(3a}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Cos[c + d*x]^3,x]
```

```
[Out] ((9*Sin[a - 3*c + b*x - 3*d*x])/(b - 3*d) + (27*Sin[a - c + b*x - d*x])/(b
- d) + Sin[3*(a - c + b*x - d*x)]/(b - d) + (9*Sin[3*a - c + 3*b*x - d*x])/
(3*b - d) + (9*Sin[3*a + c + 3*b*x + d*x])/(3*b + d) + (9*Sin[a + 3*c + b*x
 + 3*d*x])/(b + 3*d) + (27*Sin[a + c + (b + d)*x])/(b + d) + Sin[3*(a + c +
 (b + d)*x)]/(b + d))/96
```

Maple [A] time = 0.046, size = 184, normalized size = 0.9

$$\frac{3 \sin(a-3c+(b-3d)x)}{32b-96d}+\frac{9 \sin(a-c+(b-d)x)}{32b-32d}+\frac{9 \sin(a+c+(b+d)x)}{32b+32d}+\frac{3 \sin(a+3c+(b+3d)x)}{32b+96d}+\frac{\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*cos(d*x+c)^3,x)
```

```
[Out] 3/32*sin(a-3*c+(b-3*d)*x)/(b-3*d)+9/32*sin(a-c+(b-d)*x)/(b-d)+9/32*sin(a+c+
(b+d)*x)/(b+d)+3/32*sin(a+3*c+(b+3*d)*x)/(b+3*d)+1/96/(b-d)*sin((3*b-3*d)*x
-3*c+3*a)+3/32*sin(3*a-c+(3*b-d)*x)/(3*b-d)+3/32*sin(3*a+c+(3*b+d)*x)/(3*b+
d)+1/96/(b+d)*sin((3*b+3*d)*x+3*c+3*a)
```

Maxima [B] time = 1.87926, size = 3529, normalized size = 18.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*cos(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/192*(9*(3*b^5*sin(3*c) - b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) + 10*b^2*d
^3*sin(3*c) + 27*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos((3*b + d)*x + 3*a + 4
*c) - 9*(3*b^5*sin(3*c) - b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) + 10*b^2*d^3
*sin(3*c) + 27*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos((3*b + d)*x + 3*a - 2*c
) - 9*(3*b^5*sin(3*c) + b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) - 10*b^2*d^3*s
in(3*c) + 27*b*d^4*sin(3*c) + 9*d^5*sin(3*c))*cos(-(3*b - d)*x - 3*a + 4*c)
 + 9*(3*b^5*sin(3*c) + b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) - 10*b^2*d^3*si
n(3*c) + 27*b*d^4*sin(3*c) + 9*d^5*sin(3*c))*cos(-(3*b - d)*x - 3*a - 2*c)
+ 9*(9*b^5*sin(3*c) - 27*b^4*d*sin(3*c) - 10*b^3*d^2*sin(3*c) + 30*b^2*d^3*
sin(3*c) + b*d^4*sin(3*c) - 3*d^5*sin(3*c))*cos((b + 3*d)*x + a + 6*c) - 9*
(9*b^5*sin(3*c) - 27*b^4*d*sin(3*c) - 10*b^3*d^2*sin(3*c) + 30*b^2*d^3*sin(
3*c) + b*d^4*sin(3*c) - 3*d^5*sin(3*c))*cos((b + 3*d)*x + a) + (9*b^5*sin(3
*c) - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^
4*sin(3*c) - 9*d^5*sin(3*c))*cos(3*(b + d)*x + 3*a + 6*c) - (9*b^5*sin(3*c)
 - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^4*s
in(3*c) - 9*d^5*sin(3*c))*cos(3*(b + d)*x + 3*a) + 27*(9*b^5*sin(3*c) - 9*b
^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c
) - 9*d^5*sin(3*c))*cos((b + d)*x + a + 4*c) - 27*(9*b^5*sin(3*c) - 9*b^4*d
*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) -
9*d^5*sin(3*c))*cos((b + d)*x + a - 2*c) - 27*(9*b^5*sin(3*c) + 9*b^4*d*sin
(3*c) - 82*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^
5*sin(3*c))*cos(-(b - d)*x - a + 4*c) + 27*(9*b^5*sin(3*c) + 9*b^4*d*sin(3*
```

```
c) - 82*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^5*s
in(3*c))*cos(-(b - d)*x - a - 2*c) - (9*b^5*sin(3*c) + 9*b^4*d*sin(3*c) - 8
2*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^5*sin(3*c
))*cos(-3*(b - d)*x - 3*a + 6*c) + (9*b^5*sin(3*c) + 9*b^4*d*sin(3*c) - 82*
b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^5*sin(3*c))
*cos(-3*(b - d)*x - 3*a) - 9*(9*b^5*sin(3*c) + 27*b^4*d*sin(3*c) - 10*b^3*d
^2*sin(3*c) - 30*b^2*d^3*sin(3*c) + b*d^4*sin(3*c) + 3*d^5*sin(3*c))*cos(-(
b - 3*d)*x - a + 6*c) + 9*(9*b^5*sin(3*c) + 27*b^4*d*sin(3*c) - 10*b^3*d^2*
sin(3*c) - 30*b^2*d^3*sin(3*c) + b*d^4*sin(3*c) + 3*d^5*sin(3*c))*cos(-(b -
 3*d)*x - a) - 9*(3*b^5*cos(3*c) - b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) + 1
0*b^2*d^3*cos(3*c) + 27*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*sin((3*b + d)*x +
3*a + 4*c) - 9*(3*b^5*cos(3*c) - b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) + 10*
b^2*d^3*cos(3*c) + 27*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*sin((3*b + d)*x + 3*
a - 2*c) + 9*(3*b^5*cos(3*c) + b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) - 10*b^
2*d^3*cos(3*c) + 27*b*d^4*cos(3*c) + 9*d^5*cos(3*c))*sin(-(3*b - d)*x - 3*a
 + 4*c) + 9*(3*b^5*cos(3*c) + b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) - 10*b^2
*d^3*cos(3*c) + 27*b*d^4*cos(3*c) + 9*d^5*cos(3*c))*sin(-(3*b - d)*x - 3*a
- 2*c) - 9*(9*b^5*cos(3*c) - 27*b^4*d*cos(3*c) - 10*b^3*d^2*cos(3*c) + 30*b
^2*d^3*cos(3*c) + b*d^4*cos(3*c) - 3*d^5*cos(3*c))*sin((b + 3*d)*x + a + 6*
c) - 9*(9*b^5*cos(3*c) - 27*b^4*d*cos(3*c) - 10*b^3*d^2*cos(3*c) + 30*b^2*d
^3*cos(3*c) + b*d^4*cos(3*c) - 3*d^5*cos(3*c))*sin((b + 3*d)*x + a) - (9*b^
5*cos(3*c) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) +
 9*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*sin(3*(b + d)*x + 3*a + 6*c) - (9*b^5*c
os(3*c) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*
b*d^4*cos(3*c) - 9*d^5*cos(3*c))*sin(3*(b + d)*x + 3*a) - 27*(9*b^5*cos(3*c
) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*b*d^4*
cos(3*c) - 9*d^5*cos(3*c))*sin((b + d)*x + a + 4*c) - 27*(9*b^5*cos(3*c) -
9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(
3*c) - 9*d^5*cos(3*c))*sin((b + d)*x + a - 2*c) + 27*(9*b^5*cos(3*c) + 9*b^
4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c)
 + 9*d^5*cos(3*c))*sin(-(b - d)*x - a + 4*c) + 27*(9*b^5*cos(3*c) + 9*b^4*d
*cos(3*c) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) +
9*d^5*cos(3*c))*sin(-(b - d)*x - a - 2*c) + (9*b^5*cos(3*c) + 9*b^4*d*cos(3
*c) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) + 9*d^5*
cos(3*c))*sin(-3*(b - d)*x - 3*a + 6*c) + (9*b^5*cos(3*c) + 9*b^4*d*cos(3*c
) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) + 9*d^5*co
s(3*c))*sin(-3*(b - d)*x - 3*a) + 9*(9*b^5*cos(3*c) + 27*b^4*d*cos(3*c) - 1
0*b^3*d^2*cos(3*c) - 30*b^2*d^3*cos(3*c) + b*d^4*cos(3*c) + 3*d^5*cos(3*c))
*sin(-(b - 3*d)*x - a + 6*c) + 9*(9*b^5*cos(3*c) + 27*b^4*d*cos(3*c) - 10*b
^3*d^2*cos(3*c) - 30*b^2*d^3*cos(3*c) + b*d^4*cos(3*c) + 3*d^5*cos(3*c))*si
n(-(b - 3*d)*x - a))/(9*b^6*cos(3*c)^2 + 9*b^6*sin(3*c)^2 - 9*(cos(3*c)^2 +
 sin(3*c)^2)*d^6 + 91*(b^2*cos(3*c)^2 + b^2*sin(3*c)^2)*d^4 - 91*(b^4*cos(3
*c)^2 + b^4*sin(3*c)^2)*d^2)
```

Fricas [A] time = 0.591744, size = 535, normalized size = 2.74

$$\left(\left(18\,b^{5}-2\,b^{3}d^{2}+\left(9\,b^{5}-82\,b^{3}d^{2}+9\,bd^{4}\right)\cos\left(bx+a\right)^{2}\right)\cos\left(dx+c\right)^{3}-6\left(20\,b^{3}d^{2}+\left(b^{3}d^{2}-9\,bd^{4}\right)\cos\left(bx+a\right)^{2}\right)\cos\left(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*cos(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(((18*b^5 - 2*b^3*d^2 + (9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cos(b*x + a)^2)*
cos(d*x + c)^3 - 6*(20*b^3*d^2 + (b^3*d^2 - 9*b*d^4)*cos(b*x + a)^2)*cos(d*
x + c))*sin(b*x + a) + (120*b^2*d^3*cos(b*x + a) + 2*(b^2*d^3 - 9*d^5)*cos(
b*x + a)^3 - ((9*b^4*d - 82*b^2*d^3 + 9*d^5)*cos(b*x + a)^3 + 6*(9*b^4*d -
```

```
b^2*d^3)*cos(b*x + a))*cos(d*x + c)^2)*sin(d*x + c))/(9*b^6 - 91*b^4*d^2 +
91*b^2*d^4 - 9*d^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*cos(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11687, size = 244, normalized size = 1.25

$$\frac{\sin\left(3\,bx + 3\,dx + 3\,a + 3\,c\right)}{96\,(b + d)} + \frac{3\,\sin\left(3\,bx + dx + 3\,a + c\right)}{32\,(3\,b + d)} + \frac{3\,\sin\left(3\,bx - dx + 3\,a - c\right)}{32\,(3\,b - d)} + \frac{\sin\left(3\,bx - 3\,dx + 3\,a - 3\,c\right)}{96\,(b - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*cos(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/96*sin(3*b*x + 3*d*x + 3*a + 3*c)/(b + d) + 3/32*sin(3*b*x + d*x + 3*a +
c)/(3*b + d) + 3/32*sin(3*b*x - d*x + 3*a - c)/(3*b - d) + 1/96*sin(3*b*x -
 3*d*x + 3*a - 3*c)/(b - d) + 3/32*sin(b*x + 3*d*x + a + 3*c)/(b + 3*d) + 9
/32*sin(b*x + d*x + a + c)/(b + d) + 9/32*sin(b*x - d*x + a - c)/(b - d) +
3/32*sin(b*x - 3*d*x + a - 3*c)/(b - 3*d)
```

3.247 $\int \cos(a+bx)\tan^3(c+bx)\,dx$

Optimal. Leaf size=72

$$\frac{3\sin(a-c)\tanh^{-1}(\sin(bx+c))}{2b}+\frac{\cos(a-c)\sec(bx+c)}{b}-\frac{\sin(a-c)\tan(bx+c)\sec(bx+c)}{2b}+\frac{\cos(a+bx)}{b}$$

```
[Out] Cos[a + b*x]/b + (Cos[a - c]*Sec[c + b*x])/b + (3*ArcTanh[Sin[c + b*x]]*Sin
[a - c])/(2*b) - (Sec[c + b*x]*Sin[a - c]*Tan[c + b*x])/(2*b)
```

Rubi [A] time = 0.0779723, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {4579, 4576, 2638, 3770, 2606, 8, 2611}

$$\frac{3\sin(a-c)\tanh^{-1}(\sin(bx+c))}{2b}+\frac{\cos(a-c)\sec(bx+c)}{b}-\frac{\sin(a-c)\tan(bx+c)\sec(bx+c)}{2b}+\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Tan[c + b*x]^3,x]
```

```
[Out] Cos[a + b*x]/b + (Cos[a - c]*Sec[c + b*x])/b + (3*ArcTanh[Sin[c + b*x]]*Sin
[a - c])/(2*b) - (Sec[c + b*x]*Sin[a - c]*Tan[c + b*x])/(2*b)
```

Rule 4579

```
Int[Cos[v_]*Tan[w_]^(n_.), x_Symbol] :> Int[Sin[v]*Tan[w]^(n - 1), x] - Dis
t[Sin[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rule 4576

```
Int[Sin[v_]*Tan[w_]^(n_.), x_Symbol] :> -Int[Cos[v]*Tan[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\tan^3(c+bx)\,dx &= -\left(\sin(a-c)\int \sec(c+bx)\tan^2(c+bx)\,dx\right)+\int \sin(a+bx)\tan^2(c+bx)\,dx\\ &= -\frac{\sec(c+bx)\sin(a-c)\tan(c+bx)}{2b}+\cos(a-c)\int \sec(c+bx)\tan(c+bx)\,dx+\frac{1}{2}\text{si}\\ &= \frac{\tanh^{-1}(\sin(c+bx))\sin(a-c)}{2b}-\frac{\sec(c+bx)\sin(a-c)\tan(c+bx)}{2b}+\frac{\cos(a-c)\,\text{Sul}}{}\\ &= \frac{\cos(a+bx)}{b}+\frac{\cos(a-c)\sec(c+bx)}{b}+\frac{3\tanh^{-1}(\sin(c+bx))\sin(a-c)}{2b}-\frac{\sec(c+b}{}\end{aligned}$$

Mathematica [A] time = 0.36262, size = 70, normalized size = 0.97

$$\frac{\sec^2(bx+c)(2\cos(a-bx-2c)+\cos(a+3bx+2c)+5\cos(a+bx))+12\sin(a-c)\tanh^{-1}\left(\cos(c)\tan\left(\frac{bx}{2}\right)+\sin(c)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Tan[c + b*x]^3,x]
```

```
[Out] ((2*Cos[a - 2*c - b*x] + 5*Cos[a + b*x] + Cos[a + 2*c + 3*b*x])*Sec[c + b*x
]^2 + 12*ArcTanh[Sin[c] + Cos[c]*Tan[(b*x)/2]]*Sin[a - c])/(4*b)
```

Maple [C] time = 0.099, size = 181, normalized size = 2.5

$$\frac{e^{i(bx+a)}}{2b}+\frac{e^{-i(bx+a)}}{2b}+\frac{3e^{i(3bx+5a+2c)}+e^{i(3bx+3a+4c)}+e^{i(bx+5a)}+3e^{i(bx+3a+2c)}}{2b\left(e^{2i(bx+a+c)}+e^{2ia}\right)^2}+\frac{3\ln\left(e^{i(bx+a)}+ie^{i(a-c)}\right)\sin(a-c)}{2b}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*tan(b*x+c)^3,x)
```

```
[Out] 1/2*exp(I*(b*x+a))/b+1/2/b*exp(-I*(b*x+a))+1/2/b/(exp(2*I*(b*x+a+c))+exp(2*
I*a))^2*(3*exp(I*(3*b*x+5*a+2*c))+exp(I*(3*b*x+3*a+4*c))+exp(I*(b*x+5*a))+3
*exp(I*(b*x+3*a+2*c)))+3/2/b*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))*sin(a-c)-3/2
/b*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))*sin(a-c)
```

Maxima [B] time = 2.03128, size = 1386, normalized size = 19.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(cos(5*b*x + a + 4*c) + 2*cos(3*b*x + a + 2*c) + cos(b*x + a))*cos(6
*b*x + 2*a + 4*c) + 2*(5*cos(4*b*x + 2*a + 2*c) + 2*cos(4*b*x + 4*c) + 2*co
s(2*b*x + 2*a) + 5*cos(2*b*x + 2*c) + 1)*cos(5*b*x + a + 4*c) + 10*(2*cos(3
*b*x + a + 2*c) + cos(b*x + a))*cos(4*b*x + 2*a + 2*c) + 4*(2*cos(3*b*x + a
 + 2*c) + cos(b*x + a))*cos(4*b*x + 4*c) + 4*(2*cos(2*b*x + 2*a) + 5*cos(2*
b*x + 2*c) + 1)*cos(3*b*x + a + 2*c) + 4*cos(2*b*x + 2*a)*cos(b*x + a) + 10
*cos(2*b*x + 2*c)*cos(b*x + a) + 3*(cos(5*b*x + a + 4*c)^2*sin(-a + c) + 4*
cos(3*b*x + a + 2*c)^2*sin(-a + c) + 4*cos(3*b*x + a + 2*c)*cos(b*x + a)*si
n(-a + c) + cos(b*x + a)^2*sin(-a + c) + sin(5*b*x + a + 4*c)^2*sin(-a + c)
 + 4*sin(3*b*x + a + 2*c)^2*sin(-a + c) + 4*sin(3*b*x + a + 2*c)*sin(b*x +
a)*sin(-a + c) + sin(b*x + a)^2*sin(-a + c) + 2*(2*cos(3*b*x + a + 2*c)*sin
(-a + c) + cos(b*x + a)*sin(-a + c))*cos(5*b*x + a + 4*c) + 2*(2*sin(3*b*x
+ a + 2*c)*sin(-a + c) + sin(b*x + a)*sin(-a + c))*sin(5*b*x + a + 4*c))*lo
g((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2
 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos
(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)
) + 2*(sin(5*b*x + a + 4*c) + 2*sin(3*b*x + a + 2*c) + sin(b*x + a))*sin(6*
b*x + 2*a + 4*c) + 2*(5*sin(4*b*x + 2*a + 2*c) + 2*sin(4*b*x + 4*c) + 2*sin
(2*b*x + 2*a) + 5*sin(2*b*x + 2*c))*sin(5*b*x + a + 4*c) + 10*(2*sin(3*b*x
+ a + 2*c) + sin(b*x + a))*sin(4*b*x + 2*a + 2*c) + 4*(2*sin(3*b*x + a + 2*
c) + sin(b*x + a))*sin(4*b*x + 4*c) + 4*(2*sin(2*b*x + 2*a) + 5*sin(2*b*x +
 2*c))*sin(3*b*x + a + 2*c) + 4*sin(2*b*x + 2*a)*sin(b*x + a) + 10*sin(2*b*
x + 2*c)*sin(b*x + a) + 2*cos(b*x + a))/(b*cos(5*b*x + a + 4*c)^2 + 4*b*cos
(3*b*x + a + 2*c)^2 + 4*b*cos(3*b*x + a + 2*c)*cos(b*x + a) + b*cos(b*x + a
)^2 + b*sin(5*b*x + a + 4*c)^2 + 4*b*sin(3*b*x + a + 2*c)^2 + 4*b*sin(3*b*x
 + a + 2*c)*sin(b*x + a) + b*sin(b*x + a)^2 + 2*(2*b*cos(3*b*x + a + 2*c) +
 b*cos(b*x + a))*cos(5*b*x + a + 4*c) + 2*(2*b*sin(3*b*x + a + 2*c) + b*sin
(b*x + a))*sin(5*b*x + a + 4*c))
```

Fricas [B] time = 0.570641, size = 986, normalized size = 13.69

$$\frac{16 \cos(bx+a)^3 \cos(-2a+2c) - 4\left(4 \cos(bx+a)^2+1\right) \sin(bx+a) \sin(-2a+2c) - 4\left(\cos(-2a+2c)-5\right) \cos(bx + \ldots}{8\left(2 b \cos(bx+a)^2 \ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(16*cos(b*x + a)^3*cos(-2*a + 2*c) - 4*(4*cos(b*x + a)^2 + 1)*sin(b*x +
 a)*sin(-2*a + 2*c) - 4*(cos(-2*a + 2*c) - 5)*cos(b*x + a) + 3*sqrt(2)*(2*(
cos(-2*a + 2*c)^2 - 1)*cos(b*x + a)*sin(b*x + a) + (2*cos(b*x + a)^2*cos(-2
*a + 2*c) - cos(-2*a + 2*c) + 1)*sin(-2*a + 2*c))*log(-(2*cos(b*x + a)^2*co
s(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) + 2*sqrt(2)*((c
os(-2*a + 2*c) + 1)*sin(b*x + a) + cos(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-
2*a + 2*c) + 1) - cos(-2*a + 2*c) - 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) -
2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) + 1))/sqrt(co
s(-2*a + 2*c) + 1))/(2*b*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*b*cos(b*x + a)*
sin(b*x + a)*sin(-2*a + 2*c) - b*cos(-2*a + 2*c) + b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)\tan(bx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*tan(b*x + c)^3, x)
```

3.248 $\int \cos(a+bx)\tan^2(c+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\sin(a-c)\sec(bx+c)}{b}+\frac{\cos(a-c)\tanh^{-1}(\sin(bx+c))}{b}-\frac{\sin(a+bx)}{b}$$

```
[Out] (ArcTanh[Sin[c + b*x]]*Cos[a - c])/b - (Sec[c + b*x]*Sin[a - c])/b - Sin[a
+ b*x]/b
```

Rubi [A] time = 0.0391638, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4579, 4576, 2637, 3770, 2606, 8}

$$-\frac{\sin(a-c)\sec(bx+c)}{b}+\frac{\cos(a-c)\tanh^{-1}(\sin(bx+c))}{b}-\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Tan[c + b*x]^2,x]
```

```
[Out] (ArcTanh[Sin[c + b*x]]*Cos[a - c])/b - (Sec[c + b*x]*Sin[a - c])/b - Sin[a
+ b*x]/b
```

Rule 4579

```
Int[Cos[v_]*Tan[w_]^(n_.), x_Symbol] :> Int[Sin[v]*Tan[w]^(n - 1), x] - Dis
t[Sin[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rule 4576

```
Int[Sin[v_]*Tan[w_]^(n_.), x_Symbol] :> -Int[Cos[v]*Tan[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos(a+bx)\tan^2(c+bx)\,dx = -(\sin(a-c)\int \sec(c+bx)\tan(c+bx)\,dx) + \int \sin(a+bx)\tan(c+bx)\,dx$$
$$= \cos(a-c)\int \sec(c+bx)\,dx - \frac{\sin(a-c)\,\text{Subst}(\int 1\,dx, x, \sec(c+bx))}{b} - \int \cos(a+b$$
$$= \frac{\tanh^{-1}(\sin(c+bx))\cos(a-c)}{b} - \frac{\sec(c+bx)\sin(a-c)}{b} - \frac{\sin(a+bx)}{b}$$

Mathematica [C] time = 0.097455, size = 111, normalized size = 2.41

$$-\frac{\sin(a-c)\sec(bx+c)}{b} - \frac{2i\cos(a-c)\tan^{-1}\left(\frac{(\sin(c)+i\cos(c))\left(\sin(c)\cos\left(\frac{bx}{2}\right)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\sin(c)\cos\left(\frac{bx}{2}\right)}\right)}{b} - \frac{\sin(a)\cos(bx)}{b} - \frac{\cos(a)\sin(b}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Tan[c + b*x]^2,x]
```

```
[Out] ((-2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x)/2]*Sin[c] + Cos[c]*Sin[(b*x)
/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b - (Cos[b
*x]*Sin[a])/b - (Sec[c + b*x]*Sin[a - c])/b - (Cos[a]*Sin[b*x])/b
```

Maple [C] time = 0.085, size = 149, normalized size = 3.2

$$\frac{\frac{i}{2}e^{i(bx+a)}}{b} - \frac{\frac{i}{2}e^{-i(bx+a)}}{b} - \frac{i\left(-e^{i(bx+3a)}+e^{i(bx+a+2c)}\right)}{b\left(e^{2i(bx+a+c)}+e^{2ia}\right)} - \frac{\ln\left(e^{i(bx+a)}-ie^{i(a-c)}\right)\cos(a-c)}{b} + \frac{\ln\left(e^{i(bx+a)}+ie^{i(a-c)}\right)\cos(a-c}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*tan(b*x+c)^2,x)
```

```
[Out] 1/2*I*exp(I*(b*x+a))/b-1/2*I/b*exp(-I*(b*x+a))-I/b/(exp(2*I*(b*x+a+c))+exp(
2*I*a))*(-exp(I*(b*x+3*a))+exp(I*(b*x+a+2*c)))-1/b*ln(exp(I*(b*x+a))-I*exp(
I*(a-c)))*cos(a-c)+1/b*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))*cos(a-c)
```

Maxima [B] time = 1.96875, size = 710, normalized size = 15.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((sin(3*b*x + a + 2*c) + sin(b*x + a))*cos(4*b*x + 2*a + 2*c) - 3*(sin(
2*b*x + 2*a) - sin(2*b*x + 2*c))*cos(3*b*x + a + 2*c) - (cos(3*b*x + a + 2*
c)^2*cos(-a + c) + 2*cos(3*b*x + a + 2*c)*cos(b*x + a)*cos(-a + c) + cos(b*
x + a)^2*cos(-a + c) + cos(-a + c)*sin(3*b*x + a + 2*c)^2 + 2*cos(-a + c)*s
in(3*b*x + a + 2*c)*sin(b*x + a) + cos(-a + c)*sin(b*x + a)^2)*log((cos(b*x
 + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 + 2*cos(b
```

```
*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)*sin(b*
x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)) - (cos(3
*b*x + a + 2*c) + cos(b*x + a))*sin(4*b*x + 2*a + 2*c) + (3*cos(2*b*x + 2*a
) - 3*cos(2*b*x + 2*c) - 1)*sin(3*b*x + a + 2*c) - 3*cos(b*x + a)*sin(2*b*x
 + 2*a) + 3*cos(b*x + a)*sin(2*b*x + 2*c) + 3*cos(2*b*x + 2*a)*sin(b*x + a)
 - 3*cos(2*b*x + 2*c)*sin(b*x + a) - sin(b*x + a))/(b*cos(3*b*x + a + 2*c)^
2 + 2*b*cos(3*b*x + a + 2*c)*cos(b*x + a) + b*cos(b*x + a)^2 + b*sin(3*b*x
+ a + 2*c)^2 + 2*b*sin(3*b*x + a + 2*c)*sin(b*x + a) + b*sin(b*x + a)^2)
```

Fricas [B] time = 0.535624, size = 857, normalized size = 18.63

$$\frac{4\left(\cos\left(-2\,a+2\,c\right)+1\right)\cos\left(bx+a\right)\sin\left(bx+a\right)+\frac{\sqrt{2}\left(\left(\cos(-2\,a+2\,c)+1\right)\sin(bx+a)\sin(-2\,a+2\,c)-\left(\cos(-2\,a+2\,c)^{2}+2\,\cos(-2\,a+2\,c)+1\right)\cos(bx+a\cdots\right.}{\cdots}}{4\left(b\sin\left(bx+a\right)\sin\left(-2\,a\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*((cos(-2*a
 + 2*c) + 1)*sin(b*x + a)*sin(-2*a + 2*c) - (cos(-2*a + 2*c)^2 + 2*cos(-2*a
 + 2*c) + 1)*cos(b*x + a))*log((2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*
x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*sin(
b*x + a) + cos(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2
*a + 2*c) - 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x +
 a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) + 1))/sqrt(cos(-2*a + 2*c) + 1) + 4*(
cos(b*x + a)^2 - 2)*sin(-2*a + 2*c))/(b*sin(b*x + a)*sin(-2*a + 2*c) - (b*c
os(-2*a + 2*c) + b)*cos(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(a+bx\right)\tan^{2}\left(bx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)*tan(b*x + c)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(bx+a\right)\tan\left(bx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*tan(b*x + c)^2, x)
```

3.249 $\int \cos(a+bx)\tan(c+bx)\,dx$

Optimal. Leaf size=30

$$-\frac{\sin(a-c)\tanh^{-1}(\sin(bx+c))}{b}-\frac{\cos(a+bx)}{b}$$

```
[Out] -(Cos[a + b*x]/b) - (ArcTanh[Sin[c + b*x]]*Sin[a - c])/b
```

Rubi [A] time = 0.0176683, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4579, 2638, 3770}

$$-\frac{\sin(a-c)\tanh^{-1}(\sin(bx+c))}{b}-\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Tan[c + b*x],x]
```

```
[Out] -(Cos[a + b*x]/b) - (ArcTanh[Sin[c + b*x]]*Sin[a - c])/b
```

Rule 4579

```
Int[Cos[v_]*Tan[w_]^(n_.), x_Symbol] :> Int[Sin[v]*Tan[w]^(n - 1), x] - Dis
t[Sin[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\tan(c+bx)\,dx &= -(\sin(a-c)\int \sec(c+bx)\,dx)+\int \sin(a+bx)\,dx\\ &= -\frac{\cos(a+bx)}{b}-\frac{\tanh^{-1}(\sin(c+bx))\sin(a-c)}{b}\end{aligned}$$

Mathematica [C] time = 0.0595658, size = 93, normalized size = 3.1

$$\frac{2i\sin(a-c)\tan^{-1}\left(\frac{(\sin(c)+i\cos(c))\left(\sin(c)\cos\left(\frac{bx}{2}\right)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\sin(c)\cos\left(\frac{bx}{2}\right)}\right)}{b}+\frac{\sin(a)\sin(bx)}{b}-\frac{\cos(a)\cos(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Tan[c + b*x],x]
```

```
[Out] -((Cos[a]*Cos[b*x])/b) + ((2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x)/2]*S
in[c] + Cos[c]*Sin[(b*x)/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Sin[c])
]*Sin[a - c])/b + (Sin[a]*Sin[b*x])/b
```

Maple [C] time = 0.066, size = 97, normalized size = 3.2

$$-\frac{e^{i(bx+a)}}{2b}-\frac{e^{-i(bx+a)}}{2b}+\frac{\ln\left(e^{i(bx+a)}-ie^{i(a-c)}\right)\sin(a-c)}{b}-\frac{\ln\left(e^{i(bx+a)}+ie^{i(a-c)}\right)\sin(a-c)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*tan(b*x+c),x)
```

```
[Out] -1/2*exp(I*(b*x+a))/b-1/2/b*exp(-I*(b*x+a))+1/b*ln(exp(I*(b*x+a))-I*exp(I*(
a-c)))*sin(a-c)-1/b*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))*sin(a-c)
```

Maxima [B] time = 1.86161, size = 177, normalized size = 5.9

$$-\frac{\log\left(\frac{\cos(bx+2c)^2+\cos(c)^2-2\cos(c)\sin(bx+2c)+\sin(bx+2c)^2+2\cos(bx+2c)\sin(c)+\sin(c)^2}{\cos(bx+2c)^2+\cos(c)^2+2\cos(c)\sin(bx+2c)+\sin(bx+2c)^2-2\cos(bx+2c)\sin(c)+\sin(c)^2}\right)\sin(-a+c)+2\cos(bx+a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x
+ 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2
 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + s
in(c)^2))*sin(-a + c) + 2*cos(b*x + a))/b
```

Fricas [B] time = 0.53063, size = 531, normalized size = 17.7

$$\frac{\frac{\sqrt{2}\log\left(\frac{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\frac{2\sqrt{2}((\cos(-2a+2c)+1)\sin(bx+a)+\cos(bx+a)\sin(-2a+2c))}{\sqrt{\cos(-2a+2c)+1}}-\cos(-2a+2c)-3}{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\cos(-2a+2c)+1}\right)\sin(-2a+2c)}{\sqrt{\cos(-2a+2c)+1}}-4\cos(b}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*log((2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x
 + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*sin(b*x + a) + cos
(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2*a + 2*c) - 3)
/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a +
 2*c) - cos(-2*a + 2*c) + 1))*sin(-2*a + 2*c)/sqrt(cos(-2*a + 2*c) + 1) - 4
*cos(b*x + a))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(a+bx)\tan(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c),x)
```

```
[Out] Integral(cos(a + b*x)*tan(b*x + c), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)\tan(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*tan(b*x + c), x)
```

3.250 $\int \cos(a+bx)\cot(c+bx)\,dx$

Optimal. Leaf size=29

$$\frac{\cos(a+bx)}{b}-\frac{\cos(a-c)\tanh^{-1}(\cos(bx+c))}{b}$$

```
[Out] -((ArcTanh[Cos[c + b*x]]*Cos[a - c])/b) + Cos[a + b*x]/b
```

Rubi [A] time = 0.0209471, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4577, 2638, 3770}

$$\frac{\cos(a+bx)}{b}-\frac{\cos(a-c)\tanh^{-1}(\cos(bx+c))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cot[c + b*x],x]
```

```
[Out] -((ArcTanh[Cos[c + b*x]]*Cos[a - c])/b) + Cos[a + b*x]/b
```

Rule 4577

```
Int[Cos[v_]*Cot[w_]^(n_.), x_Symbol] :> -Int[Sin[v]*Cot[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\cot(c+bx)\,dx &= \cos(a-c)\int \csc(c+bx)\,dx-\int \sin(a+bx)\,dx\\ &= -\frac{\tanh^{-1}(\cos(c+bx))\cos(a-c)}{b}+\frac{\cos(a+bx)}{b}\end{aligned}$$

Mathematica [C] time = 0.0508546, size = 94, normalized size = 3.24

$$-\frac{2i\cos(a-c)\tan^{-1}\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{\sin(c)\cos\left(\frac{bx}{2}\right)+i\cos(c)\cos\left(\frac{bx}{2}\right)}\right)}{b}-\frac{\sin(a)\sin(bx)}{b}+\frac{\cos(a)\cos(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cot[c + b*x],x]
```

```
[Out] ((-2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c]*Cos[(b*x)/2] - Sin[c]*Sin[(b*x)
/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b + (Cos[a
]*Cos[b*x])/b - (Sin[a]*Sin[b*x])/b
```

Maple [C] time = 0.079, size = 93, normalized size = 3.2

$$\frac{e^{i(bx+a)}}{2b} + \frac{e^{-i(bx+a)}}{2b} - \frac{\ln\left(e^{i(bx+a)} + e^{i(a-c)}\right)\cos(a-c)}{b} + \frac{\ln\left(e^{i(bx+a)} - e^{i(a-c)}\right)\cos(a-c)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cot(b*x+c),x)
```

```
[Out] 1/2*exp(I*(b*x+a))/b+1/2/b*exp(-I*(b*x+a))-1/b*ln(exp(I*(b*x+a))+exp(I*(a-c
)))*cos(a-c)+1/b*ln(exp(I*(b*x+a))-exp(I*(a-c)))*cos(a-c)
```

Maxima [B] time = 1.18219, size = 142, normalized size = 4.9

$$-\frac{\cos(-a+c)\log\left(\cos(bx)^2 + 2\cos(bx)\cos(c) + \cos(c)^2 + \sin(bx)^2 - 2\sin(bx)\sin(c) + \sin(c)^2\right) - \cos(-a+c)\log}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(cos(-a + c)*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^
2 - 2*sin(b*x)*sin(c) + sin(c)^2) - cos(-a + c)*log(cos(b*x)^2 - 2*cos(b*x)
*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) - 2*cos(b*x
 + a))/b
```

Fricas [B] time = 0.518793, size = 510, normalized size = 17.59

$$\frac{\sqrt{2}\sqrt{\cos(-2a+2c)+1}\log\left(-\frac{2\cos(bx+a)^2\cos(-2a+2c) - 2\cos(bx+a)\sin(bx+a)\sin(-2a+2c) - \frac{2\sqrt{2}((\cos(-2a+2c)+1)\cos(bx+a) - \sin(bx+a)\sin(-2a+}{\sqrt{\cos(-2a+2c)+1}}}{2\cos(bx+a)^2\cos(-2a+2c) - 2\cos(bx+a)\sin(bx+a)\sin(-2a+2c) - \cos(-2a+2c) - 1}\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*sqrt(cos(-2*a + 2*c) + 1)*log(-(2*cos(b*x + a)^2*cos(-2*a + 2*
c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a + 2
*c) + 1)*cos(b*x + a) - sin(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c)
+ 1) - cos(-2*a + 2*c) + 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x +
 a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) - 1)) + 4*cos(b*x + a))/
b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(a+bx)\cot(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c),x)
```

```
[Out] Integral(cos(a + b*x)*cot(b*x + c), x)
```

Giac [B] time = 1.16001, size = 316, normalized size = 10.9

$$-\frac{\frac{\left(\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{3}-\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^{2}-\tan\left(\frac{1}{2}c\right)^{3}+\tan\left(\frac{1}{2}c\right)\right)\log\left(\left|\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}c\right)-1\right|\right)}{\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{3}+\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)+\tan\left(\frac{1}{2}c\right)^{3}+\tan\left(\frac{1}{2}c\right)}-\frac{\left(\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}-\tan\left(\frac{1}{2}a\right)^{2}+\cdots}{\tan\left(\frac{1}{2}a\cdots\right.}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c),x, algorithm="giac")
```

```
[Out] -((tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)^2*tan(1/2*c) + 4*tan(1/2*a)*tan(1
/2*c)^2 - tan(1/2*c)^3 + tan(1/2*c))*log(abs(tan(1/2*b*x)*tan(1/2*c) - 1))/
(tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c) + tan(1/2*c)^3 + tan(1
/2*c)) - (tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c
) - tan(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x) + tan(1/2*c)))/(tan(1/2*a)^2*tan
(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + 2*(2*tan(1/2*b*x)*tan(1/2*a)
 + tan(1/2*a)^2 - 1)/((tan(1/2*b*x)^2 + 1)*(tan(1/2*a)^2 + 1)))/b
```

3.251 $\int \cos(a+bx)\cot^2(c+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\cos(a-c)\csc(bx+c)}{b}+\frac{\sin(a-c)\tanh^{-1}(\cos(bx+c))}{b}-\frac{\sin(a+bx)}{b}$$

```
[Out] -((Cos[a - c]*Csc[c + b*x])/b) + (ArcTanh[Cos[c + b*x]]*Sin[a - c])/b - Sin
[a + b*x]/b
```

Rubi [A] time = 0.0413559, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4577, 4578, 2637, 3770, 2606, 8}

$$-\frac{\cos(a-c)\csc(bx+c)}{b}+\frac{\sin(a-c)\tanh^{-1}(\cos(bx+c))}{b}-\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cot[c + b*x]^2,x]
```

```
[Out] -((Cos[a - c]*Csc[c + b*x])/b) + (ArcTanh[Cos[c + b*x]]*Sin[a - c])/b - Sin
[a + b*x]/b
```

Rule 4577

```
Int[Cos[v_]*Cot[w_]^(n_.), x_Symbol] :> -Int[Sin[v]*Cot[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Rule 4578

```
Int[Cot[w_]^(n_.)*Sin[v_], x_Symbol] :> Int[Cos[v]*Cot[w]^(n - 1), x] + Dis
t[Sin[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos(a+bx)\cot^2(c+bx)\,dx = \cos(a-c)\int \cot(c+bx)\csc(c+bx)\,dx - \int \cot(c+bx)\sin(a+bx)\,dx$$
$$= -\frac{\cos(a-c)\,\text{Subst}(\int 1\,dx, x, \csc(c+bx))}{b} - \sin(a-c)\int \csc(c+bx)\,dx - \int \cos(a+bx)$$
$$= -\frac{\cos(a-c)\csc(c+bx)}{b} + \frac{\tanh^{-1}(\cos(c+bx))\sin(a-c)}{b} - \frac{\sin(a+bx)}{b}$$

Mathematica [C] time = 0.0985741, size = 112, normalized size = 2.43

$$-\frac{\cos(a-c)\csc(bx+c)}{b} + \frac{2i\sin(a-c)\tan^{-1}\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{\sin(c)\cos\left(\frac{bx}{2}\right)+i\cos(c)\cos\left(\frac{bx}{2}\right)}\right)}{b} - \frac{\sin(a)\cos(bx)}{b} - \frac{\cos(a)\sin(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cot[c + b*x]^2,x]
```

```
[Out] -((Cos[a - c]*Csc[c + b*x])/b) - (Cos[b*x]*Sin[a])/b + ((2*I)*ArcTan[((Cos[
c] - I*Sin[c])*(Cos[c]*Cos[(b*x)/2] - Sin[c]*Sin[(b*x)/2]))/(I*Cos[c]*Cos[(
b*x)/2] + Cos[(b*x)/2]*Sin[c])]*Sin[a - c])/b - (Cos[a]*Sin[b*x])/b
```

Maple [C] time = 0.095, size = 145, normalized size = 3.2

$$\frac{\frac{i}{2}e^{i(bx+a)}}{b} - \frac{\frac{i}{2}e^{-i(bx+a)}}{b} + \frac{i\left(e^{i(bx+3a)}+e^{i(bx+a+2c)}\right)}{b\left(-e^{2i(bx+a+c)}+e^{2ia}\right)} - \frac{\ln\left(e^{i(bx+a)}-e^{i(a-c)}\right)\sin(a-c)}{b} + \frac{\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)\sin(a-c)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cot(b*x+c)^2,x)
```

```
[Out] 1/2*I*exp(I*(b*x+a))/b-1/2*I/b*exp(-I*(b*x+a))+I/b/(-exp(2*I*(b*x+a+c))+exp
(2*I*a))*(exp(I*(b*x+3*a))+exp(I*(b*x+a+2*c)))-1/b*ln(exp(I*(b*x+a))-exp(I*
(a-c)))*sin(a-c)+1/b*ln(exp(I*(b*x+a))+exp(I*(a-c)))*sin(a-c)
```

Maxima [B] time = 1.3287, size = 828, normalized size = 18.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((sin(3*b*x + a + 2*c) - sin(b*x + a))*cos(4*b*x + 2*a + 2*c) + 3*(sin(
2*b*x + 2*a) + sin(2*b*x + 2*c))*cos(3*b*x + a + 2*c) - (cos(3*b*x + a + 2*
c)^2*sin(-a + c) - 2*cos(3*b*x + a + 2*c)*cos(b*x + a)*sin(-a + c) + cos(b*
x + a)^2*sin(-a + c) + sin(3*b*x + a + 2*c)^2*sin(-a + c) - 2*sin(3*b*x + a
 + 2*c)*sin(b*x + a)*sin(-a + c) + sin(b*x + a)^2*sin(-a + c))*log(cos(b*x)
^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)
```

```
^2) + (cos(3*b*x + a + 2*c)^2*sin(-a + c) - 2*cos(3*b*x + a + 2*c)*cos(b*x
+ a)*sin(-a + c) + cos(b*x + a)^2*sin(-a + c) + sin(3*b*x + a + 2*c)^2*sin(
-a + c) - 2*sin(3*b*x + a + 2*c)*sin(b*x + a)*sin(-a + c) + sin(b*x + a)^2*
sin(-a + c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2
*sin(b*x)*sin(c) + sin(c)^2) - (cos(3*b*x + a + 2*c) - cos(b*x + a))*sin(4*
b*x + 2*a + 2*c) - (3*cos(2*b*x + 2*a) + 3*cos(2*b*x + 2*c) - 1)*sin(3*b*x
+ a + 2*c) - 3*cos(b*x + a)*sin(2*b*x + 2*a) - 3*cos(b*x + a)*sin(2*b*x + 2
*c) + 3*cos(2*b*x + 2*a)*sin(b*x + a) + 3*cos(2*b*x + 2*c)*sin(b*x + a) - s
in(b*x + a))/(b*cos(3*b*x + a + 2*c)^2 - 2*b*cos(3*b*x + a + 2*c)*cos(b*x +
 a) + b*cos(b*x + a)^2 + b*sin(3*b*x + a + 2*c)^2 - 2*b*sin(3*b*x + a + 2*c
)*sin(b*x + a) + b*sin(b*x + a)^2)
```

Fricas [B] time = 0.543556, size = 856, normalized size = 18.61

$$\frac{4\left(\cos\left(-2\,a+2\,c\right)+1\right)\cos\left(bx+a\right)^{2}-4\,\cos\left(bx+a\right)\sin\left(bx+a\right)\sin\left(-2\,a+2\,c\right)+\frac{\sqrt{2}\left(\left(\cos(-2\,a+2\,c)+1\right)\sin(bx+a)\sin(-2\,a+2\,c}{}}{4\left(b\cos\left(bx+a\right)\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(cos(-2*a + 2*c) + 1)*cos(b*x + a)^2 - 4*cos(b*x + a)*sin(b*x + a)*s
in(-2*a + 2*c) + sqrt(2)*((cos(-2*a + 2*c) + 1)*sin(b*x + a)*sin(-2*a + 2*c
) - (cos(-2*a + 2*c)^2 - 1)*cos(b*x + a))*log(-(2*cos(b*x + a)^2*cos(-2*a +
 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a
+ 2*c) + 1)*cos(b*x + a) - sin(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*
c) + 1) - cos(-2*a + 2*c) + 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*
x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) - 1))/sqrt(cos(-2*a +
 2*c) + 1) - 8*cos(-2*a + 2*c) - 8)/(b*cos(b*x + a)*sin(-2*a + 2*c) + (b*co
s(-2*a + 2*c) + b)*sin(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(a+bx\right)\cot^{2}\left(bx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)*cot(b*x + c)**2, x)
```

Giac [B] time = 1.23013, size = 846, normalized size = 18.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)*ta
n(1/2*c) - tan(1/2*c)^2)*log(abs(tan(1/2*b*x)*tan(1/2*c) - 1))/(tan(1/2*a)^
2*tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c) + tan(1/2*c)^3 + tan(1/2*c)) - 4*(
tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c)
)*log(abs(tan(1/2*b*x) + tan(1/2*c)))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*
a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*b*x)^3*tan(1/2*a)^2*tan(1/2*c)^4 - 6*ta
n(1/2*b*x)^3*tan(1/2*a)^2*tan(1/2*c)^2 + 4*tan(1/2*b*x)^3*tan(1/2*a)*tan(1/
2*c)^3 - 6*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*b*x)^3*tan(1/
2*c)^4 + tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*b*x)^3*tan(1/2*a)
^2 - 4*tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*c) + 6*tan(1/2*b*x)^2*tan(1/2*a)^2
*tan(1/2*c) + 6*tan(1/2*b*x)^3*tan(1/2*c)^2 + 2*tan(1/2*b*x)*tan(1/2*a)^2*t
an(1/2*c)^2 + 6*tan(1/2*b*x)^2*tan(1/2*c)^3 + 12*tan(1/2*b*x)*tan(1/2*a)*ta
n(1/2*c)^3 - 2*tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*b*x)*tan(1/2*c)^4 - tan(
1/2*b*x)^3 + tan(1/2*b*x)*tan(1/2*a)^2 - 6*tan(1/2*b*x)^2*tan(1/2*c) - 12*t
an(1/2*b*x)*tan(1/2*a)*tan(1/2*c) + 2*tan(1/2*a)^2*tan(1/2*c) - 2*tan(1/2*b
*x)*tan(1/2*c)^2 - 16*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*c)^3 - tan(1/2*b*
x) - 2*tan(1/2*c))/((tan(1/2*b*x)^4*tan(1/2*c) + tan(1/2*b*x)^3*tan(1/2*c)^
2 - tan(1/2*b*x)^3 + tan(1/2*b*x)*tan(1/2*c)^2 - tan(1/2*b*x) - tan(1/2*c))
*(tan(1/2*a)^2*tan(1/2*c) + tan(1/2*c))))/b
```

3.252 $\int \cos(a+bx)\cot^3(c+bx)\,dx$

Optimal. Leaf size=73

$$\frac{3\cos(a-c)\tanh^{-1}(\cos(bx+c))}{2b}+\frac{\sin(a-c)\csc(bx+c)}{b}-\frac{\cos(a-c)\cot(bx+c)\csc(bx+c)}{2b}-\frac{\cos(a+bx)}{b}$$

```
[Out] (3*ArcTanh[Cos[c + b*x]]*Cos[a - c])/(2*b) - Cos[a + b*x]/b - (Cos[a - c]*C
ot[c + b*x]*Csc[c + b*x])/(2*b) + (Csc[c + b*x]*Sin[a - c])/b
```

Rubi [A] time = 0.07673, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {4577, 4578, 2638, 3770, 2606, 8, 2611}

$$\frac{3\cos(a-c)\tanh^{-1}(\cos(bx+c))}{2b}+\frac{\sin(a-c)\csc(bx+c)}{b}-\frac{\cos(a-c)\cot(bx+c)\csc(bx+c)}{2b}-\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cot[c + b*x]^3,x]
```

```
[Out] (3*ArcTanh[Cos[c + b*x]]*Cos[a - c])/(2*b) - Cos[a + b*x]/b - (Cos[a - c]*C
ot[c + b*x]*Csc[c + b*x])/(2*b) + (Csc[c + b*x]*Sin[a - c])/b
```

Rule 4577

```
Int[Cos[v_]*Cot[w_]^(n_.), x_Symbol] :> -Int[Sin[v]*Cot[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Rule 4578

```
Int[Cot[w_]^(n_.)*Sin[v_], x_Symbol] :> Int[Cos[v]*Cot[w]^(n - 1), x] + Dis
t[Sin[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \cos(a+bx)\cot^3(c+bx)\,dx = \cos(a-c)\int \cot^2(c+bx)\csc(c+bx)\,dx - \int \cot^2(c+bx)\sin(a+bx)\,dx$$
$$= -\frac{\cos(a-c)\cot(c+bx)\csc(c+bx)}{2b} - \frac{1}{2}\cos(a-c)\int \csc(c+bx)\,dx - \sin(a-c)\int \cot($$
$$= \frac{\tanh^{-1}(\cos(c+bx))\cos(a-c)}{2b} - \frac{\cos(a-c)\cot(c+bx)\csc(c+bx)}{2b} - \cos(a-c)\int \csc($$
$$= \frac{3\tanh^{-1}(\cos(c+bx))\cos(a-c)}{2b} - \frac{\cos(a+bx)}{b} - \frac{\cos(a-c)\cot(c+bx)\csc(c+bx)}{2b} + $$

Mathematica [A] time = 0.335982, size = 71, normalized size = 0.97

$$\frac{\csc^2(bx+c)(2\cos(a-bx-2c)+\cos(a+3bx+2c)-5\cos(a+bx))+12\cos(a-c)\tanh^{-1}\left(\cos(c)-\sin(c)\tan\left(\frac{bx}{2}\right)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cot[c + b*x]^3,x]
```

```
[Out] (12*ArcTanh[Cos[c] - Sin[c]*Tan[(b*x)/2]]*Cos[a - c] + (2*Cos[a - 2*c - b*x
] - 5*Cos[a + b*x] + Cos[a + 2*c + 3*b*x])*Csc[c + b*x]^2)/(4*b)
```

Maple [C] time = 0.104, size = 179, normalized size = 2.5

$$-\frac{e^{i(bx+a)}}{2b} - \frac{e^{-i(bx+a)}}{2b} - \frac{-3e^{i(3bx+5a+2c)}+e^{i(3bx+3a+4c)}+e^{i(bx+5a)}-3e^{i(bx+3a+2c)}}{2b\left(-e^{2i(bx+a+c)}+e^{2ia}\right)^2} + \frac{3\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)\cos(a-c)}{2b} - \frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cot(b*x+c)^3,x)
```

```
[Out] -1/2*exp(I*(b*x+a))/b-1/2/b*exp(-I*(b*x+a))-1/2/b/(-exp(2*I*(b*x+a+c))+exp(
2*I*a))^2*(-3*exp(I*(3*b*x+5*a+2*c))+exp(I*(3*b*x+3*a+4*c))+exp(I*(b*x+5*a)
)-3*exp(I*(b*x+3*a+2*c)))+3/2/b*ln(exp(I*(b*x+a))+exp(I*(a-c)))*cos(a-c)-3/
2/b*ln(exp(I*(b*x+a))-exp(I*(a-c)))*cos(a-c)
```

Maple [B] time = 1.34055, size = 1693, normalized size = 23.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(cos(5*b*x + a + 4*c) - 2*cos(3*b*x + a + 2*c) + cos(b*x + a))*cos(
6*b*x + 2*a + 4*c) - 2*(5*cos(4*b*x + 2*a + 2*c) - 2*cos(4*b*x + 4*c) - 2*c
os(2*b*x + 2*a) + 5*cos(2*b*x + 2*c) - 1)*cos(5*b*x + a + 4*c) + 10*(2*cos(
3*b*x + a + 2*c) - cos(b*x + a))*cos(4*b*x + 2*a + 2*c) - 4*(2*cos(3*b*x +
a + 2*c) - cos(b*x + a))*cos(4*b*x + 4*c) - 4*(2*cos(2*b*x + 2*a) - 5*cos(2
*b*x + 2*c) + 1)*cos(3*b*x + a + 2*c) + 4*cos(2*b*x + 2*a)*cos(b*x + a) - 1
0*cos(2*b*x + 2*c)*cos(b*x + a) - 3*(cos(5*b*x + a + 4*c)^2*cos(-a + c) + 4
*cos(3*b*x + a + 2*c)^2*cos(-a + c) - 4*cos(3*b*x + a + 2*c)*cos(b*x + a)*c
os(-a + c) + cos(b*x + a)^2*cos(-a + c) + cos(-a + c)*sin(5*b*x + a + 4*c)^
2 + 4*cos(-a + c)*sin(3*b*x + a + 2*c)^2 - 4*cos(-a + c)*sin(3*b*x + a + 2*
c)*sin(b*x + a) + cos(-a + c)*sin(b*x + a)^2 - 2*(2*cos(3*b*x + a + 2*c)*co
s(-a + c) - cos(b*x + a)*cos(-a + c))*cos(5*b*x + a + 4*c) - 2*(2*cos(-a +
c)*sin(3*b*x + a + 2*c) - cos(-a + c)*sin(b*x + a))*sin(5*b*x + a + 4*c))*l
og(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(
c) + sin(c)^2) + 3*(cos(5*b*x + a + 4*c)^2*cos(-a + c) + 4*cos(3*b*x + a +
2*c)^2*cos(-a + c) - 4*cos(3*b*x + a + 2*c)*cos(b*x + a)*cos(-a + c) + cos(
b*x + a)^2*cos(-a + c) + cos(-a + c)*sin(5*b*x + a + 4*c)^2 + 4*cos(-a + c)
*sin(3*b*x + a + 2*c)^2 - 4*cos(-a + c)*sin(3*b*x + a + 2*c)*sin(b*x + a) +
 cos(-a + c)*sin(b*x + a)^2 - 2*(2*cos(3*b*x + a + 2*c)*cos(-a + c) - cos(b
*x + a)*cos(-a + c))*cos(5*b*x + a + 4*c) - 2*(2*cos(-a + c)*sin(3*b*x + a
+ 2*c) - cos(-a + c)*sin(b*x + a))*sin(5*b*x + a + 4*c))*log(cos(b*x)^2 - 2
*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) +
2*(sin(5*b*x + a + 4*c) - 2*sin(3*b*x + a + 2*c) + sin(b*x + a))*sin(6*b*x
+ 2*a + 4*c) - 2*(5*sin(4*b*x + 2*a + 2*c) - 2*sin(4*b*x + 4*c) - 2*sin(2*b
*x + 2*a) + 5*sin(2*b*x + 2*c))*sin(5*b*x + a + 4*c) + 10*(2*sin(3*b*x + a
+ 2*c) - sin(b*x + a))*sin(4*b*x + 2*a + 2*c) - 4*(2*sin(3*b*x + a + 2*c) -
 sin(b*x + a))*sin(4*b*x + 4*c) - 4*(2*sin(2*b*x + 2*a) - 5*sin(2*b*x + 2*c
))*sin(3*b*x + a + 2*c) + 4*sin(2*b*x + 2*a)*sin(b*x + a) - 10*sin(2*b*x +
2*c)*sin(b*x + a) + 2*cos(b*x + a))/(b*cos(5*b*x + a + 4*c)^2 + 4*b*cos(3*b
*x + a + 2*c)^2 - 4*b*cos(3*b*x + a + 2*c)*cos(b*x + a) + b*cos(b*x + a)^2
+ b*sin(5*b*x + a + 4*c)^2 + 4*b*sin(3*b*x + a + 2*c)^2 - 4*b*sin(3*b*x + a
 + 2*c)*sin(b*x + a) + b*sin(b*x + a)^2 - 2*(2*b*cos(3*b*x + a + 2*c) - b*c
os(b*x + a))*cos(5*b*x + a + 4*c) - 2*(2*b*sin(3*b*x + a + 2*c) - b*sin(b*x
 + a))*sin(5*b*x + a + 4*c))
```

Fricas [B] time = 0.560044, size = 1040, normalized size = 14.25

$$-\frac{16\cos(bx+a)^3\cos(-2a+2c) - 4\left(4\cos(bx+a)^2+1\right)\sin(bx+a)\sin(-2a+2c) - 4(\cos(-2a+2c)+5)\cos(b}{8(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(16*cos(b*x + a)^3*cos(-2*a + 2*c) - 4*(4*cos(b*x + a)^2 + 1)*sin(b*x
+ a)*sin(-2*a + 2*c) - 4*(cos(-2*a + 2*c) + 5)*cos(b*x + a) + 3*sqrt(2)*(2*
(cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*(cos(-2
*a + 2*c)^2 + cos(-2*a + 2*c))*cos(b*x + a)^2 + cos(-2*a + 2*c)^2 + 2*cos(-
2*a + 2*c) + 1)*log((2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(
b*x + a)*sin(-2*a + 2*c) + 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*cos(b*x + a) -
sin(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2*a + 2*c) +
 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*
```

```
a + 2*c) - cos(-2*a + 2*c) - 1))/sqrt(cos(-2*a + 2*c) + 1))/(2*b*cos(b*x +
a)^2*cos(-2*a + 2*c) - 2*b*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - b*co
s(-2*a + 2*c) - b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.38402, size = 1300, normalized size = 17.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(12*(tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)^2*tan(1/2*c) + 4*tan(1/2*a)
*tan(1/2*c)^2 - tan(1/2*c)^3 + tan(1/2*c))*log(abs(tan(1/2*b*x)*tan(1/2*c)
- 1))/(tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c) + tan(1/2*c)^3 +
 tan(1/2*c)) - 12*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*
tan(1/2*c) - tan(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x) + tan(1/2*c)))/(tan(1/2
*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + 16*(2*tan(1/2*b*x)*
tan(1/2*a) + tan(1/2*a)^2 - 1)/((tan(1/2*b*x)^2 + 1)*(tan(1/2*a)^2 + 1)) +
(2*tan(1/2*b*x)^3*tan(1/2*a)^2*tan(1/2*c)^7 + tan(1/2*b*x)^2*tan(1/2*a)^2*t
an(1/2*c)^8 + 6*tan(1/2*b*x)^3*tan(1/2*a)^2*tan(1/2*c)^5 + 2*tan(1/2*b*x)^2
*tan(1/2*a)^2*tan(1/2*c)^6 - 2*tan(1/2*b*x)^3*tan(1/2*c)^7 - 4*tan(1/2*b*x)
^2*tan(1/2*a)*tan(1/2*c)^7 - 2*tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*c)^7 - tan
(1/2*b*x)^2*tan(1/2*c)^8 - 6*tan(1/2*b*x)^3*tan(1/2*a)^2*tan(1/2*c)^3 + 16*
tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*c)^4 - 22*tan(1/2*b*x)^2*tan(1/2*a)^2*tan
(1/2*c)^4 - 6*tan(1/2*b*x)^3*tan(1/2*c)^5 + 20*tan(1/2*b*x)^2*tan(1/2*a)*ta
n(1/2*c)^5 - 14*tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*c)^5 - 2*tan(1/2*b*x)^2*t
an(1/2*c)^6 + 16*tan(1/2*b*x)*tan(1/2*a)*tan(1/2*c)^6 + 2*tan(1/2*a)^2*tan(
1/2*c)^6 + 2*tan(1/2*b*x)*tan(1/2*c)^7 - 2*tan(1/2*b*x)^3*tan(1/2*a)^2*tan(
1/2*c) + 2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*c)^2 + 6*tan(1/2*b*x)^3*tan(
1/2*c)^3 - 20*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*c)^3 + 14*tan(1/2*b*x)*tan(
1/2*a)^2*tan(1/2*c)^3 + 22*tan(1/2*b*x)^2*tan(1/2*c)^4 - 16*tan(1/2*b*x)*ta
n(1/2*a)*tan(1/2*c)^4 + 12*tan(1/2*a)^2*tan(1/2*c)^4 + 14*tan(1/2*b*x)*tan(
1/2*c)^5 - 8*tan(1/2*a)*tan(1/2*c)^5 - 2*tan(1/2*c)^6 + tan(1/2*b*x)^2*tan(
1/2*a)^2 + 2*tan(1/2*b*x)^3*tan(1/2*c) + 4*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/
2*c) + 2*tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*c) - 2*tan(1/2*b*x)^2*tan(1/2*c)
^2 + 16*tan(1/2*b*x)*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)^2
- 14*tan(1/2*b*x)*tan(1/2*c)^3 + 8*tan(1/2*a)*tan(1/2*c)^3 - 12*tan(1/2*c)^
4 - tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*c) - 2*tan(1/2*c)^2)/((tan(1/2*
a)^2*tan(1/2*c)^2 + tan(1/2*c)^2)*(tan(1/2*b*x)^2*tan(1/2*c) + tan(1/2*b*x)
*tan(1/2*c)^2 - tan(1/2*b*x) - tan(1/2*c))^2))/b
```

3.253 $\int \cos(a+bx)\tan(c+dx)\,dx$

Optimal. Leaf size=134

$$-\frac{e^{-i(a+bx)}\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2i(c+dx)}\right)}{b}+\frac{e^{i(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{b}{2d},\frac{b}{2d}+1,-e^{2i(c+dx)}\right)}{b}+$$

```
[Out] 1/(2*b*E^(I*(a + b*x))) - E^(I*(a + b*x))/(2*b) - Hypergeometric2F1[1, -b/(
2*d), 1 - b/(2*d), -E^((2*I)*(c + d*x))]/(b*E^(I*(a + b*x))) + (E^(I*(a + b
*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), -E^((2*I)*(c + d*x))])/b
```

Rubi [A] time = 0.119283, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4560, 2194, 2251}

$$-\frac{e^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2i(c+dx)}\right)}{b}+\frac{e^{i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;-e^{2i(c+dx)}\right)}{b}+\frac{e^{-i(a+bx)}}{2b}-\frac{e^{i(a+bx)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Tan[c + d*x],x]
```

```
[Out] 1/(2*b*E^(I*(a + b*x))) - E^(I*(a + b*x))/(2*b) - Hypergeometric2F1[1, -b/(
2*d), 1 - b/(2*d), -E^((2*I)*(c + d*x))]/(b*E^(I*(a + b*x))) + (E^(I*(a + b
*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), -E^((2*I)*(c + d*x))])/b
```

Rule 4560

```
Int[Cos[(a_.) + (b_.)*(x_)]*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[-(I/(
E^(I*(a + b*x))*2)) - (I*E^(I*(a + b*x)))/2 + I/(E^(I*(a + b*x))*(1 + E^(2*
I*(c + d*x)))) + (I*E^(I*(a + b*x)))/(1 + E^(2*I*(c + d*x))), x] /; FreeQ[{
a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\tan(c+dx)\,dx &= \int \left(-\frac{1}{2}ie^{-i(a+bx)}-\frac{1}{2}ie^{i(a+bx)}+\frac{ie^{-i(a+bx)}}{1+e^{2i(c+dx)}}+\frac{ie^{i(a+bx)}}{1+e^{2i(c+dx)}}\right)dx\\ &= -\left(\frac{1}{2}i\int e^{-i(a+bx)}\,dx\right)-\frac{1}{2}i\int e^{i(a+bx)}\,dx+i\int \frac{e^{-i(a+bx)}}{1+e^{2i(c+dx)}}\,dx+i\int \frac{e^{i(a+bx)}}{1+e^{2i(c+dx)}}\,dx\\ &= \frac{e^{-i(a+bx)}}{2b}-\frac{e^{i(a+bx)}}{2b}-\frac{e^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2i(c+dx)}\right)}{b}+\frac{e^{i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};1+\frac{b}{2d}\right.}{b}\end{aligned}$$

Mathematica [A] time = 1.8844, size = 142, normalized size = 1.06

$$\frac{e^{-i(a+bx)}\left((b-2d)\left(2e^{2i(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{b}{2d},\frac{b}{2d}+1,-e^{2i(c+dx)}\right)-e^{2i(a+bx)}-1\right)+2be^{2i(c+dx)}\text{Hypergeometric}\right.}{2b(b-2d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Tan[c + d*x],x]
```

```
[Out] (2*b*E^((2*I)*(c + d*x))*Hypergeometric2F1[1, 1 - b/(2*d), 2 - b/(2*d), -E^
((2*I)*(c + d*x))] + (b - 2*d)*(-1 - E^((2*I)*(a + b*x)) + 2*E^((2*I)*(a +
b*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), -E^((2*I)*(c + d*x))]))/(2
*b*(b - 2*d)*E^(I*(a + b*x)))
```

Maple [F] time = 0.175, size = 0, normalized size = 0.

$$\int \cos(bx+a)\tan(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*tan(d*x+c),x)
```

```
[Out] int(cos(b*x+a)*tan(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)\tan(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)*tan(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(\cos(bx+a)\tan(dx+c),x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)*tan(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(a+bx)\tan(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)*tan(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)\tan(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*tan(d*x + c), x)
```

3.254 $\int \cos(a+bx)\cot(c+dx)\,dx$

Optimal. Leaf size=130

$$\frac{e^{-i(a+bx)}\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},e^{2i(c+dx)}\right)}{b}-\frac{e^{i(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{b}{2d},\frac{b}{2d}+1,e^{2i(c+dx)}\right)}{b}-\frac{e^{-i(a+bx)}}{2b}$$

```
[Out] -1/(2*b*E^(I*(a + b*x))) + E^(I*(a + b*x))/(2*b) + Hypergeometric2F1[1, -b/
(2*d), 1 - b/(2*d), E^((2*I)*(c + d*x))]/(b*E^(I*(a + b*x))) - (E^(I*(a + b
*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), E^((2*I)*(c + d*x))])/b
```

Rubi [A] time = 0.120141, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4558, 2194, 2251}

$$\frac{e^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2i(c+dx)}\right)}{b}-\frac{e^{i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;e^{2i(c+dx)}\right)}{b}-\frac{e^{-i(a+bx)}}{2b}+\frac{e^{i(a+bx)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cot[c + d*x],x]
```

```
[Out] -1/(2*b*E^(I*(a + b*x))) + E^(I*(a + b*x))/(2*b) + Hypergeometric2F1[1, -b/
(2*d), 1 - b/(2*d), E^((2*I)*(c + d*x))]/(b*E^(I*(a + b*x))) - (E^(I*(a + b
*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), E^((2*I)*(c + d*x))])/b
```

Rule 4558

```
Int[Cos[(a_.) + (b_.)*(x_)]*Cot[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[I/(E^
(I*(a + b*x))*2) + (I*E^(I*(a + b*x)))/2 - I/(E^(I*(a + b*x))*(1 - E^(2*I*(
c + d*x)))) - (I*E^(I*(a + b*x)))/(1 - E^(2*I*(c + d*x))), x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\cot(c+dx)\,dx &= \int\left(\frac{1}{2}ie^{-i(a+bx)}+\frac{1}{2}ie^{i(a+bx)}-\frac{ie^{-i(a+bx)}}{1-e^{2i(c+dx)}}-\frac{ie^{i(a+bx)}}{1-e^{2i(c+dx)}}\right)dx\\ &=\frac{1}{2}i\int e^{-i(a+bx)}\,dx+\frac{1}{2}i\int e^{i(a+bx)}\,dx-i\int\frac{e^{-i(a+bx)}}{1-e^{2i(c+dx)}}\,dx-i\int\frac{e^{i(a+bx)}}{1-e^{2i(c+dx)}}\,dx\\ &=-\frac{e^{-i(a+bx)}}{2b}+\frac{e^{i(a+bx)}}{2b}+\frac{e^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2i(c+dx)}\right)}{b}-\frac{e^{i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};1+\frac{b}{2d};e^{2i}\right.}{b}\end{aligned}$$

Mathematica [A] time = 1.7423, size = 108, normalized size = 0.83

$$\frac{e^{-i(a+bx)}\left(-2e^{2i(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{b}{2d},\frac{b}{2d}+1,e^{2i(c+dx)}\right)+2\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},e^{2i(c+dx)}\right)+\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cot[c + d*x],x]
```

```
[Out] (-1 + E^((2*I)*(a + b*x)) + 2*Hypergeometric2F1[1, -b/(2*d), 1 - b/(2*d), E
^((2*I)*(c + d*x))] - 2*E^((2*I)*(a + b*x))*Hypergeometric2F1[1, b/(2*d), 1
 + b/(2*d), E^((2*I)*(c + d*x))])/(2*b*E^(I*(a + b*x)))
```

Maple [F] time = 0.204, size = 0, normalized size = 0.

$$\int \cos(bx+a)\cot(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cot(d*x+c),x)
```

```
[Out] int(cos(b*x+a)*cot(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)\cot(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)*cot(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cos(bx+a)\cot(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)*cot(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(a+bx)\cot(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)*cot(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)\cot(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*cot(d*x + c), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal, debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=", ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
    )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
    ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
    type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
    ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
   (flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
```

```
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0], Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1], Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0], Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```